Name ______________________ Date __________ Class __________

SECTION 4A

Ready To Go On? Skills Intervention

4-1 Matrices and Data

Find these vocabulary words in Lesson 4-1 and the Multilingual Glossary.

Vocabulary

matrix dimensions entry address scalar

Displaying Data in Matrix Form

Use the table of data to answer each question.

Plain	Color	With Borders
10	14	18
15	20	26
25	30	34

a. Display the data in the form of a matrix, *C*.

b. What are the dimensions of Matrix *C*?

c. What is the entry at c_{13}? What does it represent?

d. What is the address of the entry that has the value 26?

a) Find the value in the first column of the first row of the table. Write this value in the first column of the first row of the matrix. Do the same for the other values.

COLUMNS

$$C = \begin{bmatrix} 10 & \_\_\_\_ & \_\_\_\_ \\ 15 & \_\_\_\_ & \_\_\_\_ \\ 25 & \_\_\_\_ & \_\_\_\_ \end{bmatrix}$$

ROWS

b) How many rows are in the table? ____ How many columns? ____

Matrix *C* is a ____ (rows) $\times$ ____ (columns) matrix.

c) What does the 1 in c_{13} describe? __________ What does the 3 in c_{13} describe? __________ What is the value in c_{13}? ____

d) Find the value 26 in the matrix. What row is this value in? ____

What column is it in? ____ What is the address of 26? ____

Finding Matrix Sums and Differences

Find *A* + *B*. $A = \begin{bmatrix} 3 & -1 \\ 0 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 5 & -2 \\ 2 & 0 \end{bmatrix}$

Add the values in *B* that correspond to the values in *A*.

$$\begin{bmatrix} 3 + 5 & -1 + (-2) \\ 0 + \_\_\_\_ & 4 + \_\_\_\_ \end{bmatrix} = \begin{bmatrix} 8 & -3 \\ \_\_\_\_ & \_\_\_\_ \end{bmatrix}$$

Corresponding values:
Row 1 Column 1 of *A* corresponds to
Row 1 Column 1 of *B*.

Name ____ Date ___________ Class ____________

SECTION 4A

Ready To Go On? Skills Intervention

4-2 Multiplying Matrices

Find these vocabulary words in Lesson 4-2 and the Multilingual Glossary.

Vocabulary			
matrix product	square matrix	main diagonal	multiplicative identity matrix

Multiplying Matrices

Tell whether each product is defined. If so, give its dimensions.

A. $P_{3\times2}$ and $Q_{1\times4}$; PQ

What are the inner dimensions? 2 and ____ Are they equal? ____

Is the matrix product defined? ____

B. $R_{2\times4}$ and $S_{4\times1}$; RS

What are the inner dimensions? ____ and 4 Are they equal? ____

Is the matrix product defined? ____ What are the dimensions of RS? ______

Finding the Matrix Product

Find the product *AB*.

COLUMN 1

ROW 1

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 3 & 5 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 0 \\ -3 & 1 \\ 2 & 5 \end{bmatrix} \quad AB = \begin{bmatrix} \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ \end{bmatrix}$$

What are the dimensions of A and B? A: ____ $\times$ ____ B: ____ $\times$ ____

What are the dimensions of the product AB? ____ $\times$ ____

Multiply row 1 of A by column 1 of B. $2(4) + 0(-3) + -1(2) =$ ____

Place the answer in row 1, column 1 of the product matrix.

Multiply row 1 of A by column 2 of B. ____$(0) + 0$____ $+ -1(5) =$ ____

Place the answer in row 1, column 2 of the product matrix.

Multiply row 2 of A by column 1 of B. 2 of B. ____$(4) +$ ____$(-3) +$ ____$(2) =$ ____

Place the answer in row 2, column 1 of the product matrix.

Multiply row 2 of A by column 2 of B. 3(____) + ____ (____) + ____ (____) = ____

Place the answer in row 2, column 2 of the product matrix.

Name ______________________________ Date __________ Class __________

SECTION **4A**

Ready To Go On? Skills Intervention

4-3 Using Matrices to Transform Geometric Figures

Find these vocabulary words in Lesson 4-3 and the Multilingual Glossary.

Vocabulary		
translation matrix	reflection matrix	rotation matrix

Using Matrices to Translate a Figure

Translate △*ABC* with coordinates *A*(1, 1), *B*(−1, 2), and *C*(3, 4) two units left and three units up. Find the coordinates of the vertices of the image, and graph.

What are the *x*-coordinates of the vertices of △*ABC*? 1, ____, and ____

What are the *y*-coordinates of the vertices of △*ABC*? 1, ____, and ____

Write the *x*-coordinates in the first row of the matrix and the *y*-coordinates in the second row.

$$P = \begin{bmatrix} 1 & \underline{\qquad} & \underline{\qquad} \\ 1 & \underline{\qquad} & \underline{\qquad} \end{bmatrix} \begin{array}{l} \leftarrow x\text{-coordinates} \\ \leftarrow y\text{-coordinates} \end{array}$$

You are translating △*ABC* 2 units left and 3 units up. Therefore, each *x*-coordinate will move 2 units left and each *y*-coordinate 3 units up.

Complete the translation matrix, *T*.

$$T = \begin{bmatrix} -2 & \underline{\qquad} & \underline{\qquad} \\ 3 & \underline{\qquad} & \underline{\qquad} \end{bmatrix} \begin{array}{l} \leftarrow x\text{-coordinates} \\ \leftarrow y\text{-coordinates} \end{array}$$

Add the matrices together to find the vertices of the translated image.

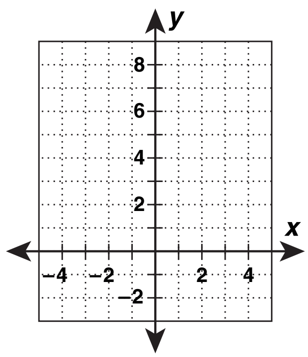
$$P + T = \begin{bmatrix} 1 + (-2) & \underline{\qquad\qquad} & \underline{\qquad\qquad} \\ 1 + 3 & \underline{\qquad\qquad} & \underline{\qquad\qquad} \end{bmatrix}$$

$$= \begin{bmatrix} \underline{\qquad} & \underline{\qquad} & 1 \\ 4 & \underline{\qquad} & \underline{\qquad} \end{bmatrix}$$

What are the coordinates of the vertices of the translated triangle?

A′ (−1, 4), *B*′ (____, ____), and *C*′ (____, ____).

Graph the translated triangle on the grid.



Name ____ Date ___________ Class ____________

SECTION 4A

Ready To Go On? Quiz

4-1 Matrices and Data

Use the table for Exercises 1–4.

Toddlers in Gymnastics Classes			
	Morning	**Afternoon**	**Evening**
Monday	8	6	12
Wednesday	6	4	10
Friday	11	15	19

1. Display the data in the form of a matrix, *M*.

$$M = \begin{bmatrix} \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \end{bmatrix}$$

2. What are the dimensions of M? ____ $\times$ ____

3. What is the value of the matrix entry with the address m_{13}? ____

4. What is the address of the entry that has the value 4? ____

Use the matrices below for Exercises 5–8. Evaluate if possible.

$$A = \begin{bmatrix} 2 & -5 \\ 0 & 3 \\ 1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \quad C = \begin{bmatrix} 3 & -2 \\ 1 & 3 \\ 4 & -1 \end{bmatrix} \quad D = \begin{bmatrix} 6 & 3 & -2 \\ -2.5 & 1 & -1 \end{bmatrix}$$

5. $A + C$

____

6. $2B$

____

7. $\frac{1}{2}C - D$

____

8. $C - 3A$

____

4-2 Multiplying Matrices

Use the matrices named below for Exercises 9–12. Tell whether each product is defined. If so, give its dimensions.

