Name ______________________________ Date __________ Class __________

LESSON 5-6

Challenge

Relating Roots and Coefficients of a Quadratic Equation

The general solution of the quadratic equation $ax^2 + bx + c = 0$ can be written in terms of the coefficients a, b, and c, and this solution is known as the Quadratic Formula. You can explore some other relationships between the roots and the coefficients.

1. Complete the table below.

	Equation	Roots	Sum of the Roots	Product of the Roots
a.	$x^2 - 6x + 8 = 0$			
b.	$x^2 - 7x + 12 = 0$			
c.	$x^2 + 2x - 35 = 0$			
d.	$4x^2 - 8x + 3 = 0$			
e.	$9x^2 + 3x - 2 = 0$			

2. Refer to the table above. Let the roots of the quadratic equation $ax^2 + bx + c = 0$ be represented by r_1 and r_2.

a. Express the sum of the roots in terms of the coefficients of the equation. $r_1 + r_2 =$ ____________

b. Express the product of the roots in terms of the coefficients of the equation. $r_1r_2 =$ ____________

Use the relationships between roots and coefficients that you wrote in Exercise 2. Verify your answer by solving the equation.

3. Write a quadratic equation whose roots are $2 + \sqrt{5}$ and $2 - \sqrt{5}$. ____________

4. The sum of the roots of $5x^2 - kx - 3 = 0$ is equal to the product of the roots. Determine the value of k. ____________

5. Without solving, decide which numbers are the roots of $9x^2 - 6x - 1$.

A. $1 \pm \sqrt{2}$ **B.** $1 \pm \sqrt{3}$ **C.** $\frac{1 \pm \sqrt{2}}{3}$ **D.** $\frac{1 \pm \sqrt{3}}{2}$ ____________

6. Which of these equations has $\frac{-5 \pm \sqrt{17}}{2}$ as its solutions?

A. $x^2 + 5x + 2 = 0$ **B.** $x^2 + 5x - 2 = 0$

C. $x^2 - 5x - 2 = 0$ **D.** $x^2 - 5x + 2 = 0$ ____________

LESSON 5-6

Reteach

The Quadratic Formula (continued)

The **discriminant** of $ax^2 + bx + c = 0$ $(a \neq 0)$ is $b^2 - 4ac$.

Use the discriminant to determine the number of roots of a quadratic equation. A quadratic equation can have 2 real solutions, 1 real solution, or 2 complex solutions.

Find the type and number of solutions.

$2x^2 - 5x = 3$	$x^2 + 10x = -25$	$3x^2 - 4x = -2$
Write the equation in standard form:	Write the equation in standard form:	Write the equation in standard form:
$2x^2 - 5x - 3 = 0$	$x^2 + 10x + 25 = 0$	$3x^2 - 4x + 2 = 0$
$a = 2, b = -5, c = -3$	$a = 1, b = 10, c = 25$	$a = 3, b = -4, c = 2$
Evaluate the discriminant:	Evaluate the discriminant:	Evaluate the discriminant:
$b^2 - 4ac$	$b^2 - 4ac$	$b^2 - 4ac$
$(-5)^2 - 4(2)(-3)$	$(10)^2 - 4(1)(25)$	$(-4)^2 - 4(3)(2)$
$25 + 24$	$100 - 100$	$16 - 24$
49	0	-8
When $b^2 - 4ac > 0$, the equation has 2 real solutions.	When $b^2 - 4ac = 0$, the equation has 1 real solution.	When $b^2 - 4ac < 0$, the equation has 2 complex solutions.

Find the type and number of solutions for each equation.

3. $x^2 - 12x = -36$

$x^2 - 12x + 36 = 0$

$a =$ 1, $b =$ −12, $c =$ 36

$b^2 - 4ac =$

0

Classify solutions:

1 real solution

4. $x^2 - 4x = -7$

$x^2 - 4x + 7 = 0$

$a = 1, b = -4, c = 7$

$b^2 - 4ac =$

−12

Classify solutions:

2 complex solutions

5. $x^2 - 7x = -3$

$x^2 - 7x + 3 = 0$

$a = 1, b = -7, c = 3$

$b^2 - 4ac =$

37

Classify solutions:

2 real solutions

LESSON 5-6

Challenge

Relating Roots and Coefficients of a Quadratic Equation

The general solution of the quadratic equation $ax^2 + bx + c = 0$ can be written in terms of the coefficients a, b, and c, and this solution is known as the Quadratic Formula. You can explore some other relationships between the roots and the coefficients.

