Name ______________________________ Date __________ Class __________

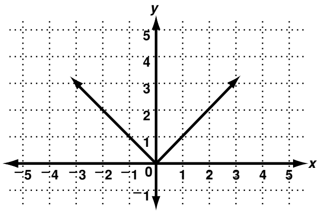
LESSON 1-8

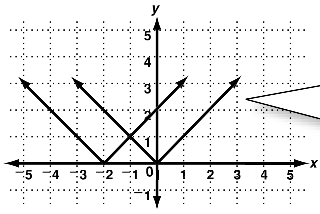
Reteach
Exploring Transformations

A **translation** moves a point, figure, or function right, left, up, or down.

Horizontal Translation (right or left)	**Vertical Translation** (up or down)
The *x*-coordinate changes. $(x, y) \rightarrow (x + h, y)$	The *y*-coordinate changes. $(x, y) \rightarrow (x, y + k)$

Translate the function $y = f(x)$ left 2 units.



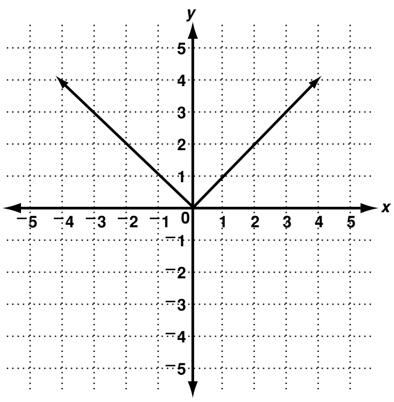


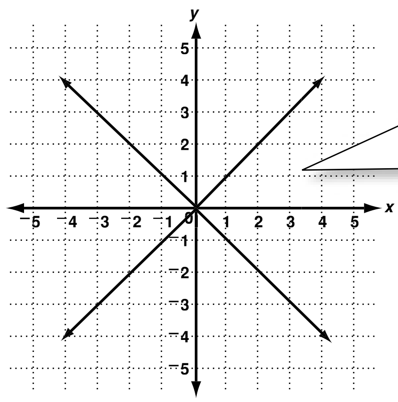
Move each point 2 units left. Connect the points. $(x, y) \rightarrow (x - 2, y)$

A **reflection** flips a point, figure, or function across a line.

Reflection Across *y*-axis	**Reflection Across *x*-axis**
The *x*-coordinate changes. $(x, y) \rightarrow (-x, y)$	The *y*-coordinate changes. $(x, y) \rightarrow (x, -y)$

Reflect the function $y = f(x)$ across the *x*-axis.

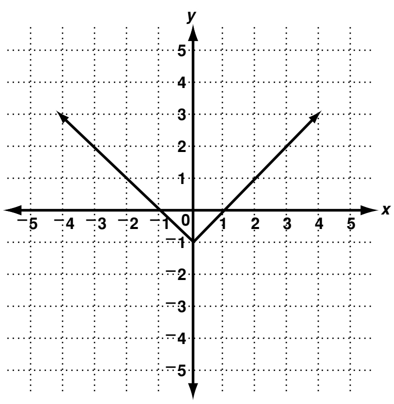




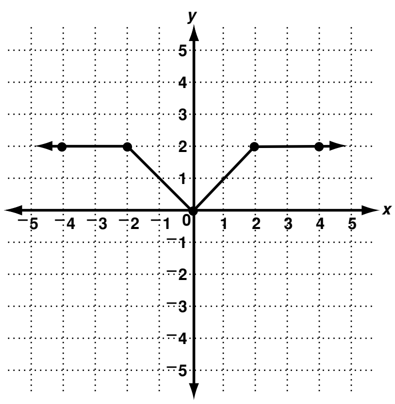
Flip each point across the axis. Connect the points. $(x, y) \rightarrow (x, -y)$

Perform each transformation of $y = f(x)$.

1. translation up 2 units



2. reflection across *x*-axis



Name ______________________________________ Date ___________ Class ____________

LESSON 1-8

Reteach

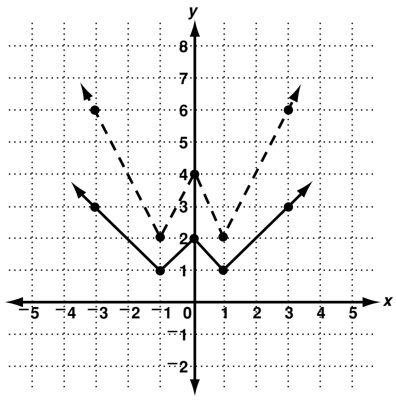
Exploring Transformations (continued)

In a stretch or a compression, the new figure has a different shape than the original.

