Name _____ Date _____ Class _____

LESSON 2-8

Reteach

Solving Absolute-Value Equations and Inequalities

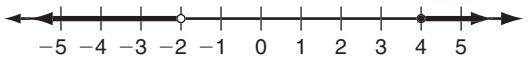
To solve compound inequalities, solve both inequalities. Then graph.

Solve $x + 6 < 4$ or $2x \geq 8$.

$x + 6 < 4$ OR $2x \geq 8$

$x < -2$ OR $x \geq 4$

This inequality uses OR. Its graph has two parts.

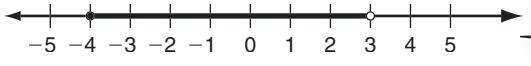


Solve $x - 2 < 1$ and $-3x \leq 12$.

$x - 2 < 1$ AND $-3x \leq 12$

$x < 3$ AND $x \geq -4$

Reverse the inequality when dividing by a negative number.

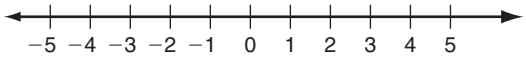


This inequality uses AND. Its graph has one part.

Solve and graph each compound inequality.

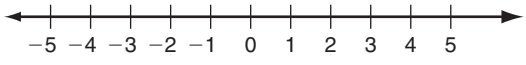
1. $x + 3 < 2$ or $\frac{1}{2}x > 1$

$x <$ ____ OR $x >$ ____



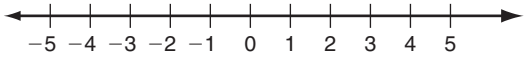
2. $-6x \leq 18$ and $x + 6 \leq 6$

$x \geq$ ____ AND $x \leq$ ____



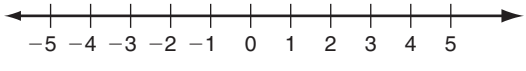
3. $x - 4 < -7$ or $-4x \leq 4$

_____ OR _____



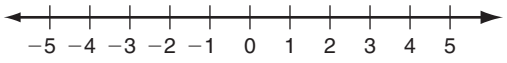
4. $-3x \leq 6$ and $x + 2 < 5$

_____ AND _____



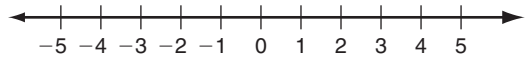
5. $3x < 12$ and $-3x < 12$

_____



6. $\frac{1}{2}x - 2 \leq 0$ or $2 - \frac{1}{2}x \leq -1$

_____



Name _____ Date _____ Class _____

LESSON 2-8

Reteach

Solving Absolute-Value Equations and Inequalities (continued)

Solving absolute-value inequalities is like solving compound inequalities.

Solve: $\|x\| < 2$ Solution: $-2 < x < 2$	Solve: $\|x\| \leq 2$ Solution: $-2 \leq x \leq 2$
Solve: $\|x\| > 2$ Solution: $x < -2$ OR $x > 2$	Solve: $\|x\| \geq 2$ Solution: $x \leq -2$ OR $x \geq 2$

Remember:
$|x| = x$ if $x \geq 0$

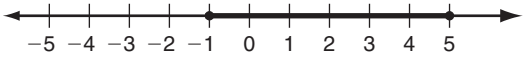
Solve $|x - 2| \leq 3$.

Use the solutions from the table to write the inequalities

$$-3 \leq x - 2 \leq 3$$

$$-3 + 2 \leq x - 2 + 2 \leq 3 + 2 \quad \textit{Add 2.}$$

$$-1 \leq x \leq 5 \quad \textit{Simplify.}$$

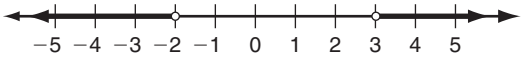


Solve $|2x - 1| > 5$.

$$2x - 1 > 5 \quad \text{OR} \quad 2x - 1 < -5$$

$$2x > 6 \quad \text{OR} \quad 2x < -4 \quad \textit{Add 1.}$$

$$x > 3 \quad \text{OR} \quad x < -2 \quad \textit{Divide by 2.}$$

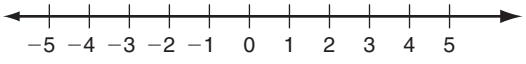


Solve and graph.

5. $|x + 3| < 2$

____ $< x + 3 <$ ____

____ $< x <$ ____

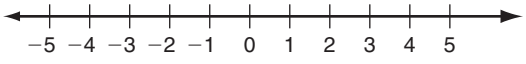


6. $|2x + 1| \geq 3$

$2x + 1 \geq$ ____ OR $2x + 1 \leq$ ____

$2x \geq$ ____ OR $2x \leq$ ____

$x \geq$ ____ OR $x \leq$ ____



LESSON 2-8

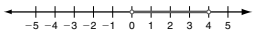
Practice A

Solving Absolute-Value Equations and Inequalities

Solve each compound inequality. Then graph the solution.

