

4-4 Determinants and Cramer's Rule

Determinant of a 2×2 Matrix

WORDS	NUMBERS	ALGEBRA
The determinant of a 2 by 2 matrix is the difference of the products of the diagonals	$\det \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} =$ $+ \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = (1)(4)$ $- (3)(2) = -2$	$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} =$ $+ \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$

Cramer's Rule for Two Equations

$$\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases} \text{ has solutions } x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{D}, y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{D},$$

where $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$.

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Determinants and Cramer's Rule (continued)

Cramer's Rule for Three Equations

The system $\begin{cases} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{cases}$ has solutions given by

$$x = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{D}, \quad y = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{D}, \quad z = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{D}$$

where $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $D \neq 0$.