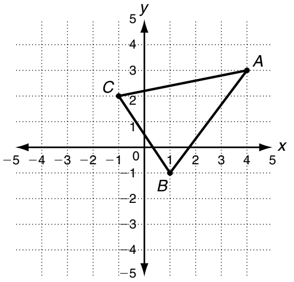
Name ______________________________________ Date ___________ Class ____________

LESSON 4-3

Reteach

Using Matrices to Transform Geometric Figures

A matrix can define a polygon in the coordinate plane.



Vertices of ΔABC:

$A(4, 3)$, $B(1, -1)$, $C(-1, 2)$

Write each pair of coordinates in a column.

Matrix for ΔABC: $\begin{bmatrix} 4 & 1 & -1 \\ 3 & -1 & 2 \end{bmatrix}$

x-coordinates

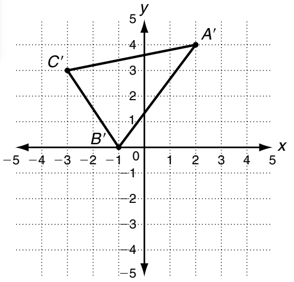
y-coordinates

To translate ΔABC 2 units left and 1 unit up, add a translation matrix to the matrix for ΔABC.

Translation matrix: $\begin{bmatrix} -2 & -2 & -2 \\ 1 & 1 & 1 \end{bmatrix}$

The *x*-coordinates are translated 2 units left.

The *y*-coordinates are translated 1 unit up.



Add the matrices to find the vertices of the translated image.

$$\begin{bmatrix} 4 & 1 & -1 \\ 3 & -1 & 2 \end{bmatrix} + \begin{bmatrix} -2 & -2 & -2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 & -3 \\ 4 & 0 & 3 \end{bmatrix}$$

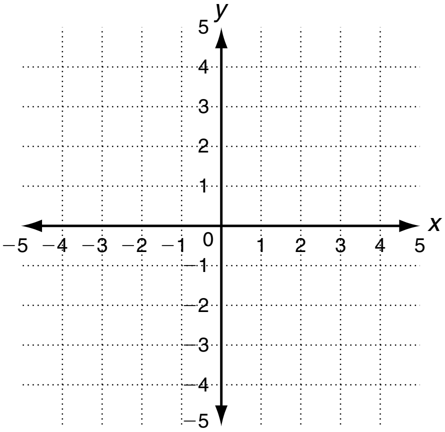
Translated image, $A'(2, 4)$, $B'(-1, 0)$, $C'(-3, 3)$.

Solve.

1. ΔDEF has vertices $D(0, 3)$, $E(-2, 0)$, and $F(1, -2)$. Write the matrix for ΔDEF. ____________________

2. Write the translation matrix to translate ΔDEF 3 units right and 2 units down. ____________________

3. Add the matrices to find the coordinates of the vertices of the image $\Delta D'E'F'$. Then graph $\Delta D'E'F'$.



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Name ______________________________________ Date ___________ Class ____________

LESSON
4-3

Reteach

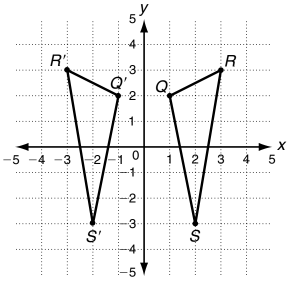
Using Matrices to Transform Geometric Figures (continued)

To reflect a figure across an axis, multiply by a reflection matrix.

ΔQRS has vertices $Q(1, 2)$, $R(3, 3)$, and $S(2, -3)$.

To reflect ΔQRS across the **y-axis**, multiply by the matrix $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$.

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 2 & 3 & -3 \end{bmatrix} = \begin{bmatrix} -1 & -3 & -2 \\ 2 & 3 & -3 \end{bmatrix}$$



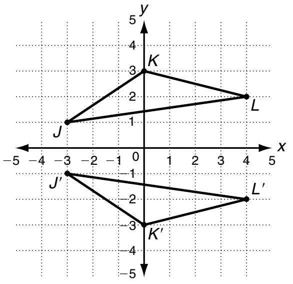
The x-coordinates are multiplied by -1.

