4-4 Determinants and Cramer's Rule

Example 1 Finding the Determinant of a 2 × 2 Matrix

Find the determinant of each matrix.

A. $\begin{bmatrix} 1 & 5 \\ 4 & 8 \end{bmatrix}$

$$\begin{vmatrix} 1 & 5 \\ 4 & 8 \end{vmatrix} = 1(8) - 4(5) = 8 - 20 = -12$$

Find the difference of the cross products.

The determinant is -12.

B. $\begin{bmatrix} \frac{1}{2} & -1 \\ -\frac{1}{2} & 2 \end{bmatrix}$

$$\begin{vmatrix} \frac{1}{2} & -1 \\ -\frac{1}{2} & 2 \end{vmatrix} = \frac{1}{2}(2) - \left(-\frac{1}{2}\right)(-1) = 1 - \frac{1}{2} = \frac{1}{2}$$

The determinant is $\frac{1}{2}$.

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Example 2 Using Cramer's Rule for Two Equations

Use Cramer's rule to solve each system of equations.

A. $\begin{cases} 2x + y = 10 \\ 3x - 2y = 8 \end{cases}$

Step 1 Find D, the determinant of the coefficient matrix.

$$\begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix}$$

$$D = \begin{vmatrix} 2 & 1 \\ 3 & -2 \end{vmatrix} = 2(-2) - 3(1) = -7$$

$D \neq 0$, so the system is consistent.

Step 2 Solve for each variable by replacing the coefficients of that variable with the constants as shown below.

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{D} = \frac{\begin{vmatrix} 10 & 1 \\ 8 & -2 \end{vmatrix}}{-7} = 4$$

$$y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{D} = \frac{\begin{vmatrix} 2 & 10 \\ 3 & 8 \end{vmatrix}}{-7} = 2$$

The solution is $(4, 2)$.

B. $\begin{cases} 3x + 5 = 2y \\ 15 - 6y = -9x \end{cases}$

Step 1 Write the equations in standard form. $\begin{cases} 3x - 2y = -5 \\ 9x - 6y = -15 \end{cases}$

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Example 2 Using Cramer's Rule for Two Equations (continued)

Step 2 Find the determinant of the coefficient matrix.

$$D = \begin{vmatrix} 3 & -2 \\ 9 & -6 \end{vmatrix} = 3(-6) - 9(-2) = 0$$

$D = 0$, so the system is either inconsistent or dependent. Check the numerators for x and y to see if either is 0.

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{0} \Rightarrow \begin{vmatrix} -5 & -2 \\ -15 & -6 \end{vmatrix} = 0 \qquad y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{0} \Rightarrow \begin{vmatrix} 3 & -5 \\ 9 & -15 \end{vmatrix} = 0$$

Since at least one numerator is 0, the system is dependent and has infinitely many solutions.

ADDITIONAL EXAMPLES

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Example 3 Finding the Determinant of a 3 × 3 Matrix

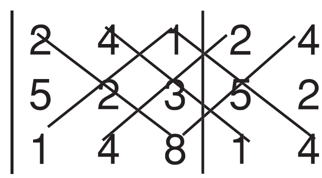
Find the determinant of *M*.

$$M = \begin{bmatrix} 2 & 4 & 1 \\ 5 & 2 & 3 \\ 1 & 4 & 8 \end{bmatrix}$$

$$\det M = \begin{vmatrix} 2 & 4 & 1 \\ 5 & 2 & 3 \\ 1 & 4 & 8 \end{vmatrix}, \text{ so write } \left|\begin{matrix} 2 & 4 & 1 \\ 5 & 2 & 3 \\ 1 & 4 & 8 \end{matrix}\right|\begin{matrix} 2 & 4 \\ 5 & 2 \\ 1 & 4 \end{matrix}$$

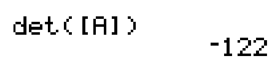
Step 1 Multiply each "down" diagonal and add.

$2(2)(8) + 4(3)(1) + 1(5)(4) = 64$



Step 2 Multiply each "up" diagonal and add.

$1(2)(1) + 4(3)(2) + 8(5)(4) = 186$



Step 3 Find the difference of the sums.

$64 - 186 = -122$

The determinant is -122.

Check Use a calculator.

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Example 4 Nutrition Application

A nutritionist creates a diet for a long-distance runner that includes 3400 Calories from 680 grams of food, with half the Calories coming from carbohydrates. Use the table on page 273 to write and solve a system using Cramer's rule.

The diet will include p grams of protein, c grams of carbohydrates, and f grams of fat.

$4p + 4c + 9f = 3400$ *Equation for total Calories*

$p + c + f = 680$ *Total grams of food*

$4c = 1700$ *Calories from carbohydrates, $\frac{1}{2}(3400) = 1700$*

Use a calculator.

$$D = \begin{vmatrix} 4 & 4 & 9 \\ 1 & 1 & 1 \\ 0 & 4 & 0 \end{vmatrix} = 20$$

$$P = \frac{\begin{vmatrix} 3400 & 4 & 9 \\ 680 & 1 & 1 \\ 1700 & 4 & 0 \end{vmatrix}}{D} \qquad c = \frac{\begin{vmatrix} 4 & 3400 & 9 \\ 1 & 680 & 1 \\ 0 & 1700 & 0 \end{vmatrix}}{D} \qquad f = \frac{\begin{vmatrix} 4 & 4 & 3400 \\ 1 & 1 & 680 \\ 0 & 4 & 1700 \end{vmatrix}}{D}$$

$$P = \frac{2380}{20} \qquad c = \frac{8500}{20} \qquad f = \frac{2720}{20}$$

$$P = 119 \qquad c = 425 \qquad f = 136$$

The diet includes 119 grams of protein, 425 grams of carbohydrates, and 136 grams of fat.