Ext. Networks and Matrices

Example 1 Representing a Network with an Adjacency Matrix

In the network shown below, find the number of ways to go from *A* to *F* with exactly three stops in between (4-step paths).

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q01

First, write the adjacency matrix Q that represents the network. This adjacency matrix shows the number of 1-step paths.

From: \ To:	*A*	*B*	*C*	*D*	*E*	*F*
A	0	1	0	1	0	0
B	0	0	0	1	1	1
C	0	1	0	0	0	1
D	0	0	0	0	1	0
E	1	1	1	0	0	0
F	0	1	0	0	1	0

Because there is a 1-step path (an arrow) from A to B, put a 1 in row 1 column 2.

Because there is no 1-step path (an arrow) from B to A, put a 0 in row 2 column 1.

The fourth power of this adjacency matrix shows the number of 4-step paths (with one stop at a vertex in between).

$$Q^4 = \begin{bmatrix} 2 & 6 & 2 & 5 & 3 & 5 \\ 2 & 9 & 2 & 6 & 10 & 6 \\ 3 & 8 & 3 & 5 & 5 & 5 \\ 1 & 3 & 1 & 2 & 6 & 2 \\ 6 & 10 & 6 & 4 & 7 & 4 \\ 3 & 7 & 3 & 5 & 8 & 5 \end{bmatrix}$$

From: \ To:	*A*	*B*	*C*	*D*	*E*	*F*
A	2	6	2	5	3	5
B	2	9	2	6	10	6
C	3	8	3	5	5	5
D	1	3	1	2	6	2
E	6	10	6	4	7	4
F	3	7	3	5	8	5

Q^4 shows that there are five 4-step paths from *A* to *F*.