Name ______________________________ Date __________ Class __________

LESSON 5-6

# Reteach

## *The Quadratic Formula*

The Quadratic Formula is another way to find the roots of a quadratic equation or the zeros of a quadratic function.

Find the zeros of $f(x) = x^2 - 6x - 11$.

**Step 1** Set $f(x) = 0$. $x^2 - 6x - 11 = 0$

**Step 2** Write the Quadratic Formula. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

**Step 3** Substitute values for $a$, $b$, and $c$ into the Quadratic Formula.
$a = 1$, $b = -6$, $c = -11$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-11)}}{2(1)}$$

**Step 4** Simplify.

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-11)}}{2(1)} = \frac{6 \pm \sqrt{36 + 44}}{2} = \frac{6 \pm \sqrt{80}}{2}$$

**Step 5** Write in simplest form.

$$x = \frac{6 \pm \sqrt{80}}{2} = 3 \pm \frac{\sqrt{80}}{2} = 3 \pm \frac{\sqrt{(16)(5)}}{2} = 3 \pm \frac{4\sqrt{5}}{2} = 3 \pm 2\sqrt{5}$$

Remember to divide both terms of the numerator by 2 to simplify.

**Find the zeros of each function using the Quadratic Formula.**

**1.** $f(x) = x^2 + x - 1$

$x^2 + x - 1 = 0$

$a =$ ____, $b =$ ____, $c =$ ____

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-(\_\_\_) \pm \sqrt{(\_\_\_)^2 - 4(\_\_\_)(\_\_\_)}}{2(\_\_\_)}$

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**2.** $f(x) = x^2 - 6x + 6$

________________________

$a =$ ____, $b =$ ____, $c =$ ____

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

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Name ______________________________ Date __________ Class __________

LESSON 5-6

# Reteach

## *The Quadratic Formula (continued)*

The **discriminant** of $ax^2 + bx + c = 0$ $(a \neq 0)$ is $b^2 - 4ac$.

Use the discriminant to determine the number of roots of a quadratic equation. A quadratic equation can have 2 real solutions, 1 real solution, or 2 complex solutions.

Find the type and number of solutions.

| $2x^2 - 5x = 3$ | $x^2 + 10x = -25$ | $3x^2 - 4x = -2$ |
|---|---|---|
| Write the equation in standard form: | Write the equation in standard form: | Write the equation in standard form: |
| $2x^2 - 5x - 3 = 0$ | $x^2 + 10x + 25 = 0$ | $3x^2 - 4x + 2 = 0$ |
| $a = 2$, $b = -5$, $c = -3$ | $a = 1$, $b = 10$, $c = 25$ | $a = 3$, $b = -4$, $c = 2$ |
| Evaluate the discriminant: | Evaluate the discriminant: | Evaluate the discriminant: |
| $b^2 - 4ac$ | $b^2 - 4ac$ | $b^2 - 4ac$ |
| $(-5)^2 - 4(2)(-3)$ | $(10)^2 - 4(1)(25)$ | $(-4)^2 - 4(3)(2)$ |
| $25 + 24$ | $100 - 100$ | $16 - 24$ |
| $49$ | $0$ | $-8$ |
| When $b^2 - 4ac > 0$, the equation has 2 real solutions. | When $b^2 - 4ac = 0$, the equation has 1 real solution. | When $b^2 - 4ac < 0$, the equation has 2 complex solutions. |

**Find the type and number of solutions for each equation.**

**3.** $x^2 - 12x = -36$

$x^2 - 12x + 36 = 0$

$a =$ ___, $b =$ _____, $c =$ ____

$b^2 - 4ac =$

______________________

Classify solutions:

______________________

**4.** $x^2 - 4x = -7$

______________________

______________________

$b^2 - 4ac =$

______________________

Classify solutions:

______________________

**5.** $x^2 - 7x = -3$

______________________

______________________

$b^2 - 4ac =$

______________________

Classify solutions:

______________________

LESSON 5-6

## Practice A

### The Quadratic Formula

**Find the zeros of each function by using the Quadratic Formula,** $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

