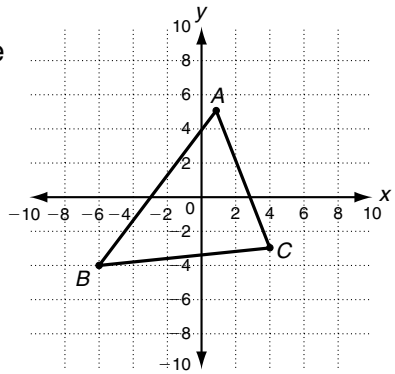


LESSON **Reading Strategies**
4-3 Use Graphic Aids

Geometric figures in the coordinate plane such as triangle ABC can be described using matrices. The top row of matrix T is made up of the x -coordinates of points A , B , and C , and the bottom row is made up of the y -coordinates. Each column represents an ordered pair.

$$T = \begin{bmatrix} 1 & -6 & 4 \\ 5 & -4 & -3 \end{bmatrix}$$

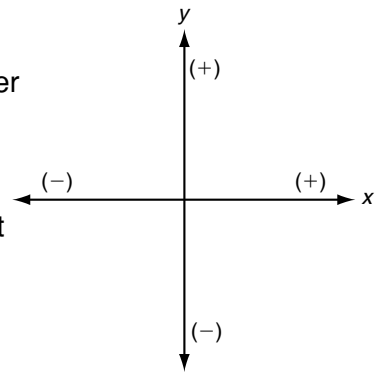


You can also use matrices to transform figures in different ways. To find the coordinates of the translation of triangle ABC 2 units left and 3 units up, find the sum of matrix T and a **translation matrix**.

$$\begin{bmatrix} 1 & -6 & 4 \\ 5 & -4 & -3 \end{bmatrix} + \begin{bmatrix} -2 & -2 & -2 \\ 3 & 3 & 3 \end{bmatrix} = T'$$

Translation matrix

In the translation matrix, the upper row contains the direction and distance that each x -coordinate will be translated. A positive number translates a point to the right and a negative number translates a point to the left. So -2 indicates that the point will shift 2 units left. The bottom row represents the direction and distance that each y -coordinate will be translated. A positive number translates a point up and a negative number translates a point down. So 3 indicates that the point will shift 3 units up.



Answer each question.

1. What does the matrix T' describe?

2. What are the coordinates of the translated triangle $A'B'C'$?

3. Write a translation matrix to shift triangle ABC 1 unit right and 4 units down.

4. What operation would you use on matrix T to reduce or enlarge triangle ABC ? Explain.

LESSON Reteach

4-3 Using Matrices to Transform Geometric Figures (continued)

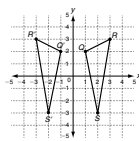
To reflect a figure across an axis, multiply by a reflection matrix.

$\triangle QRS$ has vertices $Q(1, 2)$, $R(3, 3)$, and $S(2, -3)$.

To reflect $\triangle QRS$ across the y -axis, multiply

$$\text{by the matrix } \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 3 & -3 \end{bmatrix} = \begin{bmatrix} -1 & -3 & -2 \\ 2 & 3 & -3 \end{bmatrix}$$



The x -coordinates are multiplied by -1 .

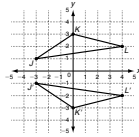
The y -coordinates do not change.

$\triangle JKL$ has vertices $J(-3, 1)$, $K(0, 3)$, and $L(4, 2)$.

To reflect $\triangle JKL$ across the x -axis, multiply

$$\text{by the matrix } \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -3 & 0 & 4 \\ 1 & 3 & 2 \end{bmatrix} = \begin{bmatrix} -3 & 0 & 4 \\ -1 & -3 & -2 \end{bmatrix}$$



The x -coordinates do not change.

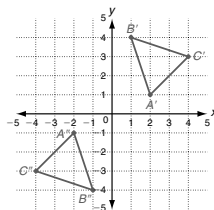
The y -coordinates are multiplied by -1 .

$\triangle ABC$ has vertices $A(-2, 1)$, $B(-1, 4)$, and $C(-4, 3)$. Use a reflection matrix to solve. Then graph each reflection on the plane.

4. Reflect $\triangle ABC$ across the y -axis.

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & -1 & -4 \\ 1 & 4 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 1 & 4 & 3 \end{bmatrix}$$

$A'(2, 1)$, $B'(1, 4)$, $C'(4, 3)$



5. Reflect $\triangle ABC$ across the x -axis.

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -2 & -1 & -4 \\ 1 & 4 & 3 \end{bmatrix} = \begin{bmatrix} -2 & -1 & -4 \\ -1 & -4 & -3 \end{bmatrix}$$

$A''(-2, -1)$, $B''(-1, -4)$, $C''(-4, -3)$

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LESSON Challenge

4-3 Matrix Representations of Transformations

Just as functions can be combined to create a new function in a process called composition, transformations can also be composed to form a new transformation. Triangle QRS is shown at right. This triangle can be represented in matrix form by

$$A = \begin{bmatrix} -2 & 1 & 3 \\ -1 & 3 & -2 \end{bmatrix}$$

Reflect the triangle across the y -axis. Now translate this image 2 units right and 3 units down. In matrix form, this operation looks like the following.