The y-coordinates do not change.

ΔJKL has vertices $J(-3, 1)$, $K(0, 3)$, and $L(4, 2)$.

To reflect ΔJKL across the **x-axis**, multiply by the matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -3 & 0 & 4 \\ 1 & 3 & 2 \end{bmatrix} = \begin{bmatrix} -3 & 0 & 4 \\ -1 & -3 & -2 \end{bmatrix}$$



The x-coordinates do not change.

The y-coordinates are multiplied by -1.

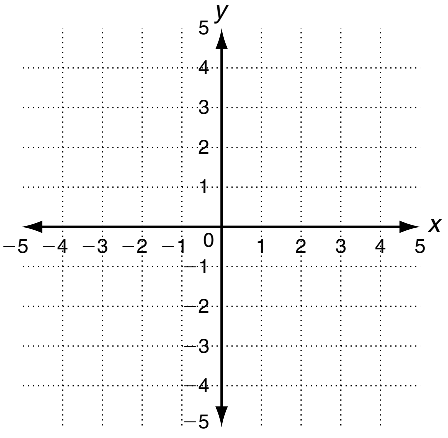
ΔABC has vertices $A(-2, 1)$, $B(-1, 4)$, and $C(-4, 3)$. Use a reflection matrix to solve. Then graph each reflection on the plane.

4. Reflect ΔABC across the y-axis.

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -2 & -1 & -4 \\ 1 & 4 & 3 \end{bmatrix}$$

____

____



5. Reflect ΔABC across the x-axis.

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -2 & -1 & -4 \\ 1 & 4 & 3 \end{bmatrix}$$

____

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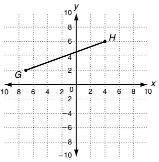
LESSON 4-3

Practice A

Using Matrices to Transform Geometric Figures

Line segment *GH* has endpoints *G*(−7, 2) and *H*(4, 6). Use line segment *GH* for Exercises 1–6.



Use a matrix to transform line segment *GH*. Find the coordinates of the image endpoints $G'H'$.

1. Translate 2 units right and 8 units down.

$$\begin{bmatrix} -7 & 4 \\ 2 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 2 \\ -8 & -8 \end{bmatrix}$$

$$= \begin{bmatrix} -7+2 & 4+(\underline{2}) \\ (\underline{2})+(\underline{-8}) & (\underline{6})+(\underline{-8}) \end{bmatrix}$$

G′(−5, −6), *H*′(6, −2)

2. Translate 5 units right and 1 unit up.

G′(−2, 3), *H*′(9, 7)

3. Translate 6 units left and 3 units down.

G′(−13, −1), *H*′(−2, 3)

4. Enlarge by a factor of 8.

$$8\begin{bmatrix} -7 & 4 \\ 2 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 8(\underline{-7}) & 8(\underline{4}) \\ \underline{8}(\underline{2}) & \underline{8}(\underline{6}) \end{bmatrix}$$

G′(−56, 16), *H*′(32, 48)

5. Enlarge by a factor of 5.

G′(−35, 10), *H*′(20, 30)

6. Reduce by a factor of 0.5.

G′(−3.5, 1), *H*′(2, 3)

Use each matrix to reflect the given point. Write the coordinates of the image. Tell which axis the point is reflected across.

7. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$; (2, −3)

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} -1(\underline{2}) + 0(\underline{-3}) \\ \underline{0}(\underline{2}) + \underline{1}(\underline{-3}) \end{bmatrix}$$

(−2, −3); *y*-axis

8. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$; (−10, 1)

(−10, −1); *x*-axis

9. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$; (5, 4)

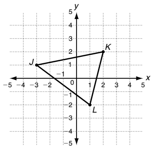
(−5, 4); *y*-axis

LESSON 4-3

Practice B

Using Matrices to Transform Geometric Figures

Triangle *JKL* has vertices *J*(−3, 1), *K*(2, 2), and *L*(1, −2).



Use a matrix to transform triangle *JKL*. Find the coordinates of the vertices of the image.

