Name ______________________ Date __________ Class __________

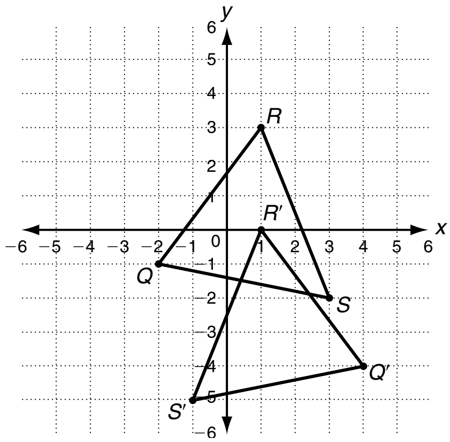
LESSON 4-3

# Challenge

## *Matrix Representations of Transformations*

Just as functions can be combined to create a new function in a process called composition, transformations can also be composed to form a new transformation. Triangle *QRS* is shown at right. This triangle can be represented in matrix form by

$$A = \begin{bmatrix} -2 & 1 & 3 \\ -1 & 3 & -2 \end{bmatrix}.$$




Reflect the triangle across the *y*-axis. Now translate this image 2 units right and 3 units down. In matrix form, this operation looks like the following.

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -2 & 1 & 3 \\ -1 & 3 & -2 \end{bmatrix} + \begin{bmatrix} 2 & 2 & 2 \\ -3 & -3 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 1 & -1 \\ -4 & 0 & -5 \end{bmatrix}$$

The result is triangle *Q'R'S'* with vertices $(4, -4)$, $(1, 0)$, and $(-1, 5)$

Be careful with the order of transformations. Transformations may or may not be commutative.

**Use matrices to transform triangle *A* with vertices at $(-2, -2)$, $(1, 2)$, and $(3, -1)$. Find the coordinates of the vertices of the final image.**

**1.** Reflect triangle *A* across the *x*-axis and translate 4 units left and 3 units up. ____________________

**2.** Enlarge triangle *A* by a factor of 2, then reflect across the *y*-axis. ____________________

**3.** Reflect triangle *A* across the line $y = x$, then translate 1 unit right and 4 units up. ____________________

**4.** Reduce triangle *A* by a factor of $\frac{1}{2}$, reflect across the line $y = -x$, and finally translate 5 units down. ____________________

**5.** Rotate triangle *A* 90° clockwise, then translate 3 units left and 2 units up. ____________________

**6.** Rotate triangle *A* 90° counterclockwise, enlarge the triangle by a factor of 3, and reflect across the *y*-axis. Finally translate 8 units right and 5 units down. ____________________

LESSON 4-3

## Reteach

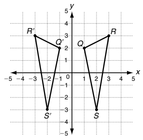
### *Using Matrices to Transform Geometric Figures (continued)*

To reflect a figure across an axis, multiply by a reflection matrix.

$\triangle QRS$ has vertices $Q(1, 2)$, $R(3, 3)$, and $S(2, -3)$.

To reflect $\triangle QRS$ across the **y-axis**, multiply by the matrix $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$.

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 2 & 3 & -3 \end{bmatrix} = \begin{bmatrix} -1 & -3 & -2 \\ 2 & 3 & -3 \end{bmatrix}$$



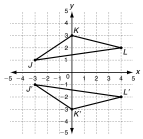

The $x$-coordinates are multiplied by $-1$.

The $y$-coordinates do not change.

$\triangle JKL$ has vertices $J(-3, 1)$, $K(0, 3)$, and $L(4, 2)$.

To reflect $\triangle JKL$ across the **x-axis**, multiply by the matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -3 & 0 & 4 \\ 1 & 3 & 2 \end{bmatrix} = \begin{bmatrix} -3 & 0 & 4 \\ -1 & -3 & -2 \end{bmatrix}$$




The $x$-coordinates do not change.

The $y$-coordinates are multiplied by $-1$.