$P_{3\times2}$ $\qquad Q_{2\times3}$ $\qquad R_{1\times3}$ $\qquad S_{3\times2}$

9. PQ ______________

10. RS ______________

11. QR __________

12. SP __________

Name ____ Date ____________ Class ____________

SECTION **4A**

Ready To Go On? Quiz continued

Use the matrices below for Exercises 13–16. Evaluate, if possible.

$$E = \begin{bmatrix} 2 & -3 & -1 \\ -1 & 0 & 5 \\ 4 & 1 & 2 \end{bmatrix} \quad F = \begin{bmatrix} -1 & 0.5 & 0.25 \end{bmatrix} \quad G = \begin{bmatrix} 3 & -2 \\ 2 & 3 \end{bmatrix} \quad H = \begin{bmatrix} 1 & 0 \\ -2 & 4 \\ 4 & -8 \end{bmatrix}$$

13. EF ________

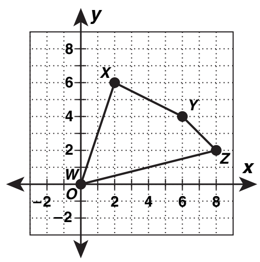
14. FH ________

15. HG ________

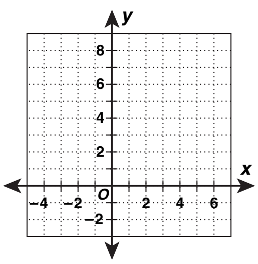
16. G^2 ________

4-3 Using Matrices to Transform Geometric Figures

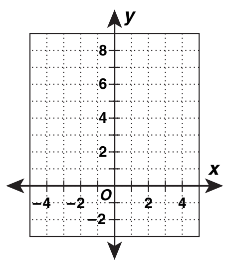
For Exercises 17–20, use polygon *WXYZ* with coordinates *W*(0, 0), *X*(2, 6), *Y*(6, 4), and *Z*(8, 2). Give the coordinates of the image and graph.



17. Translate polygon *WXYZ* two units to the left and one unit up.

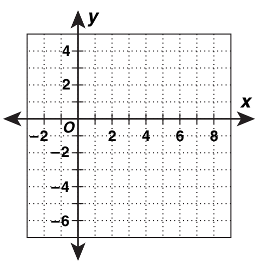


18. Reduce polygon *WXYZ* by a factor of $\frac{1}{2}$.



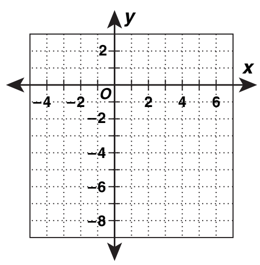
19. Use $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ to rotate polygon *WXYZ*.

Describe the image.



20. Use $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ to rotate polygon *WXYZ*.

Describe the image.



21. How does multiplying by $\begin{bmatrix} 0 & 3 \\ 3 & 0 \end{bmatrix}$ transform polygon *WXYZ*?

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SECTION 4A

Ready To Go On? Enrichment

Matrices

Use matrix operations to determine Matrix *X*.

1. $2X + 2\begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 6 \\ 0 & -2 \end{bmatrix}$ $X = \begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix}$

2. $X - 2\begin{bmatrix} 2 & 5 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 6 & -3 \\ 2 & 0 \end{bmatrix}$ $X = \begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix}$

3. $3X - \begin{bmatrix} 3 & -6 & 0 \\ 7 & 1 & 2 \end{bmatrix} = 2\begin{bmatrix} 3 & 6 & 3 \\ -8 & 1 & 2 \end{bmatrix}$ $X = \begin{bmatrix} \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \end{bmatrix}$

Solve for *x* and *y*.

4. $3[x \quad y] = [x \quad 2] + [6 \quad y]$ $x =$ ____ and $y =$ ____

5. $\begin{bmatrix} x + 4 & 2 \\ 5 & -y \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 6 & -1 \end{bmatrix} = \begin{bmatrix} -2 & 5 \\ 11 & 9 \end{bmatrix}$ $x =$ ____ and $y =$ ______

6. $\begin{bmatrix} 2x + 4 & -3 \\ 1 & x \end{bmatrix} + \begin{bmatrix} y & 10 \\ 4 & 2y \end{bmatrix} = \begin{bmatrix} 6 & 7 \\ 5 & -2 \end{bmatrix}$ $x =$ ____ and $y =$ ____

Name ______________________________ Date __________ Class __________

Ready To Go On? Skills Intervention

SECTION **4B**

4-4 Determinants and Cramer's Rule

Find these vocabulary words in Lesson 4-4 and the Multilingual Glossary.

Vocabulary		
determinant	coefficient matrix	Cramer's rule

Finding the Determinant of a Matrix

Find the determinant of each matrix.

A. $A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$

What is the product of the first diagonal? $4 \times$ ____ $=$ ____

What is the product of the second diagonal? $3 \times$ ____ $=$ ____

det A $=$ first diagonal $-$ second diagonal $=$ ____ $-$ ____ $=$ ______

B. $A = \begin{bmatrix} 2 & -3 & 1 \\ -1 & 4 & 0 \\ 3 & -2 & 4 \end{bmatrix}$ Write the matrix so that the first two columns are repeated to the right.

$A = \begin{bmatrix} 2 & -3 & 1 \\ -1 & 4 & 0 \\ 3 & -2 & 4 \end{bmatrix} \begin{matrix} 2 & \_\_\_\_ \\ \_\_\_\_ & 4 \\ \_\_\_\_ & \_\_\_\_ \end{matrix}$

Multiply the numbers in the first downward diagonal. $2(4)(4) =$ ____

Multiply the numbers in the second downward diagonal. $-3($____$)($____$) =$ ____

Multiply the numbers in the third downward diagonal. $1($____$)($____$) =$ ____

Add the products of each diagonal. $32 +$ ____ $+$ ____ $=$ ____

Multiply the numbers in the first upward diagonal. $3(4)(1) =$ ____

Multiply the numbers in the second upward diagonal. $-2($____$)($____$) =$ ____

Multiply the numbers in the third upward diagonal. $4($____$)($____$) =$ ____

Add the products of each diagonal. ____ $+ 0 +$ ____ $=$ ____

det A $=$ sum of downward diagonals $-$ sum of upward diagonal

$=$ ____ $-$ ____

$=$ ____

Name ______________________________ Date __________ Class __________

SECTION 4B

Ready To Go On? Skills Intervention

4-5 Matrix Inverses and Solving Systems

Find these vocabulary words in Lesson 4-5 and the Multilingual Glossary.

Vocabulary			
multiplicative inverse matrix	matrix equation	variable matrix	constant matrix

Finding the Inverse of a 2 × 2 Matrix

Find the inverse of the matrix $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$, if it is defined.

$$\begin{array}{cc} a & b \\ \multicolumn{2}{c}{A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}} \\ c & d \end{array}$$

Find the determinant. (1)(____) − 2(____) = ____

Does the matrix have an inverse? ____

The inverse $A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Substitute values. $A^{-1} = \dfrac{1}{\boxed{}}\begin{bmatrix} 4 & \boxed{} \\ -2 & \boxed{} \end{bmatrix}$

Multiply the fraction by each value in the matrix. $A^{-1} = \begin{bmatrix} -\frac{1}{2}(4) & -\frac{1}{2}\boxed{} \\ -\frac{1}{2}(-2) & -\frac{1}{2}\boxed{} \end{bmatrix} = \begin{bmatrix} -2 & \boxed{} \\ 1 & \boxed{} \end{bmatrix}$

Solving Systems Using Inverse Matrices

Write and solve the matrix equation for the system $\begin{cases} x + 3y = 9 \\ 2x - y = -10 \end{cases}$.