Horizontal Stretch (away from y-axis)	The x-coordinate changes. $(x, y) \to (bx, y)$; $\lvert b\rvert > 1$
Vertical Stretch (away from x-axis)	The y-coordinate changes. $(x, y) \to (x, ay)$; $\lvert a\rvert > 1$
Horizontal Compression (toward the y-axis)	The x-coordinate changes. $(x, y) \to (bx, y)$; $0 < \lvert b\rvert < 1$
Vertical Compression (toward the x-axis)	The y-coordinate changes. $(x, y) \to (x, ay)$; $0 < \lvert a\rvert < 1$

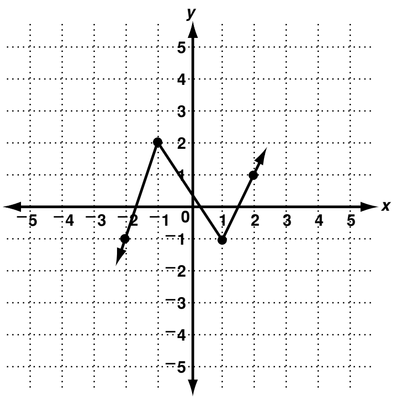
Perform a vertical stretch of the function $y = f(x)$ by a factor of 2.
In a vertical stretch $(x, y) \to (x, ay)$. In this case, $a = 2$.

Original Figure (solid line)	x	$2y$	**Stretched Figure** (dashed line)
$(-3, 3)$	-3	6	$(-3, 6)$
$(-1, 1)$	-1	2	$(-1, 2)$
$(0, 2)$	0	4	$(0, 4)$
$(1, 1)$	1	2	$(1, 2)$
$(3, 3)$	3	6	$(3, 6)$

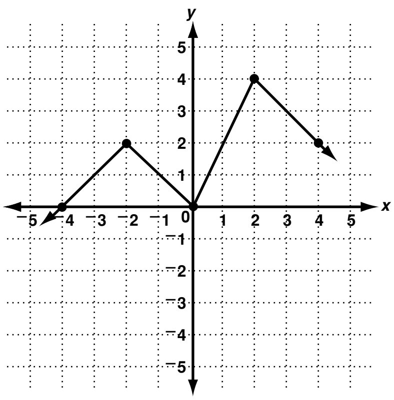


Perform each transformation of $y = f(x)$.

3. horizontal stretch by a factor of 2



4. vertical compression by a factor of $\frac{1}{2}$

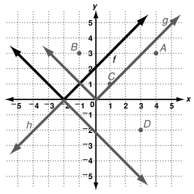


LESSON **1-8**

Practice A

Exploring Transformations

Use the graph to perform each transformation described.



1. Plot point *A* at (4, 3). Translate point *A* left 5 units. Label this point *B*. Give the coordinates of point *B*.

   (−1, 3)

2. Plot point *C* at (1, 1). Translate point *C* right 2 units and down 3 units. Label this point *D*. Give the coordinates of point *D*.

   (3, −2)

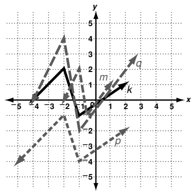
3. Transform $y = f(x)$ by translating it right 2 units. Label the new function *g*. Compare the points that make up the 2 functions. Which coordinate changes, *x* or *y*?

   x-coordinate

4. Transform $y = f(x)$ by reflecting it across the *x*-axis. Label the new function *h*. Which coordinate changes, *x* or *y*?

   y-coordinate

Use the graph to perform each transformation described.



5. Transform $y = k(x)$ by compressing it horizontally by a factor of $\frac{1}{2}$. Label the new function *m*. Which coordinate is multiplied by $\frac{1}{2}$, *x* or *y*?

   x-coordinate

6. Transform $y = k(x)$ by translating it down 3 units. Label the new function *p*. What happens to the *y*-coordinate in each new ordered pair?

   It is 3 less than the original *y*-coordinate.

