Glencoe Mathematics

Algebra 2

Chapter 12
Resource Masters

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Consumable Workbooks Many of the worksheets contained in the Chapter Resource Masters are available as consumable workbooks in both English and Spanish.

	ISBN10	ISBN13
Study Guide and Intervention Workbook	0-07-877355-5	978-0-07-877355-6
Skills Practice Workbook	0-07-877357-1	978-0-07-877357-0
Practice Workbook	0-07-877358-X	978-0-07-877358-7
Word Problem Practice Workbook	0-07-877360-1	978-0-07-877360-0
Spanish Versions		
Study Guide and Intervention Workbook	0-07-877356-3	978-0-07-877356-3
Practice Workbook	0-07-877359-8	978-0-07-877359-4

Answers for Workbooks The answers for Chapter 12 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

StudentWorks Plus™ This CD-ROM includes the entire Student Edition test along with the English workbooks listed above.

TeacherWorks Plus™ All of the materials found in this booklet are included for viewing, printing, and editing in this CD-ROM.

Spanish Assessment Masters (ISBN10: 0-07-0-07-877361-X, ISBN13: 978-0-07-877361-7) These masters contain a Spanish version of Chapter 12 Test Form 2A and Form 2C.

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Contents

Teacher's Guide to Using the *Chapter 12 Resource Masters*

The *Chapter 12 Resource Masters* includes the core materials needed for Chapter 12. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing on the *TeacherWorks Plus*™ CD-ROM.

Chapter Resources

Student-Built Glossary (pages 1–2) These masters are a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar. Give this to students before beginning Lesson 12-1. Encourage them to add these pages to their mathematics study notebooks. Remind them to complete the appropriate words as they study each lesson.

Anticipation Guide (pages 3–4) This master, presented in both English and Spanish, is a survey used before beginning the chapter to pinpoint what students may or may not know about the concepts in the chapter. Students will revisit this survey after they complete the chapter to see if their perceptions have changed.

Lesson Resources

Lesson Reading Guide Get Ready for the Lesson extends the discussion from the beginning of the Student Edition lesson. Read the Lesson asks students to interpret the context of and relationships among terms in the lesson. Finally, Remember What You Learned asks students to summarize what they have learned using various representation techniques. Use as a study tool for note taking or as an informal reading assignment. It is also a helpful tool for ELL (English Language Learners).

Study Guide and Intervention These masters provide vocabulary, key concepts, additional worked-out examples and Check Your Progress exercises to use as a reteaching activity. It can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice This master focuses more on the computational nature of the lesson. Use as an additional practice option or as homework for second-day teaching of the lesson.

Practice This master closely follows the types of problems found in the Exercises section of the Student Edition and includes word problems. Use as an additional practice option or as homework for second-day teaching of the lesson.

Word Problem Practice This master includes additional practice in solving word problems that apply the concepts of the lesson. Use as an additional practice or as homework for second-day teaching of the lesson.

Enrichment These activities may extend the concepts of the lesson, offer an historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. They are written for use with all levels of students.

Graphing Calculator, Scientific Calculator, or Spreadsheet Activities These activities present ways in which technology can be used with the concepts in some lessons of this chapter. Use as an alternative approach to some concepts or as an integral part of your lesson presentation.

Assessment Options

The assessment masters in the *Chapter 12 Resource Masters* offer a wide range of assessment tools for formative (monitoring) assessment and summative (final) assessment.

Student Recording Sheet This master corresponds with the standardized test practice at the end of the chapter.

Pre-AP Rubric This master provides information for teachers and students on how to assess performance on open-ended questions.

Quizzes Four free-response quizzes offer assessment at appropriate intervals in the chapter.

Mid-Chapter Test This 1-page test provides an option to assess the first half of the chapter. It parallels the timing of the Mid-Chapter Quiz in the Student Edition and includes both multiple-choice and free-response questions.

Vocabulary Test This test is suitable for all students. It includes a list of vocabulary words and 10 questions to assess students' knowledge of those words. This can also be used in conjunction with one of the leveled chapter tests.

Leveled Chapter Tests

- *Form 1* contains multiple-choice questions and is intended for use with below grade level students.
- *Forms 2A and 2B* contain multiple-choice questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- *Forms 2C and 2D* contain free-response questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- *Form 3* is a free-response test for use with above grade level students.

All of the above mentioned tests include a free-response Bonus question.

Extended-Response Test Performance assessment tasks are suitable for all students. Sample answers and a scoring rubric are included for evaluation.

Standardized Test Practice These three pages are cumulative in nature. It includes three parts: multiple-choice questions with bubble-in answer format, griddable questions with answer grids, and short-answer free-response questions.

Answers

- The answers for the Anticipation Guide and Lesson Resources are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment masters.

NAME _____ DATE_____ PERIOD _____

12 Student-Built Glossary

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 12. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Algebra Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
binomial experiment		
combination		
compound event		
dependent and independent events		
event		
inclusive (ihn·KLOO·sihv) events		
measure of variation		
mutually (MYOO·chuh·lee) exclusive events		
normal distribution		
outcome		

(continued on the next page)

NAME _____ DATE_____ PERIOD _____

12 Student-Built Glossary

Vocabulary Term	Found on Page	Definition/Description/Example
permutation (PUHR·myoo·TAY·shuhn)		
probability		
probability distribution		
random		
random variable		
relative-frequency histogram		
sample space		
simple event		
standard deviation		
unbiased sample		
uniform distribution		
univariate data		
variance (VEHR·ee·uhn(t)s)		

NAME _____ DATE_____ PERIOD _____

12 Anticipation Guide

Probability and Statistics

Step 1 ***Before you begin Chapter 12***

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

STEP 1 A, D, or NS	Statement	STEP 2 A or D
	1. A sample space is a partial list of possible outcomes of an experiment.	
	2. Two events are called independent if choosing one does not affect choosing the other.	
	3. According to the Fundamental Counting Principle, if one event can occur in 6 ways and another event can occur in 3 ways, then the events together can occur in 6 + 3 or 9 ways.	
	4. Since order is not important in a combination, an outcome ab is the same as an outcome ba.	
	5. The odds of an event occurring can be expressed as a ratio of the number of successes to the total number of outcomes.	
	6. If two events are dependent, then the probability of both events occurring is the product of the probabilities of each event.	
	7. Two events are *mutually exclusive* if they cannot occur at the same time.	
	8. If a set of data contains outliers, the median would be a good choice to represent the set.	
	9. Measures of variation are the differences between consecutive values in the set.	
	10. The curve representing a normal distribution is symmetric.	
	11. The Binomial Theorem can be used to find probabilities only when there are two possible outcomes.	
	12. Asking people in a music store how many hours they spend listening to music to determine how many hours people in the city listen to music is an example of an unbiased survey.	

Step 2 ***After you complete Chapter 12***

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

NOMBRE _____ FECHA _____ PERÍODO ___

12 Ejercicios preparatorios

Probabilidad y estadística

PASO 1 ***Antes de comenzar el Capítulo 12***

- Lee cada enunciado.
- Decide si estás de acuerdo (A) o en desacuerdo (D) con el enunciado.
- Escribe A o D en la primera columna O si no estás seguro(a) de la respuesta, escribe NS (No estoy seguro(a).

PASO 1 A, D o NS	Enunciado	PASO 2 A o D
	1. Un espacio muestral es una lista parcial de resultados posibles de un experimento.	
	2. A dos eventos se les llama independientes si al escoger uno, no afecta escoger el otro.	
	3. Según el principio fundamental de contar, si un evento puede ocurrir de 6 maneras y otro evento puede ocurrir de 3 maneras, los eventos juntos pueden ocurrir de 6 + 3 ó 9 maneras.	
	4. Dado que el orden no es importante en una combinación, un resultado ab es lo mismo que un resultado ba.	
	5. Las posibilidades de que ocurra un evento se pueden expresar como la razón del número de éxitos al número total de resultados.	
	6. Si dos eventos son dependientes, entonces la probabilidad de que ocurran ambos eventos es el producto de las probabilidades de cada evento.	
	7. Dos eventos son *mutuamente exclusivos* si no pueden ocurrir al mismo tiempo.	
	8. Si un conjunto de datos contiene valores atípicos, la mediana sería una buena opción para representar el conjunto.	
	9. Las medidas de variación son las diferencias entre valores consecutivos en el conjunto.	
	10. La curva que representa una distribución normal es simétrica.	
	11. Se puede usar el teorema del binomio para calcular probabilidades sólo cuando existen dos resultados posibles.	
	12. Preguntarle a la gente en una disquería cuántas horas escuchan música, para determinar así el número de horas que la gente en la ciudad escucha música, es un ejemplo de una encuesta insesgada.	

PASO 2 ***Después de completar el Capítulo 12***

- Vuelve a leer cada enunciado y completa la última columna con una A o una D.
- ¿Cambió cualquiera de tus opiniones sobre los enunciados de la primera columna?
- En una hoja de papel aparte, escribe un ejemplo de por qué estás en desacuerdo con los enunciados que marcaste con una D.