Write the coefficients in the first matrix. Write the variables in a second matrix. Write the constants in a third matrix.

$$\begin{array}{ccc} \mathbf{A} & \boldsymbol{X} & \mathbf{B} \\ \begin{bmatrix} \boxed{} & \boxed{} \\ 2 & -1 \end{bmatrix} & \begin{bmatrix} \boxed{} \\ y \end{bmatrix} & \begin{bmatrix} 9 \\ \boxed{} \end{bmatrix} \end{array}$$

$\det A = \boxed{}(-1) - 2(\boxed{}) = \boxed{}$

$A^{-1} = \dfrac{1}{\boxed{}}\begin{bmatrix} -1 & \boxed{} \\ -2 & \boxed{} \end{bmatrix} = \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix}$

$$\begin{array}{cccc} \boldsymbol{X} & & \boldsymbol{A^{-1}} & \boldsymbol{B} \end{array}$$

Solve. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix}\begin{bmatrix} 9 \\ \boxed{} \end{bmatrix} = \begin{bmatrix} \boxed{} \\ \boxed{} \end{bmatrix}$ Substitute the inverse and the constant matrices.

What is the solution? (____, ____)

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SECTION **4B**

Ready To Go On? Problem Solving Intervention

4-5 Matrix Inverses and Solving Systems

Matrix equations, $AX = B$ can be used to solve systems of equations; A is the coefficient matrix, B is the constant matrix and X is the variable matrix.

You are writing a proposal for buying and selling office equipment as a system of equations. Let x = the price of a laptop, y = the price of a printer, and z = the price of a fax machine. What is the price of each type of equipment?

$$\begin{cases} -y + 2z = 470 \\ x + y - z = 430 \\ 2x + 3z = 2410 \end{cases}$$

Understand the Problem

1. Which equation represents selling a printer and buying two fax machines?

____

2. Which equation represents buying a laptop and a printer, and selling a fax machine? ____________________

Make a Plan

3. Write a matrix to represent the coefficients of the system.

$$A = \begin{bmatrix} 0 & -1 & \boxed{} \\ \boxed{} & \boxed{} & -1 \\ \boxed{} & 0 & \boxed{} \end{bmatrix}$$

4. Write a matrix to represent the variables of the system.

$$X = \begin{bmatrix} \boxed{} \\ y \\ \boxed{} \end{bmatrix}$$

5. Write a matrix to represent the constants of the system.

$$B = \begin{bmatrix} 470 \\ \boxed{} \\ \boxed{} \end{bmatrix}$$

Solve

6. Write the matrix equation for the system as $AX = B$.

$$\begin{bmatrix} 0 & -1 & \boxed{} \\ \boxed{} & \boxed{} & -1 \\ \boxed{} & 0 & \boxed{} \end{bmatrix} \begin{bmatrix} \boxed{} \\ y \\ \boxed{} \end{bmatrix} = \begin{bmatrix} 470 \\ \boxed{} \\ \boxed{} \end{bmatrix}$$

7. Use the inverse function on your graphing calculator to find A^{-1} and rewrite the equation to solve for X.

$$X = A^{-1}B = \begin{bmatrix} 3 & 3 & \boxed{} \\ \boxed{} & \boxed{} & 2 \\ \boxed{} & -2 & \boxed{} \end{bmatrix} \begin{bmatrix} 470 \\ \boxed{} \\ \boxed{} \end{bmatrix} = \begin{bmatrix} \boxed{} \\ 210 \\ \boxed{} \end{bmatrix}$$

8. How much is a laptop? ______ a printer? ______ a fax machine? ______

Look Back

9. Substitute the values you calculated into one of the equations in the system. Try

$x + y - z = 430$. (____) + (____) − (____) = 430 Do you yield the correct result? ____

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SECTION 4B

Ready To Go On? Skills Intervention

4-6 Row Operations and Augmented Matrices

Find these vocabulary words in Lesson 4-6 and the Multilingual Glossary.

Vocabulary			
augmented matrix	row operation	row reduction	reduced row-echelon form

Solving Systems with an Augmented Matrix

Write the augmented matrix, and solve. $\begin{cases} 6x + y = 15 \\ 3x + 2y = 12 \end{cases}$

$$\begin{array}{c} \begin{array}{ccc} x\text{-coefficients} & y\text{-coefficients} & \text{constants} \end{array} \\ \left[\begin{array}{cc|c} 6 & 1 & 15 \\ \square & 2 & \square \end{array}\right] \end{array}$$

Write the augmented matrix.

Multiply every value in row 2 by 2. $\left[\begin{array}{cc|c} 6 & 1 & 15 \\ \square & \square & \square \end{array}\right]$

Subtract row 1 from row 2. $\square - 6 = \square$, $\square - 1 = 3$, $\square - 15 = \square$

Write the result in row 2. $\left[\begin{array}{cc|c} 6 & 1 & 15 \\ \square & 3 & \square \end{array}\right]$

Multiply row 1 by 3. $\left[\begin{array}{cc|c} \square & \square & \square \\ 0 & 3 & 9 \end{array}\right]$

Subtract row 2 from row 1. $\square - 0 = \square$, $\square - 3 = 0$, $\square - 9 = \square$

Write the result in row 1. $\left[\begin{array}{cc|c} \square & 0 & \square \\ 0 & 3 & 9 \end{array}\right]$

Divide row 1 by 18 and row 2 by 3. $\left[\begin{array}{cc|c} 1 & 0 & \square \\ 0 & 1 & \square \end{array}\right]$

The solution is $x =$ ____ and $y =$ ____.

Check your answer by substituting the values of x and y into each of the original equations in the system.

$6x + y = 15$ $\qquad$ $3x + 2y = 12$

6(____) + (____) = 15 $\qquad$ 3(____) + 2(____) = 12

____ = 15 $\qquad$ ____ = 12

Name ______________________________ Date __________ Class __________

SECTION 4B

Ready To Go On? Problem Solving Intervention

4-6 Row Operations and Augmented Matrices

The system of equations represents the costs of three baskets of art supplies. Use a to represent the cost of one box of pencils, b to represent the cost of one box of paints, and c to represent the cost of one bundle of brushes. Find the cost of each type of supply.

$$\begin{cases} 2a + 2b + c + 2.20 = 10.65 \\ 3a + 2b + 2c + 2.20 = 12.63 \\ 4a + 3b + 2c + 2.20 = 16.11 \end{cases}$$

Understand the Problem

1. What are the three types of art supplies? ______________________________
2. Which equation represents 3 boxes of pencils, 2 boxes of paints, and 2 bundles of brushes? ____________________
3. The cost of the basket into which the supplies are placed is included in each equation. What is the cost of each basket? ____________________
4. What information are you trying to find? ____________________________________

Make a Plan

5. Rewrite the equations with the constants on one side.

$$\begin{array}{r} 2a + 2b + c + 2.20 = 10.65 \\ \underline{-2.20 \quad -2.20} \\ \rule{6cm}{0.4pt} \end{array} \qquad \begin{array}{r} 3a + 2b + 2c + 2.20 = 12.63 \\ \underline{-2.20 \quad -2.20} \\ \rule{6cm}{0.4pt} \end{array} \qquad \begin{array}{r} 4a + 3b + 2c + 2.20 = 16.11 \\ \underline{-2.20 \quad -2.20} \\ \rule{6cm}{0.4pt} \end{array}$$

6. Use the equations to write an augmented matrix. $A^{-1} = \left[\begin{array}{ccc|c} \square & 2 & 1 & 8.45 \\ \square & 2 & \square & 10.43 \\ 4 & \square & 2 & \square \end{array}\right]$

Solve

7. Enter the 3×4 augmented matrix into your calculator as A.
8. Press 2nd x^{-1} on your calculator. Then select MATH. Move down the list to B:rref. Find the reduced row-echelon form of the augmented matrix.
9. How much does one box of pencils cost? ______ one box of paints? ______ one bundle of brushes? ______

Look Back

10. Substitute the values for a, b, and c into one of the equations in the system.

    2(______) + 2(______) + (______) + 2.20 = 10.65

    Do your values yield the correct result? ______

Name ______________________ Date __________ Class __________

SECTION 4B

Ready To Go On? Quiz

4-4 Determinants and Cramer's Rule

Find the determinant of each matrix.

