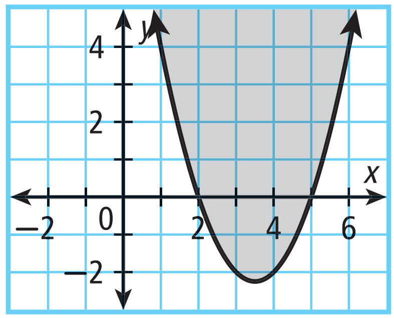
5-7 Solving Quadratic Inequalities

Example 1 Graphing Quadratic Inequalities in Two Variables

Graph $y \geq x^2 - 7x + 10$.



Step 1 Graph the boundary of the related parabola $y = x^2 - 7x + 10$ with a solid curve. Its y-intercept is 10, its vertex is $(3.5, -2.25)$, and its x-intercepts are 2 and 5.

Step 2 Shade above the parabola because the solution consists of y-values greater than those on the parabola for corresponding x-values.

Check Use a test point to verify the solution region.

$y \geq x^2 - 7x + 10$

$0 \geq (4)^2 - 7(4) + 10$ *Try (4, 0).*

$0 \geq 16 - 28 + 10$

$0 \geq -2$ ✔

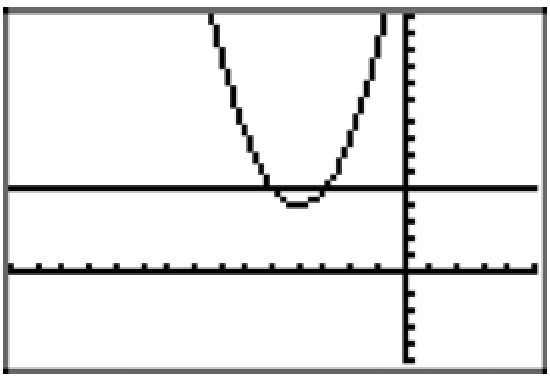
5-7 Solving Quadratic Inequalities

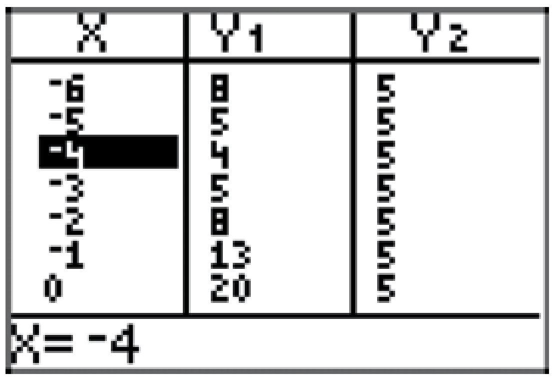
Example 2 Solving Quadratic Inequalities by using tables and graphs

Solve each inequality by using tables and graphs.

A. $x^2 + 8x + 20 \geq 5$

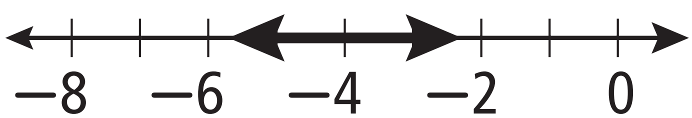
Use a graphing calculator to graph each side of the inequality. Set **Y1** equal to $x^2 + 8x + 20$ and **Y2** equal to 5. Identify the values of x for which **Y1** $\geq$ **Y2**.





The parabola is at or above the line when x is less than or equal to -5 or greater than or equal to -3. So, the solution set is $x \leq -5$ or $x \geq -3$ or $(-\infty, -5) \cup (-3, \infty)$. The table supports your answer.

The number line shows the solution set.

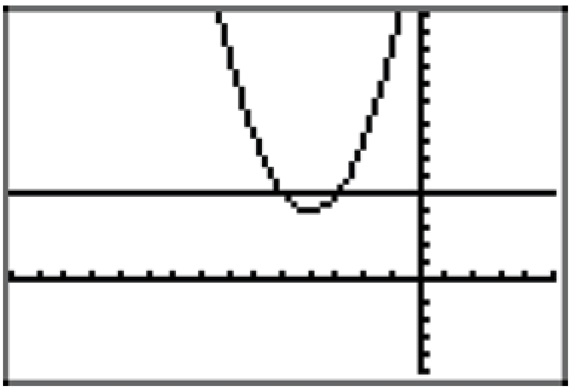


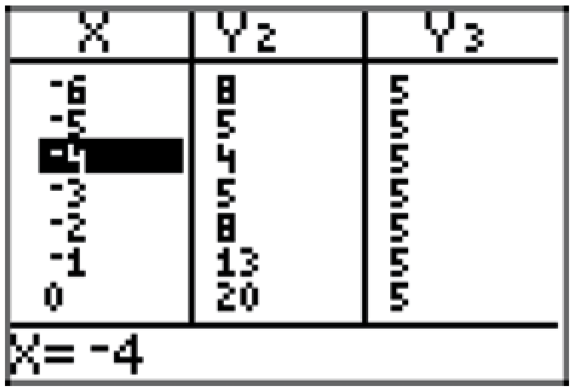
5-7 Solving Quadratic Inequalities

Example 2 Solving Quadratic Inequalities by using tables and graphs (continued)

B. $x^2 + 8x + 20 < 5$

Use a graphing calculator to graph each side of the inequality. Set **Y1** equal to $x^2 + 8x + 20$ and **Y2** equal to 5. Identify the values of x for which **Y1** < **Y2**.



