Name ____ Date ___________ Class ____________

LESSON 4-4

Reading Strategies
Analyze Information

Every square matrix has a determinant. The determinant can be positive, negative, or 0. The determinant of a 2×2 matrix is the difference of the product of the diagonals. Always subtract from the diagonal that starts in the upper left of the matrix.

Matrix	Determinant
$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$	$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$
$J = \begin{bmatrix} 4 & -1 \\ -3 & 6 \end{bmatrix}$	$\begin{vmatrix} 4 & -1 \\ -3 & 6 \end{vmatrix} = 4(6) - (-3)(-1) = 24 - 3 = 21$
$K = \begin{bmatrix} -2 & 7 \\ 4 & 9 \end{bmatrix}$	$\begin{vmatrix} -2 & 7 \\ 4 & 9 \end{vmatrix} = -2(9) - 4(7) = -18 - 28 = -46$
$L = \begin{bmatrix} 8 & 4 \\ 16 & 8 \end{bmatrix}$	$\begin{vmatrix} 8 & 4 \\ 16 & 8 \end{vmatrix} = 8(8) - 16(4) = 64 - 64 = 0$

Answer each question.

1. Complete so that each matrix has a positive determinant.
   a. $\begin{bmatrix} 6 & 9 \\ 5 & \_\_\_ \end{bmatrix}$ **b.** $\begin{bmatrix} \_\_\_ & -5 \\ \_\_\_ & -8 \end{bmatrix}$

2. Complete so that each matrix has a negative determinant.
   a. $\begin{bmatrix} -4 & 3 \\ \_\_\_ & 7 \end{bmatrix}$ **b.** $\begin{bmatrix} \frac{1}{4} & -\frac{1}{2} \\ \_\_\_ & \_\_\_ \end{bmatrix}$

3. Complete so that each matrix has a determinant of 0.
   a. $\begin{bmatrix} -5 & 7 \\ \_\_\_ & \_\_\_ \end{bmatrix}$ **b.** $\begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & -3 \end{bmatrix}$

4. Complete so that each matrix has a determinant of −1.
   a. $\begin{bmatrix} 3 & \_\_\_ \\ -4 & \_\_\_ \end{bmatrix}$ **b.** $\begin{bmatrix} 5 & \_\_\_ \\ \_\_\_ & -9 \end{bmatrix}$

5. Matrix *W* has a determinant of 0. What do you know about the dimensions and the entries of matrix *W*?

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LESSON 4-4

Reteach

Determinants and Cramer's Rule (continued)

Use **Cramer's rule** to solve a system of linear equations. $\begin{cases} x + y = 2 \\ y + 7 = 2x \end{cases}$

Step 1 Write the equations in standard form, $ax + by = c$.

$$x + y = 2$$
$$2x - y = 7$$

Step 2 Write the coefficient matrix of the system of equations. Then find the determinant of the coefficient matrix.

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} \quad \text{Coefficient matrix}$$

$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = 1(-1) - 2(1) = -3 \quad \text{Determinant}$$

Step 3 Solve for x and y using Cramer's rule. Remember to divide by the determinant.

The coefficients of x in the coefficient matrix are replaced by the constant terms.

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{D} = \frac{\begin{vmatrix} 2 & 1 \\ 7 & -1 \end{vmatrix}}{-3} = \frac{-2-7}{-3} = \frac{-9}{-3} = 3$$

The coefficients of y in the coefficient matrix are replaced by the constant terms.

$$y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{D} = \frac{\begin{vmatrix} 1 & 2 \\ 2 & 7 \end{vmatrix}}{-3} = \frac{7-4}{-3} = \frac{3}{-3} = -1$$

The solution is $(3, -1)$.

Use Cramer's rule to solve each system of equations.

7. $\begin{cases} 2x + y = -1 \\ 4x + y = -5 \end{cases}$

$D = \begin{vmatrix} 2 & 1 \\ 4 & 1 \end{vmatrix} =$ **−2**

$x = \dfrac{\begin{vmatrix} -1 & 1 \\ -5 & 1 \end{vmatrix}}{D} =$ **−2**

$y = \dfrac{\begin{vmatrix} 2 & -1 \\ 4 & -5 \end{vmatrix}}{D} =$ **3**

8. $\begin{cases} x - y = 1 \\ 3x - 2y = 4 \end{cases}$

$D =$ **1**

$x =$ **2**

$y =$ **1**

9. $\begin{cases} y - x = 3 \\ 2x - 2 = y \end{cases}$

$D =$ **−1**

$x =$ **5**

$y =$ **8**

10. $\begin{cases} 3y = 4x + 7 \\ 9 - 6x = 2y \end{cases}$

$x =$ **$\frac{1}{2}$**

$y =$ **3**

LESSON 4-4

Challenge

Determinant Variations with Matrix Operations

What happens to the determinant of a matrix as the entries in the matrix are changed? Certain changes affect the value of the determinant and others do not. Those operations that do not change the determinant are called invariant operations. The determinant of matrix R is -36. Interchange the first and second rows to get matrix S.

$$R = \begin{bmatrix} 1 & 0 & 4 \\ -3 & 2 & 5 \\ 0 & 2 & -1 \end{bmatrix} \qquad S = \begin{bmatrix} -3 & 2 & 5 \\ 1 & 0 & 4 \\ 0 & 2 & -1 \end{bmatrix}$$

