Name ______________________________ Date ___________ Class ____________

LESSON 5-4

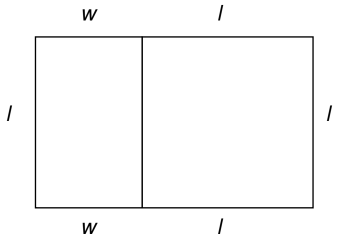
Challenge

The Golden Ratio

The ancient Greeks constructed rectangles called golden rectangles because they were thought to be pleasing to the eye. A rectangle is considered golden if the dimensions of the rectangle are in a certain ratio.

$$\frac{l}{w} = \frac{l + w}{l}$$

The ratio $\frac{l}{w}$ is called the golden ratio. A golden rectangle with length l and width w has the property that if it is joined to a square of side length l to form a larger rectangle, the length-to-width ratio of the larger rectangle is the same as that of the original rectangle.



Solve.

1. a. Clear the equation of fractions and collect all the terms that contain variables on the left side of the equation.

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b. Complete the square and solve for l in terms of w. Ignore the negative solution since l must be a positive number. Use the result to find both the exact value of $\frac{l}{w}$ and a decimal approximation.

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2. Measure the length and width of a credit card and calculate the ratio of the length and width. Does this closely approximate the golden ratio? ____________________

3. a. In the Fibonacci Sequence, {1, 1, 2, 3, 5, 8, 13, 21, 34, . . .}, each term from the third term on is the sum of the previous two terms. Make a list of values of the ratio of a term and it predecessor.

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b. What decimal value do these ratios approximate as the list is continued? ____________________

4. Consider the continued fraction $1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ldots}}}$.

Make a table of decimal values for this fraction when 1 fraction is used, then 2, then 3, and so on. Round the values to the nearest thousandth. What value do the fractions seem to approach?

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LESSON **5-4**

Reteach

Completing the Square (continued)

You can use a process called **completing the square** to rewrite a quadratic of the form $x^2 + bx$ as a perfect square trinomial.

To complete the square of $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$.	*Think:* Multiply the coefficient of x by $\frac{1}{2}$. Then square it.	$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

Complete the square: $x^2 - 8x + ?$.

Step 1 Identify b, the coefficient of x: $b = -8$.

Step 2 Find $\left(\frac{b}{2}\right)^2$: $\left(\frac{b}{2}\right)^2 = \left(\frac{-8}{2}\right)^2 = (-4)^2 = 16$

Step 3 Add $\left(\frac{b}{2}\right)^2$: $x^2 - 8x + 16$

Step 4 Factor: $x^2 - 8x + 16 = (x - 4)^2$

Use $\frac{b}{2}$ as a factor.

Check: $(x - 4)^2 = (x - 4)(x - 4)$

$= x^2 - 8x + 16$ ✓

Complete each square and factor.

5. $x^2 + 9x + ?$

$b = 9$, so $\frac{b}{2} = \frac{9}{2}$

$\left(\frac{b}{2}\right)^2 = \frac{81}{4}$

$x^2 + 9x + \frac{81}{4}$

$\left(x + \frac{9}{2}\right)^2$

6. $x^2 - 4x + ?$

$b = -4$, so $\frac{b}{2} = -2$

$\left(\frac{b}{2}\right)^2 = 4$

$x^2 - 4x + 4$

$(x - 2)^2$

7. $x^2 - 10x + ?$

$x^2 - 10x + 25$

$(x - 5)^2$

8. $x^2 + 3x + ?$

$x^2 + 3x + \frac{9}{4}$

$\left(x + \frac{3}{2}\right)^2$

LESSON **5-4**

Challenge

The Golden Ratio

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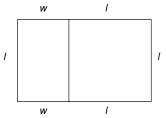
The ratio $\frac{l}{w}$ is called the golden ratio. A golden rectangle with length l and width w has the property that if it is joined to a square of side length l to form a larger rectangle, the length-to-width ratio of the larger rectangle is the same as that of the original rectangle.



Solve.

1. a. Clear the equation of fractions and collect all the terms that contain variables on the left side of the equation.

$l^2 = wl + w^2,\ l^2 - wl - w^2 = 0$

b. Complete the square and solve for l in terms of w. Ignore the negative solution since l must be a positive number. Use the result to find both the exact value of $\frac{l}{w}$ and a decimal approximation.

$l^2 - wl = w^2,\ l^2 - wl + \left(\frac{w}{2}\right)^2 = w^2 + \left(\frac{w}{2}\right)^2,$

$\left(l - \frac{w}{2}\right)^2 = \frac{5w^2}{4},\ l - \frac{w}{2} = \frac{\sqrt{5}w}{2},\ l = \frac{1 + \sqrt{5}}{2}w,\ l \cong 1.618w$

2. Measure the length and width of a credit card and calculate the ratio of the length and width. Does this closely approximate the golden ratio? Possible answer: The ratio of length to width is about 1.588, a little less than the golden ratio.

