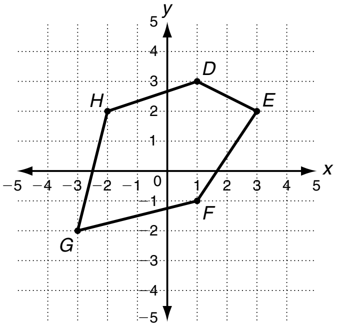
Name ______________________________________ Date ___________ Class ____________

LESSON **4-3**

Practice C

Using Matrices to Transform Geometric Figures

Use a matrix to transform figure *DEFGH* with coordinates *D*(1, 3), *E*(3, 2), *F*(1, −1), *G*(−3, −2), and *H*(−2, 2). Give the transformation matrix or scalar and the coordinates of the image.



1. Translate 9 units left and 4 units up.

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2. Reduce by a factor of 0.1. ____

3. Rotation 90° clockwise ____

4. Rotation 90° counterclockwise ____

5. Reflection across the *x*-axis ____

6. Reflection across the *y*-axis ____

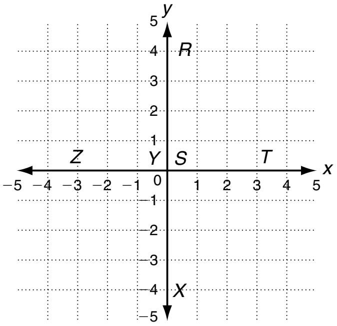
7. Reflection across $y = x$ ____

Solve.

8. Yung Li drew triangle *RST* with coordinates *R*(0, 4), *S*(0, 0), and *T*(3, 0). Then she drew triangle *XYZ* with coordinates *X*(0, −4), *Y*(0, 0), and *Z*(−3, 0).

   a. Graph triangles *RST* and *XYZ.*

   b. Write a coordinate matrix to represent each triangle.



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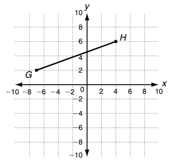
   c. Use the matrices to show the transformation of triangle *RST* into triangle *XYZ.* ______________________________

LESSON **4-3**

Practice A

Using Matrices to Transform Geometric Figures

Line segment *GH* has endpoints *G*(−7, 2) and *H*(4, 6). Use line segment *GH* for Exercises 1–6.



Use a matrix to transform line segment *GH*. Find the coordinates of the image endpoints *G*' *H*'.

1. Translate 2 units right and 8 units down.

$$\begin{bmatrix}-7 & 4\\ 2 & 6\end{bmatrix}+\begin{bmatrix}2 & 2\\ -8 & -8\end{bmatrix}$$

$$=\begin{bmatrix}-7+2 & 4+(\underline{2})\\ (\underline{2})+(\underline{-8}) & (\underline{6})+(\underline{-8})\end{bmatrix}$$

G′(−5, −6), *H*′(6, −2)

2. Translate 5 units right and 1 unit up.

G′(−2, 3), *H*′(9, 7)

3. Translate 6 units left and 3 units down.

G′(−13, −1), *H*′(−2, 3)

4. Enlarge by a factor of 8.

$$8\begin{bmatrix}-7 & 4\\ 2 & 6\end{bmatrix}$$

$$=\begin{bmatrix}8(\underline{-7}) & 8(\underline{4})\\ \underline{8}\,(\underline{2}) & \underline{8}\,(\underline{6})\end{bmatrix}$$

G′(−56, 16), *H*′(32, 48)

5. Enlarge by a factor of 5.

G′(−35, 10), *H*′(20, 30)

6. Reduce by a factor of 0.5.

G′(−3.5, 1), *H*′(2, 3)

Use each matrix to reflect the given point. Write the coordinates of the image. Tell which axis the point is reflected across.