1. $\begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix}$ ______

2. $\begin{bmatrix} \frac{1}{3} & 6 \\ 0 & \frac{3}{4} \end{bmatrix}$ ______

3. $\begin{bmatrix} 0.25 & 1.5 \\ -2.5 & 8.0 \end{bmatrix}$ ______

4. $\begin{bmatrix} -1 & 3 & 2 \\ 2 & -1 & 0 \\ 1 & 2 & -3 \end{bmatrix}$ ______

Use Cramer's rule to solve.

5. $\begin{cases} y = 2x + 1 \\ 2x - 4y = 1 \end{cases}$

____

6. $\begin{cases} 2x - 2y = 2 \\ y - x - 1 = 0 \end{cases}$

____

7. $\begin{cases} 2x + 3y = 6 \\ y = 1 - x \end{cases}$

____

8. $\begin{cases} 2x - 2y + z = -5 \\ x + 2y = 3z + 3 \\ z = 2x + 1 \end{cases}$

____

4-5 Matrix Inverses and Solving Systems

Find the inverse of each matrix, if it is defined.

9. $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ ______________

10. $\begin{bmatrix} 0.5 & 1.5 \\ -2 & 4 \end{bmatrix}$ ______________

11. $\begin{bmatrix} -2 & 1 \\ -\frac{1}{2} & \frac{1}{4} \end{bmatrix}$ ______________

12. $\begin{bmatrix} 0 & -1 & 2 \\ 1 & 1 & -1 \\ 2 & 0 & 3 \end{bmatrix}$ ______________

Write the matrix equation for the system, and solve, if possible.

13. $\begin{cases} y = 2x - 4.5 \\ 5y - x = 0 \end{cases}$

____

14. $\begin{cases} 3x - 4y = 5 \\ 2(x + y) + 6 = 0 \end{cases}$

____

15. $\begin{cases} 5x + 4y = 19 \\ 10x + 8y = 12 \end{cases}$

____

16. $\begin{cases} 4x + 5y = 2z + 5 \\ 3x + 4y = z + 3 \\ x + 3y = 5z + 12 \end{cases}$

____

SECTION **4B**

Ready To Go On? Quiz continued

17. You are starting a business renting bouncy houses for children's parties. You are deciding which equipment to purchase. Use x as the price of a bouncy slide, y as the price of a bouncy castle, and z as the price of a bouncy maze. What is the price of each type of equipment?

$$\begin{cases} 2x + y + 3z = 8000 \\ x + 3y + 2z = 7250 \\ 3x + 2y + z = 7250 \end{cases}$$

slide = $________

castle = $________

maze = $________

4-6 Row Operations and Augmented Matrices

Write the augmented matrix, and use row reduction to solve, if possible.

18. $\begin{cases} 2x + 3y = 1 \\ x = y + 3 \end{cases}$

____

19. $\begin{cases} 3x + y = 8 \\ 10x - y = 5 \end{cases}$

____

20. $\begin{cases} 6x - 2y = 16 \\ 3x = 8 + y \end{cases}$

____

21. $\begin{cases} 4x + 2y - 5 = 0 \\ x - y = \frac{1}{2} \end{cases}$

____

22. The system of equations represents the cost of bakery bouquets. Use a to represent the cost of cookie flowers, b the cost of cupcake flowers, and c the cost of chocolate flowers. Find the cost of each type of flower.

$$\begin{cases} 2a + b + 2c + 2.10 = 12.60 \\ 3a + 2b + c + 2.10 = 16.35 \\ 4a + 3b + 3c + 2.10 = 24.60 \end{cases}$$

cookie flower = $________

cupcake flower = $________

chocolate flower = $________

Name ______________________________ Date __________ Class __________

SECTION 4B

Ready To Go On? Enrichment

Inverse Matrices

Use the information provided to decode the three-word message made up of three-letter words encrypted in the matrices below.

$$\begin{bmatrix} 161 \\ 145 \\ 062 \end{bmatrix} \qquad \begin{bmatrix} 170 \\ 150 \\ 061 \end{bmatrix} \qquad \begin{bmatrix} 113 \\ 109 \\ 045 \end{bmatrix}$$

The words are encrypted by first assigning each letter of the alphabet a number. $A = 1$, $B = 2$, $C = 3$, and so on.

The three-letter words are multiplied by the matrix $M = \begin{bmatrix} 6 & 5 & 2 \\ 5 & 5 & 2 \\ 2 & 2 & 1 \end{bmatrix}$.

The message can be decoded by multiplying the coded message by the inverse of the encoding matrix.

What is the inverse matrix? $M^{-1} = \begin{bmatrix} \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \end{bmatrix}$

What word is coded by $\begin{bmatrix} 161 \\ 145 \\ 062 \end{bmatrix}$? ______________________________

What word is coded by $\begin{bmatrix} 170 \\ 150 \\ 061 \end{bmatrix}$? ______________________________

What word is coded by $\begin{bmatrix} 113 \\ 109 \\ 045 \end{bmatrix}$? ______________________________

The message is:

___ ___ ___ ___ ___ ___ ___ ___ ___.

SECTION 3B

Ready To Go On? Enrichment

Linear Systems in Two Dimensions

The formula for the distance, d, between two points (x_1, y_1) and (x_2, y_2) in a two-dimensional coordinate plane is $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

There is a similar formula for the distance, d, between two points in three-dimensional space. If (x_1, y_1, z_1) and (x_2, y_2, z_2) are two points in three-dimensional space, then the formula for the distance, d, between the points is $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$. For example, the distance, d, between $(3, 5, -2)$ and $(6, -2, 4)$ is:

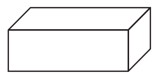
$$d = \sqrt{(3-6)^2 + (5-(-2))^2 + (-2-4)^2}$$
$$= \sqrt{(-3)^2 + (7)^2 + (-6)^2}$$
$$= \sqrt{9 + 49 + 36} = \sqrt{94} \text{ or about 9.7 units.}$$

Find the distance between each pair of points in three-dimensional space.

1. (1, 2, 3) and (4, 5, 6) ≈ 5.2
2. (0, 0, 0) and (2, 2, 2) ≈ 3.5
3. (5, 2, 6) and (5, 2, −9) 15
4. (3, 8, 1) and (3, −5, 1) 13
5. Look at Exercises 3 and 4. In each pair of points, how are the coordinates the same and how are they different? Can you relate that to the distance between each pair of points?

   Two coordinates are the same for each point. To find the distance, find the distance between the coordinates that are not the same.

6. A box is 12 centimeters long, 8 centimeters deep, and 3 centimeters tall. Calculate the length of the longest rod that can fit in the box.



   a. Use (0, 0, 0) as the coordinates of one corner of the box. What are the coordinates of the opposite corner of the box? (12, 8, 3) or (12, 3, 8)

   b. What is the length of the box from one corner to the opposite corner? $\approx$14.7 cm

7. What is the length of the longest rod that can fit in each box with the given dimensions?

   a. 15 in. by 12 in. by 8 in. $\approx$20.8 in.
   b. 20 cm by 10 cm by 2 cm $\approx$22.4 cm
   c. a cube with edge 20 cm $\approx$34.6 cm
   d. a cube with edge 24 in. $\approx$41.6 in.

SECTION 4A

Ready To Go On? Skills Intervention

4-1 Matrices and Data

Find these vocabulary words in Lesson 4-1 and the Multilingual Glossary.

Vocabulary				
matrix	dimensions	entry	address	scalar

Displaying Data in Matrix Form
Use the table of data to answer each question.

Plain	Color	With Borders
10	14	18
15	20	26
25	30	34

a. **Display the data in the form of a matrix, *C*.**

b. **What are the dimensions of Matrix *C*?**

c. **What is the entry at c_{13}? What does it represent?**

d. **What is the address of the entry that has the value 26?**

a) Find the value in the first column of the first row of the table. Write this value in the first column of the first row of the matrix. Do the same for the other values.

