Name ______________________ Date __________ Class __________

LESSON 2-8

Challenge

Relating the Length of a Solution Interval to a Coefficient

Changing the value of a coefficient in an absolute-value linear inequality results in a change in the solution interval.

Solve.

1. $|ax + b| \le c$, where $a > 0$ and $c > 0$.

a. Solve the inequality for x in terms of a, b, and c. ______________________

b. Verify that your solution is equivalent to $\frac{-(b + c)}{a} \le x \le \frac{c - b}{a}$.

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Apply the general solution to solve each inequality.

2. $|2x + 3| \le 5$ ______________________ **3.** $|4x + 3| \le 5$ ______________________

Refer to the inequalities in Exercises 2 and 3.

4. a. Compare the values of a, b, and c in the two inequalities.

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b. How does the value of a affect the length of the solution interval?

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c. Predict the solution interval for the inequality $|8x + 3| \le 5$. ______________________

d. Use the general solution to determine if your prediction was correct.

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e. What is the relationship between the solution interval and the coefficient of x in this absolute-value inequality?

____

Solve.

5. a. Use the general solution to solve $|3x - 6| \le 21$. ______________________

b. Predict the solution interval of $|6x - 6| \le 21$. ______________________

c. Predict the solution interval of $|12x - 6| \le 21$. ______________________

LESSON **2-8**

Reteach

Solving Absolute-Value Equations and Inequalities (continued)

Solving absolute-value inequalities is like solving compound inequalities.

Solve: $\lvert x\rvert < 2$ Solution: $-2 < x < 2$	Solve: $\lvert x\rvert \le 2$ Solution: $-2 \le x \le 2$
Solve: $\lvert x\rvert > 2$ Solution: $x < -2$ OR $x > 2$	Solve: $\lvert x\rvert \ge 2$ Solution: $x \le -2$ OR $x \ge 2$

Remember: $\lvert x\rvert = x$ if $x \ge 0$

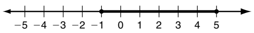
Solve $\lvert x - 2\rvert \le 3$.

Use the solutions from the table to write the inequalities

$-3 \le x - 2 \le 3$

$-3 + 2 \le x - 2 + 2 \le 3 + 2$ *Add 2.*

$-1 \le x \le 5$ *Simplify.*

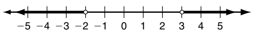


Solve $\lvert 2x - 1\rvert > 5$.

$2x - 1 > 5$ OR $2x - 1 < -5$

$2x > 6$ OR $2x < -4$ *Add 1.*

$x > 3$ OR $x < -2$ *Divide by 2.*

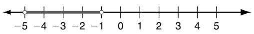


Solve and graph.

5. $\lvert x + 3\rvert < 2$

$\underline{-2} < x + 3 < \underline{2}$

$\underline{-5} < x < \underline{-1}$

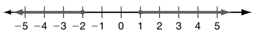


6. $\lvert 2x + 1\rvert \ge 3$

$2x + 1 \ge \underline{3}$ OR $2x + 1 \le \underline{-3}$

$2x \ge \underline{2}$ OR $2x \le \underline{-4}$

$x \ge \underline{1}$ OR $x \le \underline{-2}$



LESSON **2-8**

Challenge

Relating the Length of a Solution Interval to a Coefficient

Changing the value of a coefficient in an absolute-value linear inequality results in a change in the solution interval.

Solve.

1. $\lvert ax + b\rvert \le c$, where $a > 0$ and $c > 0$.

   a. Solve the inequality for x in terms of a, b, and c. $\frac{-c-b}{a} \le x \le \frac{c-b}{a}$

   b. Verify that your solution is equivalent to $\frac{-(b+c)}{a} \le x \le \frac{c-b}{a}$.

   Possible answer: The solution of the absolute-value inequality gives $x \le \frac{c-b}{a}$ and $x \ge \frac{-c-b}{a}$. Read the second inequality from right to left and combine the two inequalities into a single inequality.

Apply the general solution to solve each inequality.

2. $\lvert 2x + 3\rvert \le 5$ $-4 \le x \le 1$

3. $\lvert 4x + 3\rvert \le 5$ $-2 \le x \le \frac{1}{2}$

Refer to the inequalities in Exercises 2 and 3.

4. a. Compare the values of a, b, and c in the two inequalities.

   The values of b and c are the same in both inequalities. The value of a has increased from the first inequality to the second.

   b. How does the value of a affect the length of the solution interval?