NAME _____ DATE_____ PERIOD _____

12-1 Lesson Reading Guide

The Counting Principle

Get Ready for the Lesson

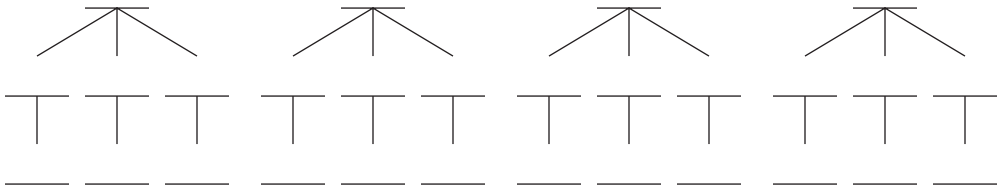
Read the introduction to Lesson 12-1 in your textbook.

Assume that all Florida license plates have three letters followed by three digits, and that there are no rules against using the same letter or number more than once. How many choices are there for each letter? for each digit?

Read the Lesson

1. Shamim is signing up for her classes. Most of her classes are required, but she has two electives. For her arts class, she can chose between Art, Band, Chorus, or Drama. For her language class, she can choose between French, German, and Spanish.

a. To organize her choices, Shamim decides to make a tree diagram. Let A, B, C, and D represent Art, Band, Chorus, and Drama, and F, G, and S represent French, German, and Spanish. Complete the following diagram.



b. How could Shamim have found the number of possible combinations without making a tree diagram?

2. A jar contains 6 red marbles, 4 blue marbles, and 3 yellow marbles. Indicate whether the events described are *dependent* or *independent*.

a. A marble is drawn out of the jar and is not replaced. A second marble is drawn.

b. A marble is drawn out of the jar and is put back in. The jar is shaken. A second marble is drawn.

Remember What You Learned

3. One definition of *independent* is "not determined or influenced by someone or something else." How can this definition help you remember the difference between *independent* and *dependent* events?

NAME _____ DATE_____ PERIOD _____

12-1 Study Guide and Intervention

The Counting Principle

Independent Events If the outcome of one event does not affect the outcome of another event and vice versa, the events are called **independent events**.

Fundamental Counting Principle	If event *M* can occur in *m* ways and is followed by event *N* that can occur in *n* ways, then the event *M* followed by the event *N* can occur in $m \cdot n$ ways.

Example **FOOD For the Breakfast Special at the Country Pantry, customers can choose their eggs scrambled, fried, or poached, whole wheat or white toast, and either orange, apple, tomato, or grapefruit juice. How many different Breakfast Specials can a customer order?**

A customer's choice of eggs does not affect his or her choice of toast or juice, so the events are independent. There are 3 ways to choose eggs, 2 ways to choose toast, and 4 ways to choose juice. By the Fundamental Counting Principle, there are $3 \cdot 2 \cdot 4$ or 24 ways to choose the Breakfast Special.

Exercises

Solve each problem.

1. The Palace of Pizza offers small, medium, or large pizzas with 14 different toppings available. How many different one-topping pizzas do they serve?

2. The letters A, B, C, and D are used to form four-letter passwords for entering a computer file. How many passwords are possible if letters can be repeated?

3. A restaurant serves 5 main dishes, 3 salads, and 4 desserts. How many different meals could be ordered if each has a main dish, a salad, and a dessert?

4. Marissa brought 8 T-shirts and 6 pairs of shorts to summer camp. How many different outfits consisting of a T-shirt and a pair of shorts does she have?

5. There are 6 different packages available for school pictures. The studio offers 5 different backgrounds and 2 different finishes. How many different options are available?

6. How many 5-digit even numbers can be formed using the digits 4, 6, 7, 2, 8 if digits can be repeated?

7. How many license plate numbers consisting of three letters followed by three numbers are possible when repetition is allowed?

8. How many 4-digit positive even integers are there?

NAME _____ DATE_____ PERIOD _____

12-1 Study Guide and Intervention *(continued)*

The Counting Principle

Dependent Events If the outcome of an event *does* affect the outcome of another event, the two events are said to be **dependent**. The Fundamental Counting Principle still applies.

Example **ENTERTAINMENT** **The guests at a sleepover brought 8 videos. They decided they would only watch 3 videos. How many orders of 3 different videos are possible?**

After the group chooses to watch a video, they will not choose to watch it again, so the choices of videos are dependent events.

There are 8 choices for the first video. That leaves 7 choices for the second. After they choose the first 2 videos, there are 6 remaining choices. Thus by the Fundamental Counting Principle, there are $8 \cdot 7 \cdot 6$ or 336 orders of 3 different videos.

Exercises

Solve each problem.

1. Three students are scheduled to give oral reports on Monday. In how many ways can their presentations be ordered?

2. In how many ways can the first five letters of the alphabet be arranged if each letter is used only once?

3. In how many different ways can 4 different books be arranged on the shelf?

4. How many license plates consisting of three letters followed by three numbers are possible when no repetition is allowed?

5. Sixteen teams are competing in a soccer match. Gold, silver, and bronze medals will be awarded to the top three finishers. In how many ways can the medals be awarded?

6. In a word-building game each player picks 7 letter tiles. If Julio's letters are all different, how many 3-letter combinations can he make out of his 7 letters?

7. The editor has accepted 6 articles for the news letter. In how many ways can the 6 articles be ordered?

8. There are 10 one-hour workshops scheduled for the open house at the greenhouse. There is only one conference room available. In how many ways can the workshops be ordered?

9. The top 5 runners at the cross-country meet will receive trophies. If there are 22 runners in the race, in how many ways can the trophies be awarded?

NAME _____ DATE_____ PERIOD _____

12-1 Skills Practice

The Counting Principle

State whether the events are *independent* or *dependent*.

1. finishing in first, second, or third place in a ten-person race

2. choosing a pizza size and a topping for the pizza

3. Seventy-five raffle tickets are placed in a jar. Three tickets are then selected, one after the other, without replacing a ticket after it is chosen.

4. The 232 members of the freshman class all vote by secret ballot for the class representative to the Student Senate.

Solve each problem.

5. A surveying firm plans to buy a color printer for printing its maps. It has narrowed its choice to one of three models. Each of the models is available with either 32 megabytes of random access memory (RAM), 64 megabytes of RAM, or 128 megabytes of RAM. From how many combinations of models and RAM does the firm have to choose?

6. How many arrangements of three letters can be formed from the letters of the word *MATH* if any letter will not be used more than once?

7. Allan is playing the role of Oliver in his school's production of *Oliver Twist*. The wardrobe crew has presented Allan with 5 pairs of pants and 4 shirts that he can wear. How many possible costumes consisting of a pair of pants and a shirt does Allan have to choose from?

8. The 10-member steering committee that is preparing a study of the public transportation needs of its town will select a chairperson, vice-chairperson, and secretary from the committee. No person can serve in more than one position. In how many ways can the three positions be filled?

9. Jeanine has decided to buy a pickup truck. Her choices include either a V-6 engine or a V-8 engine, a standard cab or an extended cab, and 2-wheel drive or 4-wheel drive. How many possible models does she have to choose from?

10. A mail-order company that sells gardening tools offers rakes in two different lengths. Customers can also choose either a wooden, plastic, or fiberglass handle for the rake. How many different kinds of rakes can a customer buy?

11. A Mexican restaurant offers chicken, beef, or vegetarian fajitas wrapped with either corn or flour tortillas, and topped with either mild, medium, or hot salsa. How many different choices of fajitas does a customer have?

NAME _____ DATE_____ PERIOD _____

12-1 Practice

The Counting Principle

State whether the events are *independent* or *dependent*.

1. choosing an ice cream flavor and choosing a topping for the ice cream

2. choosing an offensive player of the game and a defensive player of the game in a professional football game

3. From 15 entries in an art contest, a camp counselor chooses first, second, and third place winners.

4. Jillian is selecting two more courses for her block schedule next semester. She must select one of three morning history classes and one of two afternoon math classes.

Solve each problem.

5. A briefcase lock has 3 rotating cylinders, each containing 10 digits. How many numerical codes are possible?

6. A golf club manufacturer makes irons with 7 different shaft lengths, 3 different grips, 5 different lies, and 2 different club head materials. How many different combinations are offered?

7. There are five different routes that a commuter can take from her home to the office. In how many ways can she make a round trip if she uses a different route coming than going?

8. In how many ways can the four call letters of a radio station be arranged if the first letter must be W or K and no letters repeat?

9. How many 7-digit phone numbers can be formed if the first digit cannot be 0 or 1, and any digit can be repeated?

10. How many 7-digit phone numbers can be formed if the first digit cannot be 0, and any digit can be repeated?

11. How many 7-digit phone numbers can be formed if the first digit cannot be 0 or 1, and if no digit can be repeated?

12. How many 7-digit phone numbers can be formed if the first digit cannot be 0, and if no digit can be repeated?

13. How many 6-character passwords can be formed if the first character is a digit and the remaining 5 characters are letters that can be repeated?

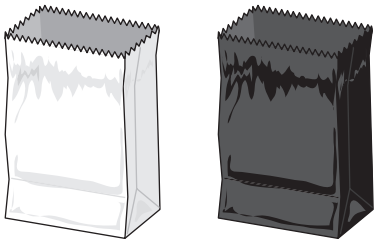
14. How many 6-character passwords can be formed if the first and last characters are digits and the remaining characters are letters? Assume that any character can be repeated.