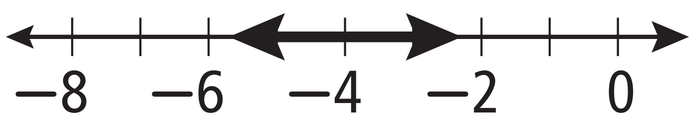


X	Y2	Y3
-6	8	5
-5	5	5
-4	4	5
-3	5	5
-2	8	5
-1	13	5
0	20	5

X= -4

The parabola is below the line when x is between -5 and -3. So, the solution set is $-5 < x < -3$ or $(-5, -3)$. The table supports your answer.

The number line shows the solution set.



5-7 Solving Quadratic Inequalities

Example 3 Solving Quadratic Inequalities by Using Algebra

Solve the inequality $x^2 - 10x + 18 \le -3$ by using algebra.

Step 1 Write the related equation.

$x^2 - 10x + 18 = -3$

Step 2 Solve the equation for x to find the critical values.

$x^2 - 10x + 21 = 0$ *Write in standard form.*

$(x - 3)(x - 7) = 0$ *Factor.*

$x - 3 = 0$ or $x - 7 = 0$ *Zero Product Property*

$x = 3$ or $x = 7$ *Solve for x.*

The critical values are 3 and 7. The critical values divide the number line into three intervals: $x < 3$, $3 < x < 7$, $x > 7$.

Step 3 Test an x-value in each interval.

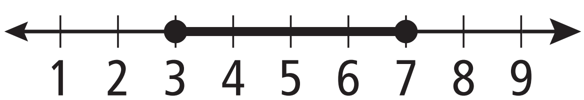
$x^2 - 10x + 18 \le -3$

$(2)^2 - 10(2) + 18 \le -3$ ✖ *Try x = 2.*

$(4)^2 - 10(4) + 18 \le -3$ ✔ *Try x = 4.*

$(8)^2 - 10(8) + 18 \le -3$ ✖ *Try x = 8.*

Shade the solution regions on the number line. Use solid circles for the critical values because the inequality contains them. The solution is $3 \le x \le 7$ or $(3, 7)$



5-7 Solving Quadratic Inequalities

Example 4 Problem-Solving Application

The monthly profit *P* of a small business that sells bicycle helmets can be modeled by the function $P(x) = -8x^2 + 600x - 4200$, where x is the average selling price of a helmet. What range of selling prices will generate a monthly profit of at least $6000?

1. Understand the Problem

The answer will be the average price of a helmet required for a profit that is greater than or equal to $6000.

List the important information:

- The profit must be at least $6000.
- The function for the business's profit is $P(x) = -8x^2 + 600x - 4200$.

2. Make a Plan

Write an inequality showing profit greater than or equal to $6000. Then solve the inequality by using algebra.

5-7 Solving Quadratic Inequalities

Example 4 Problem-Solving Application (continued)

3. Solve

Write the inequality.

$-8x^2 + 600x - 4200 \geq 6000$

Find the critical values by solving the related equation.

$-8x^2 + 600x - 4200 = 6000$ *Write as an equation.*

$-8x^2 + 600x - 10200 = 0$ *Write in standard form.*

$-8(x^2 - 75x + 1275) = 0$ *Factor out −8 to simplify.*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-75) \pm \sqrt{(-75)^2 - 4(1)(1275)}}{2(1)}$$

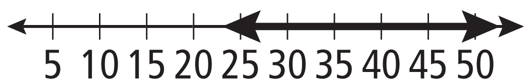
Use the Quadratic Formula.

$$= \frac{75 \pm 5\sqrt{21}}{2}$$

Simplify.

$x \approx 26.05$ or $x \approx 48.95$

Test an x-value in each of the three regions formed by the critical x-values.



$-8(25)^2 + 600(25) - 4200 \geq 6000$ *Try x = 25.*

$5800 \geq 6000$ ✖

$-8(45)^2 + 600(45) - 4200 \geq 6000$ *Try x = 45.*

$6600 \geq 6000$ ✔

$-8(50)^2 + 600(50) - 4200 \geq 6000$ *Try x = 50.*

$4600 \geq 6000$ ✖

Write the solution as an inequality. The solution is approximately $26.05 \leq x \leq 48.95$.

5-7 Solving Quadratic Inequalities

Example 4 Problem-Solving Application (continued)

For a profit of at least $6000, the average price of a helmet needs to be between $26.05 and $48.95, inclusive.

4. Look Back

Enter $y = -8x^2 + 600x - 4200$ into a graphing calculator, and create a table of values. The table shows that integer values of x between 26.05 and 48.95 inclusive result in y-values greater than or equal to 6000.