1. $f(x) = x^2 + 4$

   $x^2 + 0x + 4 = 0$

   $x = \dfrac{-0 \pm \sqrt{0^2 - 4 \cdot 1 \cdot 4}}{2 \cdot 1}$

   $x = \dfrac{\pm\sqrt{-16}}{2}$

   **$x = \pm 2i$**

2. $f(x) = 2x^2 - 5x + 3$

   $2x^2 - 5x + 3 = 0$

   $x = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot (2) \cdot (3)}}{2 \cdot 2}$

   $x = \dfrac{5 \pm \sqrt{25 - 24}}{4}$

   **$x = 1, 1.5$**

3. $f(x) = x^2 + 2x + 4$ **$x = -1 \pm i\sqrt{3}$**

4. $f(x) = x^2 + 2x$ **$x = 0, -2$**

**Find the value of the discriminant for each function.**

5. $f(x) = x^2 + x + 4$ **$-15$**

6. $f(x) = -2x^2 + 3x - 1$ **1**

7. $f(x) = 3x^2 + 6x + 3$ **0**

**Find the type and number of solutions for each equation.**

8. $x^2 + 2x + 1 = 0$ **One real solution**

9. $2x^2 + x - 4 = 0$ **Two real solutions**

10. $2x^2 + 4x + 3 = 0$ **Two nonreal complex solutions**

11. $2x^2 - 5x + 3 = 0$ **Two real solutions**

**Solve.**

12. The length of a rectangle is 3 feet longer than its width. The area of the rectangle is 270 square feet.

    a. What is the width of the rectangle? **15 ft**

    b. What is the width of the rectangle if the area is only 160 square feet? **11 ft**



LESSON 5-6

## Practice B

### The Quadratic Formula

**Find the zeros of each function by using the Quadratic Formula.**

1. $f(x) = x^2 + 10x + 9$ **$x = -9, -1$**

2. $g(x) = 2x^2 + 4x - 12$ **$x = -1 \pm \sqrt{7}$**

3. $h(x) = 3x^2 - 3x + \frac{3}{4}$ **$x = 0.5$**

4. $f(x) = x^2 + 2x - 3$ **$x = -3, 1$**

5. $g(x) = 2x^2 + 3x + 1$ **$x = -1, -0.5$**

6. $g(x) = x^2 + 5x + -3$ **$x = \dfrac{-5 \pm \sqrt{37}}{2}$**

**Find the type and number of solutions for each equation.**

7. $x^2 - 3x = -8$ **Two nonreal solutions**

8. $x^2 + 4x = -3$ **Two real solutions**

9. $2x^2 - 12x = -18$ **One real solution**

**Solve.**

10. A newspaper delivery person in a car is tossing folded newspapers from the car window to driveways. The speed of the car is 30 feet per second, and the driver does not slow down. The newspapers are tossed horizontally from a height of 4 feet above the ground. The height of the papers as they are thrown can be modeled by $y = -16t^2 + 4$, and the distance they travel to the driveway is $d = 30t$.

    a. How long does it take for a newspaper to land? **0.5 s**

    b. From how many feet before the driveway must the papers be thrown? **15 ft**

    c. The delivery person starts to throw the newspapers at an angle and the height of the papers as they travel can now be modeled by $y = -16t^2 + 12t + 4$. How long does it take the papers to reach the ground now? **1 s**