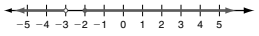
1. $2x < 8$ and $x + 3 > 3$

   $x < 4$ and $x > 0$



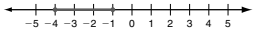
2. $x - 13 \geq -15$ or $4x < -12$

   $x \geq -2$ or $x < -3$



3. $4x \geq -16$ and $x + 1 \leq 0$

   $x \geq -4$ and $x \leq -1$



Solve each equation.

4. $|3x| = 36$

   $3x = 36$ or $3x = -36$

   $x = 12$ or $x = -12$

5. $|x| - 7 = -1$

   $x = 6$ or $x = -6$

6. $|8x| - 13 = 11$

   $x = 3$ or $x = -3$

Determine whether each inequality is a conjunction or a disjunction and whether you would use *and* or *or*.

7. $|4x| + 10 > 30$

   Disjunction, or

8. $|5x + 11| < 21$

   Conjunction, and

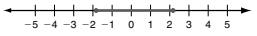
9. $3|x - 1| \geq 6$

   Disjunction, or

Solve each inequality. Then graph the solution.

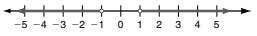
10. $\frac{|3x - 1|}{2} \leq 3$

    $-\frac{5}{3} \leq x \leq \frac{7}{3}$



11. $5|2x| > 10$

    $x < -1$ or $x > 1$



12. Phil told his friend that if you subtract 12 from his age and then take the absolute value, you'll get an answer of 3. How old is Phil?

    9 or 15

LESSON 2-8

Practice B

Solving Absolute-Value Equations and Inequalities

Solve each equation.

1. $|2x + 1| = 7$

   $x = 3$ or $x = -4$

2. $|-7x| = 28$

   $x = \pm 4$

3. $3|3x| - 7 = 2$

   $x = \pm 1$

4. $|2x - 5| = 5$

5. $2|x + 1| = 14$

   $x = 6$ or $x = -8$

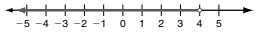
6. $|4 - x| + 2 = 9$

   $x = -3$ or $x = 11$

Solve each inequality or compound inequality. Then graph the solution.

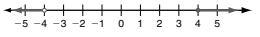
7. $-4x + 2 > -10$ and $5x - 12 < 8$

   $x < 4$



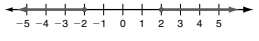
8. $3x - 4 \geq 8$ or $-x + 12 > 16$

   $x \geq 4$ or $x < -4$



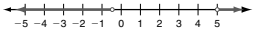
9. $|9x| \geq 18$

   $x \leq -2$ or $x \geq 2$



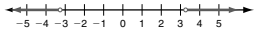
10. $|3x - 7| > 8$

    $x < -\frac{1}{3}$ or $x > 5$



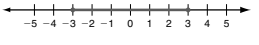
11. $|0.3x| > 1$

    $x < -\frac{10}{3}$ or $x > \frac{10}{3}$



12. $|7x| - 12 \leq 9$

    $x \geq -3$ and $x \leq 3$



Solve.

13. Any measurement is accurate within ± 0.5 of the measurement unit. For example, if you measure your pencil to the nearest inch, your measurement could be 0.5 inch too long or 0.5 inch too short. Write an absolute-value inequality that shows the maximum and minimum actual measure of a nail measured to be 4.4 centimeters to the nearest 0.1 centimeter.

    $|m - 4.4| \leq 0.05$

LESSON 2-8

Practice C

Solving Absolute-Value Equations and Inequalities

Solve each equation.

1. $|2x - 3| = 15$

   $x = -6$ or $x = 9$

2. $\frac{1}{2}|x + 9| = 1$

   $x = -11$ or $x = -7$

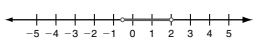
3. $11 - |4 - x| = 4$

   $x = -3$ or $x = 11$

Solve and graph.

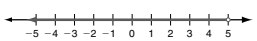
4. $5(7 - 2x) < 40$ and $5x + 2 < 12$

   $\frac{-1}{2} < x < 2$



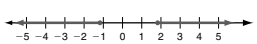
5. $\frac{7x - 10}{6} \leq 3$ or $3x + 2 > 5x - 8$

   $x < 5$



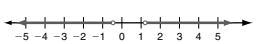
6. $\left|\frac{4x - 1}{6}\right| \geq 1$

   $x \leq -\frac{5}{4}$ or $x \geq \frac{7}{4}$



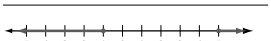
7. $-3|5x - 2| < -12$

   $x < -\frac{2}{5}$ or $x > \frac{6}{5}$



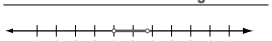
8. $2|3x - 6| + 6 \geq 24$

   $x \leq -1$ or $x \geq 5$



9. $\frac{|9x + 1|}{4} < 2$

   $x > -1$ and $x < \frac{7}{9}$



Solve.