7. Transform $y = k(x)$ by stretching it vertically by a factor of 2. Label the new function *q*. Which coordinate is multiplied by 2, *x* or *y*?

   y-coordinate

8. Describe how the coordinates of a function change when it is translated 2 units to the left and 4 units up.

   (x, y) becomes $(x - 2, y + 4)$.

9. Describe how the coordinates of a function change when you vertically compress a function by a factor of $\frac{2}{3}$.

   (x, y) becomes $\left(x, \frac{2}{3}y\right)$.

LESSON **1-8**

Practice B

Exploring Transformations

Perform the given translation on the point (2, 5) and give the coordinates of the translated point.

1. left 3 units

   (−1, 5)

2. down 6 units

   (2, −1)

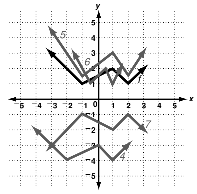
3. right 4 units, up 2 units

   (6, 7)

Use the table to perform each transformation of $y = f(x)$. Use the same coordinate plane as the original function.

4. translation left 1 unit, down 5 units

$x - 1$	x	y	$y - 5$
−4	−3	3	−2
−2	−1	1	−4
0	1	2	−3
1	2	1	−4
2	3	2	−3



5. vertical stretch factor of $\frac{3}{2}$

x	y	$\frac{3}{2}y$
−3	3	$\frac{9}{2}$
−1	1	$\frac{3}{2}$
1	2	3
2	1	$\frac{3}{2}$
3	2	3

6. horizontal compression factor of $\frac{1}{2}$

$\frac{1}{2}x$	x	y
$-\frac{3}{2}$	−3	3
$-\frac{1}{2}$	−1	1
$\frac{1}{2}$	1	2
1	2	1
$\frac{3}{2}$	3	2

7. reflection across *x*-axis

x	y	$-y$
−3	3	−3
−1	1	−1
1	2	−2
2	1	−1
3	2	−2

Solve.

8. George has a goal for the number of computers he wants to sell each month for the next 6 months at his computer store. He draws a graph to show his projected profits for that period. Then he decides to discount the prices by 10%. How will this affect his profits? Identify the transformation to his graph and describe how to find the ordered pairs for the transformation.

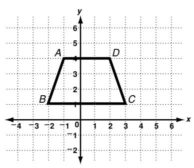
   Profits are reduced by 10%; vertical compression; $(x, 0.9y)$.

LESSON **1-8**

Practice C

Exploring Transformations

Transform trapezoid *ABCD* as indicated. Estimate the area of each transformed trapezoid as compared to the area of trapezoid *ABCD*.



1. reflection across the *x*-axis

   Areas are equal.

2. horizontal compression by a factor of $\frac{1}{2}$

   Area is $\frac{1}{2}$ of original trapezoid.

3. horizontal stretch by a factor of 2

   Area is doubled.

4. vertical compression by a factor of $\frac{1}{2}$

   Area is $\frac{1}{2}$ of original trapezoid.

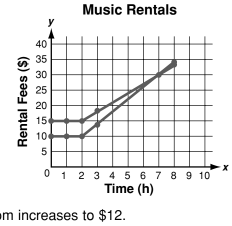
5. vertical stretch by a factor of $\frac{3}{2}$

   Area is $\frac{3}{2}$ of original trapezoid.

Tucci's House of Music rents practice space and musical instruments. Use of a practice room costs $10 for the first 2 hours and $4 for each additional hour. An electric guitar rents for $15 for the first 2 hours and $3 for each additional hour.

6. Sketch a graph of two functions, one for the cost of renting a practice room and another for the cost of renting an electric guitar.

Music Rentals



Identify the transformation of the original graphs represented by the following changes.

7. The charge for the first 2 hours' rental of a practice room increases to $12.

   Translation

8. As a special promotion, Tucci's House of Music cuts the practice room charges by 50% for first-time users.

   Vertical compression

9. The cost of renting a guitar increases to $30 for the first 4 hours and $6 for each additional hour.

   Horizontal stretch and translation

LESSON **1-8**

Reteach

Exploring Transformations

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Horizontal Translation (right or left)	**Vertical Translation** (up or down)
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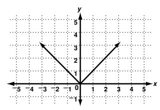
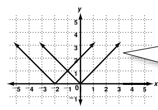
Translate the function $y = f(x)$ left 2 units.