3. a. In the Fibonacci Sequence, $\{1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots\}$, each term from the third term on is the sum of the previous two terms. Make a list of values of the ratio of a term and it predecessor.

$1, 2, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \frac{34}{21}, \frac{55}{34}, \ldots$

b. What decimal value do these ratios approximate as the list is continued? The golden ratio, 1.618

4. Consider the continued fraction $1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ldots}}}$.

Make a table of decimal values for this fraction when 1 fraction is used, then 2, then 3, and so on. Round the values to the nearest thousandth. What value do the fractions seem to approach?

2, 1.5, 1.667, 1.6, 1.625, 1.615, 1.619, 1.618; they seem to approach the golden ratio.

LESSON **5-4**

Problem Solving

Completing the Square

Sean and Mason run out of gas while fishing from their boat in the bay. They set off an emergency flare with an initial vertical velocity of 30 meters per second. The height of the flare in meters can be modeled by $h(t) = -5t^2 + 30t$, where t represents the number of seconds after launch.

1. Sean thinks the flare should reach at least 15 meters to be seen from the shore. They want to know how long the flare will take to reach this height.

a. Write an equation to determine how long it will take the flare to reach 15 meters. $15 = -5t^2 + 30t$

b. Simplify the function so you can complete the square. $t^2 - 6t = -3$

c. Solve the equation by completing the square. $t = 0.6, 5.4$

d. Mason thinks that the flare will reach 15 meters in 5.4 seconds. Is he correct? Explain.

Possible answer: He is partially correct. The flare will first reach 15 meters at 0.6 second after firing and then again at 5.4 seconds. (The function has two solutions.)

e. Sean thinks the flare will reach 15 meters sooner, but then the flare will stay above 15 meters for about 5 seconds. Is he correct? Explain.

Possible answer: He is correct. The flare will first reach 15 meters at 0.6 second after firing. Also, the difference between 5.4 and 0.6 seconds (the two solutions) is 4.8 seconds, which is about 5 seconds.

2. Sean wants to know how high the flare will reach above the surface of the water.

a. Write the function in vertex form, factoring so the coefficient of t^2 is 1. $h(t) = -5(t^2 - 6t + 9) + 45$

b. Complete the square using the vertex form of the function. $h(t) = -5(t - 3)^2 + 45$

c. How high will the flare reach? The constant term; 45 meters

Choose the letter for the best answer.

3. Use the vertex form of the function to determine how long after firing the flare it will reach its maximum height.

[✓ **A**] 3 s

B 5 s

C 9 s

D 15 s

4. The boys fire a similar flare from the deck 5 meters above the water level. Which statement is correct?

A The flare will reach 45 m in 3 s.

[✓ **B**] The flare will reach 50 m in 3 s.

C The flare will reach 45 m in 3.5 s.

D The flare will reach 50 m in 3.5 s.

LESSON **5-4**

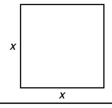
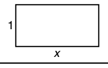
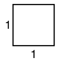
Reading Strategy

Use a Model

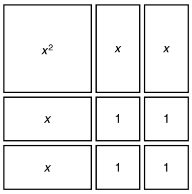
Just as some numbers are perfect squares, some quadratic expressions are perfect squares.

$(x + 2)^2 = x^2 + 4x + 4$ Perfect square

You can model this expression with the area of algebra tiles. You can use three types of tiles.

Algebraic Term	Type of Tile	Model
x^2	Square with sides x	
x	Rectangle with sides x and 1	
1	Square with sides 1	

$(x + 2)^2 = x^2 + 4x + 4$



You can make a quadratic expression $x^2 + bx$ into a perfect square.

Not a Perfect Square		Add		Perfect Square
$x^2 + bx$	➔	$\left(\frac{b}{2}\right)^2$	➔	$x^2 + bx + \frac{b^2}{4} = \left(x + \frac{1}{2}b\right)^2$
$x^2 + 8x$	➔	$\left(\frac{8}{2}\right)^2 = 4^2 = 16$	➔	$x^2 + 8x + 16 = (x + 4)^2$

Answer each question.

1. Circle the expressions that are perfect squares.

[✓ $(x - 1)^2$] $x^2 + 2x + 2$ [✓ $(4x - 5)^2$] [✓ $x^2 + 6x + 9$] $x^2 + x + 1$ [✓ x^2]

2. a. What would you add to the expression $x^2 - 4x$ to make it a perfect square? 4

b. Write this expression as a perfect square. $(x - 2)^2$

3. a. Use algebra tiles to draw a model for the expression $x^2 + 2x + 1$.

b. Write this expression as a perfect square. What are the sides of the square created in the model?

$(x + 1)^2$; the side of the model square is $x + 1$.