10. Ben says that there is no solution for this absolute-value inequality. Is he correct? If not, solve the inequality. Explain how you know you are correct.

    $32 + \frac{|x - 7|}{13} < 7$

    Possible answer: Ben is correct. There is no solution. When the inequality is simplified, the result is an inequality that sets the absolute value of an expression less than a negative number. Since absolute values are always positive, this is never true.

LESSON 2-8

Reteach

Solving Absolute-Value Equations and Inequalities

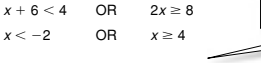
To solve compound inequalities, solve both inequalities. Then graph.

Solve $x + 6 < 4$ or $2x \geq 8$.

$x + 6 < 4$ OR $2x \geq 8$

$x < -2$ OR $x \geq 4$

This inequality uses OR. Its graph has two parts.

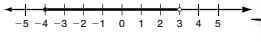


Solve $x - 2 < 1$ and $-3x \leq 12$.

$x - 2 < 1$ AND $-3x \leq 12$

$x < 3$ AND $x \geq -4$

Reverse the inequality when dividing by a negative number.

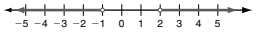


This inequality uses AND. Its graph has one part.

Solve and graph each compound inequality.

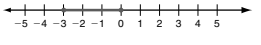
1. $x + 3 < 2$ or $\frac{1}{2}x > 1$

   $x < -1$ OR $x > 2$



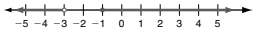
2. $-6x \leq 18$ and $x + 6 \leq 6$

   $x \geq -3$ AND $x \leq 0$



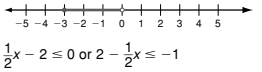
3. $x - 4 < -7$ or $-4x \leq 4$

   $x < -3$ OR $x \geq -1$



4. $-3x \leq 6$ and $x + 2 < 5$

   $x \geq -2$ AND $x < 3$



5. $3x < 12$ and $-3x < 12$

   $x < 4$ and $x > -4$

6. $\frac{1}{2}x - 2 \leq 0$ or $2 - \frac{1}{2}x \leq -1$

   $x \leq 4$ or $x \geq 5$

LESSON 2-8

Reteach

Solving Absolute-Value Equations and Inequalities (continued)

Solving absolute-value inequalities is like solving compound inequalities.

Solve: $\|x\| < 2$ Solution: $-2 < x < 2$	Solve: $\|x\| \le 2$ Solution: $-2 \le x \le 2$
Solve: $\|x\| > 2$ Solution: $x < -2$ OR $x > 2$	Solve: $\|x\| \ge 2$ Solution: $x \le -2$ OR $x \ge 2$

Remember: $|x| = x$ if $x \ge 0$

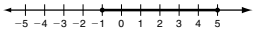
Solve $|x - 2| \le 3$.

Use the solutions from the table to write the inequalities

$-3 \le x - 2 \le 3$

$-3 + 2 \le x - 2 + 2 \le 3 + 2$ *Add 2.*

$-1 \le x \le 5$ *Simplify.*

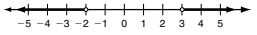


Solve $|2x - 1| > 5$.

$2x - 1 > 5$ OR $2x - 1 < -5$

$2x > 6$ OR $2x < -4$ *Add 1.*

$x > 3$ OR $x < -2$ *Divide by 2.*

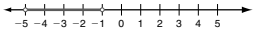


Solve and graph.

5. $|x + 3| < 2$

$\underline{-2} < x + 3 < \underline{2}$

$\underline{-5} < x < \underline{-1}$

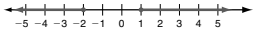


6. $|2x + 1| \ge 3$

$2x + 1 \ge \underline{3}$ OR $2x + 1 \le \underline{-3}$

$2x \ge \underline{2}$ OR $2x \le \underline{-4}$

$x \ge \underline{1}$ OR $x \le \underline{-2}$



LESSON 2-8

Challenge

Relating the Length of a Solution Interval to a Coefficient

Changing the value of a coefficient in an absolute-value linear inequality results in a change in the solution interval.

Solve.

1. $|ax + b| \le c$, where $a > 0$ and $c > 0$.

a. Solve the inequality for x in terms of a, b, and c. $\frac{-c - b}{a} \le x \le \frac{c - b}{a}$

b. Verify that your solution is equivalent to $\frac{-(b + c)}{a} \le x \le \frac{c - b}{a}$.

Possible answer: The solution of the absolute-value inequality gives $x \le \frac{c - b}{a}$ and $x \ge \frac{-c - b}{a}$. Read the second inequality from right to left and combine the two inequalities into a single inequality.

