CHAPTER 4

Vocabulary

The table contains important vocabulary terms from Chapter 4. As you work through the chapter, fill in the page number, definition, and a clarifying example.

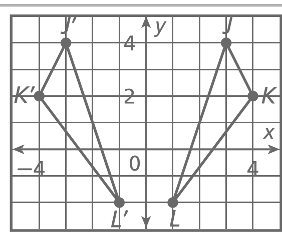
Term	Page	Definition	Clarifying Example
augmented matrix			
coefficient matrix			
constant matrix			
determinant			
dimension of a matrix			
matrix equation			

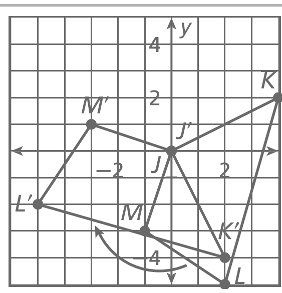
Vocabulary

The table contains important vocabulary terms from Chapter 4. As you work through the chapter, fill in the page number, definition, and a clarifying example.

Term	Page	Definition	Clarifying Example
augmented matrix	287	**A matrix that consists of the coefficients and the constant terms in a system of linear equations.**	**System of equations** $3x + 2y = 5$, $2x - 3y = 1$ **Augmented matrix** $\left(\begin{array}{cc\|c} 3 & 2 & 5 \\ 2 & -3 & 1 \end{array}\right)$
coefficient matrix	271	**The matrix of the coefficients of the variables in a linear system of equations.**	**System of equations** $2x + 3y = 11$, $5x - 4y = 16$ **Coefficient matrix** $\begin{bmatrix} 2 & 3 \\ 5 & -4 \end{bmatrix}$
constant matrix	279	**The matrix of the constants in a linear system of equations.**	**System of equations** $2x + 3y = 11$, $5x - 4y = 16$ **Constant matrix** $\begin{bmatrix} 11 \\ 16 \end{bmatrix}$
determinant	270	**A real number associated with a square matrix. The determinant of** $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ **is** $\lvert A \rvert = ad - bc$.	$\begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix} = 2 \cdot 4 - (-1) \cdot 3 = 11$ $\begin{vmatrix} 1 & 4 & 0 \\ 2 & 5 & 3 \\ 3 & 0 & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} 5 & 3 \\ 0 & 1 \end{vmatrix} - 4 \cdot \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} +$ $0 \cdot \begin{vmatrix} 2 & 5 \\ 3 & 0 \end{vmatrix} = 33$
dimension of a matrix	246	**A matrix with** m **rows and** n **columns has dimensions** $m \times n$, **read "**m **by** n.	$\begin{bmatrix} -3 & 2 & 1 & -1 \\ 4 & 0 & -5 & 2 \end{bmatrix}$ **Dimensions** 2×4
matrix equation	279	**An equation of the form** $AX = B$, **where** A **is the coefficient matrix,** X **is the variable matrix, and** B **is the constant matrix of a system of equations.**	**System of equations:** $2x + 3y = 7$, $4x - 6y = 5$ **Matrix equation:** $\begin{bmatrix} 2 & 3 \\ 4 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 5 \end{bmatrix}$

Term	Page	Definition	Clarifying Example
matrix product			
multiplicative identity matrix			
reflection matrix			
rotation matrix			
square matrix			

<table>
<tr><th>Term</th><th>Page</th><th>Definition</th><th>Clarifying Example</th></tr>
<tr><td>matrix product</td><td>253</td><td>The product of two matrices, where each entry in P_{ij} is the sum of the products of consecutive entries in row i in matrix A and column j in matrix B.</td><td>$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 1(5) + 2(7) & 1(6) + 2(8) \\ 3(5) + 4(7) & 3(6) + 4(8) \end{bmatrix}$</td></tr>
<tr><td>multiplicative identity matrix</td><td>255</td><td>A square matrix with 1 in every entry of the main diagonal and 0 in every other entry.</td><td>$6 \cdot 1 = 6$
$\begin{pmatrix} -2 & 5 \\ 7 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 5 \\ 7 & -1 \end{pmatrix}$</td></tr>
<tr><td>reflection matrix</td><td>263</td><td>A matrix used to reflect a figure across a specified line of symmetry.</td><td>