LESSON 5-6

## Practice C

### The Quadratic Formula

**Find the zeros of each function by using the Quadratic Formula.**

1. $f(x) = x^2 + 8x - 3$ **$x = -4 \pm \sqrt{19}$**

2. $g(x) = 2x^2 - 6x - 1$ **$x = \dfrac{3 \pm \sqrt{11}}{2}$**

3. $h(x) = x^2 - x + 12$ **$x = \dfrac{1 \pm i\sqrt{47}}{2}$**

4. $f(x) = -2x^2 - 5x + 20$ **$x = -\dfrac{5 \pm \sqrt{185}}{4}$**

5. $f(x) = -2x^2 + 6x - 2$ **$x = \dfrac{3 \pm \sqrt{5}}{2}$**

6. $f(x) = 3x^2 - 10x + 4$ **$x = \dfrac{5 \pm \sqrt{13}}{3}$**

**Find the type and number of solutions for each equation.**

7. $2x^2 + 7 = -4x$ **Two nonreal solutions**

8. $x^2 - 3 = -6x$ **Two real solutions**

9. $4x^2 + 4 = -8x$ **One real solution**

**Solve.**

10. The height $h(t)$ measured in feet of an object dropped by an astronaut on the moon can be approximated by $h(t) = h_0 - 2.7t^2$, where $h_0$ is the height from which the object was dropped. About how long would it take an object to fall to the surface of the moon $(h = 0)$ if it were dropped by an astronaut from a height of 6 feet? **About 1.49 s**

11. The height in feet, $h$, of a base jumper jumping off a cliff is given by the equation $h = 3t^2 - 700t + 2000$, where $t$ is the time in seconds. The horizontal distance that he travels from the cliff is given by $d = 13t$.

    a. How long does it take the base jumper from the time he jumps $(t = 0)$ until he hits ground $(h = 0)$? **2.9 s**

    b. When he reaches the ground, how far away is he from the base of the cliff? **37.7 ft**

12. A path of uniform width surrounds a rectangular garden that is 5m wide and 12m long. The area of the path is $168\text{m}^2$. Find the width of the path. **3.5 m**



LESSON 5-6

## Reteach

### The Quadratic Formula

The Quadratic Formula is another way to find the roots of a quadratic equation or the zeros of a quadratic function.

Find the zeros of $f(x) = x^2 - 6x - 11$.

**Step 1** Set $f(x) = 0$. $x^2 - 6x - 11 = 0$

**Step 2** Write the Quadratic Formula. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

**Step 3** Substitute values for $a$, $b$, and $c$ into the Quadratic Formula.
$a = 1$, $b = -6$, $c = -11$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-11)}}{2(1)}$$

**Step 4** Simplify.

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-11)}}{2(1)} = \frac{6 \pm \sqrt{36 + 44}}{2} = \frac{6 \pm \sqrt{80}}{2}$$

**Step 5** Write in simplest form.

$$x = \frac{6 \pm \sqrt{80}}{2} = 3 \pm \frac{\sqrt{80}}{2} = 3 \pm \frac{\sqrt{(16)(5)}}{2} = 3 \pm \frac{4\sqrt{5}}{2} = 3 \pm 2\sqrt{5}$$

Remember to divide both terms of the numerator by 2 to simplify.

**Find the zeros of each function using the Quadratic Formula.**

1. $f(x) = x^2 + x - 1$

   $x^2 + x - 1 = 0$

   $a = 1$, $b = 1$, $c = -1$

   $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

   $x = \dfrac{-(1) \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)}$

   **$x = \dfrac{-1 \pm \sqrt{1 + 4}}{2}$**

   **$x = \dfrac{-1 \pm \sqrt{5}}{2}$**

2. $f(x) = x^2 - 6x + 6$

   **$x^2 - 6x + 6 = 0$**

   $a = 1$, $b = -6$, $c = 6$

   $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

   **$x = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(6)}}{2(1)}$**

   **$x = \dfrac{6 \pm \sqrt{36 - 24}}{2}$**

   **$x = 3 \pm \sqrt{3}$**

LESSON 5-6

## Reteach

### The Quadratic Formula (continued)

The **discriminant** of $ax^2 + bx + c = 0$ $(a \neq 0)$ is $b^2 - 4ac$.

Use the discriminant to determine the number of roots of a quadratic equation. A quadratic equation can have 2 real solutions, 1 real solution, or 2 complex solutions.