7. $\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}$; (2, −3)

$$\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}\cdot\begin{bmatrix}2\\ -3\end{bmatrix}$$

$$=\begin{bmatrix}-1(\underline{2})+0(\underline{-3})\\ \underline{0}\,(\underline{2})+\underline{1}\,(\underline{-3})\end{bmatrix}$$

(−2, −3); *y*-axis

8. $\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$; (−10, 1)

(−10, −1); *x*-axis

9. $\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}$; (5, 4)

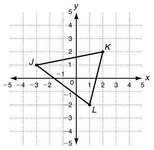
(−5, 4); *y*-axis

LESSON **4-3**

Practice B

Using Matrices to Transform Geometric Figures

Triangle *JKL* has vertices *J*(−3, 1), *K*(2, 2), and *L*(1, −2).



Use a matrix to transform triangle *JKL*. Find the coordinates of the vertices of the image.

1. Translate 5 units right, 6 units down.

J′(2, −5), *K*′(7, −4), *L*′(6, −8)

2. Translate 2 units left, 4 units up.

J′(−5, 5), *K*′(0, 6), *L*′(−1, 2)

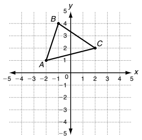
3. Enlarge by a factor of 7.

J′(−21, 7), *K*′(14, 14), *L*′(7, −14)

4. Reduce by a factor of 0.25.

J′(−0.75, 0.25), *K*′(0.5, 0.5), *L*′(0.25, −0.5)

Reflect or rotate triangle *ABC* with vertices *A*(−2, 1), *B*(−1, 4), and *C*(2, 2). Find the coordinates of the vertices of the image. Describe the transformation.



5. $\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}$

A′(2, 1), *B*′(1, 4), *C*′(−2, 2); reflection across the *y*-axis

6. $\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$

A′(1, 2), *B*′(4, 1), *C*′(2, −2); 90° clockwise rotation

7. $\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$

A′(−1, −2), *B*′(−4, −1), *C*′(−2, 2); 90° counterclockwise rotation

8. $\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$

A′(−2, −1), *B*′(−1, −4), *C*′(2, −2); reflection across the *x*-axis

Solve.

9. **a.** Natalie drew a figure with vertices *H*(−3, −2), *O*(−3, 3), *U*(0, 5), *S*(3, 3), *E*(3, −2) to use as a pattern on a sweatshirt. Write a matrix that defines the figure.

$$\begin{bmatrix}-3 & -3 & 0 & 3 & 3\\ -2 & 3 & 5 & 3 & -2\end{bmatrix}$$

b. Natalie wants to enlarge the figure by a factor of 5. Describe a method she can use.

Multiply each entry in the matrix by 5.

c. What are the coordinates of Natalie's enlarged figure?

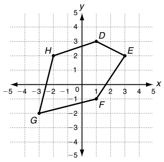
H′ (−15, −10) *O*′ (−15, 15) *U*′ (0, 25) *S*′ (15, 15) *E*′ (15, −10)

LESSON **4-3**

Practice C

Using Matrices to Transform Geometric Figures

Use a matrix to transform figure *DEFGH* with coordinates *D*(1, 3), *E*(3, 2), *F*(1, −1), *G*(−3, −2), and *H*(−2, 2). Give the transformation matrix or scalar and the coordinates of the image.



1. Translate 9 units left and 4 units up.

$$\begin{bmatrix}-9 & -9 & -9 & -9 & -9\\ 4 & 4 & 4 & 4 & 4\end{bmatrix}$$

D′(−8, 7), *E*′(−6, 6), *F*′(−8, 3), *G*′(−12, 2), *H*′(−11, 6)

2. Reduce by a factor of 0.1.

scalar 0.1; *D*′(0.1, 0.3), *E*′(0.3, 0.2), *F*′(0.1, −0.1), *G*′(−0.3, −0.2), *H*′(−0.2, 0.2)

3. Rotation 90° clockwise

$\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$; *D*′(3, −1), *E*′(2, −3), *F*′(−1, −1), *G*′(−2, 3), *H*′(2, 2)