NAME _____ DATE_____ PERIOD _____

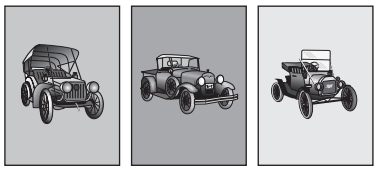
12-1 Word Problem Practice

The Counting Principle

1. CANDY Amy, Bruce, and Carol can choose one piece of candy from either a white or black bag. The white bag contains various chocolates. The black bag contains small bags of jelly beans. Amy picks from the white bag and Bruce and Carol both pick from the black bag. Describe whether each of the picks is related as dependent or independent events.



2. PHOTOS Morgan has three pictures that she would like to display side by side.



In how many different ways can the pictures be displayed?

3. COMBINATION LOCKS Eric uses a combination lock for his locker. The lock uses a three number secret code. Each number ranges from 1 to 35, inclusive. How many different combinations are possible with Eric's lock?

4. TRUE OR FALSE Faith is preparing a true or false quiz for her biology class. How many different answer keys can there be for a 10 question true or false quiz?

WEBSITES For Exercises 5-8, use the following information.

Greg is registering to use a website. The website requires him to choose an 8 character alphanumeric password that is not case-sensitive. In other words, for each character, he can choose one of the 26 letters A through Z or one of the 10 digits 0 through 9.

5. How many different passwords are possible?

6. Greg decides to use a password that does not contain any repeated characters. How many different passwords are possible with this constraint?

7. Suppose Greg chooses to use only letters with possible repeats. How many different passwords would be possible?

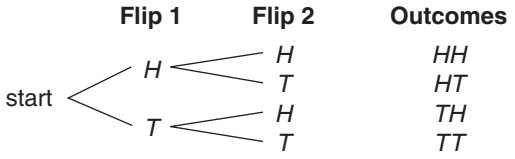
8. If Greg's password begins with his first name and ends with his birth month and date, how many possibilities would need to be checked to find his password?

NAME _____ DATE_____ PERIOD _____

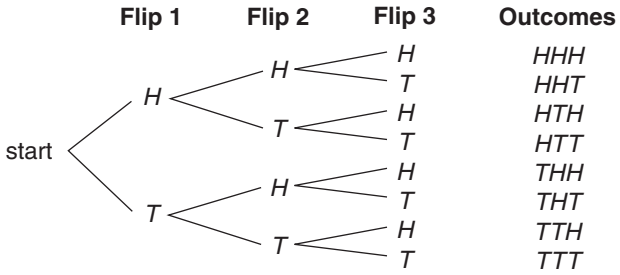
12-1 Enrichment

Tree Diagrams and the Power Rule

If you flip a coin once, there are two possible outcomes: heads showing (H) or tails showing (T). The tree diagram to the right shows the four (2^2) possible outcomes if you flip a coin twice.

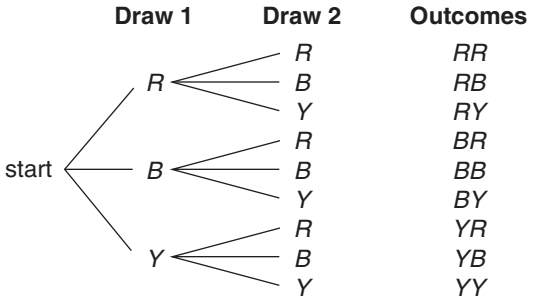


Example 1 **Draw a tree diagram to show all the possible outcomes for flipping a coin three times. List the outcomes.**



There are eight (2^3) possible outcomes. With each extra flip, the number of outcomes doubles. With 4 flips, there would be sixteen (2^4) outcomes.

Example 2 **In a cup there are a red, a blue, and a yellow marble. How many possible outcomes are there if you draw one marble at random, replace it, and then draw another?**



There are nine (3^2) possible outcomes.

The Power Rule for the number of outcomes states that if an experiment is repeated n times, and if there are b possible outcomes each time, there are b^n total possible outcomes.

Find the total number of possible outcomes for each experiment. Use tree diagrams to help you.

1. flipping a coin 5 times

2. doing the marble experiment 6 times

3. flipping a coin 8 times

4. rolling a 6-sided die 2 times

5. rolling a 6-sided die 3 times

6. rolling a 4-sided die 2 times

7. rolling a 4-sided die 3 times

8. rolling a 12-sided die 2 times

Lesson 12-1

NAME _____ DATE_____ PERIOD _____

12-2 Lesson Reading Guide

Permutations and Combinations

Get Ready for the Lesson

Read the introduction to Lesson 12-2 in your textbook.

Suppose that 20 students enter a math contest. In how many ways can first, second, and third places be awarded? (Write your answer as a product. Do not calculate the product.)

Read the Lesson

1. Indicate whether each situation involves a *permutation* or a *combination*.

a. choosing five students from a class to work on a special project

b. arranging five pictures in a row on a wall

c. drawing a hand of 13 cards from a 52-card deck

d. arranging the letters of the word *algebra*

2. Write an expression that can be used to calculate each of the following.

a. number of combinations of n distinct objects taken r at a time

b. number of permutations of n objects of which p are alike and q are alike

c. number of permutations of n distinct objects taken r at a time

3. Five cards are drawn from a standard deck of cards. Suppose you are asked to determine how many possible hands consist of one heart, two diamonds, and two spades.

a. Which of the following would you use to solve this problem: *Fundamental Counting Principle*, *permutations*, or *combinations*? (More than one of these may apply.)

b. Write an expression that involves the notation $P(n, r)$ and/or $C(n, r)$ that you would use to solve this problem. (Do not do any calculations.)

Remember What You Learned

4. Many students have trouble knowing when to use permutations and when to use combinations to solve counting problems. How can the idea of *order* help you to remember the difference between permutations and combinations?

NAME _____ DATE_____ PERIOD _____

12-2 Study Guide and Intervention

Permutations and Combinations

Permutations When a group of objects or people are arranged in a certain order, the arrangement is called a **permutation**.

Permutations	The number of permutations of n distinct objects taken r at a time is given by $P(n, r) = \frac{n!}{(n-r)!}$.
Permutations with Repetitions	The number of permutations of n objects of which p are alike and q are alike is $\frac{n!}{p!q!}$.

The rule for permutations with repetitions can be extended to any number of objects that are repeated.

Example **From a list of 20 books, each student must choose 4 books for book reports. The first report is a traditional book report, the second a poster, the third a newspaper interview with one of the characters, and the fourth a timeline of the plot. How many different orderings of books can be chosen?**

Since each book report has a different format, order is important. You must find the number of permutations of 20 objects taken 4 at a time.

$$P(n, r) = \frac{n!}{(n-r)!}$$ Permutation formula

$$P(20, 4) = \frac{20!}{(20-4)!}$$ $n = 20$, $r = 4$

$$= \frac{20!}{16!}$$ Simplify.

$$= \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot \cancel{16} \cdot \cancel{15} \cdot \ldots \cdot \cancel{1}}{\cancel{16} \cdot \cancel{15} \cdot \ldots \cdot \cancel{1}}$$ Divide by common factors.

$$= 116{,}280$$

Books for the book reports can be chosen 116,280 ways.

Exercises

Evaluate each expression.

1. $P(6, 3)$ **2.** $P(8, 5)$ **3.** $P(9, 4)$ **4.** $P(11, 6)$

How many different ways can the letters of each word be arranged?

5. MOM **6.** MONDAY **7.** STEREO

8. SCHOOL The high school chorus has been practicing 12 songs, but there is time for only 5 of them at the spring concert. How may different orderings of 5 songs are possible?

Lesson 12-2

NAME _____ DATE_____ PERIOD _____

12-2 Study Guide and Intervention *(continued)*

Permutations and Combinations

Combinations An arrangement or selection of objects in which order is *not* important is called a combination.

Combinations	The number of combinations of n distinct objects taken r at a time is given by $C(n, r) = \frac{n!}{(n-r)!r!}$.

Example 1 **SCHOOL How many groups of 4 students can be selected from a class of 20?**

Since the order of choosing the students is not important, you must find the number of combinations of 20 students taken 4 at a time.

$$C(n, r) = \frac{n!}{(n-r)!r!} \quad \text{Combination formula}$$

$$C(20, 4) = \frac{20!}{(20-4)!4!} \quad n = 20,\ r = 4$$

$$= \frac{20!}{16!4!} \text{ or } 4845$$

There are 4845 possible ways to choose 4 students.

Example 2 **In how many ways can you choose 1 vowel and 2 consonants from a set of 26 letter tiles? (Assume there are 5 vowels and 21 consonants.)**

By the Fundamental Counting Principle, you can multiply the number of ways to select one vowel and the number of ways to select 2 consonants. Only the letters chosen matter, not the order in which they were chosen, so use combinations.

$C(5, 1)$ One of 5 vowels are drawn.

$C(21, 2)$ Two of 21 consonants are drawn.

$$C(5, 1) \cdot C(21, 2) = \frac{5!}{(5-1)!1!} \cdot \frac{21!}{(21-2)!2!} \quad \text{Combination formula}$$

$$= \frac{5!}{4!} \cdot \frac{21!}{19!2!} \quad \text{Subtract.}$$

$$= 5 \cdot 210 \text{ or } 1050 \quad \text{Simplify.}$$

There are 1050 combinations of 1 vowel and 2 consonants.

