4-2 Multiplying Matrices

Example 1 Identifying Matrix Products

Tell whether each product is defined. If so, give its dimensions.

A. $A_{3 \times 4}$ **and** $B_{4 \times 2}$; AB

$$\begin{array}{ccc} A & B & AB \\ 3 \times 4 & 4 \times 2 = & 3 \times 2 \text{ matrix} \end{array}$$

The inner dimensions are equal $(4 = 4)$, so the matrix product is defined. The dimensions of the product are the outer numbers, 3×2.

B. $C_{1 \times 4}$ **and** $D_{3 \times 4}$; CD

$$\begin{array}{cc} C & D \\ 1 \times 4 & 3 \times 4 \end{array}$$

The inner dimensions are not equal $(4 \neq 3)$, so the matrix product is not defined. ✖

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Example 2 Finding the Matrix Product

$$W = \begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \quad X = \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix} \quad Y = \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix}$$

Find each product, if possible.

A. ***WX***

Check the dimensions. W is 3×2, X is 2×3. WX is defined and is 3×3.

Multiply row 1 of W and column 1 of X as shown. Place the result in wx_{11}.

$$WX = \begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}$$

3(4) − 2(5)

Multiply row 1 of W and column 2 of X as shown. Place the result in wx_{12}.

$$\begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 19 & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}$$

3(7) − 2(1)

Multiply row 1 of W and column 3 of X as shown. Place the result in wx_{13}.

$$\begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 19 & -4 \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}$$

3(−2) − 2(−1)

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Example 2 Finding the Matrix Product (continued)

Multiply row 2 of W and column 1 of X as shown. Place the result in wx_{21}.

$$\begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 19 & -4 \\ 4 & ? & ? \\ ? & ? & ? \end{bmatrix} \quad 1(4) + 0(5)$$

Multiply row 2 of W and column 2 of X as shown. Place the result in wx_{22}.

$$\begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 19 & -4 \\ 4 & 7 & ? \\ ? & ? & ? \end{bmatrix} \quad 1(7) + 0(1)$$

Multiply row 2 of W and column 3 of X as shown. Place the result in wx_{23}.

$$\begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 19 & -4 \\ 4 & 7 & -2 \\ ? & ? & ? \end{bmatrix} \quad 1(-2) + 0(-1)$$

Multiply row 3 of W and column 1 of X as shown. Place the result in wx_{31}.

$$\begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 19 & -4 \\ 4 & 7 & -2 \\ 3 & ? & ? \end{bmatrix} \quad 2(4) - 1(5)$$

Multiply row 3 of W and column 2 of X as shown. Place the result in wx_{32}.

$$\begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 19 & -4 \\ 4 & 7 & -2 \\ 3 & 13 & ? \end{bmatrix} \quad 2(7) - 1(1)$$

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Example 2 Finding the Matrix Product (continued)

Multiply row 3 of W and column 3 of X as shown. Place the result in wx_{33}.

$$\begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 4 & 7 & -2 \\ 5 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 19 & -4 \\ 4 & 7 & -2 \\ 3 & 13 & -3 \end{bmatrix}$$

$2(-2) - 1(-1)$

$$WX = \begin{bmatrix} 2 & 19 & -4 \\ 4 & 7 & -2 \\ 3 & 13 & -3 \end{bmatrix}$$

B. ***XW***

Check the dimensions. X is 2×3, and W is 3×2 so the product is defined and is 2×2.

$$XW = \begin{bmatrix} 4(3) + 7(1) - 2(2) & 4(-2) + 7(0) - 2(-1) \\ 5(3) + 1(1) - 1(2) & 5(-2) + 1(0) - 1(-1) \end{bmatrix} = \begin{bmatrix} 15 & -6 \\ 14 & -9 \end{bmatrix}$$

C. ***XY***

Check the dimensions. $2 \times 3 \;\; 2 \times 2$. The product is not defined. The matrices cannot be multiplied in this order.

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Example 3 Business Application

Two stores held sales on their videos and DVDs, with prices as shown. Use the sales data to determine how much money each store brought in from the sale on Saturday.

Sales Price		
	Videos	DVDs
Video World	\$8.95	\$11.95
Star Movies	\$7.50	\$12.50

Total Sales			
	Fri	Sat	Sun
Videos	23	31	25
DVDs	40	48	42

Use a product matrix to find the sales of each store for each day.

$$\begin{bmatrix} 8.95 & 11.95 \\ 7.50 & 12.50 \end{bmatrix} \begin{bmatrix} 23 & 31 & 25 \\ 40 & 48 & 42 \end{bmatrix} =$$

$$\begin{bmatrix} 8.95(23)+11.95(40) & 8.95(31)+11.95(48) & 8.95(25)+11.95(42) \\ 7.50(23)+12.50(40) & 7.50(31)+12.50(48) & 7.50(25)+12.50(42) \end{bmatrix}$$

$$\begin{array}{c c} & \begin{array}{ccc} \text{Fri} & \text{Sat} & \text{Sun} \end{array} \\ = & \begin{bmatrix} 683.85 & 851.05 & 725.65 \\ 672.50 & 832.50 & 712.50 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Video World} \\ \text{Star Movies} \end{array}$$

On Saturday, Video World made \$851.05 and Star Movies made \$832.50.

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Example 4 Finding Powers of Square Matrices

$$P = \begin{bmatrix} 4 & 0 \\ 2 & 3 \end{bmatrix} \quad Q = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 3 \\ 2 & 1 & 2 \end{bmatrix}$$

Evaluate, if possible.

A. P^3

$$P^3 = \begin{bmatrix} 4 & 0 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4(4)+0(2) & 4(0)+0(3) \\ 2(4)+3(2) & 2(0)+3(3) \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 2 & 3 \end{bmatrix}$$

```
[A]^3
      [[64 0 ]
       [74 27]]
```

$$= \begin{bmatrix} 16 & 0 \\ 14 & 9 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 16(4)+0(2) & 16(0)+0(3) \\ 14(4)+9(2) & 14(0)+9(3) \end{bmatrix}$$

$$= \begin{bmatrix} 64 & 0 \\ 74 & 27 \end{bmatrix}$$

Check Use a calculator.

B. Q^2

For large matrices, use a graphing calculator.

```
[A]
       [[1  2 9]
        [-1 2 3]
        [2  1 2]]

[A]²
    [[17 11 33]
     [5  1  -3]
     [5  6  25]]
```

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