1. Translate 5 units right, 6 units down.

J′(2, −5), *K*′(7, −4), *L*′(6, −8)

2. Translate 2 units left, 4 units up.

J′(−5, 5), *K*′(0, 6), *L*′(−1, 2)

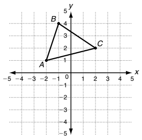
3. Enlarge by a factor of 7.

J′(−21, 7), *K*′(14, 14), *L*′(7, −14)

4. Reduce by a factor of 0.25.

J′(−0.75, 0.25), *K*′(0.5, 0.5), *L*′(0.25, −0.5)

Reflect or rotate triangle *ABC* with vertices *A*(−2, 1), *B*(−1, 4), and *C*(2, 2). Find the coordinates of the vertices of the image. Describe the transformation.



5. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

A′(2, 1), *B*′(1, 4), *C*′(−2, 2); reflection across the *y*-axis

6. $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

A′(1, 2), *B*′(4, 1), *C*′(2, −2); 90° clockwise rotation

7. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

A′(−1, −2), *B*′(−4, −1), *C*′(−2, 2); 90° counterclockwise rotation

8. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

A′(−2, −1), *B*′(−1, −4), *C*′(2, −2); reflection across the *x*-axis

Solve.

9. a. Natalie drew a figure with vertices *H*(−3, −2), *O*(−3, 3), *U*(0, 5), *S*(3, 3), *E*(3, −2) to use as a pattern on a sweatshirt. Write a matrix that defines the figure.

$$\begin{bmatrix} -3 & -3 & 0 & 3 & 3 \\ -2 & 3 & 5 & 3 & -2 \end{bmatrix}$$

b. Natalie wants to enlarge the figure by a factor of 5. Describe a method she can use.

Multiply each entry in the matrix by 5.

c. What are the coordinates of Natalie's enlarged figure?

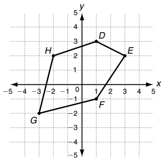
H′ (−15, −10) *O*′ (−15, 15) *U*′ (0, 25) *S*′ (15, 15) *E*′ (15, −10)

LESSON 4-3

Practice C

Using Matrices to Transform Geometric Figures

Use a matrix to transform figure *DEFGH* with coordinates *D*(1, 3), *E*(3, 2), *F*(1, −1), *G*(−3, −2), and *H*(−2, 2). Give the transformation matrix or scalar and the coordinates of the image.



1. Translate 9 units left and 4 units up.

$$\begin{bmatrix} -9 & -9 & -9 & -9 & -9 \\ 4 & 4 & 4 & 4 & 4 \end{bmatrix}$$

D′(−8, 7), *E*′(−6, 6), *F*′(−8, 3), *G*′(−12, 2), *H*′(−11, 6)

2. Reduce by a factor of 0.1.

scalar 0.1; *D*′(0.1, 0.3), *E*′(0.3, 0.2), *F*′(0.1, −0.1), *G*′(−0.3, −0.2), *H*′(−0.2, 0.2)

3. Rotation 90° clockwise

$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$; *D*′(3, −1), *E*′(2, −3), *F*′(−1, −1), *G*′(−2, 3), *H*′(2, 2)

4. Rotation 90° counterclockwise

$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$; *D*′(−3, 1), *E*′(−2, 3), *F*′(1, 1), *G*′(2, −3), *H*′(−2, −2)

5. Reflection across the *x*-axis

$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$; *D*′(1, −3), *E*′(3, −2), *F*′(1, 1), *G*′(−3, 2), *H*′(−2, −2)

6. Reflection across the *y*-axis

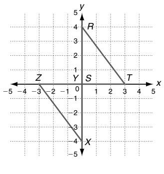
$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$; *D*′(−1, 3), *E*′(−3, 2), *F*′(−1, −1), *G*′(3, −2), *H*′(2, 2)

7. Reflection across $y = x$

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$; *D*′(3, 1), *E*′(2, 3), *F*′(−1, 1), *G*′(−2, −3), *H*′(2, −2)

Solve.

