Name ______________________________ Date ___________ Class ____________

LESSON 4-2

Reteach

Multiplying Matrices

Use the dimensions to decide whether matrices can be multiplied.

To multiply two matrices, the number of columns in *A*

must equal the number of rows in *B*.

Matrices: $A \times B = AB$

Dimensions: $m \times \boldsymbol{n}$ $\boldsymbol{n} \times p$ $m \times p$

Remember, with matrices, *AB* is NOT the same as *BA*.

Inner dimensions are equal: $n = n$.

Outer dimensions give the dimensions of the product.

To determine which products are defined, check the dimensions.

$$A = \begin{bmatrix} 3 & 5 & 1 \\ -2 & 0 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 0 & 3 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$$

A: 2×3 *B*: 3×2 *C*: 2×2

AB: 2×3 and 3×2, so *AB* is defined and has dimensions 2×2.

Inner dimensions are equal.

AC: 2×3 and 2×2, so *AC* is not defined.

Inner dimensions are NOT equal.

Use the following matrices for Exercises 1–3. Tell whether each product is defined. If so, give its dimensions.

$$A = \begin{bmatrix} -1 & 0 \\ 2 & -2 \end{bmatrix} \qquad B = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \qquad C = \begin{bmatrix} 4 & 3 \end{bmatrix}$$

1. *AB*

A: 2×2

B: 2×1

Product defined?

____

____

2. *BC*

B: ______________

C: ______________

Product defined?

____

____

3. *AC*

A: ______________

C: ______________

Product defined?

____

____

Name ______________________ Date __________ Class __________

LESSON 4-2

Reteach

Multiplying Matrices (continued)

To find a matrix product, first make sure the product is defined.

Find AB. $A = \begin{bmatrix} 3 & 5 & 1 \\ -2 & 0 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 0 & 3 \end{bmatrix}$

A is 2×3 and *B* is 3×2. The product is a 2×2 matrix.

Step 1: Multiply row 1 entries of *A* by column 1 entries of *B*. The sum is the first entry in the product.

$$\begin{bmatrix} 3 & 5 & 1 \\ -2 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 3(1) + 5(-1) + 1(0) & ? \\ ? & ? \end{bmatrix} = \begin{bmatrix} -2 & ? \\ ? & ? \end{bmatrix}$$

Step 2: Multiply row 1 entries of *A* by column 2 entries of *B*. Add.

$$\begin{bmatrix} 3 & 5 & 1 \\ -2 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 3(2) + 5(4) + 1(3) \\ ? & ? \end{bmatrix} = \begin{bmatrix} -2 & 29 \\ ? & ? \end{bmatrix}$$

Step 3: Multiply row 2 entries of *A* by column 1 entries of *B*. Add.

$$\begin{bmatrix} 3 & 5 & 1 \\ -2 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 29 \\ -2(1) + 0(-1) + (-1)(0) & ? \end{bmatrix} = \begin{bmatrix} -2 & 29 \\ -2 & ? \end{bmatrix}$$

Step 4: Multiply row 2 entries of *A* by column 2 entries of *B*. Add.

$$\begin{bmatrix} 3 & 5 & 1 \\ -2 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 29 \\ -2 & -2(2) + 0(4) + (-1)(3) \end{bmatrix} = \begin{bmatrix} -2 & 29 \\ -2 & -7 \end{bmatrix}$$

Find each product.

4. $\begin{bmatrix} 3 \\ 1 \end{bmatrix}\begin{bmatrix} 4 & 3 \end{bmatrix} = \begin{bmatrix} 3(\_\_\_\_) & 3(\_\_\_\_) \\ 1(\_\_\_\_) & 1(\_\_\_\_) \end{bmatrix} = \begin{bmatrix} \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ \end{bmatrix}$

5. $\begin{bmatrix} -1 & 0 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -4 & 0 \end{bmatrix} = \begin{bmatrix} -1(1) + 0(-4) & -1(\_\_\_\_) + 0(\_\_\_\_) \\ 3(\_\_\_\_) + -2(\_\_\_\_) & 3(\_\_\_\_) + (-2)(\_\_\_\_) \end{bmatrix} = \begin{bmatrix} \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ \end{bmatrix}$

6. $\begin{bmatrix} -5 & 2 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ -1 & 2 \end{bmatrix}$

LESSON 4-2

Practice A

Multiplying Matrices

Tell whether each product is defined. If so, give its dimensions.