   As a increases, the length of the solution interval decreases.

   c. Predict the solution interval for the inequality $\lvert 8x + 3\rvert \le 5$. $-1 \le x \le \frac{1}{4}$

   d. Use the general solution to determine if your prediction was correct.

   $\frac{-5-3}{8} \le x \le \frac{5-3}{8} = \frac{-8}{8} \le x \le \frac{2}{8} = -1 \le x \le \frac{1}{4}$

   e. What is the relationship between the solution interval and the coefficient of x in this absolute-value inequality?

   Possible answer: When the coefficient of x is doubled, the solution interval is reduced by $\frac{1}{2}$ of the units.

Solve.

5. a. Use the general solution to solve $\lvert 3x - 6\rvert \le 21$. $\frac{-21+6}{3} = -5 \le x \le \frac{21+6}{3} = 9$

   b. Predict the solution interval of $\lvert 6x - 6\rvert \le 21$. $-2.5 \le x \le 4.5$

   c. Predict the solution interval of $\lvert 12x - 6\rvert \le 21$. $-1.25 \le x \le 2.25$

LESSON **2-8**

Problem Solving

Solving Absolute-Value Equations and Inequalities

Gita's science class is making a set of posters about North American wildlife. The table shows some of the data collected.

North American Wildlife		
Weight Groups (kg)	**Animal**	**Daily Food Requirement (kg)**
W_1 135–450	Grizzly bear	10.5
	Polar bear	9.9
	Black bear	3.9
W_2 10–90	Mule deer	2.8
	Arctic wolf	2.3
	River otter	0.8
W_3 3–8	Nutria	0.38
	Opossum	0.19
	Rabbit	0.18

1. What is the center of each weight group?

   a. W_1 292.5

   b. W_2 50

   c. W_3 5.5

2. Express each weight group as an absolute-value expression.

   a. W_1 $\lvert W_1 - 292.5\rvert \le 157.5$

   b. W_2 $\lvert W_2 - 50\rvert \le 40$

   c. W_3 $\lvert W_3 - 5.5\rvert \le 2.5$

3. Write inequalities to show the amount of food required each day for animals in each weight group.

   a. W_1 $f \ge 3.9$ and $f \le 10.5$

   b. W_2 $f \ge 0.8$ and $f \le 2.8$

   c. W_3 $f \ge 0.18$ and $f \le 0.38$

4. Gita wants to use the term *disjunction* or *conjunction* on her poster showing the inequalities. Which term should she use? Why?

   Conjunction; Possible answer: the compound statement uses the term *and*.

5. Les includes the following on his poster:
   Solve this equation to find the number of kilograms of food consumed each day by an animal in one of the weight groups:

   $\lvert f - 7.2\rvert \le 3.3$.

   Find the solution.

   $3.9 \le f \le 10.5$

6. Write an absolute-value inequality to represent the maximum weight difference between a grizzly bear, g, and a black bear, b.

   $\lvert g - b\rvert \le 315$

LESSON **2-8**

Reading Strategies

Understand Vocabulary

Equations and inequalities can be combined to make compound statements. **Disjunctions** and **conjunctions** are two types of compound statements.

Compound Statement	Definition and Symbol	Example
Disjunction	Two statements joined by the word *or*	$x > 1$ or $x \le -2$
	Symbol: ∪	
Conjunction	Two statements joined by the word *and*	$x > 0$ and $x \le 6$
	Symbol: ∩	

Answer each question.

1. $x > 1$ or $x \le 2$

   a. Is the compound statement true for $x = 6$? Explain.

   Yes; since $x = 6$ makes the first inequality in the disjunction true, the compound statement is also true.

   b. Is the compound statement true for $x = 0$? Explain.

   No; $x = 0$ makes both inequalities false, so the compound statement is also false.

   c. For which values of x is the disjunction false?

   $-2 < x \le 1$; all x-values within this range make both inequalities false.

2. $x > 0$ and $x \le 6$

   a. Describe the values of x for which the conjunction is true.

   The conjunction is true for all numbers greater than 0 and less than or equal to 6.

   b. Describe the values of x for which the conjunction is false?

   The conjunction is false for all numbers less than or equal to 0 and all numbers greater than 6.

3. $\lvert x\rvert > 5$

   a. Describe in words the values of x for which the inequality is true. Then write a compound statement for those values of x.

   All number greater than 5 or all numbers less than -5; $x > 5$ or $x < -5$

   b. Write a compound statement to show all the values of x for which the inequality is false.

   $x \ge -5$ and $x \le 5$