1. Complete the table below.

	Equation	Roots	Sum of the Roots	Product of the Roots
a.	$x^2 - 6x + 8 = 0$	4, 2	6	8
b.	$x^2 - 7x + 12 = 0$	4, 3	7	12
c.	$x^2 + 2x - 35 = 0$	5, −7	−2	−35
d.	$4x^2 - 8x + 3 = 0$	$\frac{1}{2}, \frac{3}{2}$	2	$\frac{3}{4}$
e.	$9x^2 + 3x - 2 = 0$	$\frac{1}{3}, -\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{2}{9}$

2. Refer to the table above. Let the roots of the quadratic equation $ax^2 + bx + c = 0$ be represented by r_1 and r_2.

a. Express the sum of the roots in terms of the coefficients of the equation. $r_1 + r_2 =$ $r_1 + r_2 = -\frac{b}{a}$

b. Express the product of the roots in terms of the coefficients of the equation. $r_1 r_2 =$ $r_1 r_2 = \frac{c}{a}$

Use the relationships between roots and coefficients that you wrote in Exercise 2. Verify your answer by solving the equation.

3. Write a quadratic equation whose roots are $2 + \sqrt{5}$ and $2 - \sqrt{5}$. $x^2 - 4x - 1 = 0$

4. The sum of the roots of $5x^2 - kx - 3 = 0$ is equal to the product of the roots. Determine the value of k. $k = -3$

5. Without solving, decide which numbers are the roots of $9x^2 - 6x - 1$.

A. $1 \pm \sqrt{2}$ **B.** $1 \pm \sqrt{3}$ **C.** $\frac{1 \pm \sqrt{2}}{3}$ **D.** $\frac{1 \pm \sqrt{3}}{2}$ C

6. Which of these equations has $\frac{-5 \pm \sqrt{17}}{2}$ as its solutions?

A. $x^2 + 5x + 2 = 0$ **B.** $x^2 + 5x - 2 = 0$

C. $x^2 - 5x - 2 = 0$ **D.** $x^2 - 5x + 2 = 0$ A

LESSON 5-6

Problem Solving

The Quadratic Formula

In a shot-put event, Jenna tosses her last shot from a position of about 6 feet above the ground with an initial vertical and horizontal velocity of 20 feet per second. The height of the shot is modeled by the function $h(t) = -16t^2 + 20t + 6$, where t is the time in seconds after the toss. The horizontal distance traveled after t seconds is modeled by $d(t) = 20t$.

1. Jenna wants to know the exact distance the shot travels at a velocity of 20 feet per second.

a. Use the Quadratic Formula $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to solve the height function for t. $t = -0.25, 1.5$

b. Use the value for t and the distance function to find the distance her shot travels. 30 ft

2. Jenna is working to improve her performance. She makes a table to show how the horizontal distance varies with velocity. Complete the table.

	Velocity (ft/s)	Formula	Time (s)	Distance (ft)
a.	22	$t = \frac{-22 \pm \sqrt{(22)^2 - 4(-16)(6)}}{2(-16)}$	$t = -0.23, 1.61$	35.4 ft
b.	25		$t = -0.21, 1.77$	44.3 ft
c.	28		$t = -0.19, 1.94$	54.3 ft

Jenna has not reached her full potential yet. Her goal is to toss the shot from a height of 6 feet 6 inches with a vertical and horizontal velocity of 30 feet per second. Choose the letter for the best answer.

3. If she achieves her goal, how long will her shot stay in the air?

- **A** 1.65 s
- **B** 1.87 s
- [✓ **C**] 2.07 s
- **D** 2.27 s

4. If she achieves her goal, what horizontal distance will the shot travel?

- **A** 41.4 ft
- **B** 56.1 ft
- [✓ **C**] 62.1 ft
- **D** 68.1 ft

LESSON 5-6

Reading Strategy

Graphic Organizer

The Quadratic Formula can be used to solve any quadratic equation.

Definition When the equation is in the form $ax^2 + bx + c = 0$ The quadratic formula is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**Facts** In a quadratic equation, the expression under the square root sign, $b^2 - 4ac$, is known as the **discriminant**. It tells you about the roots of the equation. $b^2 - 4ac > 0$: two real roots $b^2 - 4ac < 0$: two complex roots $b^2 - 4ac = 0$: one real root
Example $x^2 - x - 6 = 0$ $a = 1, b = -1, c = -6$ $x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-6)}}{2(1)}$ $x = 3, x = -2$	**Find the number of roots.** $b^2 - 4ac$ $(-1)^2 - 4(1)(-6)$ $1 + 24 = 25$ $25 > 0$ There are two real roots.

Use the equation $2x^2 - 6x - 9 = 0$ to answer the following questions.

1. Write the values of a, b, and c.

$a = 2, b = -6, c = -9$

2. Find the value of the discriminant.

$(-6)^2 - 4(2)(-9) = 108$

3. Does this quadratic equation have real or complex roots?

Since the discriminant is positive, the equation has two real roots.

4. Does the graph of the related quadratic function $f(x) = 2x^2 - 6x - 9$ intersect the x-axis? Explain how you know.

Yes; since the equation has two real roots, the related function has two zeros.

5. What are the solutions to this equation?

$$x = \frac{-(-6) \pm \sqrt{108}}{2(2)} = \frac{3 \pm 3\sqrt{3}}{2}$$