2-8 Solving Absolute-Value Equations and Inequalities

Example 1 Solving Compound Inequalities

Solve each compound inequality. Then graph the solution set.

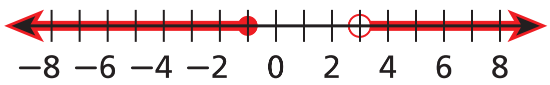
A. $6y < -24$ OR $y + 5 \geq 3$

Solve both inequalities for y.

$6y < -24$ or $y + 5 \geq 3$

$y < -4$ $\qquad$ $y \geq -2$

The solution set is all points that satisfy $\{y \mid y < -4 \textit{ or } y \geq -2\}$.



$(-\infty, -4) \cup [-2, \infty)$

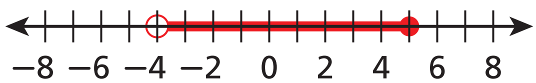
B. $\frac{1}{2}c \geq -2$ AND $2c + 1 < 1$

Solve both inequalities for c.

$\frac{1}{2}c \geq -2$ *and* $2c + 1 < 1$

$c \geq -4$ $\qquad$ $c < 0$

The solution set is the set of points that satisfy both $c \geq -4$ and $c < 0$.



$[-4, 0)$

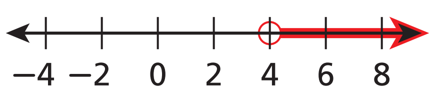
C. $x - 5 < -2$ OR $-2x \leq -10$

Solve both inequalities for x.

$x - 5 < -2$ *or* $-2x \leq -10$

$x < 3$ $\qquad$ $x \geq 5$

The solution set is all points that satisfy $\{x \mid x < 3 \textit{ or } x \geq 5\}$.



$(-\infty, 3) \cup [5, \infty)$

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Example 2 Solving Absolute-Value Equations

Solve each equation.

A. $|-3 + k| = 10$

This can be read as "the distance from k to −3 is 10."

$-3 + k = 10$ or $-3 + k = -10$

Rewrite the absolute value as a disjunction.

$k = 13$ or $k = -7$

Add 3 to both sides of each equation.

B. $\left|\frac{x}{4}\right| - 6 = -2$

$\left|\frac{x}{4}\right| = 4$

Isolate the absolute-value expression.

$\frac{x}{4} = 4$ or $\frac{x}{4} = -4$

Rewrite the absolute value as a disjunction.

$x = 16$ or $x = -16$

Multiply both sides of each equation by 4.

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Example 3 Solving Absolute-Value Inequalities with Disjunctions

Solve each inequality. Then graph the solutions set.

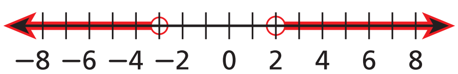
A. $|-4q + 2| \geq 10$

$-4q + 2 \geq 10$ *or* $-4q + 2 \leq -10$ — *Rewrite the absolute value as a disjunction.*

$-4q \geq 8$ or $-4q \leq -12$ — *Subtract 2 from both sides of each inequality.*

$q \leq -2$ or $q \geq 3$ — *Divide both sides of each inequality by -4 and reverse the inequality symbols.*

$\{q \mid q \leq -2 \text{ or } q \geq 3\}$



$(-\infty, -2) \cup (-3, \infty)$

To check, you can test a point in each of the three regions.

$|-4(-3) + 2| \geq 10$ $\quad |-4(0) + 2| \geq 10$ $\quad |-4(4) + 2| \geq 10$

$|14| \geq 10$ ✔ $\quad |2| \geq 10$ ✖ $\quad |-14| \geq 10$ ✔

B. $|0.5r| - 3 \geq -3$

$|0.5r| \geq 0$ — *Isolate the absolute-value expression.*

$0.5r \geq 0$ or $0.5r \leq 0$ — *Rewrite the absolute value as a disjunction.*

$r \geq 0$ or $r \leq 0$ — *Divide both sides of each inequality by 0.5.*

$(-\infty, \infty)$

The solution set is *all real numbers*, $\mathbb{R}$.

ADDITIONAL EXAMPLES

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Example 4 Solving Absolute-Value Inequalities with Conjunctions

Solve each inequality. Then graph the solution set.

A. $\dfrac{|2x + 7|}{3} \leq 1$

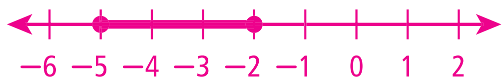
$|2x + 7| \leq 3$ *Multiply both sides by 3.*

$2x + 7 \leq 3$ and $2x + 7 \geq -3$ *Rewrite the absolute value as a conjunction.*

$2x \leq -4$ and $2x \geq -10$ *Subtract 7 from both sides of each inequality.*

$x \leq -2$ and $x \geq -5$ *Divide both sides of each inequality by 2.*

The solution set is $\{x | -5 \leq x \leq -2\}$.



B. $-\frac{1}{2}|p - 2| \geq 3$

$|p - 2| \leq -6$ *Multiply both sides by −2, and reverse the inequality symbol.*

$p - 2 \leq -6$ and $p - 2 \geq 6$ *Rewrite the absolute value as a conjunction.*

$p \leq -4$ and $p \geq 8$ *Add 2 to both sides of each inequality.*

Because no real number satisfies both $p \leq -4$ and $p \geq 8$, there is *no solution*. The solution set is $\varnothing$.