5-4 Completing the Square

Example 1 Solving Equations by Using the Square Root Property

Solve each equation.

A. $4x^2 + 11 = 59$

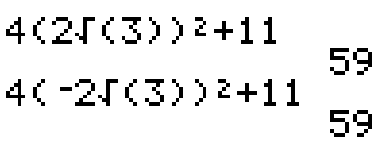
$4x^2 = 48$ *Subtract 11 from both sides.*

$x^2 = 12$ *Divide both sides by 4 to isolate the square term.*

$x = \pm\sqrt{12}$ *Take the square root of both sides.*

$x = \pm 2\sqrt{3}$ *Simplify.*

Check Use a graphing calculator.



B. $x^2 + 12x + 36 = 28$

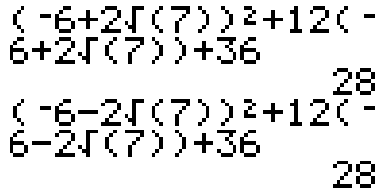
$(x + 6)^2 = 28$ *Factor the perfect square trinomial.*

$x + 6 = \pm\sqrt{28}$ *Take the square root of both sides.*

$x = -6 \pm \sqrt{28}$ *Subtract 6 from both sides.*

$x = -6 \pm 2\sqrt{7}$ *Simplify.*

Check Use a graphing calculator.



5-4 Completing the Square

Example 2 Completing the Square

Complete the square for each expression. Write the resulting expression as a binomial squared.

A. $x^2 - 14x + \square$

$\left(\frac{-14}{2}\right)^2 = (-7)^2 = 49$ *Find* $\left(\frac{b}{2}\right)^2$.

$x^2 - 14x + 49$ *Add.*

$(x - 7)^2$ *Factor.*

Check Find the square of the binomial.

$$(x - 7)^2 = (x - 7)(x - 7)$$
$$= x^2 - 14x + 49$$

B. $x^2 + 9x + \square$

$\left(\frac{9}{2}\right)^2 = \frac{81}{4}$ *Find* $\left(\frac{b}{2}\right)^2$.

$x^2 + 9x + \frac{81}{4}$ *Add.*

$\left(x + \frac{9}{2}\right)^2$ *Factor.*

Check Find the square of the binomial.

$$\left(x + \frac{9}{2}\right)^2 = \left(x + \frac{9}{2}\right)\left(x + \frac{9}{2}\right)$$
$$= x^2 + 9x + \frac{81}{4}$$

5-4 Completing the Square

Example 3 Solving a Quadratic Equation by Completing the Square

Solve each equation by completing the square.

A. $x^2 = 12x - 20$

$x^2 - 12x = -20$ *Collect variable terms on one side.*

$x^2 - 12x + \square = -20 + \square$ *Set up to complete the square.*

$x^2 - 12x + \left(-\frac{12}{2}\right)^2 = -20 + \left(-\frac{12}{2}\right)^2$ *Add $\left(\frac{b}{2}\right)^2$ to both sides.*

$x^2 - 12x + 36 = -20 + 36$ *Simplify.*

$(x - 6)^2 = 16$ *Factor.*

$x - 6 = \pm\sqrt{16}$ *Take the square root of both sides.*

$x - 6 = \pm 4$ *Simplify.*

$x - 6 = 4$ or $x - 6 = -4$ *Solve for x.*

$x = 10$ or $x = 2$

B. $18x + 3x^2 = 45$

$x^2 + 6x = 15$ *Divide both sides by 3.*

$x^2 + 6x + \square = 15 + \square$ *Set up to complete the square.*

$x^2 + 6x + \left(\frac{6}{2}\right)^2 = 15 + \left(\frac{6}{2}\right)^2$ *Add $\left(\frac{b}{2}\right)^2$ to both sides.*

$x^2 + 6x + 9 = 15 + 9$ *Simplify.*

$(x + 3)^2 = 24$ *Factor.*

$x + 3 = \pm\sqrt{24}$ *Take the square root of both sides.*

$x = -3 \pm 2\sqrt{6}$ *Simplify.*

5-4 Completing the Square

Example 4 Writing a Quadratic Function in Vertex Form

Write each function in vertex form, and identify its vertex.

A. $f(x) = x^2 + 16x - 12$

$f(x) = (x^2 + 16x + \square) - 12 - \square$ *Set up to complete the square.*

$f(x) = \left[x^2 + 16x + \left(\frac{16}{2}\right)^2\right] - 12 - \left(\frac{16}{2}\right)^2$ *Add and subtract* $\left(\frac{b}{2}\right)^2$.

$f(x) = (x + 8)^2 - 76$ *Simplify and factor.*

Because $h = -8$ and $k = -76$, the vertex is $(-8, -76)$.

Check Use the axis of symmetry formula to confirm the vertex.

$$x = -\frac{b}{2a} = -\frac{16}{2(1)} = -8$$

$$y = f(-8) = (-8)^2 + 16(-8) - 12 = -76 \checkmark$$

5-4 Completing the Square

Example 4 Writing a Quadratic Function in Vertex Form (continued)

B. $g(x) = 3x^2 - 18x + 7$

$g(x) = 3(x^2 - 6x) + 7$ *Factor so the coefficient of x^2 is 1.*

$g(x) = 3(x^2 - 6x + \square) + 7 - \square$ *Set up to complete the square.*

$g(x) = 3\left[x^2 - 6x + \left(-\frac{6}{2}\right)^2\right] + 7 - 3\left(-\frac{6}{2}\right)^2$ *Add $\left(\frac{b}{2}\right)^2$. Because $\left(\frac{b}{2}\right)^2$ is multiplied by 3, you must subtract $3\left(\frac{b}{2}\right)^2$.*

$g(x) = 3(x - 3)^2 - 20$ *Simplify and factor.*

Because $h = 3$ and $k = -20$, the vertex is $(3, -20)$.

Check A graph of the function on a graphing calculator supports your answer.