1. $A_{3 \times 4}$ and $B_{4 \times 6}$; AB — 3×6
2. $C_{4 \times 2}$ and $D_{2 \times 1}$; CD — 4×1
3. $E_{5 \times 2}$ and $F_{5 \times 3}$; EF — No

Use the following matrices for Exercises 4–7. Evaluate, if possible.

$$A = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 5 \\ 2 & 1 \\ 0 & 3 \\ 1 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 8 & 1 & 0 & 1 \\ 0 & 2 & 3 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 4 & 5 & 0 & 2 \\ 0 & 1 & 3 & 0 \\ 1 & 2 & 1 & 2 \\ 5 & 0 & 1 & 4 \end{bmatrix}$$

4. $A^2 = A \times A = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix} \times \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 2(2) + 0(1) & 2(\underline{0}) + 0(\underline{3}) \\ 1(\underline{2}) + 3(\underline{1}) & 1(\underline{0}) + 3(\underline{3}) \end{bmatrix} \qquad \begin{bmatrix} 4 & 0 \\ 5 & 9 \end{bmatrix}$$

5. CB — $\begin{bmatrix} 3 & 41 \\ 5 & 11 \end{bmatrix}$

6. BA — $\begin{bmatrix} 5 & 15 \\ 5 & 3 \\ 3 & 9 \\ 2 & 0 \end{bmatrix}$

7. CD — $\begin{bmatrix} 37 & 41 & 4 & 20 \\ 8 & 8 & 10 & 10 \end{bmatrix}$

Solve.

8. Julie and Steve are playing the games at the arcade. The first table shows the number of each type of ticket they won. Find the total number of points they each won.

Tickets Won			
Player	**Red**	**Yellow**	**Blue**
Julie	15	6	2
Steve	17	3	4

Ticket Point Values	
Ticket	**Points**
Red	5
Yellow	10
Blue	25

a. Write a matrix that represents the data in each table.

$$\begin{bmatrix} 15 & 6 & 2 \\ 17 & 3 & 4 \end{bmatrix}, \begin{bmatrix} 5 \\ 10 \\ 25 \end{bmatrix}$$

b. Find the product matrix.

$$\begin{bmatrix} 185 \\ 215 \end{bmatrix}$$

c. How many points did each player win? — Julie 185, Steve 215

LESSON 4-2

Practice B

Multiplying Matrices

Tell whether each product is defined. If so, give its dimensions.

1. $P_{3 \times 3}$ and $Q_{3 \times 4}$; PQ — 3×4
2. $R_{3 \times 8}$ and $S_{4 \times 3}$; SR — 4×8
3. $W_{2 \times 5}$ and $X_{2 \times 5}$; WX — No

Use the following matrices for Exercises 4–7. Evaluate, if possible.

$$E = \begin{bmatrix} -4 & 1 \\ -2 & 2 \end{bmatrix} \quad F = \begin{bmatrix} 1 & 0 \\ 4 & -3 \\ -2 & 6 \\ -1 & 5 \end{bmatrix} \quad G = \begin{bmatrix} -4 & 0 & 3 & 5 \\ 1 & -2 & 0 & 0 \end{bmatrix} \quad H = \begin{bmatrix} 1 & -2 & -1 & 3 \\ 2 & 0 & 4 & -1 \\ 3 & 5 & -2 & 2 \\ 1 & -1 & 0 & 0 \end{bmatrix}$$

4. EG — $\begin{bmatrix} 17 & -2 & -12 & -20 \\ 10 & -4 & -6 & -10 \end{bmatrix}$

5. HF — $\begin{bmatrix} -8 & 15 \\ -5 & 19 \\ 25 & -17 \\ -3 & 3 \end{bmatrix}$

6. FG — $\begin{bmatrix} -4 & 0 & 3 & 5 \\ -19 & 6 & 12 & 20 \\ 14 & -12 & -6 & -10 \\ 9 & -10 & -3 & -5 \end{bmatrix}$

7. E^2 — $\begin{bmatrix} 14 & -2 \\ 4 & 2 \end{bmatrix}$

Solve.

