Name ______________________________ Date __________ Class __________

LESSON 4-2

Reading Strategies
Compare and Contrast

Like real numbers, matrices can be multiplied. But unlike numbers that can be multiplied in any order, matrices must be multiplied in a specific way.

Multiplication of real numbers is commutative; that is, the order does not matter.	Matrix multiplication is NOT commutative.
$4 \times 8 = 32$ and $8 \times 4 = 32$	$A = \begin{bmatrix} 2 & 5 \\ 6 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 4 \\ 7 & 8 \end{bmatrix}$
So the product $a \times b$ is the same as $b \times a$.	$A \times B = \begin{bmatrix} 37 & 48 \\ 27 & 48 \end{bmatrix}$ but $B \times A = \begin{bmatrix} 26 & 17 \\ 62 & 59 \end{bmatrix}$
$a \times b = b \times a$	$A \times B \neq B \times A$

Two matrices can be multiplied if the number of *columns* in the first matrix is the same as the number of *rows* in the second matrix. If matrix R is 2×4 and matrix S is 4×3, then RS is possible but SR is NOT possible.

Matrix R	Matrix S	Product Matrix RS
$\begin{bmatrix} 5 & 2 & -4 & 1 \\ -1 & 0 & 3 & -2 \end{bmatrix}$	$\begin{bmatrix} 3 & 0 & -2 \\ 1 & 4 & 2 \\ 0 & -3 & 1 \\ -5 & 2 & 0 \end{bmatrix}$	$\begin{bmatrix} 12 & 22 & -10 \\ 7 & -13 & 5 \end{bmatrix}$
2×4	4×3	2×3

Use matrices *D*, *E*, and *F* to answer the following questions.

$$D = \begin{bmatrix} -1 & -4 \\ 6 & 3 \end{bmatrix} \qquad E = \begin{bmatrix} -3 & 8 \\ 1 & 5 \\ 7 & -4 \end{bmatrix} \qquad F = \begin{bmatrix} 2 & -1 & 6 \\ 4 & 7 & -3 \end{bmatrix}$$

1. Can you multiply matrices D and E to give DE or ED? Explain.

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2. Can you multiply matrices D and F to give DF or FD? Explain.

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3. Explain why matrix multiplication is not commutative. Give examples.

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4. Can you multiply matrices E and F? Describe all possibilities. Give the dimensions of any resulting matrices.

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LESSON **4-2**

Reteach
Multiplying Matrices (continued)

To find a matrix product, first make sure the product is defined.

Find AB. $A = \begin{bmatrix}3 & 5 & 1\\ -2 & 0 & -1\end{bmatrix}$ $B = \begin{bmatrix}1 & 2\\ -1 & 4\\ 0 & 3\end{bmatrix}$

A is 2 × 3 and *B* is 3 × 2. The product is a 2 × 2 matrix.

Step 1: Multiply row 1 entries of *A* by column 1 entries of *B*. The sum is the first entry in the product.

$$\begin{bmatrix}3 & 5 & 1\\ -2 & 0 & -1\end{bmatrix}\begin{bmatrix}1 & 2\\ -1 & 4\\ 0 & 3\end{bmatrix} = \begin{bmatrix}3(1)+5(-1)+1(0) & ?\\ ? & ?\end{bmatrix} = \begin{bmatrix}-2 & ?\\ ? & ?\end{bmatrix}$$

Step 2: Multiply row 1 entries of *A* by column 2 entries of *B*. Add.

$$\begin{bmatrix}3 & 5 & 1\\ -2 & 0 & -1\end{bmatrix}\begin{bmatrix}1 & 2\\ -1 & 4\\ 0 & 3\end{bmatrix} = \begin{bmatrix}-2 & 3(2)+5(4)+1(3)\\ ? & ?\end{bmatrix} = \begin{bmatrix}-2 & 29\\ ? & ?\end{bmatrix}$$

Step 3: Multiply row 2 entries of *A* by column 1 entries of *B*. Add.

$$\begin{bmatrix}3 & 5 & 1\\ -2 & 0 & -1\end{bmatrix}\begin{bmatrix}1 & 2\\ -1 & 4\\ 0 & 3\end{bmatrix} = \begin{bmatrix}-2 & 29\\ -2(1)+0(-1)+(-1)(0) & ?\end{bmatrix} = \begin{bmatrix}-2 & 29\\ -2 & ?\end{bmatrix}$$

Step 4: Multiply row 2 entries of *A* by column 2 entries of *B*. Add.

$$\begin{bmatrix}3 & 5 & 1\\ -2 & 0 & -1\end{bmatrix}\begin{bmatrix}1 & 2\\ -1 & 4\\ 0 & 3\end{bmatrix} = \begin{bmatrix}-2 & 29\\ -2 & -2(2)+0(4)+(-1)(3)\end{bmatrix} = \begin{bmatrix}-2 & 29\\ -2 & -7\end{bmatrix}$$

Find each product.