Find the type and number of solutions.

| $2x^2 - 5x = 3$ | $x^2 + 10x = -25$ | $3x^2 - 4x = -2$ |
|---|---|---|
| Write the equation in standard form: | Write the equation in standard form: | Write the equation in standard form: |
| $2x^2 - 5x - 3 = 0$ | $x^2 + 10x + 25 = 0$ | $3x^2 - 4x + 2 = 0$ |
| $a = 2, b = -5, c = -3$ | $a = 1, b = 10, c = 25$ | $a = 3, b = -4, c = 2$ |
| Evaluate the discriminant: | Evaluate the discriminant: | Evaluate the discriminant: |
| $b^2 - 4ac$ | $b^2 - 4ac$ | $b^2 - 4ac$ |
| $(-5)^2 - 4(2)(-3)$ | $(10)^2 - 4(1)(25)$ | $(-4)^2 - 4(3)(2)$ |
| $25 + 24$ | $100 - 100$ | $16 - 24$ |
| $49$ | $0$ | $-8$ |
| When $b^2 - 4ac > 0$, the equation has 2 real solutions. | When $b^2 - 4ac = 0$, the equation has 1 real solution. | When $b^2 - 4ac < 0$, the equation has 2 complex solutions. |

**Find the type and number of solutions for each equation.**

**3.** $x^2 - 12x = -36$

$x^2 - 12x + 36 = 0$

$a = $ 1, $b = $ −12, $c = $ 36

$b^2 - 4ac = $ 0

Classify solutions: 1 real solution

**4.** $x^2 - 4x = -7$

$x^2 - 4x + 7 = 0$

$a = 1, b = -4, c = 7$

$b^2 - 4ac = $ −12

Classify solutions: 2 complex solutions

**5.** $x^2 - 7x = -3$

$x^2 - 7x + 3 = 0$

$a = 1, b = -7, c = 3$

$b^2 - 4ac = $ 37

Classify solutions: 2 real solutions



LESSON 5-6

## Challenge

### Relating Roots and Coefficients of a Quadratic Equation

The general solution of the quadratic equation $ax^2 + bx + c = 0$ can be written in terms of the coefficients $a$, $b$, and $c$, and this solution is known as the Quadratic Formula. You can explore some other relationships between the roots and the coefficients.

**1.** Complete the table below.

| | Equation | Roots | Sum of the Roots | Product of the Roots |
|---|---|---|---|---|
| **a.** | $x^2 - 6x + 8 = 0$ | 4, 2 | 6 | 8 |
| **b.** | $x^2 - 7x + 12 = 0$ | 4, 3 | 7 | 12 |
| **c.** | $x^2 + 2x - 35 = 0$ | 5, −7 | −2 | −35 |
| **d.** | $4x^2 - 8x + 3 = 0$ | $\frac{1}{2}, \frac{3}{2}$ | 2 | $\frac{3}{4}$ |
| **e.** | $9x^2 + 3x - 2 = 0$ | $\frac{1}{3}, -\frac{2}{3}$ | $-\frac{1}{3}$ | $-\frac{2}{9}$ |

**2.** Refer to the table above. Let the roots of the quadratic equation $ax^2 + bx + c = 0$ be represented by $r_1$ and $r_2$.

**a.** Express the sum of the roots in terms of the coefficients of the equation. $r_1 + r_2 = $ $r_1 + r_2 = -\frac{b}{a}$

**b.** Express the product of the roots in terms of the coefficients of the equation. $r_1r_2 = $ $r_1r_2 = \frac{c}{a}$

**Use the relationships between roots and coefficients that you wrote in Exercise 2. Verify your answer by solving the equation.**

**3.** Write a quadratic equation whose roots are $2 + \sqrt{5}$ and $2 - \sqrt{5}$. $x^2 - 4x - 1 = 0$

**4.** The sum of the roots of $5x^2 - kx - 3 = 0$ is equal to the product of the roots. Determine the value of $k$. $k = -3$