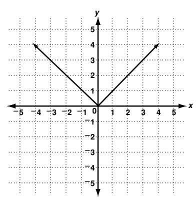


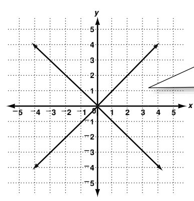
Move each point 2 units left. Connect the points. $(x, y) \rightarrow (x - 2, y)$

A **reflection** flips a point, figure, or function across a line.

Reflection Across *y*-axis	**Reflection Across *x*-axis**
The *x*-coordinate changes. $(x, y) \rightarrow (-x, y)$	The *y*-coordinate changes. $(x, y) \rightarrow (x, -y)$

Reflect the function $y = f(x)$ across the *x*-axis.

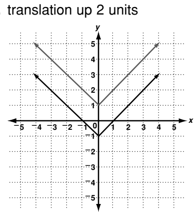




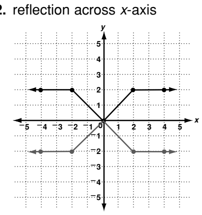
Flip each point across the axis. Connect the points. $(x, y) \rightarrow (x, -y)$

Perform each transformation of $y = f(x)$.

1. translation up 2 units



2. reflection across *x*-axis



LESSON 1-8
Reteach
Exploring Transformations (continued)

In a stretch or a compression, the new figure has a different shape than the original.

Horizontal Stretch (away from *y*-axis)	The *x*-coordinate changes. $(x, y) \rightarrow (bx, y)$; $\lvert b\rvert > 1$
Vertical Stretch (away from *x*-axis)	The *y*-coordinate changes. $(x, y) \rightarrow (x, ay)$; $\lvert a\rvert > 1$
Horizontal Compression (toward the *y*-axis)	The *x*-coordinate changes. $(x, y) \rightarrow (bx, y)$; $0 < \lvert b\rvert < 1$
Vertical Compression (toward the *x*-axis)	The *y*-coordinate changes. $(x, y) \rightarrow (x, ay)$; $0 < \lvert a\rvert < 1$

Perform a vertical stretch of the function $y = f(x)$ by a factor of 2.
In a vertical stretch $(x, y) \rightarrow (x, ay)$. In this case, $a = 2$.

Original Figure (solid line)	x	$2y$	**Stretched Figure** (dashed line)
$(-3, 3)$	-3	6	$(-3, 6)$
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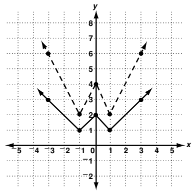
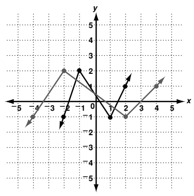
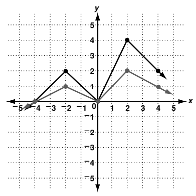


Perform each transformation of $y = f(x)$.

3. horizontal stretch by a factor of 2

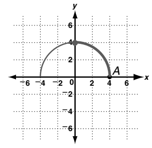


4. vertical compression by a factor of $\frac{1}{2}$



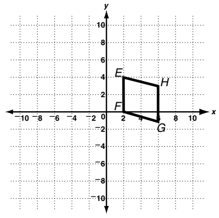
LESSON 1-8
Challenge
Turn it Around

Translations and reflections are transformations in the position of a figure. A third transformation that preserves congruence is called a **rotation**. Point *A* is at (4, 0). Move point *A* along the semicircle 90° in a counterclockwise direction. The rotated point has coordinates (0, 4). This is called a rotation of 90° counterclockwise centered at the origin.



Use the graph at right for Exercises 1–6.

Rotate figure *EFGH* 90° clockwise through the origin.



1. What are the coordinates of the vertices of the rotated figure?

   $(4, -2), (0, -2), (-1, -6), (3, -6)$

2. Write a general rule to show the result of rotating a point 90° clockwise through the origin.

   $(x, y) \rightarrow (y, -x)$

3. Write a general rule to show the result of rotating a point 90° counterclockwise through the origin.

   $(x, y) \rightarrow (-y, x)$

To rotate a figure through a point other than the origin, translate the figure so that the point of rotation is at the origin. Then perform the rotation through the origin. Finally, reverse the translation.

Rotate quadrilateral *EFGH* counterclockwise 90° through the point (2, 0). First translate the quadrilateral 2 units left to move the point of rotation, (2, 0), to the origin.