4. $\begin{bmatrix}3\\ 1\end{bmatrix}\begin{bmatrix}4 & 3\end{bmatrix} = \begin{bmatrix}3(\underline{4}) & 3(\underline{3})\\ 1(\underline{4}) & 1(\underline{3})\end{bmatrix} = \begin{bmatrix}12 & 9\\ 4 & 3\end{bmatrix}$

5. $\begin{bmatrix}-1 & 0\\ 3 & -2\end{bmatrix}\begin{bmatrix}1 & 2\\ -4 & 0\end{bmatrix} = \begin{bmatrix}-1(1)+0(-4) & -1(\underline{2})+0(\underline{0})\\ 3(\underline{1})+-2(\underline{-4}) & 3(\underline{2})+(-2)(\underline{0})\end{bmatrix} = \begin{bmatrix}-1 & -2\\ 11 & 6\end{bmatrix}$

6. $\begin{bmatrix}-5 & 2\\ 0 & -3\end{bmatrix}\begin{bmatrix}0 & 4\\ -1 & 2\end{bmatrix}$ $\begin{bmatrix}-2 & -16\\ 3 & -6\end{bmatrix}$

LESSON **4-2**

Challenge
Matrix Codes

Matrices can be used to send messages in coded form, and then another matrix can be used to decode the message. Use the code shown in the table for letters and characters.

Code	
A–Z	1–26
Comma	27
Period	28
Space	29

Take a message such as "Math is fun." Code it into numbers and the message becomes:

13 1 20 8 29 9 19 29 6 21 14 28

The message in code can be represented by matrix *M*. $\begin{bmatrix}13 & 1 & 20 & 8\\ 29 & 9 & 19 & 29\\ 6 & 21 & 14 & 28\end{bmatrix}$

Notice how the numbers in the message read from left to right. Multiply matrix *M* by the coding matrix *C*.

$$C = \begin{bmatrix}2 & 0 & 1\\ 3 & 1 & 2\\ 1 & 0 & 1\end{bmatrix},\ CM = \begin{bmatrix}2 & 0 & 1\\ 3 & 1 & 2\\ 1 & 0 & 1\end{bmatrix} \times \begin{bmatrix}13 & 1 & 20 & 8\\ 29 & 9 & 19 & 29\\ 6 & 21 & 14 & 28\end{bmatrix} = \begin{bmatrix}32 & 23 & 54 & 44\\ 80 & 54 & 107 & 109\\ 19 & 22 & 34 & 36\end{bmatrix}$$

Since many of the numbers are greater than 29, the greatest number in the code, divide each number by 29 and record only the remainder. This is called arithmetic modulo 29.

$$CM \text{ modulo } 29 = \begin{bmatrix}32 & 23 & 54 & 44\\ 80 & 54 & 107 & 109\\ 19 & 22 & 34 & 36\end{bmatrix} \text{ modulo } 29 = \begin{bmatrix}3 & 23 & 25 & 15\\ 22 & 25 & 20 & 22\\ 19 & 22 & 5 & 7\end{bmatrix}$$

This gives the coded message CWYOVYTVSVEG. To decode, multiply by the decoding matrix *D*, $\begin{bmatrix}1 & 0 & 28\\ 28 & 1 & 28\\ 28 & 0 & 2\end{bmatrix}$. This gives

$$D \times CM = \begin{bmatrix}1 & 0 & 28\\ 28 & 1 & 28\\ 28 & 0 & 2\end{bmatrix} \times \begin{bmatrix}3 & 23 & 25 & 15\\ 22 & 25 & 20 & 22\\ 19 & 22 & 5 & 7\end{bmatrix} = \begin{bmatrix}535 & 639 & 165 & 211\\ 638 & 1285 & 860 & 638\\ 122 & 688 & 710 & 434\end{bmatrix},$$

which in arithmetic modulo 29 is $\begin{bmatrix}13 & 1 & 20 & 8\\ 0 & 9 & 19 & 0\\ 6 & 21 & 14 & 28\end{bmatrix}$ and translates back to "Math is fun."