8. Jamal, Ken, and Barry are playing a baseball video game. The first table shows the number of singles, doubles, triples, and home runs each scored. Find the total number of points they each scored.

	Hits			
Player	**S**	**D**	**T**	**HR**
Jamal	3	2	0	1
Ken	2	4	0	0
Barry	0	1	3	1

Points Scored for Hits	
Hit	**Points**
Single (S)	1
Double (D)	2
Triple (T)	3
Home run (HR)	4

a. Write a matrix that represents the data in each table.

$$\begin{bmatrix} 3 & 2 & 0 & 1 \\ 2 & 4 & 0 & 0 \\ 0 & 1 & 3 & 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

b. Find the product matrix.

$$\begin{bmatrix} 11 \\ 10 \\ 15 \end{bmatrix}$$

c. How many points did each player score? — Jamal 11, Ken 10, Barry 15

LESSON 4-2

Practice C

Multiplying Matrices

Use the following matrices for Exercises 1–9. Evaluate, if possible.

$$J = \begin{bmatrix} 1 & -3 & 2 \\ 0 & -4 & 1 \\ 5 & 1 & 0 \end{bmatrix} \quad K = \begin{bmatrix} 8 & 0 & 1 \\ -2 & 1 & -1 \end{bmatrix} \quad L = \begin{bmatrix} 0 & 6 & 0 \\ -3 & 1 & 4 \\ 1 & 0 & -2 \end{bmatrix} \quad M = \begin{bmatrix} 5 & -4 \\ -3 & 3 \end{bmatrix}$$

1. JK — No

2. KJ — $\begin{bmatrix} 13 & -23 & 16 \\ -7 & 1 & -3 \end{bmatrix}$

3. JL — $\begin{bmatrix} 11 & 3 & -16 \\ 13 & -4 & -18 \\ -3 & 31 & 4 \end{bmatrix}$

4. KL — $\begin{bmatrix} 1 & 48 & -2 \\ -4 & -11 & 6 \end{bmatrix}$

5. KM — No

6. M^2 — $\begin{bmatrix} 37 & -32 \\ -24 & 21 \end{bmatrix}$

7. MK — $\begin{bmatrix} 48 & -4 & 9 \\ -30 & 3 & -6 \end{bmatrix}$

8. LJ — $\begin{bmatrix} 0 & -24 & 6 \\ 17 & 9 & -5 \\ -9 & -5 & 2 \end{bmatrix}$

9. L^2 — $\begin{bmatrix} -18 & 6 & 24 \\ 1 & -17 & -4 \\ -2 & 6 & 4 \end{bmatrix}$

Solve.

10. The tables show the prices and amounts of milk sold in a dairy store during one week.

Milk Prices	
Size	**Price**
Quart	$1.65
Half-gallon	$2.10
Gallon	$3.20

Dairy Milk Sales					
Size	**Mon**	**Tue**	**Wed**	**Thu**	**Fri**
Quart	18	21	20	25	12
Half-gallon	12	50	10	5	10
Gallon	60	55	40	60	25

a. To compare milk sales for each day, show the data as matrices.

$$\begin{bmatrix} 1.65 & 2.1 & 3.2 \end{bmatrix} \quad \begin{bmatrix} 18 & 21 & 20 & 25 & 12 \\ 12 & 50 & 10 & 5 & 10 \\ 60 & 55 & 40 & 60 & 25 \end{bmatrix}$$

b. Find the product matrix. — $\begin{bmatrix} 246.9 & 315.65 & 182 & 243.75 & 120.8 \end{bmatrix}$

c. Order the days from greatest to least in total sales. — Tue, Mon, Thu, Wed, Fri

LESSON 4-2

Reteach

Multiplying Matrices

Use the dimensions to decide whether matrices can be multiplied.