Exercises

Evaluate each expression.

1. $C(5, 3)$ **2.** $C(7, 4)$ **3.** $C(15, 7)$ **4.** $C(10, 5)$

5. PLAYING CARDS From a standard deck of 52 cards, in how many ways can 5 cards be drawn?

6. HOCKEY How many hockey teams of 6 players can be formed from 14 players without regard to position played?

7. COMMITTEES From a group of 10 men and 12 women, how many committees of 5 men and 6 women can be formed?

NAME _____ DATE_____ PERIOD _____

12-2 Skills Practice

Permutations and Combinations

Evaluate each expression.

1. $P(6, 3)$ **2.** $P(8, 2)$ **3.** $P(2, 1)$

4. $P(3, 2)$ **5.** $P(10, 4)$ **6.** $P(5, 5)$

7. $C(2, 2)$ **8.** $C(5, 3)$ **9.** $C(4, 1)$

10. $C(8, 7)$ **11.** $C(3, 2)$ **12.** $C(7, 4)$

Determine whether each situation involves a *permutation* or a *combination*. Then find the number of possibilities.

13. seating 8 students in 8 seats in the front row of the school auditorium

14. introducing the 5 starting players on the Woodsville High School basketball team at the beginning of the next basketball game

15. checking out 3 library books from a list of 8 books for a research paper

16. choosing 2 movies to rent from 5 movies

17. the first-, second-, and third-place finishers in a race with 10 contestants

18. electing 4 candidates to a municipal planning board from a field of 7 candidates

19. choosing 2 vegetables from a menu that offers 6 vegetable choices

20. an arrangement of the letters in the word *rhombus*

21. selecting 2 of 8 choices of orange juice at a store

22. placing a red rose bush, a yellow rose bush, a white rose bush, and a pink rose bush in a row in a planter

23. selecting 2 of 9 kittens at an animal rescue shelter

24. an arrangement of the letters in the word *isosceles*

NAME _____ DATE_____ PERIOD _____

12-2 Practice

Permutations and Combinations

Evaluate each expression.

1. $P(8, 6)$ **2.** $P(9, 7)$ **3.** $P(3, 3)$

4. $P(4, 3)$ **5.** $P(4, 1)$ **6.** $P(7, 2)$

7. $C(8, 2)$ **8.** $C(11, 3)$ **9.** $C(20, 18)$

10. $C(9, 9)$ **11.** $C(3, 1)$ **12.** $C(9, 3) \cdot C(6, 2)$

Determine whether each situation involves a *permutation* or a *combination*. Then find the number of possibilities.

13. selecting a 4-person bobsled team from a group of 9 athletes

14. an arrangement of the letters in the word *Canada*

15. arranging 4 charms on a bracelet that has a clasp, a front, and a back

16. selecting 3 desserts from 10 desserts that are displayed on a dessert cart in a restaurant

17. an arrangement of the letters in the word *annually*

18. forming a 2-person sales team from a group of 12 salespeople

19. making 5-sided polygons by choosing any 5 of 11 points located on a circle to be the vertices

20. seating 5 men and 5 women alternately in a row, beginning with a woman

21. STUDENT GROUPS Farmington High is planning its academic festival. All math classes will send 2 representatives to compete in the math bowl. How many different groups of students can be chosen from a class of 16 students?

22. PHOTOGRAPHY A photographer is taking pictures of a bride and groom and their 6 attendants. If she takes photographs of 3 people in a group, how many different groups can she photograph?

23. AIRLINES An airline is hiring 5 flight attendants. If 8 people apply for the job, how many different groups of 5 attendants can the airline hire?

24. SUBSCRIPTIONS A school librarian would like to buy subscriptions to 7 new magazines. Her budget, however, will allow her to buy only 4 new subscriptions. How many different groups of 4 magazines can she choose from the 7 magazines?

NAME _____ DATE_____ PERIOD _____

12-2 Word Problem Practice

Permutations and Combinations

1. WAITING IN LINE When the 12 students in Mr. Jaybird's class go to lunch, they form a single file line. Does forming a line involve a permutation or a combination of the students?

2. ART Isabel needs to select three different colors of construction paper to make a flag for a school project. She can choose from a selection of 15 different colors. In how many ways can she pick her colors?

3. SUDOKU A popular game called "Sudoku" involves square arrays of numbers. In a game of Sudoku, every entry is an integer between 1 and 9, inclusive. No number appears twice in any row or column.

7	1	8	6	9	4	2	3	5
9	2	5	7	3	1	6	4	8
4	6	3	8	5	2	7	9	1
5	9	2	1	7	3	4	8	6
8	3	1	4	6	5	9	2	7
9	7	4	2	8	9	5	1	3
3	4	9	5	1	7	8	6	2
2	8	7	3	4	6	1	5	9
1	5	6	9	2	8	3	7	4

For a game of Sudoku, how many different possibilities are there for the first row of numbers?

4. NAMES Hannah is curious to know how many different 6 letter sequences she can make using each of the letters of her name exactly once. For example, "HANNAH," "AAHHNN," and "NAHNAH" are all possible sequences. How many total sequences are possible?

METEORITES For Exercises 5 and 6, use the following information.

Over the course of several years, Kendra managed to collect 7 meteorites. Each one is unique.

5. For a school science fair, Kendra displays her meteorites in a row. How many ways are there to order the meteorites?

6. She decides to trade three of her meteorites for a telescope after the fair. How many ways can she pick out 3 meteorites from her collection?

Lesson 12-2

NAME _____ DATE_____ PERIOD _____

12-2 Enrichment

Combinations and Pascal's Triangle

Pascal's triangle is a special array of numbers invented by Blaise Pascal (1623–1662). The values in Pascal's triangle can be found using the combinations shown below.

1. Evaluate the expression in each cell of the triangle.

		C(1,0)	C(1,1)		
	C(2,0)	C(2,1)	C(2,2)		
C(3,0)	C(3,1)	C(3,2)	C(3,3)		
C(4,0)	C(4,1)	C(4,2)	C(4,3)	C(4,4)	
C(5,0)	C(5,1)	C(5,2)	C(5,3)	C(5,4)	C(5,5)

2. The pattern shows the relationship between $C(n, r)$ and Pascal's triangle. In general, it is true that $C(n, r) + C(n, r + 1) = C(n + 1, r + 1)$. Complete the proof of this property. In each step, the denominator has been given.

$$C(n, r) + C(n, r + 1) = \frac{\quad}{r!(n - r)!} + \frac{\quad}{(r + 1)!(n - r - 1)!}$$

$$= \frac{\quad}{r!(n - r)!(r + 1)} + \frac{\quad}{(r + 1)!(n - r - 1)!(n - r)}$$

$$= \frac{\quad}{(r + 1)!(n - r)!} + \frac{\quad}{(r + 1)!(n - r)!}$$

$$= \frac{\quad}{(r + 1)!(n - r)!}$$

$$= \frac{\quad}{(r + 1)!(n - r)!}$$

$$= \frac{\quad}{(r + 1)!(n - r)!}$$

$$= \frac{\quad}{(r + 1)![(n + 1) - (r + 1)]!}$$

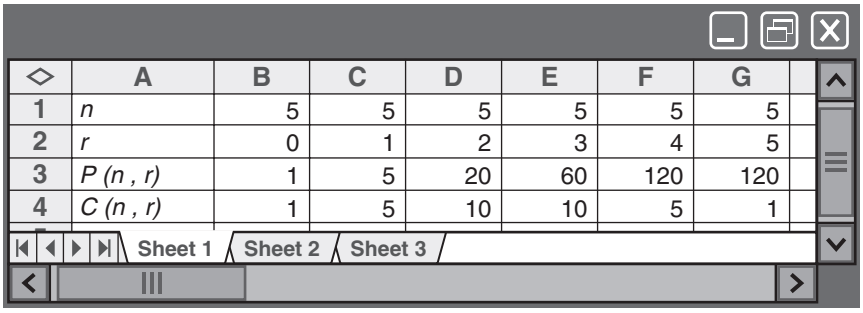
$$= C(n + 1, r + 1)$$

NAME _____ DATE_____ PERIOD _____

12-2 Spreadsheet Activity

Permutations and Combinations

You have learned the formulas for the number of permutations of n objects taken r at a time, $P(n, r)$, and the number of combinations of n objects taken r at a time, $C(n, r)$. You are going to set up a spreadsheet like the one shown below to perform analyses of these functions.



	A	B	C	D	E	F	G
1	*n*	5	5	5	5	5	5
2	*r*	0	1	2	3	4	5
3	*P (n , r)*	1	5	20	60	120	120
4	*C (n , r)*	1	5	10	10	5	1

Sheet 1 Sheet 2 Sheet 3

In the spreadsheet, the values in row 1 represent n, the values in row 2 represent r, and the formulas for $P(n, r)$ and $C(n, r)$ are in rows 3 and 4, respectively.

The formula to calculate $P(n, r)$ is =FACT(B1)/FACT(B1-B2).

FACT is a special function from the function list and should not be entered from the letters on the keyboard. Enter the formula in B3. Then drag the cursor across the row to copy the formula into cells C3 through G3.

