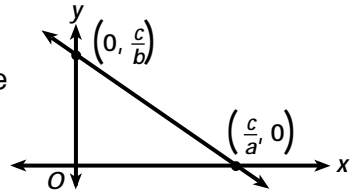


LESSON
2-3 **Challenge**
Intercepts and Triangles

Every linear equation in x and y can be written in the form $ax + by = c$, where a and b cannot both be 0. If a , b , and c are not zero, then the graph is a line that crosses both the x -axis and the y -axis at points other than the origin, such as in the diagram at right.



You can use the equation of a line to find the area of a triangle.

- a , b , and c are nonzero constants and $ax + by = c$. Show that the x -intercept of the graph is $\frac{c}{a}$ and that the y -intercept is $\frac{c}{b}$.

- Explain why a , b , and c must be nonzero in order to form a triangle whose sides are the line represented by the equation $ax + by = c$ and the coordinate axes.

- a , b , and c are positive numbers. Write a formula for the area of the triangle formed by the graph of $ax + by = c$ and the coordinate axes.

- Find the area of the right triangle formed by the graph of $4x + 5y = 20$ and the coordinate axes.
- _____

- A triangle whose sides are the graph of a line and the coordinate axes has an area of 100 square units. Write an equation of the form $ax + by = c$ for the hypotenuse of the triangle.
- _____

Solve.

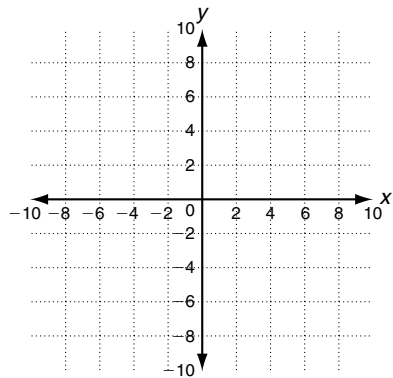
- Draw the graph of a line with x -intercept 5 and y -intercept 8.

- Find the constants a , b , and c for the line.
- _____

- Write the equation for the line.
- _____

- Write the equation in slope-intercept form. What is the slope of the line?
- _____

- What is the area of the right triangle formed by the line and the coordinate axes?
- _____



LESSON **Reteach**

2-3 Graphing Linear Functions (continued)

Use the slope and the y -intercept to graph a linear function.

To write $2y + x = 6$ in slope-intercept form, solve for y .

$$2y + x = 6$$

$$-x -x$$

$$2y = -x + 6$$

$$\frac{2y}{2} = \frac{-x + 6}{2}$$

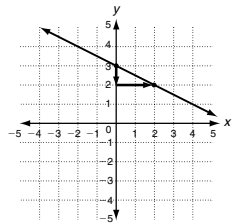
$$y = -\frac{1}{2}x + 3$$

$y = mx + b$ is the slope-intercept form. m represents the slope and b represents the y -intercept.

Compare $y = -\frac{1}{2}x + 3$ to $y = mx + b$.

$$m = -\frac{1}{2}, \text{ so the slope is } -\frac{1}{2}.$$

$$b = 3, \text{ so the } y\text{-intercept is } 3.$$



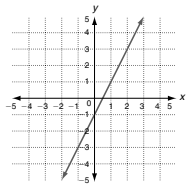
Write each function in slope-intercept form. Use m and b to graph.

3. $2x - y = 1$

a. $y = 2x - 1$

b. $m = 2$

c. $b = -1$

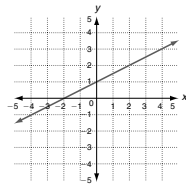


4. $y - \frac{x}{2} = 1$

a. $y = \frac{1}{2}x + 1$

b. $m = \frac{1}{2}$

c. $b = 1$



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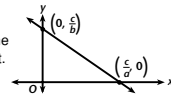
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Holt Algebra 2

LESSON **Challenge**

2-3 Intercepts and Triangles

Every linear equation in x and y can be written in the form $ax + by = c$, where a and b cannot both be 0. If a , b , and c are not zero, then the graph is a line that crosses both the x -axis and the y -axis at points other than the origin, such as in the diagram at right.



You can use the equation of a line to find the area of a triangle.

1. a , b , and c are nonzero constants and $ax + by = c$. Show that the x -intercept of the graph is $\frac{c}{a}$ and that the y -intercept is $\frac{c}{b}$.

To find the x -intercept, let $y = 0$.

$$ax + by = c$$

$$ax + b(0) = c$$

$$ax = c$$

$$x = \frac{c}{a}$$

To find the y -intercept, let $x = 0$.

$$ax + by = c$$

$$a(0) + by = c$$

$$by = c$$

$$y = \frac{c}{b}$$

2. Explain why a , b , and c must be nonzero in order to form a triangle whose sides are the line represented by the equation $ax + by = c$ and the coordinate axes.