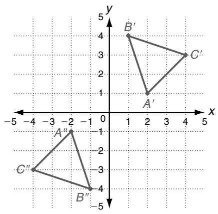
**$\triangle ABC$ has vertices $A(-2, 1)$, $B(-1, 4)$, and $C(-4, 3)$. Use a reflection matrix to solve. Then graph each reflection on the plane.**

4. Reflect $\triangle ABC$ across the $y$-axis.

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -2 & -1 & -4 \\ 1 & 4 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 4 \\ 1 & 4 & 3 \end{bmatrix}$$

$A'(2, 1)$, $B'(1, 4)$, $C'(4, 3)$




5. Reflect $\triangle ABC$ across the $x$-axis.

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -2 & -1 & -4 \\ 1 & 4 & 3 \end{bmatrix}$$

$$\begin{bmatrix} -2 & -1 & -4 \\ -1 & -4 & -3 \end{bmatrix}$$

$A''(-2, -1)$, $B''(-1, -4)$, $C''(-4, -3)$




LESSON 4-3

## Challenge

### *Matrix Representations of Transformations*

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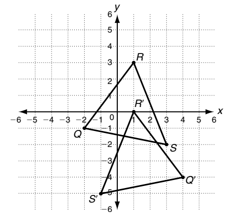



Reflect the triangle across the $y$-axis. Now translate this image 2 units right and 3 units down. In matrix form, this operation looks like the following.

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -2 & 1 & 3 \\ -1 & 3 & -2 \end{bmatrix} + \begin{bmatrix} 2 & 2 & 2 \\ -3 & -3 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 1 & -1 \\ -4 & 0 & -5 \end{bmatrix}$$

The result is triangle $Q'R'S'$ with vertices $(4, -4)$, $(1, 0)$, and $(-1, 5)$

Be careful with the order of transformations. Transformations may or may not be commutative.

**Use matrices to transform triangle $A$ with vertices at $(-2, -2)$, $(1, 2)$, and $(3, -1)$. Find the coordinates of the vertices of the final image.**

1. Reflect triangle $A$ across the $x$-axis and translate 4 units left and 3 units up. $(-6, 5)$, $(-3, 1)$, $(-1, 4)$
2. Enlarge triangle $A$ by a factor of 2, then reflect across the $y$-axis. $(4, -4)$, $(-2, 4)$, $(-6, -2)$
3. Reflect triangle $A$ across the line $y = x$, then translate 1 unit right and 4 units up. $(-1, 2)$, $(3, 5)$, $(0, 7)$
4. Reduce triangle $A$ by a factor of $\frac{1}{2}$, reflect across the line $y = -x$, and finally translate 5 units down. $(1, -4)$, $\left(-1, -5\frac{1}{2}\right)$, $\left(\frac{1}{2}, -6\frac{1}{2}\right)$
5. Rotate triangle $A$ 90° clockwise, then translate 3 units left and 2 units up. $(-5, 4)$, $(-1, 1)$, $(-4, -1)$
6. Rotate triangle $A$ 90° counterclockwise, enlarge the triangle by a factor of 3, and reflect across the $y$-axis. Finally translate 8 units right and 5 units down. $(2, -11)$, $(14, -2)$, $(5, 4)$

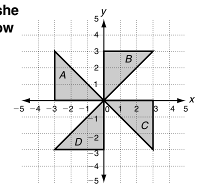



LESSON 4-3

## Problem Solving

### *Using Matrices to Transform Geometric Figures*

**Sherrill is trying to re-create the pattern of a vintage quilt she saw at an antique store. The shaded parts of the figure show the pattern of the quilt.**




1. What directions would you give Sherrill to help her draw triangle $A$ on a grid?

   Draw a triangle with vertices at $(0, 0)$, $(-3, 0)$, and $(-3, 3)$.

2. a. What transformation can Sherrill use on triangle $A$ to create triangle $B$?

   Rotate 90° clockwise

   b. What transformation matrix should she use to create triangle $B$?

   $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

3. a. What transformation can Sherrill use on triangle $A$ to create triangle $C$?