8. Yung Li drew triangle *RST* with coordinates *R*(0, 4), *S*(0, 0), and *T*(3, 0). Then she drew triangle *XYZ* with coordinates *X*(0, −4), *Y*(0, 0), and *Z*(−3, 0).



a. Graph triangles *RST* and *XYZ*.

b. Write a coordinate matrix to represent each triangle.

$$\begin{bmatrix} 0 & 0 & 3 \\ 4 & 0 & 0 \end{bmatrix}; \begin{bmatrix} 0 & 0 & -3 \\ -4 & 0 & 0 \end{bmatrix}$$

c. Use the matrices to show the transformation of triangle *RST* into triangle *XYZ*.

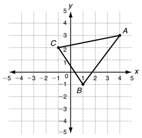
$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 3 \\ 4 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ -4 & 0 & 0 \end{bmatrix}$$

LESSON 4-3

Reteach

Using Matrices to Transform Geometric Figures

A matrix can define a polygon in the coordinate plane.



Vertices of ΔABC:

A(4, 3), *B*(1, −1), *C*(−1, 2)

Write each pair of coordinates in a column.

Matrix for ΔABC: $\begin{bmatrix} 4 & 1 & -1 \\ 3 & -1 & 2 \end{bmatrix}$

x-coordinates

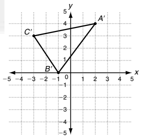
y-coordinates

To translate ΔABC 2 units left and 1 unit up, add a translation matrix to the matrix for ΔABC.

Translation matrix: $\begin{bmatrix} -2 & -2 & -2 \\ 1 & 1 & 1 \end{bmatrix}$

The *x*-coordinates are translated 2 units left.

The *y*-coordinates are translated 1 unit up.



Add the matrices to find the vertices of the translated image.

$$\begin{bmatrix} 4 & 1 & -1 \\ 3 & -1 & 2 \end{bmatrix} + \begin{bmatrix} -2 & -2 & -2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 & -3 \\ 4 & 0 & 3 \end{bmatrix}$$

Translated image, *A*′(2, 4), *B*′(−1, 0), *C*′(−3, 3).

Solve.

1. ΔDEF has vertices *D*(0, 3), *E*(−2, 0), and *F*(1, −2). Write the matrix for ΔDEF.

$$\begin{bmatrix} 0 & -2 & 1 \\ 3 & 0 & -2 \end{bmatrix}$$

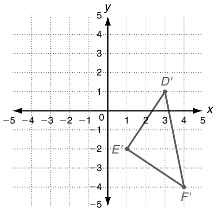
2. Write the translation matrix to translate ΔDEF 3 units right and 2 units down.

$$\begin{bmatrix} 3 & 3 & 3 \\ -2 & -2 & -2 \end{bmatrix}$$

3. Add the matrices to find the coordinates of the vertices of the image $\Delta D'E'F'$. Then graph $\Delta D'E'F'$.

$$\begin{bmatrix} 3 & 1 & 4 \\ 1 & -2 & -4 \end{bmatrix}$$

D′(3, 1), *E*′(1, −2), *F*′(4, −4);



LESSON 4-3

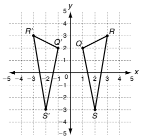
Reteach

Using Matrices to Transform Geometric Figures (continued)

To reflect a figure across an axis, multiply by a reflection matrix.

ΔQRS has vertices $Q(1, 2)$, $R(3, 3)$, and $S(2, -3)$.

To reflect ΔQRS across the ***y*-axis**, multiply by the matrix $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$.



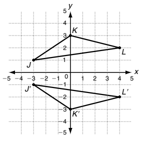
$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 2 & 3 & -3 \end{bmatrix} = \begin{bmatrix} -1 & -3 & -2 \\ 2 & 3 & -3 \end{bmatrix}$$

The *x*-coordinates are multiplied by −1.

The *y*-coordinates do not change.

ΔJKL has vertices $J(-3, 1)$, $K(0, 3)$, and $L(4, 2)$.