1. a. Calculate the determinant of matrix S. **36**
   b. Interchange the second and third rows of matrix R. Find the determinant of the new matrix. How do they compare? **The determinants are opposites of one another.**
   c. Make a conjecture about how the value of a matrix changes if you interchange two rows of the matrix. **Possible answer: If two rows of a matrix are interchanged, the determinant changes sign.**
2. a. Multiply the first row of matrix R by 3. Now find the new determinant. How does this value compare to the original determinant? **Determinant is −108, which is 3 times the original determinant of *R*.**
   b. Try this with another 3 × 3 matrix. Then make a conjecture about how the determinant changes when a row is multiplied by a constant.
   Possible answer: If a row of a matrix is multiplied by a constant, the value of the determinant is multiplied by that same constant.
3. a. Multiply matrix R by 3 to get matrix T. Find the determinant of matrix T. **−972**
   b. Now create a 2 × 2 matrix and find its determinant. Multiply the matrix by 4 and find the determinant again. Write a conjecture about how the determinant of a matrix changes when the matrix is multiplied by a constant.
   The determinant of T is 27 (3^3) times the determinant of R. The determinant of the second matrix will be 16 (4^2) times greater. Possible answer: If an $n \times n$ matrix is multiplied by a constant k, the determinant will be multiplied by k^n.
4. a. Use matrix R and add twice the first row to the second row. This becomes the new second row. Write the new matrix U. Find its determinant. **$U = \begin{bmatrix} 1 & 0 & 4 \\ -1 & 2 & 13 \\ 0 & 2 & -1 \end{bmatrix}$; the determinant of U is −36.**
   b. Try this with another 3 × 3 matrix. What conjecture can you make about how this operation affects the determinant?
   Possible answer: This is an invariant operation. Adding a multiple of one row to another row does not change the determinant.

LESSON 4-4

Problem Solving

Determinants and Cramer's Rule

As Kristin prepares for a triathlon, she makes a chart of her exercise time, along with the calories burned each day. Part of her chart is shown in the table below. How many calories per hour does she burn for each activity?

Triathlon Training Record				
Day	Swimming (h)	Cycling (h)	Running (h)	Calories Burned
Friday	1.5	2.0	0.5	2450
Saturday	2.5	3.0	1.5	4310
Sunday	2.0	1.5	1.6	3150

1. Write a system of equations that relates Kristin's exercise time to the number of calories burned each day. Use s, c, and r for the calories burned per hour for the three activities.
   $$\begin{cases} 1.5s + 2c + 0.5r = 2450 \\ 2.5s + 3c + 1.5r = 4310 \\ 2s + 1.5c + 1.6r = 3150 \end{cases}$$
2. Write the coefficient matrix for the system of equations.
   $$D = \begin{bmatrix} 1.5 & 2.0 & 0.5 \\ 2.5 & 3.0 & 1.5 \\ 2.0 & 1.5 & 1.6 \end{bmatrix}$$
3. What is the value, D, for the determinant of the coefficient matrix? **$D = 0.7$**
4. Use Cramer's rule to solve this system of equations. Give the values for s, c, and r. **$s = 590$; $c = 620$; $r = 650$**

Choose the letter for the best answer.

5. Ty has a bag of pennies, nickels, and dimes. He has 10 times as many pennies as dimes. He has a total of 52 coins and twice as many nickels as dimes. Which coefficient matrix could you use to solve this problem?

   (A) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -10 \\ 0 & 1 & -2 \end{bmatrix}$ C $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -10 \\ 0 & -1 & 2 \end{bmatrix}$

   B $\begin{bmatrix} 1 & 1 & 1 \\ 10 & 0 & -1 \\ 0 & 2 & -1 \end{bmatrix}$ D $\begin{bmatrix} 1 & 1 & 1 \\ 10 & 0 & -1 \\ 0 & -2 & 1 \end{bmatrix}$

6. Phyllis collects silver dollars and Kennedy half-dollars. She has 5 times as many half-dollars as dollar coins. She has a total of 192 coins. Which solution could you use to find the number of silver dollars Phyllis has?

   A $\dfrac{\begin{vmatrix} 192 & 1 \\ 0 & -5 \end{vmatrix}}{-6}$ C $\dfrac{\begin{vmatrix} 1 & 192 \\ -5 & 0 \end{vmatrix}}{-6}$

   (B) $\dfrac{\begin{vmatrix} 1 & 192 \\ 1 & 0 \end{vmatrix}}{-6}$ D $\dfrac{\begin{vmatrix} 192 & 1 \\ 0 & 1 \end{vmatrix}}{-6}$

LESSON 4-4

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1. Complete so that each matrix has a positive determinant. **a.** $\begin{bmatrix} 6 & 9 \\ 5 & \mathbf{8} \end{bmatrix}$ **b.** $\begin{bmatrix} \mathbf{-3} & -5 \\ \mathbf{-2} & -8 \end{bmatrix}$
2. Complete so that each matrix has a negative determinant. **a.** $\begin{bmatrix} -4 & 3 \\ \mathbf{-5} & 7 \end{bmatrix}$ **b.** $\begin{bmatrix} \frac{1}{4} & -\frac{1}{2} \\ \mathbf{-10} & \mathbf{8} \end{bmatrix}$
3. Complete so that each matrix has a determinant of 0. **a.** $\begin{bmatrix} -5 & 7 \\ \mathbf{-5} & \mathbf{7} \end{bmatrix}$ **b.** $\begin{bmatrix} \mathbf{-6} & \mathbf{3} \\ \mathbf{6} & -3 \end{bmatrix}$
4. Complete so that each matrix has a determinant of −1. **a.** $\begin{bmatrix} 3 & \mathbf{-4} \\ -4 & \mathbf{5} \end{bmatrix}$ **b.** $\begin{bmatrix} 5 & \mathbf{11} \\ \mathbf{-4} & -9 \end{bmatrix}$
5. Matrix W has a determinant of 0. What do you know about the dimensions and the entries of matrix W?
   It must be a square matrix and the products of the diagonals are equal.