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 & 3 \\ -1 & 3 & -2 \end{bmatrix} + \begin{bmatrix} 2 & 2 & 2 \\ -3 & -3 & -3 \end{bmatrix}$$

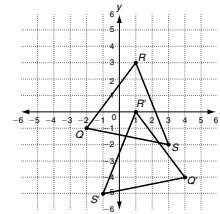
$$= \begin{bmatrix} 4 & 1 & -1 \\ -4 & 0 & -5 \end{bmatrix}$$

The result is triangle $Q'R'S'$ with vertices $(4, -4)$, $(1, 0)$, and $(-1, 5)$

Be careful with the order of transformations. Transformations may or may not be commutative.

Use matrices to transform triangle A with vertices at $(-2, -2)$, $(1, 2)$, and $(3, -1)$. Find the coordinates of the vertices of the final image.

1. Reflect triangle A across the x -axis and translate 4 units left and 3 units up. $(-6, 5)$, $(-3, 1)$, $(-1, 4)$
2. Enlarge triangle A by a factor of 2, then reflect across the y -axis. $(4, -4)$, $(-2, 4)$, $(-6, -2)$
3. Reflect triangle A across the line $y = x$, then translate 1 unit right and 4 units up. $(-1, 2)$, $(3, 5)$, $(0, 7)$
4. Reduce triangle A by a factor of $\frac{1}{2}$, reflect across the line $y = -x$, and finally translate 5 units down. $(1, -4)$, $(-1, -5\frac{1}{2})$, $(\frac{1}{2}, -6\frac{1}{2})$
5. Rotate triangle A 90° clockwise, then translate 3 units left and 2 units up. $(-5, 4)$, $(-1, 1)$, $(-4, -1)$
6. Rotate triangle A 90° counterclockwise, enlarge the triangle by a factor of 3, and reflect across the y -axis. Finally translate 8 units right and 5 units down. $(2, -11)$, $(14, -2)$, $(5, 4)$



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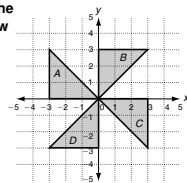
LESSON Problem Solving

4-3 Using Matrices to Transform Geometric Figures

Sherrill is trying to re-create the pattern of a vintage quilt she saw at an antique store. The shaded parts of the figure show the pattern of the quilt.

1. What directions would you give Sherrill to help her draw triangle A on a grid?

Draw a triangle with vertices at $(0, 0)$, $(-3, 0)$, and $(-3, 3)$.



2. a. What transformation can Sherrill use on triangle A to create triangle B ?

Rotate 90° clockwise

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Rotate 180°

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

3. a. What transformation can Sherrill use on triangle A to create triangle C ?

Rotate 90° counterclockwise

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

- b. What transformation matrix should she use to create triangle C ?

4. a. What transformation can Sherrill use on triangle A to create triangle D ?

- b. What transformation matrix should she use to create triangle D ?

Choose the letter for the best answer.

5. Jesse drew a rectangle represented by $R = \begin{bmatrix} 2 & 5 & 5 & 2 \\ -3 & -3 & -5 & -5 \end{bmatrix}$. He added the transformation matrix $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 2 & 2 & 2 & 2 \end{bmatrix}$ to R and drew a second rectangle. Then he added the transformation matrix $\begin{bmatrix} -3 & -3 & -3 & -3 \\ 2 & 2 & 2 & 2 \end{bmatrix}$ to R and drew a third rectangle. Which describes the resulting figure?

- A Rectangle
 B Irregular hexagon
 C Square
 D Irregular octagon

6. Tina drew rectangle F with vertices at $(0, 0)$, $(0, 5)$, $(3, 5)$, and $(3, 0)$. She wants to transform F into a rectangle that is 6 units wide and 10 units long with the center of the rectangle located at the origin. Which list of transformations will accomplish that?

- A Rotate F 90° clockwise, rotate F 90° counterclockwise, translate F 5 units left and 3 units down
 B Reflect F over the x -axis, translate F 5 units down, rotate F 90° counterclockwise
 C Translate F 3 units left, translate F 3 units down, rotate F 90° clockwise
 D Reflect F over the y -axis, reflect F over the x -axis, rotate F by 180°

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LESSON Reading Strategies

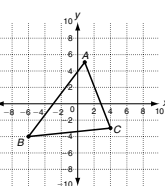
4-3 Use Graphic Aids

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Translation matrix

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Answer each question.

1. What does the matrix T' describe?
 The matrix describes the coordinates of the triangle after it has been translated 2 units left and 3 units up.
2. What are the coordinates of the translated triangle $A'B'C'$?
 $A'(-1, 8)$, $B'(-8, -1)$, $C'(2, 0)$
3. Write a translation matrix to shift triangle ABC 1 unit right and 4 units down.
 $\begin{bmatrix} 1 & 1 & 1 \\ -4 & -4 & -4 \end{bmatrix}$
4. What operation would you use on matrix T to reduce or enlarge triangle ABC ? Explain.
 Possible answer: I would use multiplication because you need to reduce or enlarge the position of each vertex by the same factor.

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