To reflect ΔJKL across the ***x*-axis**, multiply by the matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

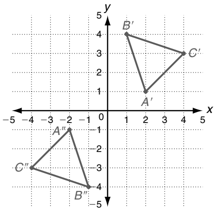


$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -3 & 0 & 4 \\ 1 & 3 & 2 \end{bmatrix} = \begin{bmatrix} -3 & 0 & 4 \\ -1 & -3 & -2 \end{bmatrix}$$

The *x*-coordinates do not change.

The *y*-coordinates are multiplied by −1.

ΔABC has vertices $A(-2, 1)$, $B(-1, 4)$, and $C(-4, 3)$. Use a reflection matrix to solve. Then graph each reflection on the plane.



4. Reflect ΔABC across the *y*-axis.

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -2 & -1 & -4 \\ 1 & 4 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 4 \\ 1 & 4 & 3 \end{bmatrix}$$

$A'(2, 1)$, $B'(1, 4)$, $C'(4, 3)$

5. Reflect ΔABC across the *x*-axis.

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -2 & -1 & -4 \\ 1 & 4 & 3 \end{bmatrix}$$

$$\begin{bmatrix} -2 & -1 & -4 \\ -1 & -4 & -3 \end{bmatrix}$$

$A''(-2, -1)$, $B''(-1, -4)$, $C''(-4, -3)$

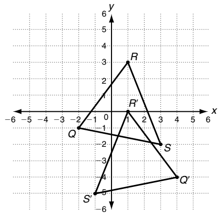
LESSON 4-3

Challenge

Matrix Representations of Transformations

Just as functions can be combined to create a new function in a process called composition, transformations can also be composed to form a new transformation. Triangle *QRS* is shown at right. This triangle can be represented in matrix form by

$$A = \begin{bmatrix} -2 & 1 & 3 \\ -1 & 3 & -2 \end{bmatrix}.$$



Reflect the triangle across the *y*-axis. Now translate this image 2 units right and 3 units down. In matrix form, this operation looks like the following.

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -2 & 1 & 3 \\ -1 & 3 & -2 \end{bmatrix} + \begin{bmatrix} 2 & 2 & 2 \\ -3 & -3 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 1 & -1 \\ -4 & 0 & -5 \end{bmatrix}$$

The result is triangle *Q′R′S′* with vertices (4, −4), (1, 0), and (−1, 5)

Be careful with the order of transformations. Transformations may or may not be commutative.

Use matrices to transform triangle *A* with vertices at (−2, −2), (1, 2), and (3, −1). Find the coordinates of the vertices of the final image.

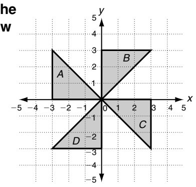
1. Reflect triangle *A* across the *x*-axis and translate 4 units left and 3 units up. (−6, 5), (−3, 1), (−1, 4)
2. Enlarge triangle *A* by a factor of 2, then reflect across the *y*-axis. (4, −4), (−2, 4),(−6, −2)
3. Reflect triangle *A* across the line $y = x$, then translate 1 unit right and 4 units up. (−1, 2), (3, 5), (0, 7)
4. Reduce triangle *A* by a factor of $\frac{1}{2}$, reflect across the line $y = -x$, and finally translate 5 units down. $(1, -4), \left(-1, -5\frac{1}{2}\right), \left(\frac{1}{2}, -6\frac{1}{2}\right)$
5. Rotate triangle *A* 90° clockwise, then translate 3 units left and 2 units up. (−5, 4), (−1, 1), (−4, −1)
6. Rotate triangle *A* 90° counterclockwise, enlarge the triangle by a factor of 3, and reflect across the *y*-axis. Finally translate 8 units right and 5 units down. (2, −11), (14, −2), (5, 4)

LESSON 4-3

Problem Solving

Using Matrices to Transform Geometric Figures

Sherrill is trying to re-create the pattern of a vintage quilt she saw at an antique store. The shaded parts of the figure show the pattern of the quilt.



1. What directions would you give Sherrill to help her draw triangle *A* on a grid?

   Draw a triangle with vertices at (0, 0), (−3, 0), and (−3, 3).

2. a. What transformation can Sherrill use on triangle *A* to create triangle *B*?

