5-6 The Quadratic Formula

Example 1 Quadratic Functions with Real Zeros

Find the zeros of $f(x) = 2x^2 - 16x + 27$ using the Quadratic Formula.

$2x^2 - 16x + 27 = 0$	*Set $f(x) = 0$.*
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	*Write the Quadratic Formula.*
$x = \dfrac{-(-16) \pm \sqrt{(-16)^2 - 4(2)(27)}}{2(2)}$	*Substitute 2 for a, −16 for b, and 27 for c.*
$x = \dfrac{16 \pm \sqrt{256 - 216}}{4} = \dfrac{16 \pm \sqrt{40}}{4}$	*Simplify.*
$x = \dfrac{16 \pm 2\sqrt{10}}{4} = 4 \pm \dfrac{\sqrt{10}}{2}$	*Write in simplest form.*

Check Solve by completing the square.

$$2x^2 - 16x + 27 = 0$$

$$2(x^2 - 8x) = -27$$

$$2(x - 4)^2 = -27 + 32$$

$$2(x - 4)^2 = 5$$

$$x = 4 \pm \frac{\sqrt{10}}{2}$$

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Example 2 Quadratic Functions with Complex Zeros

Find the zeros of $f(x) = 4x^2 + 3x + 2$ using the Quadratic Formula.

$4x^2 + 3x + 2 = 0$ — *Set $f(x) = 0$.*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ *Write the Quadratic Formula.*

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(4)(2)}}{2(4)}$$ Substitute 4 for a, 3 for b, and 2 for c.

$$x = \frac{-3 \pm \sqrt{9 - 32}}{2(4)} = \frac{-3 \pm \sqrt{-23}}{8}$$ Simplify.

$$x = \frac{-3 \pm \sqrt{-23}}{8} = -\frac{3}{8} \pm \frac{\sqrt{23}}{8}i$$ Write in terms of i.

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Example 3 Analyzing Quadratic Equations by Using the Discriminant

Find the type and number of solutions for each equation.

A. $x^2 + 36 = 12x$

$x^2 - 12x + 36 = 0$

$b^2 - 4ac$

$(-12)^2 - 4(1)(36)$

$144 - 144 = 0$

$b^2 - 4ac = 0$

The equation has one distinct real solution.

B. $x^2 + 40 = 12x$

$x^2 - 12x + 40 = 0$

$b^2 - 4ac$

$(-12)^2 - 4(1)(40)$

$144 - 160 = -16$

$b^2 - 4ac < 0$

The equation has two distinct nonreal complex solutions.

C. $x^2 + 30 = 12x$

$x^2 - 12x + 30 = 0$

$b^2 - 4ac$

$(-12)^2 - 4(1)(30)$

$144 - 120 = 24$

$b^2 - 4ac > 0$

The equation has two distinct real solutions.

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Example 4 Sports Application

An athlete on a track team throws a shot put. The height y of the shot put in feet t seconds after it is thrown is modeled by $y = -16t^2 + 24.6t + 6.5$. The horizontal distance x in feet between the athlete and the shot put is modeled by $x = 29.3t$. To the nearest foot, how far does the shot put land from the athlete?

Step 1 Use the first equation to determine how long it will take the shot put to hit the ground. Set the height of the shot put equal to 0 feet, and use the quadratic formula to solve for t.

$$y = -16t^2 + 24.6t + 6.5$$

$$0 = -16t^2 + 24.6t + 6.5$$ *Set y equal to 0.*

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ *Write the Quadratic Formula.*

$$t = \frac{-(24.6) \pm \sqrt{(24.6)^2 - 4(-16)(6.5)}}{2(-16)}$$ *Substitute –16 for a, 24.6 for b, and 6.5 for c.*

$$t = \frac{24.6 \pm \sqrt{1021.16}}{32} \approx \frac{24.6 \pm 31.96}{32}$$ *Simplify.*

$$t \approx \frac{56.56}{32} \approx 1.77 \quad \text{or} \quad t \approx \frac{-7.36}{32} \approx -0.23$$

The time cannot be negative, so the shot put hits the ground around 1.8 seconds after it is released.

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Example 4 Sports Application (continued)

Step 2 Find the horizontal distance that the shot put will have traveled in this time.

$x = 29.3t$

$x \approx 29.3(1.77)$ *Substitute 1.77 for t.*

$x \approx 51.86$

$x \approx 52$ *Simplify.*

The shot put will have traveled a horizontal distance of about 52 feet.

Check *Use substitution to check that the shot put hits the ground after about 1.77 seconds.*

$y = -16t^2 + 24.6t + 6.5$

$y = -16(1.77)^2 + 24.6(1.77) + 6.5$

$y \approx -50.13 + 43.54 + 6.5$

$y \approx -0.09$ ✔ *The height is approximately equal to 0 when $t = 1.77$.*