Note that in the matrix for the decoded message, 0 corresponds to 29.

When you code a message, be sure to use a matrix with 3 rows. Add spaces at the end of the message, if necessary.

1. Code the message "NOT NOW." **V KA.QHNT**
2. Decode the message "FE RRIQP CCDORI". **CAN I HAVE CAR**

LESSON **4-2**

Problem Solving
Multiplying Matrices

Members of the Cooking Club entered the Culinary Challenge. In this contest, the score for each entry is multiplied by an assigned degree of difficulty.

Cooking Club Members Scores			
	Appetizer	Main Course	Dessert
Beth	25	38	28
Jon	35	29	37
Lupe	20	31	39
Amy	40	32	36

Culinary Challenge Degrees of Difficulty				
	Beth	Jon	Lupe	Amy
Appetizer	3.1	2.0	3.5	1.5
Main Course	2.1	1.8	3.7	2.8
Dessert	2.3	2.4	3.0	3.5

1. Display each table as a matrix. Matrix *S* should show the scores and matrix *D* should show the degrees of difficulty.

$$S = \begin{bmatrix}25 & 38 & 28\\ 35 & 29 & 37\\ 20 & 31 & 39\\ 40 & 32 & 36\end{bmatrix};\quad D = \begin{bmatrix}3.1 & 2.0 & 3.5 & 1.5\\ 2.1 & 1.8 & 3.7 & 2.8\\ 2.3 & 2.4 & 3.0 & 3.5\end{bmatrix}$$

2. Write an equation using *S*, *D*, and product matrix *P* you could use to evaluate the final scores. **$S \times D = P$**

3. Explain how you know that matrix *S* can be multiplied by matrix *D*.
Possible answer: because matrix *S* has the same number of columns (3) as matrix *D* has rows (3); the result will be a 4 × 4 matrix.

4. Write the product matrix *P*.

$$P = \begin{bmatrix}221.7 & 185.6 & 312.1 & 241.9\\ 254.5 & 211.0 & 340.8 & 263.2\\ 216.8 & 189.4 & 301.7 & 253.3\\ 274.0 & 224.0 & 366.4 & 275.6\end{bmatrix}$$

5. Roger is writing a story for the school newspaper about the Culinary Challenge. Explain how he can use *P* to find the final scores for his story.
The numbers along the main diagonal of the product matrix give the final scores.

6. List the contestants and their final scores, in descending order.
Lupe: 301.7; Amy: 275.6; Beth: 221.7; Jon: 211

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Two matrices can be multiplied if the number of *columns* in the first matrix is the same as the number of *rows* in the second matrix. If matrix *R* is 2 × 4 and matrix *S* is 4 × 3, then *RS* is possible but *SR* is NOT possible.

Matrix *R*	Matrix *S*	Product Matrix *RS*
$\begin{bmatrix}5 & 2 & -4 & 1\\ -1 & 0 & 3 & -2\end{bmatrix}$	$\begin{bmatrix}3 & 0 & -2\\ 1 & 4 & 2\\ 0 & -3 & 1\\ -5 & 2 & 0\end{bmatrix}$	$\begin{bmatrix}12 & 22 & -10\\ 7 & -13 & 5\end{bmatrix}$
2 × 4	4 × 3	2 × 3

Use matrices *D*, *E*, and *F* to answer the following questions.

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1. Can you multiply matrices *D* and *E* to give *DE* or *ED*? Explain.
DE is not possible because matrix D has 2 columns and matrix E has 3 rows; ED is possible because matrix E has 2 columns and matrix D has 2 rows.

2. Can you multiply matrices *D* and *F* to give *DF* or *FD*? Explain.
DF is possible because matrix D has 2 columns and matrix F has 2 rows; FD is not possible because matrix F has 3 columns and matrix D has 2 rows.

3. Explain why matrix multiplication is not commutative. Give examples.
Possible answer: Matrix multiplication is not commutative because the products are not the same when the order of multiplication is reversed. Matrix DF is not the same as matrix FD.

4. Can you multiply matrices *E* and *F*? Describe all possibilities. Give the dimensions of any resulting matrices.
Yes; both EF and FE are possible. Matrix EF is 3 × 3 and matrix FE is 2 × 2.