   Rotate 90° clockwise

   b. What transformation matrix should she use to create triangle *B*?

   $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

3. a. What transformation can Sherrill use on triangle *A* to create triangle *C*?

   Rotate 180°

   b. What transformation matrix should she use to create triangle *C*?

   $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

4. a. What transformation can Sherrill use on triangle *A* to create triangle *D*?

   Rotate 90° counterclockwise

   b. What transformation matrix should she use to create triangle *D*?

   $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Choose the letter for the best answer.

5. Jesse drew a rectangle represented by $R = \begin{bmatrix} 2 & 5 & 5 & 2 \\ -3 & -3 & -5 & -5 \end{bmatrix}$. He added the transformation matrix $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 2 & 2 & 2 & 2 \end{bmatrix}$ to *R* and drew a second rectangle. Then he added the transformation matrix $\begin{bmatrix} -3 & -3 & -3 & -3 \\ 2 & 2 & 2 & 2 \end{bmatrix}$ to *R* and drew a third rectangle. Which describes the resulting figure?
   - **A** Rectangle
   - [✓ **B**] Irregular hexagon
   - **C** Square
   - **D** Irregular octagon

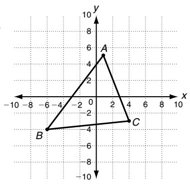
6. Tina drew rectangle *F* with vertices at (0, 0), (0, 5), (3, 5), and (3, 0). She wants to transform *F* into a rectangle that is 6 units wide and 10 units long with the center of the rectangle located at the origin. Which list of transformations will accomplish that?
   - **A** Rotate *F* 90° clockwise, rotate *F* 90° counterclockwise, translate *F* 5 units left and 3 units down
   - **B** Reflect *F* over the *x*-axis, translate *F* 5 units down, rotate *F* 90° counterclockwise
   - **C** Translate *F* 3 units left, translate *F* 3 units down, rotate *F* 90° clockwise
   - [✓ **D**] Reflect *F* over the *y*-axis, reflect *F* over the *x*-axis, rotate *F* by 180°

LESSON 4-3

Reading Strategies

Use Graphic Aids

Geometric figures in the coordinate plane such as triangle *ABC* can be described using matrices. The top row of matrix *T* is made up of the *x*-coordinates of points *A*, *B*, and *C*, and the bottom row is made up of the *y*-coordinates. Each column represents an ordered pair.

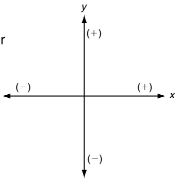


$$T = \begin{bmatrix} 1 & -6 & 4 \\ 5 & -4 & -3 \end{bmatrix}$$

You can also use matrices to transform figures in different ways. To find the coordinates of the translation of triangle *ABC* 2 units left and 3 units up, find the sum of matrix *T* and a **translation matrix**.

$$\begin{bmatrix} 1 & -6 & 4 \\ 5 & -4 & -3 \end{bmatrix} + \begin{bmatrix} -2 & -2 & -2 \\ 3 & 3 & 3 \end{bmatrix} = T'$$

Translation matrix



In the translation matrix, the upper row contains the direction and distance that each *x*-coordinate will be translated. A positive number translates a point to the right and a negative number translates a point to the left. So −2 indicates that the point will shift 2 units left. The bottom row represents the direction and distance that each *y*-coordinate will be translated. A positive number translates a point up and a negative number translates a point down. So 3 indicates that the point will shift 3 units up.

Answer each question.

1. What does the matrix *T′* describe?

   The matrix describes the coordinates of the triangle after it has been translated 2 units left and 3 units up.

2. What are the coordinates of the translated triangle *A′B′C′*?

   A'(−1, 8), *B*'(−8, −1), *C*'(2, 0)

3. Write a translation matrix to shift triangle *ABC* 1 unit right and 4 units down.

   $\begin{bmatrix} 1 & 1 & 1 \\ -4 & -4 & -4 \end{bmatrix}$

4. What operation would you use on matrix *T* to reduce or enlarge triangle *ABC*? Explain.

   Possible answer: I would use multiplication because you need to reduce or enlarge the position of each vertex by the same factor.