The formula for $C(n, r)$ is =FACT(B1)/(FACT(B1−B2)*FACT(B2)) and should be entered in cell B4. Copy the formula into cells C4 through G4.

Exercises

1. Compare the values of $P(n, r)$ and $C(n, r)$ for $n = 5$ and $r = 0$ through 5, as well as for two other choices of n and r.

2. Several identities hold for $P(n, r)$ and $C(n, r)$. Use the spreadsheet to verify the following identities by finding three examples of each.

a. $P(n, n) = P(n, n - 1)$

b. $C(n + 1, r) = C(n, r - 1) + C(n, r)$

c. $C(n, 0) + C(n, 1) + C(n, 2) + \ldots + C(n, n) = 2^n$

Lesson 12-2

NAME _____ DATE_____ PERIOD _____

12-3 Lesson Reading Guide

Probability

Get Ready for the Lesson

Read the introduction to Lesson 12-3 in your textbook.

What is the probability that a person will *not* be struck by lightning in a given year?

Read the Lesson

1. Indicate whether each of the following statements is *true* or *false*.

a. If an event can never occur, its probability is a negative number.

b. If an event is certain to happen, its probability is 1.

c. If an event can succeed in s ways and fail in f ways, then the probability of success is $\frac{s}{f}$.

d. If an event can succeed in s ways and fail in f ways, then the odds against the event are $s:f$.

e. A probability distribution is a function in which the domain is the sample space of an experiment.

2. A weather forecast says that the chance of rain tomorrow is 40%.

a. Write the probability that it will rain tomorrow as a fraction in lowest terms.

b. Write the probability that it will not rain tomorrow as a fraction in lowest terms.

c. What are the odds in favor of rain?

d. What are the odds against rain?

3. Refer to the table in Example 4 on page 646 in your textbook.

a. What other sum has the same probability as a sum of 11?

b. What are the odds of rolling a sum of 8?

c. What are the odds against rolling a sum of 9?

Remember What You Learned

4. A good way to remember something is to explain it to someone else. Suppose that your friend Roberto is having trouble remembering the difference between probability and odds. What would you tell him to help him remember this easily?

NAME _____ DATE_____ PERIOD _____

12-3 Study Guide and Intervention

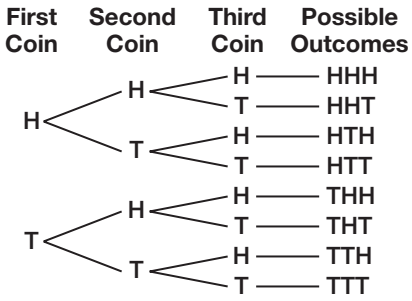
Probability

Probability and Odds In probability, a desired outcome is called a **success**; any other outcome is called a **failure**.

Probability of Success and Failure	If an event can succeed in s ways and fail in f ways, then the probabilities of success, $P(S)$, and of failure, $P(F)$, are as follows. $P(S) = \frac{s}{s+f}$ and $P(F) = \frac{f}{s+f}$.
Definition of Odds	If an event can succeed in s ways and fail in f ways, then the odds of success and of failure are as follows. Odds of success $= s:f$ Odds of failure $= f:s$

Example 1 **When 3 coins are tossed, what is the probability that at least 2 are heads?**

You can use a tree diagram to find the sample space.



Of the 8 possible outcomes, 4 have at least 2 heads. So the probability of tossing at least 2 heads is $\frac{4}{8}$ or $\frac{1}{2}$.

Example 2 **What is the probability of picking 4 fiction books and 2 biographies from a best-seller list that consists of 12 fiction books and 6 biographies?**

By the Fundamental Counting Principle, the number of successes is $C(12, 4) \cdot C(6, 2)$. The total number of selections, $s + f$, of 6 books is $C(18, 6)$.

$P(4 \text{ fiction, } 2 \text{ biography}) = \frac{C(12, 4) \cdot C(6, 2)}{C(18, 6)}$ or about 0.40

The probability of selecting 4 fiction books and 2 biographies is about 40%.

Exercises

Find the odds of an event occurring, given the probability of the event.

1. $\frac{3}{7}$ **2.** $\frac{4}{5}$ **3.** $\frac{2}{13}$ **4.** $\frac{1}{15}$

Find the probability of an event occurring, given the odds of the event.

5. 10:1 **6.** 2:5 **7.** 4:9 **8.** 8:3

One bag of candy contains 15 red candies, 10 yellow candies, and 6 green candies. Find the probability of each selection.

9. picking a red candy

10. not picking a yellow candy

11. picking a green candy

12. not picking a red candy

NAME _____ DATE_____ PERIOD _____

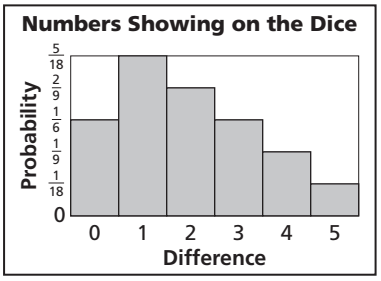
12-3 Study Guide and Intervention *(continued)*

Probability

Probability Distributions A **random variable** is a variable whose value is the numerical outcome of a random event. A **probability distribution** for a particular random variable is a function that maps the sample space to the probabilities of the outcomes in the sample space.

Example **Suppose two dice are rolled. The table and the relative-frequency histogram show the distribution of the absolute value of the difference of the numbers rolled. Use the graph to determine which outcome is the most likely. What is its probability?**

Difference	0	1	2	3	4	5
Probability	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{2}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{18}$



The greatest probability in the graph is $\frac{5}{18}$.
The most likely outcome is a difference of 1 and its probability is $\frac{5}{18}$.

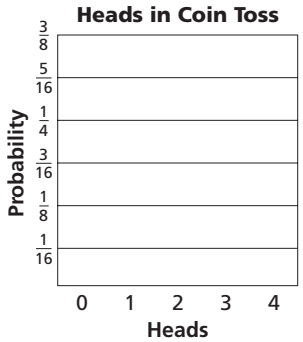
Exercises

Four coins are tossed.

1. Complete the table below to show the probability distribution of the number of heads.

Number of Heads	0	1	2	3	4
Probability					

2. Make relative-frequency distribution of the data.



NAME _____ DATE_____ PERIOD _____

12-3 Skills Practice

Probability

Ahmed is posting 2 photographs on his website. He has narrowed his choices to 4 landscape photographs and 3 portraits. If he chooses the two photographs at random, find the probability of each selection.

1. P(2 portrait)

2. P(2 landscape)

3. P(1 of each)

The Carubas have a collection of 28 video movies, including 12 westerns and 16 science fiction. Elise selects 3 of the movies at random to bring to a sleep-over at her friend's house. Find the probability of each selection.

4. P(3 westerns)

5. P(3 science fiction)

6. P(1 western and 2 science fiction)

7. P(2 westerns and 1 science fiction)

8. P(3 comedy)

9. P(2 science fiction and 2 westerns)

For Exercises 10–13, use the chart that shows the class and gender statistics for the students taking an Algebra 1 or Algebra 2 class at La Mesa High School.

If a student taking Algebra 1 or Algebra 2 is selected at random, find each probability. Express as decimals rounded to the nearest thousandth.

Class/Gender	Number
Freshman/Male	95
Freshman/Female	101
Sophomore/Male	154
Sophomore/Female	145
Junior/Male	100
Junior/Female	102

10. P(sophomore/female)

11. P(junior/male)

12. P(freshman/male)

13. P(freshman/female)

Find the odds of an event occurring, given the probability of the event.

14. $\frac{5}{8}$

15. $\frac{2}{7}$

16. $\frac{3}{5}$

17. $\frac{1}{10}$

18. $\frac{5}{6}$

19. $\frac{5}{12}$

Find the probability of an event occurring, given the odds of the event.

20. 2:1

21. 8:9

22. 4:1

23. 1:9

24. 2:7

25. 5:9

NAME _____ DATE_____ PERIOD _____

12-3 Practice

Probability

A bag contains 1 green, 4 red, and 5 yellow balls. Two balls are selected at random. Find the probability of each selection.

1. P(2 red)

2. P(1 red and 1 yellow)

3. P(1 green and 1 yellow)

4. P(2 green)

5. P(2 red and 1 yellow)

6. P(1 red and 1 green)

A bank contains 3 pennies, 8 nickels, 4 dimes, and 10 quarters. Two coins are selected at random. Find the probability of each selection.

7. P(2 pennies)

8. P(2 dimes)

9. P(1 nickel and 1 dime)

10. P(1 quarter and 1 penny)

11. P(1 quarter and 1 nickel)

12. P(2 dimes and 1 quarter)

Henrico visits a home decorating store to choose wallpapers for his new house. The store has 28 books of wallpaper samples, including 10 books of WallPride samples and 18 books of Deluxe Wall Coverings samples. The store will allow Henrico to bring 4 books home for a few days so he can decide which wallpapers he wants to buy. If Henrico randomly chooses 4 books to bring home, find the probability of each selection.