Matrix $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ was used to reflect the figure across the y-axis.</td></tr>
<tr><td>rotation matrix</td><td>264</td><td>A matrix used to rotate a figure about the origin.</td><td>

Matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ was used to rotate the figure 90° clockwise.</td></tr>
<tr><td>square matrix</td><td>255</td><td>A matrix with the same number of rows as columns.</td><td>$\begin{pmatrix} 3 & 1 & 2 \\ 22 & 3 & 7 \\ 13 & 0 & 0 \end{pmatrix}$</td></tr>
</table>

6. Get Organized Fill in the augmented matrix for a three-equation system. Then write an example of the given operation in each box. Tell whether the operation produces an equivalent system. (p. 290).

	SYSTEM OF EQUATIONS	AUGMENTED MATRIX
Interchange rows or equations		
Replace a row or equation with a multiple.		
Replace a row or equation with a sum or difference.		
Combine the above.		

6. Get Organized Fill in the augmented matrix for a three-equation system. Then write an example of the given operation in each box. Tell whether the operation produces an equivalent system. (p. 290).

	SYSTEM OF EQUATIONS	AUGMENTED MATRIX
Interchange rows or equations	❶ $x + 3y = 5$ ❷ $2x + y = 8$ ❷ $2x + y = 8$ ❶ $x + 3y = 5$	❶❷ $\left[\begin{array}{cc:c} 1 & 3 & 5 \\ 2 & 1 & 8 \end{array}\right]$ ❷❶ $\left[\begin{array}{cc:c} 2 & 1 & 8 \\ 1 & 3 & 5 \end{array}\right]$
Replace a row or equation with a multiple.	❶ $x + 3y = 5$ ❷ $2x + y = 8$ 2❶ $\rightarrow 2x + 6y = 10$ $2x + y = 8$	❶❷ $\left[\begin{array}{cc:c} 1 & 3 & 5 \\ 2 & 1 & 8 \end{array}\right]$ 2❶ $\rightarrow \left[\begin{array}{cc:c} 2 & 6 & 10 \\ 2 & 1 & 8 \end{array}\right]$
Replace a row or equation with a sum or difference.	❶ $2x + 6y = 10$ ❷ $2x + y = 8$ $2x + 6y = 10$ ❶ − ❷ $\rightarrow 5y = 2$	❶❷ $\left[\begin{array}{cc:c} 2 & 6 & 10 \\ 2 & 1 & 8 \end{array}\right]$ ❶ − ❷ $\rightarrow \left[\begin{array}{cc:c} 2 & 6 & 10 \\ 0 & 5 & 2 \end{array}\right]$
Combine the above.	❶ $x + 3y = 5$ ❷ $2x + y = 8$ $x + 3y = 5$ 2❶ − ❷ $\rightarrow 0x + 5y = 2$	❶❷ $\left[\begin{array}{cc:c} 1 & 3 & 5 \\ 2 & 1 & 8 \end{array}\right]$ 2❶ − ❷ $\rightarrow \left[\begin{array}{cc:c} 1 & 3 & 5 \\ 0 & 5 & 2 \end{array}\right]$

CHAPTER 4

Chapter Review

Know it! Note

4-1 Matrices and Data

Use the table for Exercises 1–4.