**5.** Without solving, decide which numbers are the roots of $9x^2 - 6x - 1$.

**A.** $1 \pm \sqrt{2}$ **B.** $1 \pm \sqrt{3}$ **C.** $\frac{1 \pm \sqrt{2}}{3}$ **D.** $\frac{1 \pm \sqrt{3}}{2}$ C

**6.** Which of these equations has $\frac{-5 \pm \sqrt{17}}{2}$ as its solutions?

**A.** $x^2 + 5x + 2 = 0$ **B.** $x^2 + 5x - 2 = 0$

**C.** $x^2 - 5x - 2 = 0$ **D.** $x^2 - 5x + 2 = 0$ A



LESSON 5-6

## Problem Solving

### The Quadratic Formula

**In a shot-put event, Jenna tosses her last shot from a position of about 6 feet above the ground with an initial vertical and horizontal velocity of 20 feet per second. The height of the shot is modeled by the function $h(t) = -16t^2 + 20t + 6$, where $t$ is the time in seconds after the toss. The horizontal distance traveled after $t$ seconds is modeled by $d(t) = 20t$.**

**1.** Jenna wants to know the exact distance the shot travels at a velocity of 20 feet per second.

**a.** Use the Quadratic Formula $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to solve the height function for $t$. $t = -0.25, 1.5$

**b.** Use the value for $t$ and the distance function to find the distance her shot travels. 30 ft

**2.** Jenna is working to improve her performance. She makes a table to show how the horizontal distance varies with velocity. Complete the table.

| | Velocity (ft/s) | Formula | Time (s) | Distance (ft) |
|---|---|---|---|---|
| **a.** | 22 | $t = \frac{-22 \pm \sqrt{(22)^2 - 4(-16)(6)}}{2(-16)}$ | $t = -0.23, 1.61$ | 35.4 ft |
| **b.** | 25 | | $t = -0.21, 1.77$ | 44.3 ft |
| **c.** | 28 | | $t = -0.19, 1.94$ | 54.3 ft |

**Jenna has not reached her full potential yet. Her goal is to toss the shot from a height of 6 feet 6 inches with a vertical and horizontal velocity of 30 feet per second. Choose the letter for the best answer.**

**3.** If she achieves her goal, how long will her shot stay in the air?

**A** 1.65 s
**B** 1.87 s
**(C)** 2.07 s
**D** 2.27 s

**4.** If she achieves her goal, what horizontal distance will the shot travel?

**A** 41.4 ft
**B** 56.1 ft
**(C)** 62.1 ft
**D** 68.1 ft



LESSON 5-6

## Reading Strategy

### Graphic Organizer

The Quadratic Formula can be used to solve any quadratic equation.

| **Definition** | **Facts** |
|---|---|
| When the equation is in the form $ax^2 + bx + c = 0$. The quadratic formula is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ | In a quadratic equation, the expression under the square root sign, $b^2 - 4ac$, is known as the **discriminant**. It tells you about the roots of the equation. $b^2 - 4ac > 0$ : two real roots; $b^2 - 4ac < 0$ : two complex roots; $b^2 - 4ac = 0$ : one real root |
| **Example** | **Find the number of roots.** |
| $x^2 - x - 6 = 0$; $a = 1, b = -1, c = -6$; $x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-6)}}{2(1)}$; $x = 3, x = -2$ | $b^2 - 4ac$; $(-1)^2 - 4(1)(-6)$; $1 + 24 = 25$; $25 > 0$; There are two real roots. |

**Use the equation $2x^2 - 6x - 9 = 0$ to answer the following questions.**

**1.** Write the values of $a$, $b$, and $c$.

$a = 2, b = -6, c = -9$

**2.** Find the value of the discriminant.

$(-6)^2 - 4(2)(-9) = 108$

**3.** Does this quadratic equation have real or complex roots?

Since the discriminant is positive, the equation has two real roots.

**4.** Does the graph of the related quadratic function $f(x) = 2x^2 - 6x - 9$ intersect the $x$-axis? Explain how you know.

Yes; since the equation has two real roots, the related function has two zeros.

**5.** What are the solutions to this equation?

$$x = \frac{-(-6) \pm \sqrt{108}}{2(2)} = \frac{3 \pm 3\sqrt{3}}{2}$$