13. P(4 WallPride)

14. P(2 WallPride and 2 Deluxe)

15. P(1 WallPride and 3 Deluxe)

16. P(3 WallPride and 1 Deluxe)

For Exercises 17–20, use the table that shows the range of verbal SAT scores for freshmen at a small liberal arts college. If a freshman student is chosen at random, find each probability. Express as decimals rounded to the nearest thousandth.

Range	400–449	450–499	500–549	550–559	600–649	650+
Number of Students	129	275	438	602	620	412

17. P(400–449)

18. P(550–559)

19. P(at least 650)

Find the odds of an event occurring, given the probability of the event.

20. $\frac{4}{11}$

21. $\frac{12}{13}$

22. $\frac{5}{99}$

23. $\frac{1}{1000}$

24. $\frac{5}{16}$

25. $\frac{3}{95}$

26. $\frac{9}{70}$

27. $\frac{8}{15}$

Find the probability of an event occurring, given the odds of the event.

28. 2:23

29. 2:5

30. 15:1

31. 9:7

32. 11:14

33. 1000:1

34. 12:17

35. 8:13

NAME _____ DATE_____ PERIOD _____

12-3 Word Problem Practice

Probability

1. ART The letters "A", "R", and "T" are written on three different pieces of paper. The pieces of paper are then put in a bag and mixed up. Logan picks each letter without looking and places them side by side. What is the probability that the letters spell "ART"?

2. AGE There are 24 students in Miss Mason's third grade class, all born on different days. Eleven students are boys. In the morning, the classroom is empty. One student arrives followed by another. What is the probability that when the first two students arrive, one is a boy and the other a girl?

3. DICE Jamal rolls two six-sided dice, one after the other. What is the probability that the second die shows a number larger than the first die?

4. LANGUAGES Noah cannot decide whether to learn French, German, Italian, Russian, or Chinese. He assigns each language a different number from 0 to 4. He then takes four fair coins and flips them. He decided to take the language corresponding to the number of coins that come up heads. Does Noah's method for choosing a language give each language the same chance of being chosen? Explain.

ICE CREAM For Exercises 5-7, use the following information.

A survey of the students in Mr. Orr's fifth grade class asked each student to name their favorite flavor of ice cream. The results are shown in the table below.

Flavor	Number of Students
Vanilla	10
Chocolate	9
Butternut	5
Strawberry	4
Banana	1
Coffee	1

5. A student from Mr. Orr's class is selected at random. What is the probability that the student's favorite flavor of ice cream is chocolate?

6. A student from Mr. Orr's class is selected at random. What is the probability that the student's favorite flavor of ice cream is banana?

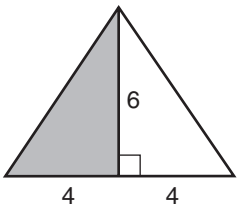
7. A student from Mr. Orr's class is selected at random. Is it more likely that the student prefers either butternut or strawberry or that the student prefers either chocolate or banana?

NAME _____ DATE_____ PERIOD _____

12-3 Enrichment

Geometric Probability

If a dart, thrown at random, hits the triangular board shown at the right, what is the chance that it will hit the shaded region? This chance, also called a probability, can be determined by comparing the area of the shaded region to the area of the board. This ratio indicates what fraction of the tosses should hit in the shaded region.



$$\frac{\text{area of shaded region}}{\text{area of triangular board}} = \frac{\frac{1}{2}(4)(6)}{\frac{1}{2}(8)(6)}$$

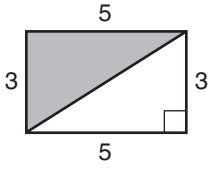
$$= \frac{12}{24} \text{ or } \frac{1}{2}$$

In general, if S is a subregion of some region R, then the probability, $P(S)$, that a point, chosen at random, belongs to subregion S is given by the following.

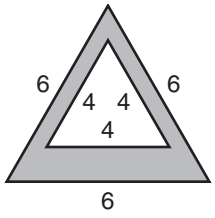
$$P(S) = \frac{\text{area of subregion } S}{\text{are of region } R}$$

Find the probability that a point, chosen at random, belongs to the shaded subregions of the following regions.

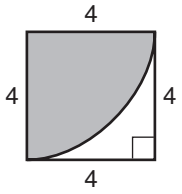
1.



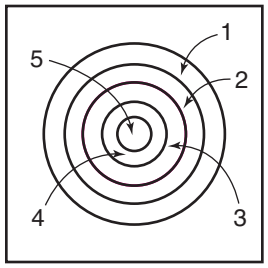
2.



3.



The dart board shown at the right has 5 concentric circles whose centers are also the center of the square board. Each side of the board is 38 cm, and the radii of the circles are 2 cm, 5 cm, 8 cm, 11 cm, and 14 cm. A dart hitting within one of the circular regions scores the number of points indicated on the board, while a hit anywhere else scores 0 points. If a dart, thrown at random, hits the board, find the probability of scoring the indicated number of points.



4. 0 points

5. 1 point

6. 2 points

7. 3 points

8. 4 points

9. 5 points

NAME _____ DATE_____ PERIOD _____

12-4 Lesson Reading Guide

Multiplying Probabilities

Get Ready for the Lesson

Read the introduction to Lesson 12-4 in your textbook.

Write the probability that Yao Ming made a field goal shot during the 2004–05 season as a fraction in lowest terms. (Your answer should not include a decimal.)

Read the Lesson

1. A bag contains 4 yellow balls, 5 red balls, 1 white ball, and 2 black balls. *A* ball is drawn from the bag and is not replaced. *A* second ball is drawn.

a. Let Y be the event "first ball is yellow" and B be the event "second ball is black." Are these events *independent* or *dependent*?

b. Tell which formula you would use to find the probability that the first ball is yellow and the second ball is black.

A. $P(Y \text{ and } B) = \dfrac{P(Y)}{P(Y) + P(B)}$

B. $P(Y \text{ and } B) = P(Y) \cdot P(B)$

C. $P(Y \text{ and } B) = P(Y) \cdot P(B \text{ following } Y)$

c. Which equation shows the correct calculation of this probability?

A. $\frac{1}{3} + \frac{2}{11} = \frac{17}{33}$ **B.** $\frac{1}{3} \cdot \frac{2}{11} = \frac{2}{33}$

C. $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$ **D.** $\frac{1}{3} \cdot \frac{1}{6} = \frac{1}{18}$

d. Which equation shows the correct calculation of the probability that if three balls are drawn in succession without replacement, all three will be red?

A. $\frac{5}{12} \cdot \frac{5}{12} \cdot \frac{5}{12} = \frac{125}{1728}$ **B.** $\frac{5}{12} \cdot \frac{4}{11} \cdot \frac{3}{10} = \frac{1}{22}$

C. $\frac{5}{12} + \frac{4}{11} + \frac{3}{10} = \frac{713}{660}$

Remember What You Learned

2. Some students have trouble remembering a lot of formulas, so they try to keep the number of formulas they have to know to a minimum. Can you learn just one formula that will allow you to find probabilities for both independent and dependent events? Explain your reasoning.

NAME _____ DATE_____ PERIOD _____

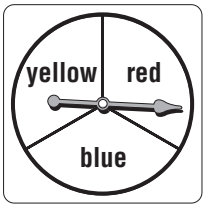
12-4 Study Guide and Intervention

Multiplying Probabilities

Probability of Independent Events

Probability of Two Independent Events	If two events, A and B, are independent, then the probability of both occurring is $P(A \text{ and } B) = P(A) \cdot P(B)$.

Example **In a board game each player has 3 different-colored markers. To move around the board the player first spins a spinner to determine which piece can be moved. He or she then rolls a die to determine how many spaces that colored piece should move. On a given turn what is the probability that a player will be able to move the yellow piece more than 2 spaces?**



Let A be the event that the spinner lands on yellow, and let B be the event that the die shows a number greater than 2. The probability of A is $\frac{1}{3}$, and the probability of B is $\frac{2}{3}$.

$P(A \text{ and } B) = P(A) \cdot P(B)$ Probability of independent events

$= \frac{1}{3} \cdot \frac{2}{3}$ or $\frac{2}{9}$ Substitute and multiply.

The probability that the player can move the yellow piece more than 2 spaces is $\frac{2}{9}$.

Exercises

A die is rolled 3 times. Find the probability of each event.

1. a 1 is rolled, then a 2, then a 3

2. a 1 or a 2 is rolled, then a 3, then a 5 or a 6

3. 2 odd numbers are rolled, then a 6

4. a number less than 3 is rolled, then a 3, then a number greater than 3

5. A box contains 5 triangles, 6 circles, and 4 squares. If a figure is removed, replaced, and a second figure is picked, what is the probability that a triangle and then a circle will be picked?

6. A bag contains 5 red marbles and 4 white marbles. A marble is selected from the bag, then replaced, and a second selection is made. What is the probability of selecting 2 red marbles?

7. A jar contains 7 lemon jawbreakers, 3 cherry jawbreakers, and 8 rainbow jawbreakers. What is the probability of selecting 2 lemon jawbreakers in succession providing the jawbreaker drawn first is then replaced before the second is drawn?

NAME _____ DATE_____ PERIOD _____

12-4 Study Guide and Intervention *(continued)*

Multiplying Probabilities

Probability of Dependent Events

Probability of Two Dependent Events	If two events, A and B, are dependent, then the probability of both events occurring is $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$.