Apply the general solution to solve each inequality.

2. $|2x + 3| \le 5$ $-4 \le x \le 1$

3. $|4x + 3| \le 5$ $-2 \le x \le \frac{1}{2}$

Refer to the inequalities in Exercises 2 and 3.

4. a. Compare the values of a, b, and c in the two inequalities.

The values of b and c are the same in both inequalities. The value of a has increased from the first inequality to the second.

b. How does the value of a affect the length of the solution interval?

As a increases, the length of the solution interval decreases.

c. Predict the solution interval for the inequality $|8x + 3| \le 5$. $-1 \le x \le \frac{1}{4}$

d. Use the general solution to determine if your prediction was correct.

$\frac{-5 - 3}{8} \le x \le \frac{5 - 3}{8} = \frac{-8}{8} \le x \le \frac{2}{8} = -1 \le x \le \frac{1}{4}$

e. What is the relationship between the solution interval and the coefficient of x in this absolute-value inequality?

Possible answer: When the coefficient of x is doubled, the solution interval is reduced by $\frac{1}{2}$ of the units.

Solve.

5. a. Use the general solution to solve $|3x - 6| \le 21$. $\frac{-21 + 6}{3} = -5 \le x \le \frac{21 + 6}{3} = 9$

b. Predict the solution interval of $|6x - 6| \le 21$. $-2.5 \le x \le 4.5$

c. Predict the solution interval of $|12x - 6| \le 21$. $-1.25 \le x \le 2.25$

LESSON 2-8

Problem Solving

Solving Absolute-Value Equations and Inequalities

Gita's science class is making a set of posters about North American wildlife. The table shows some of the data collected.

North American Wildlife		
Weight Groups (kg)	**Animal**	**Daily Food Requirement (kg)**
W_1 135–450	Grizzly bear	10.5
	Polar bear	9.9
	Black bear	3.9
W_2 10–90	Mule deer	2.8
	Arctic wolf	2.3
	River otter	0.8
W_3 3–8	Nutria	0.38
	Opossum	0.19
	Rabbit	0.18

1. What is the center of each weight group?

a. W_1 292.5

b. W_2 50

c. W_3 5.5

2. Express each weight group as an absolute-value expression.

a. W_1 $|W_1 - 292.5| \le 157.5$

b. W_2 $|W_2 - 50| \le 40$

c. W_3 $|W_3 - 5.5| \le 2.5$

3. Write inequalities to show the amount of food required each day for animals in each weight group.

a. W_1 $f \ge 3.9$ and $f \le 10.5$

b. W_2 $f \ge 0.8$ and $f \le 2.8$

c. W_3 $f \ge 0.18$ and $f \le 0.38$

4. Gita wants to use the term *disjunction* or *conjunction* on her poster showing the inequalities. Which term should she use? Why?

Conjunction; Possible answer: the compound statement uses the term *and*.

5. Les includes the following on his poster:
Solve this equation to find the number of kilograms of food consumed each day by an animal in one of the weight groups:

$|f - 7.2| \le 3.3$.

Find the solution.

$3.9 \le f \le 10.5$

6. Write an absolute-value inequality to represent the maximum weight difference between a grizzly bear, g, and a black bear, b.

$|g - b| \le 315$

LESSON 2-8

Reading Strategies

Understand Vocabulary

Equations and inequalities can be combined to make compound statements. **Disjunctions** and **conjunctions** are two types of compound statements.

Compound Statement	Definition and Symbol	Example
Disjunction	Two statements joined by the word *or*	$x > 1$ or $x \le -2$
	Symbol: $\cup$	
Conjunction	Two statements joined by the word *and*	$x > 0$ and $x \le 6$
	Symbol: $\cap$	

Answer each question.

1. $x > 1$ or $x \le 2$

a. Is the compound statement true for $x = 6$? Explain.

Yes; since $x = 6$ makes the first inequality in the disjunction true, the compound statement is also true.

b. Is the compound statement true for $x = 0$? Explain.

No; $x = 0$ makes both inequalities false, so the compound statement is also false.

c. For which values of x is the disjunction false?

$-2 < x \le 1$; all x-values within this range make both inequalities false.

2. $x > 0$ and $x \le 6$

a. Describe the values of x for which the conjunction is true.

The conjunction is true for all numbers greater than 0 and less than or equal to 6.

b. Describe the values of x for which the conjunction is false?

The conjunction is false for all numbers less than or equal to 0 and all numbers greater than 6.

3. $|x| > 5$

a. Describe in words the values of x for which the inequality is true. Then write a compound statement for those values of x.

All number greater than 5 or all numbers less than -5; $x > 5$ or $x < -5$

b. Write a compound statement to show all the values of x for which the inequality is false.

$x \ge -5$ and $x \le 5$