Possible answer: If any of a , b , or c are 0, then there is no triangle since the lengths of two of the sides are $\frac{c}{b}$ and $\frac{c}{a}$.

3. a. a , b , and c are positive numbers. Write a formula for the area of the triangle formed by the graph of $ax + by = c$ and the coordinate axes.

$$A = \frac{c^2}{2ab}$$

b. Find the area of the right triangle formed by the graph of $4x + 5y = 20$ and the coordinate axes.

10 square units

4. A triangle whose sides are the graph of a line and the coordinate axes has an area of 100 square units. Write an equation of the form $ax + by = c$ for the hypotenuse of the triangle.

Possible answer: $2x + y = 20$

Solve.

5. a. Draw the graph of a line with x -intercept 5 and y -intercept 8.

b. Find the constants a , b , and c for the line.

$$a = 8, b = 5, c = 40$$

c. Write the equation for the line.

$$8x + 5y = 40$$

d. Write the equation in slope-intercept form. What is the slope of the line?

$$y = -\frac{8}{5}x + 8; -\frac{8}{5}$$

e. What is the area of the right triangle formed by the line and the coordinate axes?

20 square units

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Holt Algebra 2

LESSON **Problem Solving**

2-3 Graphing Linear Functions

Solve

1. Nathan made a table to record the balance in his savings account when he made a deposit every other month.

| Savings Balance | | | | | | |
|-----------------|-----|-----|------|------|------|------|
| Month | 2 | 4 | 6 | 8 | 10 | 12 |
| Balance (\$) | 575 | 810 | 1025 | 1280 | 1545 | 1850 |

Is this data set linear? How do you know?

No; Possible answer: the rate of change is not constant.

2. Sally runs a landscape service business. The table shows her fee schedule.

| Landscape Services | | | | | | |
|--------------------|---|----|----|----|----|----|
| Time (h) | 1 | 2 | 3 | 4 | 5 | 6 |
| Price (\$) | 8 | 14 | 20 | 26 | 32 | 38 |

a. Why is the data set linear?

Because the rate of change is constant

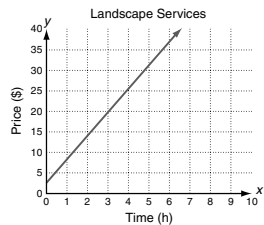
b. Find the slope of the line that passes through the points.

$$6$$

c. Graph these data.

d. Estimate the cost for 9 hours of landscape services.

$$\$56$$



Choose the letter for the best answer.

3. Jan built a skateboard ramp from her back porch to the ground. The porch is 30 inches above the ground. The ramp extends 9 feet from the base of the porch. Find the slope of the ramp.

- A 3.6 C 0.3
B 3.33 D 0.278

4. When Rafiq left home on a business trip he noted that the odometer on his car read 47,823. He drove 3 h 15 min and then noted that the odometer read 48,017. Find his average speed in miles per hour.

- A 55.6 C 61.6
B 59.7 D 63.5

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Holt Algebra 2

LESSON **Reading Strategies**

2-3 Graphic Organizer

| Definition | Facts |
|--|---|
| A function with a constant rate of change is called a linear function . $f(x) = mx + b$ m is the slope. b is the y -intercept. | The graph of a linear function is always a straight line. You can use the equation of a linear function to find its slope and intercepts: $y = mx + b$. |
| Example | Useful Hints |
| Linear function: $2x + y = 4$ Slope-intercept form of the linear function: $y = -2x + 4$ Slope = -2 y -intercept = 4 | You can use any two points on a line to draw its graph. The intercepts give you two points on the line. You can also graph a line using its slope and one point on the line. |

Complete the table

| Linear Function | Slope-Intercept Form | Slope | y -intercept |
|--------------------|----------------------|-------|----------------|
| 1. $4x + y = 7$ | $y = -4x + 7$ | -4 | 7 |
| 2. $3y - 3x = -9$ | $y = x - 3$ | 1 | -3 |
| 3. $-6x + 2y = 12$ | $y = 3x + 6$ | 3 | 6 |

Use the function $x - 2y = 4$ for Exercises 4–6.

4. What do the terms x -intercept and y -intercept mean?

Possible answer: The x -intercept is the point where the line crosses the x -axis. The y -intercept is the point where the line crosses the y -axis.

5. The function passes through the point $(2, -1)$. Describe how to use the slope to find another point on the line.

Possible answer: Plot $(2, -1)$. The slope of the line is $\frac{1}{2}$, so move 1 unit up and 2 units to the right, to $(4, 0)$.

6. Describe how to graph the function using its intercepts.

Possible answer: Plot the points $(4, 0)$ and $(0, -2)$. Draw a line through both points.

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