Example 1 **There are 7 dimes and 9 pennies in a wallet. Suppose two coins are to be selected at random, without replacing the first one. Find the probability of picking a penny and then a dime.**

Because the coin is not replaced, the events are dependent.

Thus, $P(A \text{ and } B) = P(\text{A}) \cdot P(B \text{ following } A)$.

$P(\text{penny, then dime}) = P(\text{penny}) \cdot P(\text{dime following penny})$

$$\frac{9}{16} \cdot \frac{7}{15} = \frac{21}{80}$$

The probability is $\frac{21}{80}$ or about 0.26

Example 2 **What is the probability of drawing, without replacement, 3 hearts, then a spade from a standard deck of cards?**

Since the cards are not replaced, the events are dependent. Let H represent a heart and S represent a spade.

$P(\text{H, H, H, S}) = P(\text{H}) \cdot P(\text{H following H}) \cdot P(\text{H following 2 Hs}) \cdot P(\text{S following 3 Hs})$

$$= \frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} \cdot \frac{13}{49} \text{ or about } 0.003$$

The probability is about 0.003 of drawing 3 hearts, then a spade.

Exercises

Find each probability.

1. The cup on Sophie's desk holds 4 red pens and 7 black pens. What is the probability of her selecting first a black pen, then a red one?

2. What is the probability of drawing two cards showing odd numbers from a set of cards that show the first 20 counting numbers if the first card is not replaced before the second is chosen?

3. There are 3 quarters, 4 dimes, and 7 nickels in a change purse. Suppose 3 coins are selected without replacement. What is the probability of selecting a quarter, then a dime, and then a nickel?

4. A basket contains 4 plums, 6 peaches, and 5 oranges. What is the probability of picking 2 oranges, then a peach if 3 pieces of fruit are selected at random?

5. A photographer has taken 8 black and white photographs and 10 color photographs for a brochure. If 4 photographs are selected at random, what is the probability of picking first 2 black and white photographs, then 2 color photographs?

NAME _____ DATE_____ PERIOD _____

12-4 Skills Practice

Multiplying Probabilities

A die is rolled twice. Find each probability.

1. P(5, then 6)

2. P(no 2s)

3. P(two 1s)

4. P(any number, then not 5)

5. P(4, then not 6)

6. P(not 1, then not 2)

A board game uses a set of 6 different cards. Each card displays one of the following figures: a star, a square, a circle, a diamond, a rectangle, or a pentagon. The cards are placed face down, and a player chooses two cards. Find each probability.

7. P(circle, then star), if no replacement occurs

8. P(diamond, then square), if replacement occurs

9. P(2 polygons), if replacement occurs

10. P(2 polygons), if no replacement occurs

11. P(circle, then hexagon), if no replacement occurs

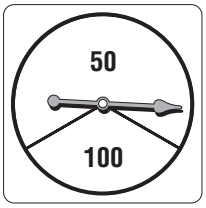
Determine whether the events are *independent* or *dependent*. Then find each probability.

12. A mixed box of herbal teabags contains 2 lemon teabags, 3 orange-mango teabags, 3 chamomile teabags, and 1 apricot-ginger teabag. Kevin chooses 2 teabags at random to bring to work with him. What is the probability that he first chooses a lemon teabag and then a chamomile teabag?

13. The chart shows the selection of olive oils that Hasha finds in a specialty foods catalog. If she randomly selects one type of oil, then randomly selects another, different oil, what is the probability that both selections are domestic, first cold pressed oils?

Type of Oil	Domestic	Imported
Pure	2	5
Cold Pressed	4	8
First Cold Pressed	7	15

For Exercises 14 and 15, two thirds of the area of the spinner earns you 50 points. Suppose you spin the spinner twice.



14. Sketch a tree diagram showing all of the possibilities. Use it to find the probability of spinning 50 points, then 100 points.

15. What is the probability that you get 100 points on each spin?

NAME _____ DATE_____ PERIOD _____

12-4 Practice

Multiplying Probabilities

A die is rolled three times. Find each probability.

1. P(three 4s)

2. P(no 4s)

3. P(2, then 3, then 1)

4. P(three different even numbers)

5. P(any number, then 5, then 5)

6. P(even number, then odd number, then 1)

There are 3 nickels, 2 dimes, and 5 quarters in a purse. Three coins are selected in succession at random. Find the probability.

7. P(nickel, then dime, then quarter), if no replacement occurs

8. P(nickel, then dime, then quarter), if replacement occurs

9. P(2 nickels, then 1 quarter), if no replacement occurs

10. P(3 dimes), if replacement occurs

11. P(3 dimes), if no replacement occurs

For Exercises 12 and 13, determine whether the events are *independent* or *dependent*. Then find each probability.

12. Serena is creating a painting. She wants to use 2 more colors. She chooses randomly from 6 shades of red, 10 shades of green, 4 shades of yellow, 4 shades of purple, and 6 shades of blue. What is the probability that she chooses 2 shades of green?

13. Kershel's mother is shopping at a bakery. The owner offers Kershel a cookie from a jar containing 22 chocolate chip cookies, 18 sugar cookies, and 15 oatmeal cookies. Without looking, Kershel selects one, drops it back in, and then randomly selects another. What is the probability that neither selection was a chocolate chip cookie?

14. METEOROLOGY The Fadeeva's are planning a 3-day vacation to the mountains. A long-range forecast reports that the probability of rain each day is 10%. Assuming that the daily probabilities of rain are independent, what is the probability that there is no rain on the first two days, but that it rains on the third day?

RANDOM NUMBERS For Exercises 15 and 16, use the following information.

Anita has a list of 20 jobs around the house to do, and plans to do 3 of them today. She assigns each job a number from 1 to 20, and sets her calculator to generate random numbers from 1 to 20, which can reoccur. Of the jobs, 3 are outside, and the rest are inside.

15. Sketch a tree diagram showing all of the possibilities that the first three numbers generated correspond to inside jobs or outside jobs. Use it to find the probability that the first two numbers correspond to inside jobs, and the third to an outside job.

16. What is the probability that the number generated corresponds to an outside job three times in a row?

NAME _____ DATE_____ PERIOD _____

12-4 Word Problem Practice

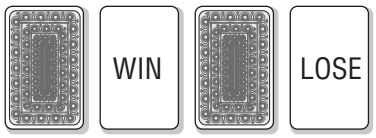
Multiplying Probabilities

1. BUSSING Portia and Quinton use the same bus stop when they go to work. They arrive at the bus stop independently of each other. The probability that Portia catches the 7:45 A.M. bus is $\frac{3}{5}$. The probability that Quinton catches the 7:45 A.M. bus is $\frac{1}{2}$. What is the probability that they both catch the 7:45 A.M. bus on the same day?

2. GOODY BAGS Ryan and Sophia are given goody bags with identical contents. The probability of reaching into either of these goody bags and pulling out a stick of chewing gum is $\frac{1}{10}$. Ryan and Sophia each reach into their own goody bag and randomly pull something out. What is the probability that they both pulled out a stick of chewing gum?

3. PENCILS A box of pencils contains 11 type 2 pencils and 5 type 3 pencils. Tara picks out a pencil from the box without looking and keeps it. Then, Upton picks out a pencil from the box without looking. What is the probability that Tara picks a type 2 pencil and Upton picks a type 3 pencil?

4. GUESSING GAMES Valerie is playing a guessing game. Four cards are placed face down before her. The hidden side of each card shows either the word "LOSE" or "WIN". Only one card is labeled "WIN". Valerie is given two chances to find the card labeled "WIN".



What is the probability that she does not pick the "win" card on her first try but does find it with her second?

WALLETS For Exercises 5 and 6, use the following information.

Wayne has 1 ten-dollar bill, 2 five-dollar bills, and 5 one-dollar bills in his wallet.

5. Wayne randomly chooses a bill from his wallet, puts it back, then picks another bill, and puts that one back too. What is the probability that both were five-dollar bills?

6. Wayne randomly pulls out a bill from his wallet, and then, without putting it back, randomly pulls a second bill from his wallet. He then puts both bills back into the wallet. What is the probability that both of the bills pulled out were five-dollar bills?

NAME _____ DATE_____ PERIOD _____

12-4 Enrichment

Conditional Probability

Suppose a pair of dice is thrown. It is known that the sum is greater than seven. Find the probability that the dice match.

The probability of an event given the occurrence of another event is called *conditional probability.* The conditional probability of event A, the dice match, given event B, their sum is greater than seven, is denoted $P(A/B)$.

There are 15 sums greater than seven and there are 36 possible pairs altogether.

$P(B) = \frac{15}{36}$

There are three matching pairs greater than seven.

$P(A \text{ and } B) = \frac{3}{36}$

$$P(A/B) = \frac{P(A \text{ and } B)}{P(B)}$$

$$P(A/B) = \frac{\frac{3}{36}}{\frac{15}{36}} \text{ or } \frac{1}{5}$$

The conditional probability is $\frac{1}{5}$.

A card is drawn from a standard deck of 52 and is found to be red. Given that event, find each of the following probabilities.