To multiply two matrices, the number of columns in *A* **must equal** the number of rows in *B*.

Matrices: $A \times B = AB$

Dimensions: $m \times \mathbf{n}$ $\quad \mathbf{n} \times p \quad m \times p$

Remember, with matrices, *AB* is NOT the same as *BA*.

Inner dimensions are equal: $n = n$.

Outer dimensions give the dimensions of the product.

To determine which products are defined, check the dimensions.

$$A = \begin{bmatrix} 3 & 5 & 1 \\ -2 & 0 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 0 & 3 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$$

A: 2×3 $\quad B$: 3×2 $\quad C$: 2×2

AB: 2×3 and 3×2, so AB is defined and has dimensions 2×2.

Inner dimensions are equal.

AC: 2×3 and 2×2, so AC is not defined.

Inner dimensions are NOT equal.

Use the following matrices for Exercises 1–3. Tell whether each product is defined. If so, give its dimensions.

$$A = \begin{bmatrix} -1 & 0 \\ 2 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 4 & 3 \end{bmatrix}$$

1. AB

   A: 2×2

   B: 2×1

   Product defined? Yes

   2×1

2. BC

   B: 2×1

   C: 1×2

   Product defined? yes

   2×2

3. AC

   A: 2×2

   C: 1×2

   Product defined? no

LESSON 4-2

Reteach
Multiplying Matrices (continued)

To find a matrix product, first make sure the product is defined.

Find *AB*. $A = \begin{bmatrix} 3 & 5 & 1 \\ -2 & 0 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 0 & 3 \end{bmatrix}$

A is 2×3 and *B* is 3×2. The product is a 2×2 matrix.

Step 1: Multiply row 1 entries of *A* by column 1 entries of *B*. The sum is the first entry in the product.

$$\begin{bmatrix} 3 & 5 & 1 \\ -2 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 3(1)+5(-1)+1(0) & ? \\ ? & ? \end{bmatrix} = \begin{bmatrix} -2 & ? \\ ? & ? \end{bmatrix}$$

Step 2: Multiply row 1 entries of *A* by column 2 entries of *B*. Add.

$$\begin{bmatrix} 3 & 5 & 1 \\ -2 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 3(2)+5(4)+1(3) \\ ? & ? \end{bmatrix} = \begin{bmatrix} -2 & 29 \\ ? & ? \end{bmatrix}$$

Step 3: Multiply row 2 entries of *A* by column 1 entries of *B*. Add.

$$\begin{bmatrix} 3 & 5 & 1 \\ -2 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 29 \\ -2(1)+0(-1)+(-1)(0) & ? \end{bmatrix} = \begin{bmatrix} -2 & 29 \\ -2 & ? \end{bmatrix}$$

Step 4: Multiply row 2 entries of *A* by column 2 entries of *B*. Add.

$$\begin{bmatrix} 3 & 5 & 1 \\ -2 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 29 \\ -2 & -2(2)+0(4)+(-1)(3) \end{bmatrix} = \begin{bmatrix} -2 & 29 \\ -2 & -7 \end{bmatrix}$$

Find each product.

4. $\begin{bmatrix} 3 \\ 1 \end{bmatrix}\begin{bmatrix} 4 & 3 \end{bmatrix} = \begin{bmatrix} 3(\underline{4}) & 3(\underline{3}) \\ 1(\underline{4}) & 1(\underline{3}) \end{bmatrix} = \begin{bmatrix} 12 & 9 \\ 4 & 3 \end{bmatrix}$

5. $\begin{bmatrix} -1 & 0 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -4 & 0 \end{bmatrix} = \begin{bmatrix} -1(1)+0(-4) & -1(\underline{2})+0(\underline{0}) \\ 3(\underline{1})+-2(\underline{-4}) & 3(\underline{2})+(-2)(\underline{0}) \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ 11 & 6 \end{bmatrix}$

6. $\begin{bmatrix} -5 & 2 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ -1 & 2 \end{bmatrix}$ $\begin{bmatrix} -2 & -16 \\ 3 & -6 \end{bmatrix}$

LESSON 4-2

Challenge
Matrix Codes

Matrices can be used to send messages in coded form, and then another matrix can be used to decode the message. Use the code shown in the table for letters and characters.