1. Display the data in the form of a matrix *M*.

Coffee Shop Muffin Orders (dozens)			
	Barb's	Dugan's	Gonzalez
Banana nut	8	15	9
Blueberry	6	10	7
Cinnamon	4	3	5

2. What are the dimensions of *M*?

3. What is the value of the matrix entry with the address M_{23}? What does it represent?

4. What is the address of the entry that has the value 6?

Use the matrices below for Exercises 5–8. Evaluate, if possible.

$$A = \begin{bmatrix} 1 & 6 \\ 3 & -5 \\ -2 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 4 & -6 \\ 1 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 2 & -1 \\ 0 & 1 \\ 5 & -2 \end{bmatrix} \quad D = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 0 & 3 \end{bmatrix}$$

5. $A + C$

6. $2B$

7. $3A - C$

8. $\frac{1}{2}C - D$

Chapter Review

Know it! Note

4-1 Matrices and Data

Use the table for Exercises 1–4.

1. Display the data in the form of a matrix M.

$$\begin{bmatrix} 8 & 15 & 9 \\ 6 & 10 & 7 \\ 4 & 3 & 5 \end{bmatrix}$$

Coffee Shop Muffin Orders (dozens)			
	Barb's	Dugan's	Gonzalez
Banana nut	8	15	9
Blueberry	6	10	7
Cinnamon	4	3	5

2. What are the dimensions of M?

3 × 3

3. What is the value of the matrix entry with the address M_{23}? What does it represent?

7; The 7 represents 7 dozen blueberry muffins that have been ordered by the Gonzalez coffee shop.

4. What is the address of the entry that has the value 6?

M_{21}

Use the matrices below for Exercises 5–8. Evaluate, if possible.

$$A = \begin{bmatrix} 1 & 6 \\ 3 & -5 \\ -2 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 4 & -6 \\ 1 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 2 & -1 \\ 0 & 1 \\ 5 & -2 \end{bmatrix} \quad D = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 0 & 3 \end{bmatrix}$$

5. $A + C$

$$\begin{bmatrix} 3 & 5 \\ 3 & -4 \\ 3 & 2 \end{bmatrix}$$

6. $2B$

$$\begin{bmatrix} 8 & -12 \\ 2 & 4 \end{bmatrix}$$

7. $3A - C$

$$\begin{bmatrix} 1 & 19 \\ 9 & -16 \\ -11 & 14 \end{bmatrix}$$

8. $\frac{1}{2}C - D$

not possible

4-2 Multiplying Matrices

Use the matrices named below for Exercises 9–12. Tell whether each product is defined. If so, give its dimensions.

$P_{4\times3}$, $Q_{3\times3}$, $R_{3\times4}$, and $S_{3\times2}$

9. *PQ*

10. *QR*

11. *RS*

12. *PS*

Use the matrices below for Exercises 13–16. Evaluate, if possible.

$L = \begin{bmatrix} 1 & -1 & 1 \\ 4 & 3 & 0 \\ -1 & 1 & -2 \end{bmatrix}$ $M = [0.5 \quad 1 \quad 0.25]$ $N = \begin{bmatrix} 1 & 3 \\ 3 & -1 \end{bmatrix}$ $P = \begin{bmatrix} -2 & 3 \\ 3 & 0 \\ 0 & -1 \end{bmatrix}$

13. *LM*

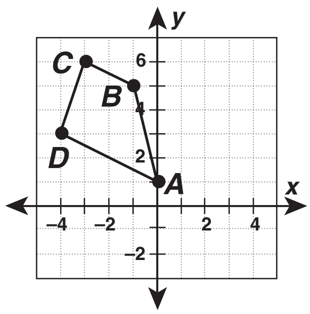
14. *MP*

15. *PN*

16. N^2

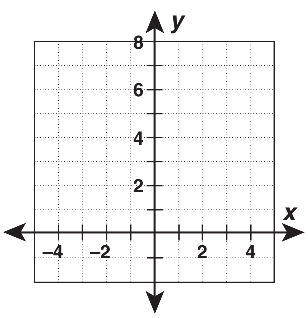
4-3 Using Matrices to Transform Geometric Figures

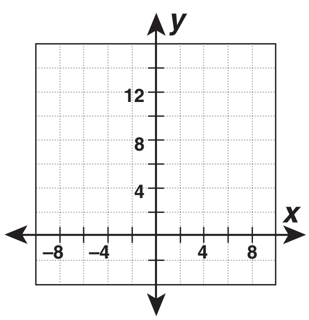
For Exercises 17–21, use polygon *ABCD* with coordinates *A*(0, 1), *B*(–1, 5), *C*(–3, 6), *D*(–4, 3). Give the coordinates of the image and graph.