4. What are the coordinates of the vertices of the translated quadrilateral?

   $(0, 4), (0, 0), (4, -1), (4, 3)$

Now, rotate the translated quadrilateral 90° counterclockwise through the origin.

5. What are the coordinates of the vertices of the rotated quadrilateral?

   $(-4, 0), (0, 0), (1, 4), (-3, 4)$

Finally, reverse the translation by moving the quadrilateral 2 units right.

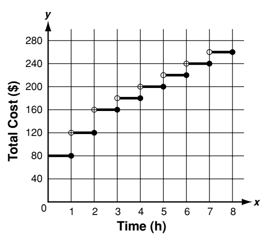
6. What are the coordinates of the vertices of the final quadrilateral?

   $(-2, 0), (2, 0), (3, 4), (-1, 4)$

LESSON 1-8
Problem Solving
Exploring Transformations

Harry is working on a budget for a concert. The graph shows the total cost of renting the hall. A cleaning fee of $40 for each rental is included in the graph. Use the graph for Exercises 1–6.

Concert Hall Rental



1. What is the cost of renting the hall for 2 hours? for 3 hours? for 6 hours? for 7 hours?

   $120; $160; $220; $240

2. What is the rate per hour not including the cleaning fee if Harry rents the hall for up to 3 hours?

   $40 per hour

3. What is the rate per hour after the first 3 hours?

   $20 per hour

4. Describe the effect on the graph if the cleaning fee were changed to $25.

   Translated down 15 units

5. The managers decide that the minimum time for which the hall can be rented is 3 hours. Describe the effect this change would have on the graph above. How would the range change?

   Possible answers: A line would go from (0, 160) to (3, 160) with no open circle; the range would not include any numbers less than 160.

6. The Art Center gives Harry a graph showing its charges. This graph is the same shape as the graph above, but every point has been translated up 10 units. What would be the effect on Harry's budget if he chose to have the concert at the Art Center?

   He would have to pay more to rent the Art Center.

Choose the letter for the best answer.

7. Martha's profits from her bagel store last year were $0.35 per dozen bagels sold. This year her profits decreased 10%. What kind of transformation does this represent?

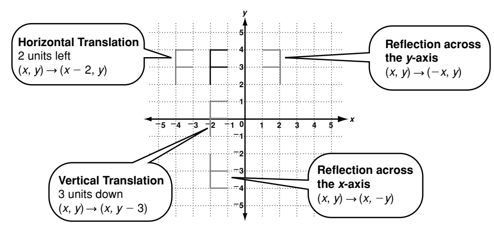
   (A) vertical compression
   B vertical stretch
   C horizontal compression
   D horizontal stretch

8. Shana drew the graph for a quadratic function. Then she did a horizontal stretch of the curve. Which transformation did she perform?

   F $(x, y) \rightarrow (x, ay)$; $\lvert a\rvert > 1$
   G $(x, y) \rightarrow (bx, y)$; $0 < \lvert b\rvert < 1$
   H $(x, y) \rightarrow (x, ay)$; $0 < \lvert a\rvert < 1$
   (J) $(x, y) \rightarrow (bx, y)$; $\lvert b\rvert > 1$

LESSON 1-8
Reading Strategies
Understand Vocabulary

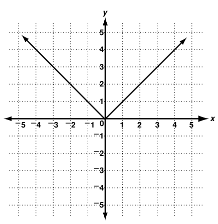
A diagram can help you connect transformation vocabulary to corresponding graphs. When you perform a transformation on a graph, the figure is moved according to the type of transformation.



1. What happens to the shape and the position of a figure during a translation or a reflection?

   The shape of the figure does not change, only the position changes.

Use the graph for Exercises 2–5.



2. How will the coordinates change if the function is translated 3 units right?

   Add 3 to each *x*-coordinate; *y*-coordinates do not change.

3. How will the coordinates change if the function is translated 5 units down?

   x-coordinates do not change; subtract 5 from each *y*-coordinate.

4. How will the coordinates change if the function is reflected across the *x*-axis?

   x-coordinates do not change; multiply each *y*-coordinate by -1.

5. How will the coordinates change if the function is translated 4 units left and 2 units up?

   Subtract 4 from each *x*-coordinate and add 2 to each *y*-coordinate.