Code	
A–Z	1–26
Comma	27
Period	28
Space	29

Take a message such as "Math is fun." Code it into numbers and the message becomes:

13 1 20 8 29 9 19 29 6 21 14 28

The message in code can be represented by matrix *M*. $\begin{bmatrix} 13 & 1 & 20 & 8 \\ 29 & 9 & 19 & 29 \\ 6 & 21 & 14 & 28 \end{bmatrix}$

Notice how the numbers in the message read from left to right. Multiply matrix *M* by the coding matrix *C*.

$$C = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}, CM = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 13 & 1 & 20 & 8 \\ 29 & 9 & 19 & 29 \\ 6 & 21 & 14 & 28 \end{bmatrix} = \begin{bmatrix} 32 & 23 & 54 & 44 \\ 80 & 54 & 107 & 109 \\ 19 & 22 & 34 & 36 \end{bmatrix}$$

Since many of the numbers are greater than 29, the greatest number in the code, divide each number by 29 and record only the remainder. This is called arithmetic modulo 29.

$$CM \text{ modulo } 29 = \begin{bmatrix} 32 & 23 & 54 & 44 \\ 80 & 54 & 107 & 109 \\ 19 & 22 & 34 & 36 \end{bmatrix} \text{ modulo } 29 = \begin{bmatrix} 3 & 23 & 25 & 15 \\ 22 & 25 & 20 & 22 \\ 19 & 22 & 5 & 7 \end{bmatrix}$$

This gives the coded message CWYOVYTVSVEG. To decode, multiply by the decoding matrix *D*, $\begin{bmatrix} 1 & 0 & 28 \\ 28 & 1 & 28 \\ 28 & 0 & 2 \end{bmatrix}$. This gives

$$D \times CM = \begin{bmatrix} 1 & 0 & 28 \\ 28 & 1 & 28 \\ 28 & 0 & 2 \end{bmatrix} \times \begin{bmatrix} 3 & 23 & 25 & 15 \\ 22 & 25 & 20 & 22 \\ 19 & 22 & 5 & 7 \end{bmatrix} = \begin{bmatrix} 535 & 639 & 165 & 211 \\ 638 & 1285 & 860 & 638 \\ 122 & 688 & 710 & 434 \end{bmatrix},$$

which in arithmetic modulo 29 is $\begin{bmatrix} 13 & 1 & 20 & 8 \\ 0 & 9 & 19 & 0 \\ 6 & 21 & 14 & 28 \end{bmatrix}$ and translates back to "Math is fun."

Note that in the matrix for the decoded message, 0 corresponds to 29.

When you code a message, be sure to use a matrix with 3 rows. Add spaces at the end of the message, if necessary.

1. Code the message "NOT NOW." **V KA.QHNT**
2. Decode the message "FE RRIQP CCDORI". **CAN I HAVE CAR**

LESSON 4-2

Problem Solving
Multiplying Matrices

Members of the Cooking Club entered the Culinary Challenge. In this contest, the score for each entry is multiplied by an assigned degree of difficulty.

Cooking Club Members Scores

	Appetizer	Main Course	Dessert
Beth	25	38	28
Jon	35	29	37
Lupe	20	31	39
Amy	40	32	36

Culinary Challenge Degrees of Difficulty

	Beth	Jon	Lupe	Amy
Appetizer	3.1	2.0	3.5	1.5
Main Course	2.1	1.8	3.7	2.8
Dessert	2.3	2.4	3.0	3.5

1. Display each table as a matrix. Matrix *S* should show the scores and matrix *D* should show the degrees of difficulty.

$$S = \begin{bmatrix} 25 & 38 & 28 \\ 35 & 29 & 37 \\ 20 & 31 & 39 \\ 40 & 32 & 36 \end{bmatrix}; \quad D = \begin{bmatrix} 3.1 & 2.0 & 3.5 & 1.5 \\ 2.1 & 1.8 & 3.7 & 2.8 \\ 2.3 & 2.4 & 3.0 & 3.5 \end{bmatrix}$$

2. Write an equation using *S*, *D*, and product matrix *P* you could use to evaluate the final scores. $S \times D = P$

3. Explain how you know that matrix *S* can be multiplied by matrix *D*.
Possible answer: because matrix *S* has the same number of columns (3) as matrix *D* has rows (3); the result will be a 4×4 matrix.