COLUMNS

$$C = \begin{bmatrix} 10 & 14 & 18 \\ 15 & 20 & 26 \\ 25 & 30 & 34 \end{bmatrix} \text{ ROWS}$$

b) How many rows are in the table? 3 How many columns? 3

Matrix C is a 3 (rows) × 3 (columns) matrix.

c) What does the 1 in c_{13} describe? Row 1 What does the 3 in c_{13} describe? Column 3 What is the value in c_{13}? 18

d) Find the value 26 in the matrix. What row is this value in? 2

What column is it in? 3 What is the address of 26? c_{32}

Finding Matrix Sums and Differences

Find *A* + *B*. $A = \begin{bmatrix} 3 & -1 \\ 0 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 5 & -2 \\ 2 & 0 \end{bmatrix}$

Add the values in B that correspond to the values in A.

Corresponding values: Row 1 Column 1 of *A* corresponds to Row 1 Column 1 of *B*.

$$\begin{bmatrix} 3+5 & -1+(-2) \\ 0+2 & 4+0 \end{bmatrix} = \begin{bmatrix} 8 & -3 \\ 2 & 4 \end{bmatrix}$$

SECTION 4A

Ready To Go On? Skills Intervention

4-2 Multiplying Matrices

Find these vocabulary words in Lesson 4-2 and the Multilingual Glossary.

Vocabulary			
matrix product	square matrix	main diagonal	multiplicative identity matrix

Multiplying Matrices
Tell whether each product is defined. If so, give its dimensions.

A. $P_{3\times2}$ and $Q_{1\times4}$; PQ

What are the inner dimensions? 2 and 1 Are they equal? No

Is the matrix product defined? No

B. $R_{2\times4}$ and $S_{4\times1}$; RS

What are the inner dimensions? 4 and 4 Are they equal? Yes

Is the matrix product defined? Yes What are the dimensions of RS? 2 × 1

Finding the Matrix Product
Find the product *AB*.

COLUMN 1

$A = \begin{bmatrix} 2 & 0 & -1 \\ 3 & 5 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 0 \\ -3 & 1 \\ 2 & 5 \end{bmatrix}$ $AB = \begin{bmatrix} 6 & -5 \\ 9 & 35 \end{bmatrix}$ ROW 1

What are the dimensions of A and B? A: 2 × 3 B: 3 × 2

What are the dimensions of the product AB? 2 × 2

Multiply row 1 of A by column 1 of B. $2(4) + 0(-3) + -1(2) =$ 6

Place the answer in row 1, column 1 of the product matrix.

Multiply row 1 of A by column 2 of B. 2(0) + 0(1) + −1(5) = −5

Place the answer in row 1, column 2 of the product matrix.

Multiply row 2 of A by column 1 of B. 2 of B. 3(4) + 5(−3) + 6(2) = 9

Place the answer in row 2, column 1 of the product matrix.

Multiply row 2 of A by column 2 of B. 3(0) + 5(1) + 6(5) = 35

Place the answer in row 2, column 2 of the product matrix.

SECTION 4A

Ready To Go On? Skills Intervention

4-3 Using Matrices to Transform Geometric Figures

Find these vocabulary words in Lesson 4-3 and the Multilingual Glossary.

Vocabulary		
translation matrix	reflection matrix	rotation matrix

Using Matrices to Translate a Figure
Translate △*ABC* with coordinates *A*(1, 1), *B*(−1, 2), and *C*(3, 4) two units left and three units up. Find the coordinates of the vertices of the image, and graph.

What are the x-coordinates of the vertices of △ABC? 1, −1, and 3

What are the y-coordinates of the vertices of △ABC? 1, 2, and 4

Write the x-coordinates in the first row of the matrix and the y-coordinates in the second row.

$$P = \begin{bmatrix} 1 & -1 & 3 \\ 1 & 2 & 4 \end{bmatrix} \begin{array}{l} \leftarrow x\text{-coordinates} \\ \leftarrow y\text{-coordinates} \end{array}$$

You are translating △ABC 2 units left and 3 units up. Therefore, each x-coordinate will move 2 units left and each y-coordinate 3 units up.

Complete the translation matrix, T.

$$T = \begin{bmatrix} -2 & -2 & -2 \\ 3 & 3 & 3 \end{bmatrix} \begin{array}{l} \leftarrow x\text{-coordinates} \\ \leftarrow y\text{-coordinates} \end{array}$$

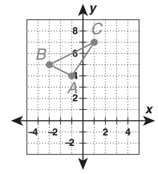
Add the matrices together to find the vertices of the translated image.

$$P + T = \begin{bmatrix} 1+(-2) & -1+-2 & 3+-2 \\ 1+3 & 2+3 & 4+3 \end{bmatrix}$$
$$= \begin{bmatrix} -1 & -3 & 1 \\ 4 & 5 & 7 \end{bmatrix}$$

What are the coordinates of the vertices of the translated triangle?

A' (−1, 4), B' (−3, 5), and C' (1, 7).

Graph the translated triangle on the grid.



SECTION **4A**

Ready To Go On? Quiz

4-1 Matrices and Data

Use the table for Exercises 1-4 .

Toddlers in Gymnastics Classes			
	Morning	Afternoon	Evening
Monday	8	6	12
Wednesday	6	4	10
Friday	11	15	19

1. Display the data in the form of a matrix, M.

$$M = \begin{bmatrix} 8 & 6 & 12 \\ 6 & 4 & 10 \\ 11 & 15 & 19 \end{bmatrix}$$

2. What are the dimensions of M? 3 × 3
3. What is the value of the matrix entry with the address m_{13}? 12
4. What is the address of the entry that has the value 4? m_{22}

Use the matrices below for Exercises 5–8. Evaluate if possible.

$$A = \begin{bmatrix} 2 & -5 \\ 0 & 3 \\ 1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \quad C = \begin{bmatrix} 3 & -2 \\ 1 & 3 \\ 4 & -1 \end{bmatrix} \quad D = \begin{bmatrix} 6 & 3 & -2 \\ -2.5 & 1 & -1 \end{bmatrix}$$

5. $A + C$ $\begin{bmatrix} 5 & -7 \\ 1 & 6 \\ 5 & -3 \end{bmatrix}$
6. $2B$ $\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$
7. $\frac{1}{2}C - D$ Not possible
8. $C - 3A$ $\begin{bmatrix} -3 & 13 \\ 1 & -6 \\ 1 & 5 \end{bmatrix}$

4-2 Multiplying Matrices

Use the matrices named below for Exercises 9-12 . Tell whether each product is defined. If so, give its dimensions.

$P_{3\times2}$ $Q_{2\times3}$ $R_{1\times3}$ $S_{3\times2}$

9. PQ Yes; 3 × 3
10. RS Yes; 1 × 2
11. QR No
12. SP No

SECTION **4A**

Ready To Go On? Quiz continued

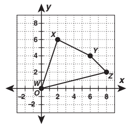
Use the matrices below for Exercises 13-16 . Evaluate, if possible.

$$E = \begin{bmatrix} 2 & -3 & -1 \\ -1 & 0 & 5 \\ 4 & 1 & 2 \end{bmatrix} \quad F = [-1 \quad 0.5 \quad 0.25] \quad G = \begin{bmatrix} 3 & -2 \\ 2 & 3 \end{bmatrix} \quad H = \begin{bmatrix} 1 & 0 \\ -2 & 4 \\ 4 & -8 \end{bmatrix}$$

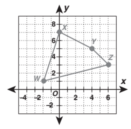
13. EF Not possible
14. FH $[-1 \quad 0]$
15. HG $\begin{bmatrix} 3 & -2 \\ 2 & 16 \\ -4 & -32 \end{bmatrix}$
16. G^2 $\begin{bmatrix} 5 & -12 \\ 12 & 5 \end{bmatrix}$

4-3 Using Matrices to Transform Geometric Figures

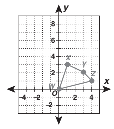
For Exercises 17-20, use polygon *WXYZ* with coordinates *W*(0, 0), *X*(2, 6), *Y*(6, 4), and *Z*(8, 2). Give the coordinates of the image and graph.



