4-3 Using Matrices to Transform Geometric Figures

Example 1 Using Matrices to Translate a Figure

Translate △ABC with coordinates A(−2, 1), B(3, 2), and C(0, −3), 3 units left and 4 units up. Find the coordinates of the vertices of the image, and graph.
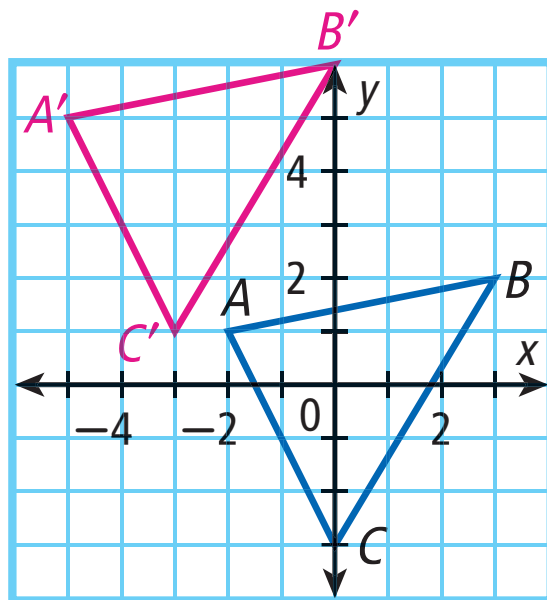
The translation matrix will have −3 in all entries in row 1 and 4 in all entries in row 2.

$$\begin{bmatrix} -3 & -3 & -3 \\ 4 & 4 & 4 \end{bmatrix} \begin{matrix} \longleftarrow x\text{-translation} \\ \longleftarrow y\text{-translation} \end{matrix}$$

$$\begin{bmatrix} -2 & 3 & 0 \\ 1 & 2 & -3 \end{bmatrix} + \begin{bmatrix} -3 & -3 & -3 \\ 4 & 4 & 4 \end{bmatrix} = \begin{bmatrix} -2-3 & 3-3 & 0-3 \\ 1+4 & 2+4 & -3+4 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & 0 & -3 \\ 5 & 6 & 1 \end{bmatrix}$$

$A'B'C'$, the image of △ABC, has coordinates $A'(-5, 5)$, $B'(0, 6)$, and $C'(-3, 1)$.
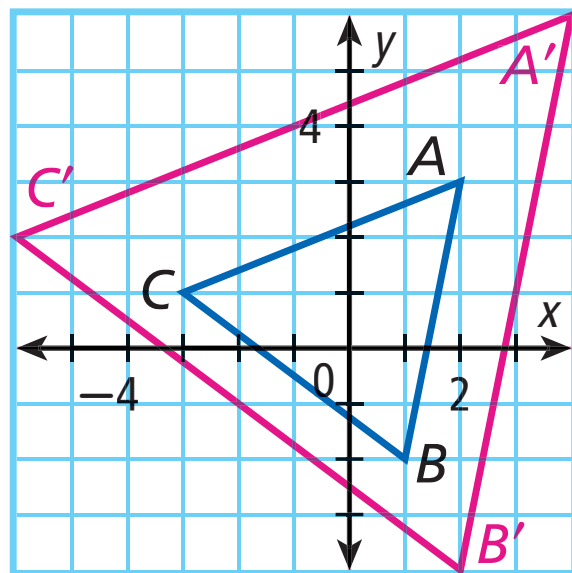
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Example 2 Using Matrices to Enlarge a Figure

Enlarge △ABC with coordinates A(2, 3), B(1, −2), and C(−3, 1), by a factor of 2. Find the coordinates of the vertices of the image, and graph.

Multiply each coordinate by 2 by multiplying each entry by 2.

$$2\begin{bmatrix} 2 & 1 & -3 \\ 3 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 2(2) & 2(1) & 2(-3) \\ 2(3) & 2(-2) & 2(1) \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & -6 \\ 6 & -4 & 2 \end{bmatrix} \begin{matrix} \longleftarrow x\text{-coordinates} \\ \longleftarrow y\text{-coordinates} \end{matrix}$$

$A'B'C'$, the image of △ABC, has coordinates $A'(4, 6)$, $B'(2, -4)$, and $C'(-6, 2)$.

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Example 3 Using Matrices to Reflect a Figure

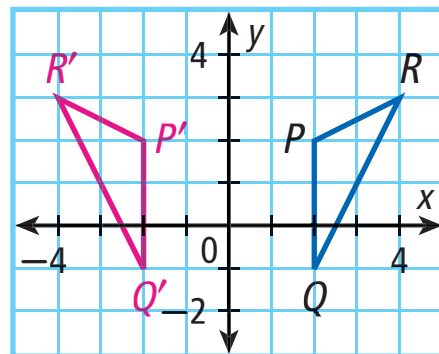**Reflect △PQR with coordinates P(2, 2), Q(2, −1), and R(4, 3) across the y-axis. Find the coordinates of the vertices of the image, and graph.**

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 & 4 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & -2 & -4 \\ 2 & -1 & 3 \end{bmatrix}$$

Each x-coordinate is multiplied by −1.

Each y-coordinate is multiplied by 1.

The coordinates of the vertices of the image are P′ (−2, 2), Q′ (−2, −1), and R′ (−4, 3).

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Example 4 Using Matrices to Rotate a Figure

Use each matrix to rotate polygon *ABCD* with coordinates *A*(0, 1), *B*(2, −4), *C*(5, 1), and *D*(2, 3) about the origin. Graph and describe the image.

A. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

B. $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 2 & 5 & 2 \\ 1 & -4 & 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 2 & 5 & 2 \\ 1 & -4 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 4 & -1 & -3 \\ 0 & 2 & 5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -4 & 1 & 3 \\ 0 & -2 & -5 & -2 \end{bmatrix}$$

The image is rotated 90° counterclockwise.

The image is rotated 90° clockwise.

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