17. Translate polygon *ABCD* 1 unit right and 2 units up.

18. Enlarge polygon *ABCD* by a factor of 2.





4-2 Multiplying Matrices

Use the matrices named below for Exercises 9–12. Tell whether each product is defined. If so, give its dimensions.

$P_{4\times3}$, $Q_{3\times3}$, $R_{3\times4}$, and $S_{3\times2}$

9. *PQ* Yes; 4 × 3

10. *QR* Yes; 3 × 4

11. *RS* not possible

12. *PS* Yes; 4 × 2

Use the matrices below for Exercises 13–16. Evaluate, if possible.

$L = \begin{bmatrix} 1 & -1 & 1 \\ 4 & 3 & 0 \\ -1 & 1 & -2 \end{bmatrix}$ $M = [0.5 \quad 1 \quad 0.25]$ $N = \begin{bmatrix} 1 & 3 \\ 3 & -1 \end{bmatrix}$ $P = \begin{bmatrix} -2 & 3 \\ 3 & 0 \\ 0 & -1 \end{bmatrix}$

13. *LM* not possible

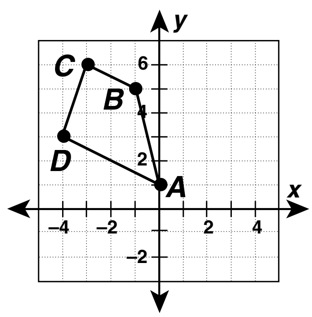
14. *MP* $[2 \quad 1.25]$

15. *PN* $\begin{bmatrix} 7 & -9 \\ 3 & 9 \\ -3 & 1 \end{bmatrix}$

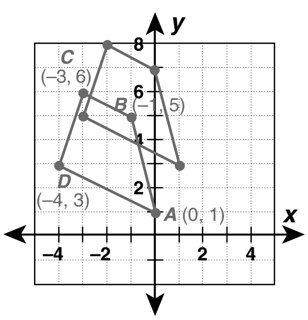
16. N^2 $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$

4-3 Using Matrices to Transform Geometric Figures

For Exercises 17–21, use polygon *ABCD* with coordinates *A*(0, 1), *B*(–1, 5), *C*(–3, 6), *D*(–4, 3). Give the coordinates of the image and graph.

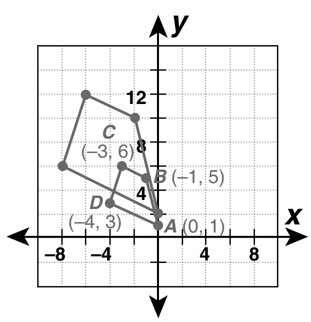


17. Translate polygon *ABCD* 1 unit right and 2 units up.



(1, 3); (0, 7); (−2, 8); (−3, 5)

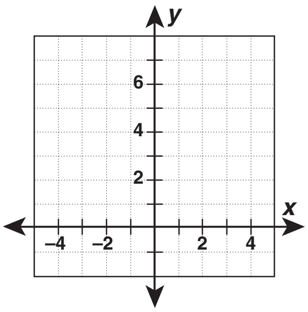
18. Enlarge polygon *ABCD* by a factor of 2.



(0, 2); (−2, 10); (−6, 12); −8, 6)

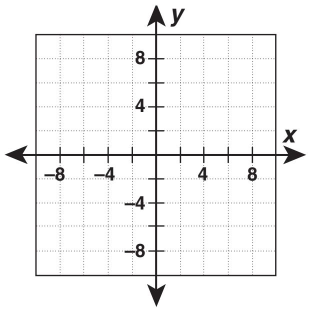
19. Use $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ to rotate polygon *ABCD*.

Describe the image.