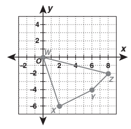
17. Translate polygon $WXYZ$ two units to the left and one unit up.



18. Reduce polygon $WXYZ$ by a factor of $\frac{1}{2}$.

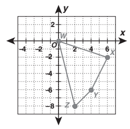


19. Use $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ to rotate polygon $WXYZ$. Describe the image.



The image is a reflection of the figure across the x-axis.

20. Use $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ to rotate polygon $WXYZ$. Describe the image.



The image is a rotation of the figure by 90° clockwise.

21. How does multiplying by $\begin{bmatrix} 0 & 3 \\ 3 & 0 \end{bmatrix}$ transform polygon $WXYZ$?

The polygon is reflected across the line $y = x$ and enlarged by a factor of 3.

SECTION **4A**

Ready To Go On? Enrichment

Matrices

Use matrix operations to determine Matrix *X*.

1. $2X + 2\begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 6 \\ 0 & -2 \end{bmatrix}$ $X = \begin{bmatrix} 0 & 5 \\ -3 & -2 \end{bmatrix}$

2. $X - 2\begin{bmatrix} 2 & 5 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 6 & -3 \\ 2 & 0 \end{bmatrix}$ $X = \begin{bmatrix} 10 & 7 \\ -4 & 8 \end{bmatrix}$

3. $3X - \begin{bmatrix} 3 & -6 & 0 \\ 7 & 1 & 2 \end{bmatrix} = 2\begin{bmatrix} 3 & 6 & 3 \\ -8 & 1 & 2 \end{bmatrix}$ $X = \begin{bmatrix} 3 & 2 & 2 \\ -3 & 1 & 2 \end{bmatrix}$

Solve for *x* and *y*.

4. $3[x \quad y] = [x \quad 2] + [6 \quad y]$ $x =$ 3 and $y =$ 1

5. $\begin{bmatrix} x+4 & 2 \\ 5 & -y \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 6 & -1 \end{bmatrix} = \begin{bmatrix} -2 & 5 \\ 11 & 9 \end{bmatrix}$ $x =$ −8 and $y =$ −10

6. $\begin{bmatrix} 2x+4 & -3 \\ 1 & x \end{bmatrix} + \begin{bmatrix} y & 10 \\ 4 & 2y \end{bmatrix} = \begin{bmatrix} 6 & 7 \\ 5 & -2 \end{bmatrix}$ $x =$ 2 and $y =$ −2

SECTION **4B**

Ready To Go On? Skills Intervention

4-4 Determinants and Cramer's Rule

Find these vocabulary words in Lesson 4-4 and the Multilingual Glossary.

Vocabulary		
determinant	coefficient matrix	Cramer's rule

Finding the Determinant of a Matrix

Find the determinant of each matrix.

A. $A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$

What is the product of the first diagonal? 4 × −1 = −4

What is the product of the second diagonal? 3 × 2 = 6

$det\ A$ = first diagonal − second diagonal = −4 − 6 = −10

B. $A = \begin{bmatrix} 2 & -3 & 1 \\ -1 & 4 & 0 \\ 3 & -2 & 4 \end{bmatrix}$ Write the matrix so that the first two columns are repeated to the right.

$$A = \begin{bmatrix} 2 & -3 & 1 \\ -1 & 4 & 0 \\ 3 & -2 & 4 \end{bmatrix} \begin{matrix} 2 & -3 \\ -1 & 4 \\ 3 & -2 \end{matrix}$$

Multiply the numbers in the first downward diagonal. 2(4)(4) = 32

Multiply the numbers in the second downward diagonal. −3(0)(3) = 0

Multiply the numbers in the third downward diagonal. 1(−1)(−2) = 2

Add the products of each diagonal. 32 + 0 + 2 = 34

Multiply the numbers in the first upward diagonal. 3(4)(1) = 12

Multiply the numbers in the second upward diagonal. −2(0)(2) = 0

Multiply the numbers in the third upward diagonal. 4(−1)(−3) = 12

Add the products of each diagonal. 12 + 0 + 12 = 24

$det\ A$ = sum of downward diagonals − sum of upward diagonal

= 34 − 24

= 10

SECTION 4B

Ready To Go On? Skills Intervention

4-5 Matrix Inverses and Solving Systems

Find these vocabulary words in Lesson 4-5 and the Multilingual Glossary.

Vocabulary			
multiplicative inverse matrix	matrix equation	variable matrix	constant matrix

Finding the Inverse of a 2 × 2 Matrix

Find the inverse of the matrix $A = \begin{bmatrix}1&3\\2&4\end{bmatrix}$, if it is defined.

Find the determinant. (1)(4) − 2(3) = −2

$$\begin{array}{ccc} & a & b \\ A = & \begin{bmatrix}1&3\\2&4\end{bmatrix} & \\ & c & d \end{array}$$

Does the matrix have an inverse? Yes

The inverse $A^{-1} = \frac{1}{\det A}\begin{bmatrix}d&-b\\-c&a\end{bmatrix}$.

Substitute values. $A^{-1} = \frac{1}{\boxed{-2}}\begin{bmatrix}4&\boxed{-3}\\-2&\boxed{1}\end{bmatrix}$

Multiply the fraction by each value in the matrix. $A^{-1} = \begin{bmatrix}-\frac{1}{2}(4) & -\frac{1}{2}\boxed{-3}\\ -\frac{1}{2}(-2) & -\frac{1}{2}\boxed{1}\end{bmatrix} = \begin{bmatrix}-2 & \boxed{\frac{3}{2}}\\ 1 & \boxed{-\frac{1}{2}}\end{bmatrix}$

Solving Systems Using Inverse Matrices

Write and solve the matrix equation for the system $\begin{cases}x + 3y = 9\\ 2x - y = -10\end{cases}$.

Write the coefficients in the first matrix.	Write the variables in a second matrix.	Write the constants in a third matrix.
A	***X***	**B**
$\begin{bmatrix}\boxed{1}&\boxed{3}\\2&-1\end{bmatrix}$	$\begin{bmatrix}\boxed{x}\\y\end{bmatrix}$	$\begin{bmatrix}9\\\boxed{-10}\end{bmatrix}$

det A = $\boxed{1}$(−1) − 2($\boxed{3}$) = $\boxed{-7}$

$A^{-1} = \frac{1}{\boxed{-7}}\begin{bmatrix}-1&\boxed{-3}\\-2&\boxed{1}\end{bmatrix} = \begin{bmatrix}\boxed{\frac{1}{7}}&\boxed{\frac{3}{7}}\\\boxed{\frac{2}{7}}&\boxed{-\frac{1}{7}}\end{bmatrix}$

$$\begin{array}{ccccc} X & & A^{-1} & B & \\ \begin{bmatrix}x\\y\end{bmatrix} & = & \begin{bmatrix}\boxed{\frac{1}{7}}&\boxed{\frac{3}{7}}\\\boxed{\frac{2}{7}}&\boxed{-\frac{1}{7}}\end{bmatrix} & \begin{bmatrix}9\\\boxed{-10}\end{bmatrix} & = \begin{bmatrix}\boxed{-3}\\\boxed{4}\end{bmatrix}\end{array}$$

Solve. Substitute the inverse and the constant matrices.

What is the solution? (−3 , 4)

SECTION 4B

Ready To Go On? Problem Solving Intervention

4-5 Matrix Inverses and Solving Systems

Matrix equations, $AX = B$ can be used to solve systems of equations; A is the coefficient matrix, B is the constant matrix and X is the variable matrix.

You are writing a proposal for buying and selling office equipment as a system of equations. Let x = the price of a laptop, y = the price of a printer, and z = the price of a fax machine. What is the price of each type of equipment?

$$\begin{cases}-y + 2z = 470\\ x + y - z = 430\\ 2x + 3z = 2410\end{cases}$$

Understand the Problem

1. Which equation represents selling a printer and buying two fax machines?
Equation 1

2. Which equation represents buying a laptop and a printer, and selling a fax machine? Equation 2

Make a Plan

3. Write a matrix to represent the coefficients of the system.
4. Write a matrix to represent the variables of the system.
5. Write a matrix to represent the constants of the system.