4. Rotation 90° counterclockwise

$\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$; *D*′(−3, 1), *E*′(−2, 3), *F*′(1, 1), *G*′(2, −3), *H*′(−2, −2)

5. Reflection across the *x*-axis

$\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$; *D*′(1, −3), *E*′(3, −2), *F*′(1, 1), *G*′(−3, 2), *H*′(−2, −2)

6. Reflection across the *y*-axis

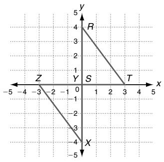
$\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}$; *D*′(−1, 3), *E*′(−3, 2), *F*′(−1, −1), *G*′(3, −2), *H*′(2, 2)

7. Reflection across $y = x$

$\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$; *D*′(3, 1), *E*′(2, 3), *F*′(−1, 1), *G*′(−2, −3), *H*′(2, −2)

Solve.

8. Yung Li drew triangle *RST* with coordinates *R*(0, 4), *S*(0, 0), and *T*(3, 0). Then she drew triangle *XYZ* with coordinates *X*(0, −4), *Y*(0, 0), and *Z*(−3, 0).



a. Graph triangles *RST* and *XYZ*.

b. Write a coordinate matrix to represent each triangle.

$$\begin{bmatrix}0 & 0 & 3\\ 4 & 0 & 0\end{bmatrix};\begin{bmatrix}0 & 0 & -3\\ -4 & 0 & 0\end{bmatrix}$$

c. Use the matrices to show the transformation of triangle *RST* into triangle *XYZ*.

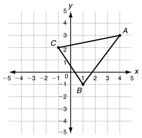
$$\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}\cdot\begin{bmatrix}0 & 0 & 3\\ 4 & 0 & 0\end{bmatrix}=\begin{bmatrix}0 & 0 & -3\\ -4 & 0 & 0\end{bmatrix}$$

LESSON **4-3**

Reteach

Using Matrices to Transform Geometric Figures

A matrix can define a polygon in the coordinate plane.



Vertices of ΔABC:

A(4, 3), *B*(1, −1), *C*(−1, 2)

Write each pair of coordinates in a column.

Matrix for ΔABC: $\begin{bmatrix}4 & 1 & -1\\ 3 & -1 & 2\end{bmatrix}$

x-coordinates

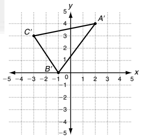
y-coordinates

To translate ΔABC 2 units left and 1 unit up, add a translation matrix to the matrix for ΔABC.

Translation matrix: $\begin{bmatrix}-2 & -2 & -2\\ 1 & 1 & 1\end{bmatrix}$

The *x*-coordinates are translated 2 units left.

The *y*-coordinates are translated 1 unit up.



Add the matrices to find the vertices of the translated image.

$$\begin{bmatrix}4 & 1 & -1\\ 3 & -1 & 2\end{bmatrix}+\begin{bmatrix}-2 & -2 & -2\\ 1 & 1 & 1\end{bmatrix}=\begin{bmatrix}2 & -1 & -3\\ 4 & 0 & 3\end{bmatrix}$$

Translated image, *A*′(2, 4), *B*′(−1, 0), *C*′(−3, 3).

Solve.

1. ΔDEF has vertices *D*(0, 3), *E*(−2, 0), and *F*(1, −2). Write the matrix for ΔDEF.

$$\begin{bmatrix}0 & -2 & 1\\ 3 & 0 & -2\end{bmatrix}$$

2. Write the translation matrix to translate ΔDEF 3 units right and 2 units down.

$$\begin{bmatrix}3 & 3 & 3\\ -2 & -2 & -2\end{bmatrix}$$

3. Add the matrices to find the coordinates of the vertices of the image $\Delta D'E'F'$. Then graph $\Delta D'E'F'$.

$$\begin{bmatrix}3 & 1 & 4\\ 1 & -2 & -4\end{bmatrix}$$

D′(3, 1), *E*′(1, −2), *F*′(4, −4);