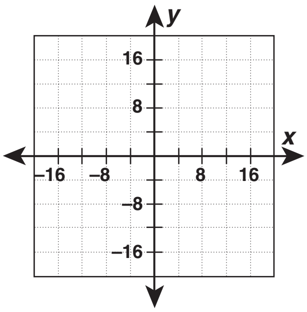


20. Use $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ to rotate polygon *ABCD*.

Describe the image.



21. How does multiplying by $\begin{bmatrix} 0 & 3 \\ 3 & 0 \end{bmatrix}$ transform polygon *ABCD*?



4-4 Determinants and Cramer's Rule

Find the determinant of each matrix.

22. $\begin{bmatrix} 4 & -1 \\ 1 & 4 \end{bmatrix}$

23. $\begin{bmatrix} \frac{1}{3} & 0 \\ 4 & \frac{2}{3} \end{bmatrix}$

24. $\begin{bmatrix} 0.2 & 1.5 \\ -0.4 & 4.0 \end{bmatrix}$

25. $\begin{bmatrix} -1 & 2 & 4 \\ -3 & -2 & -3 \\ 2 & -1 & 5 \end{bmatrix}$

Use Cramer's rule to solve.

26. $\begin{cases} x - y = 4 \\ y - x + 6 = 0 \end{cases}$

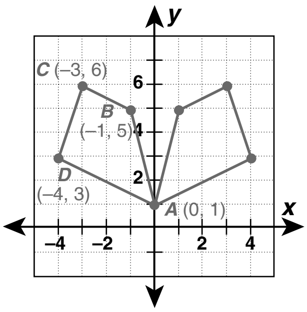
27. $\begin{cases} 2x + 5y = 14 \\ y = 7 + x \end{cases}$

28. $\begin{cases} 3x - y + z = 7 \\ 4x + 2y = 3z + 2 \\ z = x + 2 \end{cases}$

29. $\begin{cases} y = 3x - 5 \\ x - 3y = 1 \end{cases}$

19. Use $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ to rotate polygon *ABCD*.

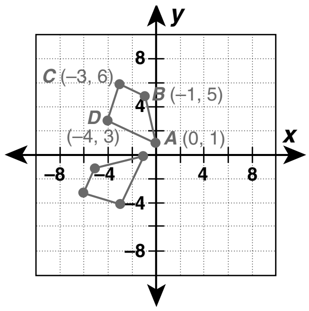
Describe the image.



(0, 1); (1, 5); (3, 6); (4, 3); reflection across the *y*-axis

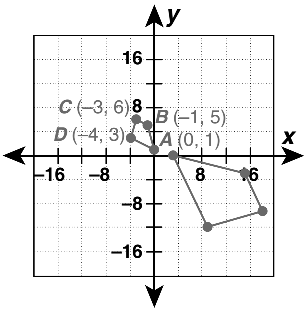
20. Use $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ to rotate polygon *ABCD*.

Describe the image.



(−1, 0); (−5, −1); (−6, −3); (−3, −4); rotation 90° counter-clockwise

21. How does multiplying by $\begin{bmatrix} 0 & 3 \\ 3 & 0 \end{bmatrix}$ transform polygon *ABCD*?



(3, 0); (15, −3); (18, −9); (9, −12)

4-4 Determinants and Cramer's Rule

Find the determinant of each matrix.

22. $\begin{bmatrix} 4 & -1 \\ 1 & 4 \end{bmatrix}$ **17**

23. $\begin{bmatrix} \frac{1}{3} & 0 \\ 4 & \frac{2}{3} \end{bmatrix}$ $\mathbf{\frac{2}{9}}$

24. $\begin{bmatrix} 0.2 & 1.5 \\ -0.4 & 4.0 \end{bmatrix}$ **1.4**

25. $\begin{bmatrix} -1 & 2 & 4 \\ -3 & -2 & -3 \\ 2 & -1 & 5 \end{bmatrix}$ **59**

Use Cramer's rule to solve.