$A = \begin{bmatrix}0&-1&\boxed{2}\\\boxed{1}&\boxed{1}&-1\\\boxed{2}&0&\boxed{3}\end{bmatrix}$ $X = \begin{bmatrix}\boxed{x}\\y\\\boxed{z}\end{bmatrix}$ $B = \begin{bmatrix}470\\\boxed{430}\\\boxed{2140}\end{bmatrix}$

Solve

6. Write the matrix equation for the system as $AX = B$. $\begin{bmatrix}0&-1&\boxed{2}\\\boxed{1}&\boxed{1}&-1\\\boxed{2}&0&\boxed{3}\end{bmatrix}\begin{bmatrix}\boxed{x}\\y\\\boxed{z}\end{bmatrix} = \begin{bmatrix}470\\\boxed{430}\\\boxed{2140}\end{bmatrix}$

7. Use the inverse function on your graphing calculator to find A^{-1} and rewrite the equation to solve for X. $X = A^{-1}B = \begin{bmatrix}3&3&\boxed{-1}\\\boxed{-5}&\boxed{-4}&2\\\boxed{-2}&-2&\boxed{1}\end{bmatrix}\begin{bmatrix}470\\\boxed{430}\\\boxed{2140}\end{bmatrix} = \begin{bmatrix}\boxed{560}\\210\\\boxed{340}\end{bmatrix}$

8. How much is a laptop? $560 a printer? $210 a fax machine? $340

Look Back

9. Substitute the values you calculated into one of the equations in the system. Try
$x + y - z = 430$. (560) + (210) − (340) = 430 Do you yield the correct result? Yes

SECTION 4B

Ready To Go On? Skills Intervention

4-6 Row Operations and Augmented Matrices

Find these vocabulary words in Lesson 4-6 and the Multilingual Glossary.

Vocabulary			
augmented matrix	row operation	row reduction	reduced row-echelon form

Solving Systems with an Augmented Matrix

Write the augmented matrix, and solve. $\begin{cases}6x + y = 15\\ 3x + 2y = 12\end{cases}$

$$\begin{array}{ccc} x\text{-coefficients} & y\text{-coefficients} & \text{constants}\end{array}$$

$\left[\begin{array}{cc:c}6&1&15\\\boxed{3}&2&\boxed{12}\end{array}\right]$ Write the augmented matrix.

Multiply every value in row 2 by 2. $\left[\begin{array}{cc:c}6&1&15\\\boxed{6}&\boxed{4}&\boxed{24}\end{array}\right]$

Subtract row 1 from row 2. $\boxed{6} - 6 = \boxed{0}$, $\boxed{4} - 1 = 3$, $\boxed{24} - 15 = \boxed{9}$

Write the result in row 2. $\left[\begin{array}{cc:c}6&1&15\\\boxed{0}&3&\boxed{9}\end{array}\right]$

Multiply row 1 by 3. $\left[\begin{array}{cc:c}\boxed{18}&\boxed{3}&\boxed{45}\\0&3&9\end{array}\right]$

Subtract row 2 from row 1. $\boxed{18} - 0 = \boxed{18}$, $\boxed{3} - 3 = 0$, $\boxed{45} - 9 = \boxed{36}$

Write the result in row 1. $\left[\begin{array}{cc:c}\boxed{18}&0&\boxed{36}\\0&3&9\end{array}\right]$

Divide row 1 by 18 and row 2 by 3. $\left[\begin{array}{cc:c}1&0&\boxed{2}\\0&1&\boxed{3}\end{array}\right]$

The solution is x = 2 and y = 3 .

Check your answer by substituting the values of x and y into each of the original equations in the system.

$6x + y = 15$	$3x + 2y = 12$
6(2) + (3) = 15	3(2) + 2(3) = 12
15 = 15	12 = 12

SECTION 4B

Ready To Go On? Problem Solving Intervention

4-6 Row Operations and Augmented Matrices

The system of equations represents the costs of three baskets of art supplies. Use a to represent the cost of one box of pencils, b to represent the cost of one box of paints, and c to represent the cost of one bundle of brushes. Find the cost of each type of supply.

$$\begin{cases}2a + 2b + c + 2.20 = 10.65\\ 3a + 2b + 2c + 2.20 = 12.63\\ 4a + 3b + 2c + 2.20 = 16.11\end{cases}$$

Understand the Problem

1. What are the three types of art supplies? pencils, paints, brushes

2. Which equation represents 3 boxes of pencils, 2 boxes of paints, and 2 bundles of brushes? Equation 2

3. The cost of the basket into which the supplies are placed is included in each equation. What is the cost of each basket? $2.20

4. What information are you trying to find? The cost of each type of supply

Make a Plan

5. Rewrite the equations with the constants on one side.

$2a + 2b + c + 2.20 = 10.65$	$3a + 2b + 2c + 2.20 = 12.63$	$4a + 3b + 2c + 2.20 = 16.11$
$-2.20 \quad -2.20$	$-2.20 \quad -2.20$	$-2.20 \quad -2.20$
$2a + 2b + c = 8.45$	$3a + 2b + 2c = 10.43$	$4a + 3b + 2c = 13.91$

6. Use the equations to write an augmented matrix. $A^{-1} = \begin{bmatrix}\boxed{2}&2&1&8.45\\\boxed{3}&2&\boxed{2}&10.43\\4&\boxed{3}&2&\boxed{13.91}\end{bmatrix}$

Solve

7. Enter the 3 × 4 augmented matrix into your calculator as A.

8. Press [2nd] [x^{-1}] on your calculator. Then select [MATH]. Move down the list to B:rref. Find the reduced row-echelon form of the augmented matrix.

9. How much does one box of pencils cost? $0.49 one box of paints? $2.99 one bundle of brushes? $1.49

Look Back

10. Substitute the values for a, b, and c into one of the equations in the system.
2(0.49) + 2(2.99) + (1.49) + 2.20 = 10.65
Do your values yield the correct result? Yes

SECTION 4B

Ready To Go On? Quiz

4-4 Determinants and Cramer's Rule

Find the determinant of each matrix.

1. $\begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix}$ **12**

2. $\begin{bmatrix} \frac{1}{3} & 6 \\ 0 & \frac{3}{4} \end{bmatrix}$ **$\frac{1}{4}$**

3. $\begin{bmatrix} 0.25 & 1.5 \\ -2.5 & 8.0 \end{bmatrix}$ **5.75**

4. $\begin{bmatrix} -1 & 3 & 2 \\ 2 & -1 & 0 \\ 1 & 2 & -3 \end{bmatrix}$ **25**

Use Cramer's rule to solve.

5. $\begin{cases} y = 2x + 1 \\ 2x - 4y = 1 \end{cases}$ **$\left(\frac{-5}{6}, \frac{-2}{3}\right)$**

6. $\begin{cases} 2x - 2y = 2 \\ y - x - 1 = 0 \end{cases}$ **No solution**

7. $\begin{cases} 2x + 3y = 6 \\ y = 1 - x \end{cases}$ **(−3, 4)**

8. $\begin{cases} 2x - 2y + z = -5 \\ x + 2y = 3z + 3 \\ z = 2x + 1 \end{cases}$ **(0, 3, 1)**

4-5 Matrix Inverses and Solving Systems

Find the inverse of each matrix, if it is defined.