4. Write the product matrix *P*.

$$P = \begin{bmatrix} 221.7 & 185.6 & 312.1 & 241.9 \\ 254.5 & 211.0 & 340.8 & 263.2 \\ 216.8 & 189.4 & 301.7 & 253.3 \\ 274.0 & 224.0 & 366.4 & 275.6 \end{bmatrix}$$

5. Roger is writing a story for the school newspaper about the Culinary Challenge. Explain how he can use *P* to find the final scores for his story.
The numbers along the main diagonal of the product matrix give the final scores.

6. List the contestants and their final scores, in descending order.
Lupe: 301.7; Amy: 275.6; Beth: 221.7; Jon: 211

LESSON 4-2

Reading Strategies
Compare and Contrast

Like real numbers, matrices can be multiplied. But unlike numbers that can be multiplied in any order, matrices must be multiplied in a specific way.

Multiplication of real numbers is commutative; that is, the order does not matter.	Matrix multiplication is NOT commutative.
$4 \times 8 = 32$ and $8 \times 4 = 32$	$A = \begin{bmatrix} 2 & 5 \\ 6 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 4 \\ 7 & 8 \end{bmatrix}$
So the product $a \times b$ is the same as $b \times a$. $a \times b = b \times a$	$A \times B = \begin{bmatrix} 37 & 48 \\ 27 & 48 \end{bmatrix}$ but $B \times A = \begin{bmatrix} 26 & 17 \\ 62 & 59 \end{bmatrix}$ $A \times B \neq B \times A$

Two matrices can be multiplied if the number of *columns* in the first matrix is the same as the number of *rows* in the second matrix. If matrix *R* is 2×4 and matrix *S* is 4×3, then *RS* is possible but *SR* is NOT possible.

Matrix *R* $\begin{bmatrix} 5 & 2 & -4 & 1 \\ -1 & 0 & 3 & -2 \end{bmatrix}$ 2×4

Matrix *S* $\begin{bmatrix} 3 & 0 & -2 \\ 1 & 4 & 2 \\ 0 & -3 & 1 \\ -5 & 2 & 0 \end{bmatrix}$ 4×3

Product Matrix *RS* $\begin{bmatrix} 12 & 22 & -10 \\ 7 & -13 & 5 \end{bmatrix}$ 2×3

Use matrices *D*, *E*, and *F* to answer the following questions.

$$D = \begin{bmatrix} -1 & -4 \\ 6 & 3 \end{bmatrix} \quad E = \begin{bmatrix} -3 & 8 \\ 1 & 5 \\ 7 & -4 \end{bmatrix} \quad F = \begin{bmatrix} 2 & -1 & 6 \\ 4 & 7 & -3 \end{bmatrix}$$

1. Can you multiply matrices *D* and *E* to give *DE* or *ED*? Explain.
***DE* is not possible because matrix *D* has 2 columns and matrix *E* has 3 rows; *ED* is possible because matrix *E* has 2 columns and matrix *D* has 2 rows.**

2. Can you multiply matrices *D* and *F* to give *DF* or *FD*? Explain.
***DF* is possible because matrix *D* has 2 columns and matrix *F* has 2 rows; *FD* is not possible because matrix *F* has 3 columns and matrix *D* has 2 rows.**

3. Explain why matrix multiplication is not commutative. Give examples.
Possible answer: Matrix multiplication is not commutative because the products are not the same when the order of multiplication is reversed. Matrix *DF* is not the same as matrix *FD*.

4. Can you multiply matrices *E* and *F*? Describe all possibilities. Give the dimensions of any resulting matrices.
Yes; both *EF* and *FE* are possible. Matrix *EF* is 3×3 and matrix *FE* is 2×2.