26. $\begin{cases} x - y = 4 \\ y - x + 6 = 0 \end{cases}$ **no solution**

27. $\begin{cases} 2x + 5y = 14 \\ y = 7 + x \end{cases}$ **(−3, 4)**

28. $\begin{cases} 3x - y + z = 7 \\ 4x + 2y = 3z + 2 \\ z = x + 2 \end{cases}$ **(2, 3, 4)**

29. $\begin{cases} y = 3x - 5 \\ x - 3y = 1 \end{cases}$ $\left(1\frac{3}{4}, \frac{1}{4}\right)$

4-5 Matrix Inverses and Solving Systems

Find the inverse matrix of each matrix, if it is defined.

30. $\begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}$

31. $\begin{bmatrix} \frac{1}{3} & -1 \\ -\frac{1}{3} & 1 \end{bmatrix}$

32. $\begin{bmatrix} \frac{1}{2} & -1 \\ 3 & -12 \end{bmatrix}$

33. $\begin{bmatrix} 0 & 2 & 0 \\ 3 & 3 & 2 \\ 2 & 5 & 1 \end{bmatrix}$

Write the matrix equation for the system, and solve, if possible.

34. $\begin{cases} 3x + 4y = 13 \\ 2x - 3y = -14 \end{cases}$

35. $\begin{cases} -4(x - 3y) + 9 \\ x - 3y = 9 \end{cases}$

36. $\begin{cases} 6x + 7y = 16 \\ 12x + 3y = -12 \end{cases}$

37. $\begin{cases} (x + y) = 3z + 1 \\ 2x - y = 9 - z \\ 3x + y = 8 + 4z \end{cases}$

38. You are writing three proposals for office furniture and copiers as a system of equations. Use x as the price for a file cabinet, y as the price for a desk, and z as the price for a copier. What is the price for each type of office furniture or a copier?

$$\begin{cases} 4x + 8y + 2z = 2920 \\ 2x + 3y + z = 1270 \\ 5x + 9y + 2x = 3285 \end{cases}$$

4-6 Row Operations and Augmented Matrices

Write the augmented matrix, and use row reduction to solve, if possible.

39. $\begin{cases} x - 3y = 8 \\ 6y = 2x + 3 \end{cases}$

40. $\begin{cases} -2x + 6y = 4 \\ 3x = 9y - 6 \end{cases}$

41. $\begin{cases} x - 4y = 9 \\ 2x + 5y = 5 \end{cases}$

42. $\begin{cases} 3x + 4y - 7 = 0 \\ 2y - 5x = 10 \end{cases}$

4-5 Matrix Inverses and Solving Systems

Find the inverse matrix of each matrix, if it is defined.

30. $\begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}$ $\begin{bmatrix} 7 & -4 \\ -5 & 3 \end{bmatrix}$

31. $\begin{bmatrix} \frac{1}{3} & -1 \\ -\frac{1}{3} & 1 \end{bmatrix}$ not defined

32. $\begin{bmatrix} \frac{1}{2} & -1 \\ 3 & -12 \end{bmatrix}$ $\begin{bmatrix} 4 & -\frac{1}{3} \\ 1 & -\frac{1}{6} \end{bmatrix}$

33. $\begin{bmatrix} 0 & 2 & 0 \\ 3 & 3 & 2 \\ 2 & 5 & 1 \end{bmatrix}$ $\begin{bmatrix} -3.5 & -1 & 2 \\ 0.5 & 0 & 0 \\ 4.5 & 2 & -3 \end{bmatrix}$

Write the matrix equation for the system, and solve, if possible.

34. $\begin{cases} 3x + 4y = 13 \\ 2x - 3y = -14 \end{cases}$ (−1, 4)

35. $\begin{cases} -4(x - 3y) + 9 \\ x - 3y = 9 \end{cases}$ no solution

36. $\begin{cases} 6x + 7y = 16 \\ 12x + 3y = -12 \end{cases}$ (−2, 4)

37. $\begin{cases} (x + y) = 3z + 1 \\ 2x - y = 9 - z \\ 3x + y = 8 + 4z \end{cases}$ (4, 0, 1)

38. You are writing three proposals for office furniture and copiers as a system of equations. Use *x* as the price for a file cabinet, *y* as the price for a desk, and *z* as the price for a copier. What is the price for each type of office furniture or a copier?