9. $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ **$\begin{bmatrix} \frac{4}{5} & -\frac{3}{5} \\ -\frac{1}{5} & \frac{2}{5} \end{bmatrix}$**

10. $\begin{bmatrix} 0.5 & 1.5 \\ -2 & 4 \end{bmatrix}$ **$\begin{bmatrix} 0.8 & -0.3 \\ 0.4 & 0.1 \end{bmatrix}$**

11. $\begin{bmatrix} -2 & 1 \\ -\frac{1}{2} & \frac{1}{4} \end{bmatrix}$ **Not defined**

12. $\begin{bmatrix} 0 & -1 & 2 \\ 1 & 1 & -1 \\ 2 & 0 & 3 \end{bmatrix}$ **$\begin{bmatrix} 3 & 3 & -1 \\ -5 & -4 & 2 \\ -2 & -2 & 1 \end{bmatrix}$**

Write the matrix equation for the system, and solve, if possible.

13. $\begin{cases} y = 2x - 4.5 \\ 5y - x = 0 \end{cases}$ $\begin{bmatrix} -2 & 1 \\ -1 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4.5 \\ 0 \end{bmatrix}$ **(2.5, 0.5)**

14. $\begin{cases} 3x - 4y = 5 \\ 2(x + y) + 6 = 0 \end{cases}$ $\begin{bmatrix} 3 & -4 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -6 \end{bmatrix}$ **(−1, −2)**

15. $\begin{cases} 5x + 4y = 19 \\ 10x + 8y = 12 \end{cases}$ **No solution**

16. $\begin{cases} 4x + 5y = 2z + 5 \\ 3x + 4y = z + 3 \\ x + 3y = 5z + 12 \end{cases}$ **[−1, 1, −2]**

SECTION 4B

Ready To Go On? Quiz continued

17. You are starting a business renting bouncy houses for children's parties. You are deciding which equipment to purchase. Use x as the price of a bouncy slide, y as the price of a bouncy castle, and z as the price of a bouncy maze. What is the price of each type of equipment?

$\begin{cases} 2x + y + 3z = 8000 \\ x + 3y + 2z = 7250 \\ 3x + 2y + z = 7250 \end{cases}$

slide = $ **1250**

castle = $ **1000**

maze = $ **1500**

4-6 Row Operations and Augmented Matrices

Write the augmented matrix, and use row reduction to solve, if possible.

18. $\begin{cases} 2x + 3y = 1 \\ x = y + 3 \end{cases}$ **$\left[\begin{array}{cc|c} 2 & 3 & 1 \\ 1 & -1 & 3 \end{array}\right]$; (2, −1)**

19. $\begin{cases} 3x + y = 8 \\ 10x - y = 5 \end{cases}$ **$\left[\begin{array}{cc|c} 3 & 1 & 8 \\ 10 & -1 & 5 \end{array}\right]$; (1, 5)**

20. $\begin{cases} 6x - 2y = 16 \\ 3x = 8 + y \end{cases}$ **The system is dependent.**

21. $\begin{cases} 4x + 2y - 5 = 0 \\ x - y = \frac{1}{2} \end{cases}$ **$\left(1, \frac{1}{2}\right)$**

22. The system of equations represents the cost of bakery bouquets. Use a to represent the cost of cookie flowers, b the cost of cupcake flowers, and c the cost of chocolate flowers. Find the cost of each type of flower.

$\begin{cases} 2a + b + 2c + 2.10 = 12.60 \\ 3a + 2b + c + 2.10 = 16.35 \\ 4a + 3b + 3c + 2.10 = 24.60 \end{cases}$

cookie flower = $ **2.25**

cupcake flower = $ **3.00**

chocolate flower = $ **1.50**

SECTION 4B

Ready To Go On? Enrichment

Inverse Matrices

Use the information provided to decode the three-word message made up of three-letter words encrypted in the matrices below.

$\begin{bmatrix} 161 \\ 145 \\ 062 \end{bmatrix}$ $\begin{bmatrix} 170 \\ 150 \\ 061 \end{bmatrix}$ $\begin{bmatrix} 113 \\ 109 \\ 045 \end{bmatrix}$

The words are encrypted by first assigning each letter of the alphabet a number. $A = 1$, $B = 2$, $C = 3$, and so on.

The three-letter words are multiplied by the matrix $M = \begin{bmatrix} 6 & 5 & 2 \\ 5 & 5 & 2 \\ 2 & 2 & 1 \end{bmatrix}$.

The message can be decoded by multiplying the coded message by the inverse of the encoding matrix.

What is the inverse matrix? $M^{-1} =$ **$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -2 \\ 0 & -2 & 5 \end{bmatrix}$**

What word is coded by $\begin{bmatrix} 161 \\ 145 \\ 062 \end{bmatrix}$? **$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -2 \\ 0 & -2 & 5 \end{bmatrix}\begin{bmatrix} 161 \\ 145 \\ 062 \end{bmatrix} = \begin{bmatrix} 16 \\ 5 \\ 20 \end{bmatrix} = \begin{bmatrix} P \\ E \\ T \end{bmatrix}$**

What word is coded by $\begin{bmatrix} 170 \\ 150 \\ 061 \end{bmatrix}$? **$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -2 \\ 0 & -2 & 5 \end{bmatrix}\begin{bmatrix} 170 \\ 150 \\ 061 \end{bmatrix} = \begin{bmatrix} 20 \\ 8 \\ 5 \end{bmatrix} = \begin{bmatrix} T \\ H \\ E \end{bmatrix}$**

What word is coded by $\begin{bmatrix} 113 \\ 109 \\ 045 \end{bmatrix}$? **$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -2 \\ 0 & -2 & 5 \end{bmatrix}\begin{bmatrix} 113 \\ 109 \\ 045 \end{bmatrix} = \begin{bmatrix} 4 \\ 15 \\ 7 \end{bmatrix} = \begin{bmatrix} D \\ O \\ G \end{bmatrix}$**

The message is:

P E T   T H E   D O G.

SECTION 5A

Ready To Go On? Skills Intervention

5-1 Using Transformations to Graph Quadratic Functions

Find these vocabulary words in Lesson 5-1 and the Multilingual Glossary.

Vocabulary			
quadratic function	parabola	vertex of a parabola	vertex form

Translating Quadratic Functions

Using the graph of $f(x)^2$ as a guide, describe the transformations, and then graph the function. $g(x) = (x + 3)^2 - 1$

$f(x - h) = (x - h)^2$ represents the general form for a horizontal shift. If $h < 0$ the graph moves left and if $h > 0$ the graph moves **right**.

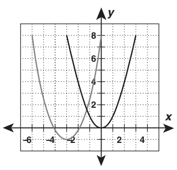
$f(x) + k = x^2 + k$ represents the general form for a vertical shift. If k is negative the graph is shifted down and if k is positive the graph is shifted **up**.

$g(x) = (x + 3)^2 - 1$

$= (x - (-3))^2 - 1$ Rewrite to identify h and k.

Because $h =$ **−3**, the graph is translated 3 units left and since $k = 1$, the graph is translated **1** unit up.

Complete the table of values and graph.



x	$f(x) = (x + 3)^2 - 1$	$(x, f(x))$
−5	$f(-5) = (-5 + 3)^2 - 1 = 3$	(−5, 3)
−4	$f(-4) = (-4 + 3)^2 - 1 =$ **0**	(−4, **0**)
−3	$f(-3) = (-3 + 3)^2 - 1 =$ **−1**	(−3, **−1**)
−2	$f(-2) = ($**−2** $+ 3)^2 - 1 =$ **0**	(−2, **0**)
−1	$f(-1) = ($**−1** $+ 3)^2 - 1 =$ **3**	(−1, **3**)

Writing Transformed Quadratic Functions

Use the description to write the quadratic function in vertex form: $f(x)^2$ is vertically stretched by a factor of 3 and translated 4 units left.

The **vertex** form of a quadratic function is $f(x) = a(x - h)^2 + k$.

The a indicates a **reflection** across the x-axis and/or a vertical **stretch** or compression. The h represents a **horizontal** translation and **k** indicates a vertical translation. Vertical stretch by 3: means **a** = 3. Translated 4 units left means $h = -$**4**.

Substitute to write the transformed function. $g(x) = a(x - h)^2 + k$

$g(x) =$ **3** $(x -$ **−4** $)^2 + 0$

$g(x) =$ **3** $(x +$ **4** $)^2$