   Rotate 180°

   b. What transformation matrix should she use to create triangle $C$?

   $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

4. a. What transformation can Sherrill use on triangle $A$ to create triangle $D$?

   Rotate 90° counterclockwise

   b. What transformation matrix should she use to create triangle $D$?

   $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

**Choose the letter for the best answer.**

5. Jesse drew a rectangle represented by $R = \begin{bmatrix} 2 & 5 & 5 & 2 \\ -3 & -3 & -5 & -5 \end{bmatrix}$. He added the transformation matrix $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 2 & 2 & 2 & 2 \end{bmatrix}$ to $R$ and drew a second rectangle. Then he added the transformation matrix $\begin{bmatrix} -3 & -3 & -3 & -3 \\ 2 & 2 & 2 & 2 \end{bmatrix}$ to $R$ and drew a third rectangle. Which describes the resulting figure?

   A Rectangle
   
   **(B)** Irregular hexagon
   
   C Square
   
   D Irregular octagon

6. Tina drew rectangle $F$ with vertices at (0, 0), (0, 5), (3, 5), and (3, 0). She wants to transform $F$ into a rectangle that is 6 units wide and 10 units long with the center of the rectangle located at the origin. Which list of transformations will accomplish that?

   A Rotate $F$ 90° clockwise, rotate $F$ 90° counterclockwise, translate $F$ 5 units left and 3 units down
   
   B Reflect $F$ over the $x$-axis, translate $F$ 5 units down, rotate $F$ 90° counterclockwise
   
   C Translate $F$ 3 units left, translate $F$ 3 units down, rotate $F$ 90° clockwise
   
   **(D)** Reflect $F$ over the $y$-axis, reflect $F$ over the $x$-axis, rotate $F$ by 180°



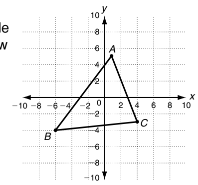

LESSON 4-3

## Reading Strategies

### *Use Graphic Aids*

Geometric figures in the coordinate plane such as triangle $ABC$ can be described using matrices. The top row of matrix $T$ is made up of the $x$-coordinates of points $A$, $B$, and $C$, and the bottom row is made up of the $y$-coordinates. Each column represents an ordered pair.

$$T = \begin{bmatrix} 1 & -6 & 4 \\ 5 & -4 & -3 \end{bmatrix}$$

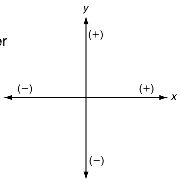



You can also use matrices to transform figures in different ways. To find the coordinates of the translation of triangle $ABC$ 2 units left and 3 units up, find the sum of matrix $T$ and a **translation matrix**.

$$\begin{bmatrix} 1 & -6 & 4 \\ 5 & -4 & -3 \end{bmatrix} + \begin{bmatrix} -2 & -2 & -2 \\ 3 & 3 & 3 \end{bmatrix} = T'$$

Translation matrix

In the translation matrix, the upper row contains the direction and distance that each $x$-coordinate will be translated. A positive number translates a point to the right and a negative number translates a point to the left. So $-2$ indicates that the point will shift 2 units left. The bottom row represents the direction and distance that each $y$-coordinate will be translated. A positive number translates a point up and a negative number translates a point down. So 3 indicates that the point will shift 3 units up.




**Answer each question.**

1. What does the matrix $T'$ describe?

   The matrix describes the coordinates of the triangle after it has been translated 2 units left and 3 units up.

2. What are the coordinates of the translated triangle $A'B'C'$?

   $A'(-1, 8)$, $B'(-8, -1)$, $C'(2, 0)$

3. Write a translation matrix to shift triangle $ABC$ 1 unit right and 4 units down.

   $\begin{bmatrix} 1 & 1 & 1 \\ -4 & -4 & -4 \end{bmatrix}$

4. What operation would you use on matrix $T$ to reduce or enlarge triangle $ABC$? Explain.

   Possible answer: I would use multiplication because you need to reduce or enlarge the position of each vertex by the same factor.