$\begin{cases} 4x + 8y + 2z = 2920 \\ 2x + 3y + z = 1270 \\ 5x + 9y + 2x = 3285 \end{cases}$

$175 file cabinet; $190 desk; $350 copier

4-6 Row Operations and Augmented Matrices

Write the augmented matrix, and use row reduction to solve, if possible.

39. $\begin{cases} x - 3y = 8 \\ 6y = 2x + 3 \end{cases}$ no solution

40. $\begin{cases} -2x + 6y = 4 \\ 3x = 9y - 6 \end{cases}$ (3*y* − 2, *y*)

41. $\begin{cases} x - 4y = 9 \\ 2x + 5y = 5 \end{cases}$ (5, −1)

42. $\begin{cases} 3x + 4y - 7 = 0 \\ 2y - 5x = 10 \end{cases}$ $\left(-1, \frac{5}{2}\right)$

43. The system of equations represents the costs of three different types of bread at a bakery. Use a to represent the cost of a loaf of honey wheat bread, b the cost of a loaf of pumpernickel, and c the cost of a loaf of raisin bread. Find the cost of each type of bread.

$$\begin{cases} 3a + 3b + c + 1.10 = 20.60 \\ 4a + 3b + 2c + 1.10 = 25.25 \\ 5a + 4b + 3c + 1.10 = 33.25 \end{cases}$$

43. The system of equations represents the costs of three different types of bread at a bakery. Use a to represent the cost of a loaf of honey wheat bread, b the cost of a loaf of pumpernickel, and c the cost of a loaf of raisin bread. Find the cost of each type of bread.

$$\begin{cases} 3a + 3b + c + 1.10 = 20.60 \\ 4a + 3b + 2c + 1.10 = 25.25 \\ 5a + 4b + 3c + 1.10 = 33.25 \end{cases}$$

$2.40 honey wheat;
$3.35 pumpernickel;
$2.25 raisin

Big Ideas

Answer these questions to summarize the important concepts from Chapter 4 in your own words.

1. Explain how you can add or subtract two matrices.

2. Explain how you can tell if two matrices can be multiplied.

3. Explain how determinants and Cramer's rule are used.

4. Explain how to solve a system of equations using the inverse of a matrix.

For more review of Chapter 4:

- Complete the Chapter 4 Study Guide and Review on pages 298–301 of your textbook.
- Complete the Ready to Go On quizzes on pages 269 and 295 of your textbook.

Big Ideas

Answer these questions to summarize the important concepts from Chapter 4 in your own words.

1. Explain how you can add or subtract two matrices.

Answers will vary. Possible answer: The two matrices must have the same number of rows and columns. If that is the case you add or subtract corresponding entries.

2. Explain how you can tell if two matrices can be multiplied.

Answers will vary. Possible answer: Two matrices can be multiplied only if the number of columns in the first matrix is equal to the number of rows in the other matrix.

3. Explain how determinants and Cramer's rule are used.

Answers will vary. Possible answer: A determinant is the difference in the product of the diagonals in a matrix. You use the determinant to help you solve a system of equations. A determinant is part of Cramer's rule and you can use Cramer's rule to help you determine if a system represented by a matrix has one solution, no solution or infinitely many solutions.

4. Explain how to solve a system of equations using the inverse of a matrix.

Answers will vary. Possible answer: You can solve a system of equations using matrices by writing the matrix equation and multiplying both sides by the inverse of the matrix.

For more review of Chapter 4:

- Complete the Chapter 4 Study Guide and Review on pages 298–301 of your textbook.
- Complete the Ready to Go On quizzes on pages 269 and 295 of your textbook.