1. P(heart)

2. P(ace)

3. P(face card)

4. P(jack or ten)

5. P(six of spades)

6. P(six of hearts)

A sports survey taken at Stirers High School shows that 48% of the respondents liked soccer, 66% liked basketball, and 38% liked hockey. Also, 30% liked soccer and basketball, 22% liked basketball and hockey and 28% liked soccer and hockey. Finally, 12% liked all three sports. Find each of the following probabilities.

7. The probability Meg likes soccer if she likes basketball.

8. The probability Biff likes basketball if he likes soccer.

9. The probability Muffy likes hockey if she likes basketball.

10. The probability Greg likes hockey and basketball if he likes soccer.

NAME _____ DATE_____ PERIOD _____

12-5 Lesson Reading Guide

Adding Probabilities

Get Ready for the Lesson

Read the introduction to Lesson 12-5 in your textbook.

Why do the percentages shown on the bar graph add up to more than 100%?

Read the Lesson

1. Indicate whether the events in each pair are *inclusive* or *mutually exclusive*.

a. Q: drawing a queen from a standard deck of cards
D: drawing a diamond from a standard deck of cards

b. J: drawing a jack from a standard deck of cards
K: drawing a king from a standard deck of cards

2. Marla took a quiz on this lesson that contained the following problem.

Each of the integers from 1 through 25 is written on a slip of paper and placed in an envelope. If one slip is drawn at random, what is the probability that it is odd or a multiple of 5?

Here is Marla's work.

$$P(\text{odd}) = \frac{13}{25} \qquad P(\text{multiple of } 5) = \frac{5}{25} \text{ or } \frac{1}{5}$$

$$P(\text{odd or multiple of } 5) = P(\text{odd}) + P(\text{multiple of } 5)$$

$$= \frac{13}{25} + \frac{5}{25} = \frac{18}{25}$$

a. Why is Marla's work incorrect?

b. Show the corrected work.

Remember What You Learned

3. Some students have trouble remembering a lot of formulas, so they try to keep the number of formulas they have to know to a minimum. Can you learn just one formula that will allow you to find probabilities for both mutually exclusive and inclusive events? Explain your reasoning.

NAME _____ DATE_____ PERIOD _____

12-5 Study Guide and Intervention

Adding Probabilities

Mutually Exclusive Events Events that cannot occur at the same time are called mutually exclusive events.

Probability of Mutually Exclusive Events	If two events, A and B, are mutually exclusive, then $P(A \text{ or } B) = P(A) + P(B)$.

This formula can be extended to any number of mutually exclusive events.

Example 1 **To choose an afternoon activity, summer campers pull slips of paper out of a hat. Today there are 25 slips for a nature walk, 35 slips for swimming, and 30 slips for arts and crafts. What is the probability that a camper will pull a slip for a nature walk or for swimming?**

These are mutually exclusive events. Note that there is a total of 90 slips.

$$P(\text{nature walk or swimming}) = P(\text{nature walk}) + P(\text{swimming})$$
$$= \frac{25}{90} + \frac{35}{90} \text{ or } \frac{2}{3}$$

The probability of a camper's pulling out a slip for a nature walk or for swimming is $\frac{2}{3}$.

Example 2 **By the time one tent of 6 campers gets to the front of the line, there are only 10 nature walk slips and 15 swimming slips left. What is the probability that more than 4 of the 6 campers will choose a swimming slip?**

$$P(\text{more than 4 swimmers}) = P(5 \text{ swimmers}) + P(6 \text{ swimmers})$$
$$= \frac{C(10, 1) \cdot C(15, 5)}{C(25, 6)} + \frac{C(10, 0) \cdot C(15, 6)}{C(25, 6)}$$
$$\approx 0.2$$

The probability of more than 4 of the campers swimming is about 0.2.

Exercises

Find each probability.

1. A bag contains 45 dyed eggs: 15 yellow, 12 green, and 18 red. What is the probability of selecting a green or a red egg?

2. The letters from the words LOVE and LIVE are placed on cards and put in a box. What is the probability of selecting an L or an O from the box?

3. A pair of dice is rolled, and the two numbers are added. What is the probability that the sum is either a 5 or a 7?

4. A bowl has 10 whole wheat crackers, 16 sesame crackers, and 14 rye crisps. If a person picks a cracker at random, what is the probability of picking either a sesame cracker or a rye crisp?

5. An art box contains 12 colored pencils and 20 pastels. If 5 drawing implements are chosen at random, what is the probability that at least 4 of them are pastels?

NAME _____ DATE_____ PERIOD _____

12-5 Study Guide and Intervention *(continued)*

Adding Probabilities

Inclusive Events

Probability of Inclusive Events	If two events, A and B, are inclusive, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

Example **What is the probability of drawing a face card or a black card from a standard deck of cards?**

The two events are inclusive, since a card can be both a face card and a black card.

$$P(\text{face card or black card}) = P(\text{face card}) + P(\text{black card}) - P(\text{black face card})$$
$$= \frac{3}{13} + \frac{1}{2} - \frac{3}{26}$$
$$= \frac{8}{13} \text{ or about } 0.62$$

The probability of drawing either a face card or a black card is about 0.62

Exercises

Find each probability.

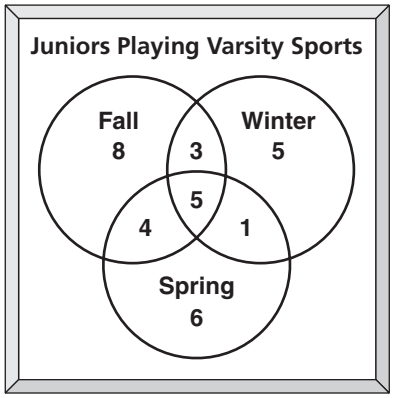
1. What is the probability of drawing a red card or an ace from a standard deck of cards?

2. Three cards are selected from a standard deck of 52 cards. What is the probability of selecting a king, a queen, or a red card?

3. The letters of the alphabet are placed in a bag. What is the probability of selecting a vowel or one of the letters from the word QUIZ?

4. A pair of dice is rolled. What is the probability that the sum is odd or a multiple of 3?

5. The Venn diagram at the right shows the number of juniors on varsity sports teams at Elmwood High School. Some athletes are on varsity teams for one season only, some athletes for two seasons, and some for all three seasons. If a varsity athlete is chosen at random from the junior class, what is the probability that he or she plays a fall or winter sport?



NAME _____ DATE_____ PERIOD _____

12-5 Skills Practice

Adding Probabilities

Eli has 10 baseball cards of 10 different players in his pocket. Three players are pitchers, 5 are outfielders, and 2 are catchers. If Eli randomly selects a card to trade, find each probability.

1. P(pitcher or outfielder) **2.** P(pitcher or catcher) **3.** P(outfielder or catcher)

A die is rolled. Find each probability.

4. P(5 or 6) **5.** P(at least a 3) **6.** P(less than 4)

Determine whether the events are *mutually exclusive* or *inclusive*. Then find the probability.

7. A die is rolled. What is the probability of rolling a 3 or a 4?

8. A die is rolled. What is the probability of rolling an even number or a 4?

9. A card is drawn from a standard deck of cards. What is the probability of drawing a king or a queen?

10. A card is drawn from a standard deck of cards. What is the probability of drawing a jack or a heart?

11. The sophomore class is selling Mother's Day plants to raise money. Susan's prize for being the top seller of plants is a choice of a book, a CD, or a video. She can choose from 6 books, 3 CDs, and 5 videos. What is the probability that Susan selects a book or a CD?

A spinner numbered 1−10 is spun. Find each probability.

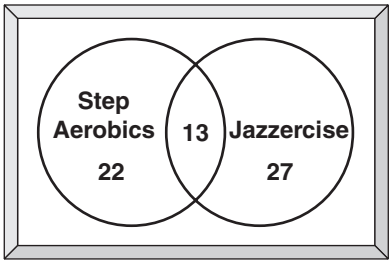
12. P(less than 5 or even) **13.** P(even or odd) **14.** P(prime or even)

Two cards are drawn from a standard deck of cards. Find each probability.

15. P(both red or both black) **16.** P(both aces or both red)

17. P(both 2s or both less than 5) **18.** P(both black or both less than 5)

For Exercises 19 and 20, use the Venn diagram that shows the number of participants in two different kinds of aerobic exercise classes that are offered at a health club. Determine each probability if a person is selected at random from the participants.



19. P(step aerobics or jazzercise, but not both)

20. P(step aerobics and jazzercise)

NAME _____ DATE_____ PERIOD _____

12-5 Practice

Adding Probabilities

An urn contains 7 white marbles and 5 blue marbles. Four marbles are selected without replacement. Find each probability.

1. P(4 white or 4 blue)

2. P(exactly 3 white)

3. P(at least 3 white)

4. P(fewer than 3 white)

5. P(3 white or 3 blue)

6. P(no white or no blue)

Jason and Maria are playing a board game in which three dice are tossed to determine a player's move. Find each probability.

7. P(two 5s)

8. P(three 5s)

9. P(at least two 5s)

10. P(no 5s)

11. P(one 5)

12. P(one 5 or two 5s)

Determine whether the events are *mutually exclusive* or *inclusive*. Then find the probability.

13. A clerk chooses 4 CD players at random for floor displays from a shipment of 24 CD players. If 15 of the players have a blue case and the rest have a red case, what is the probability of choosing 4 players with a blue case or 4 players with a red case?

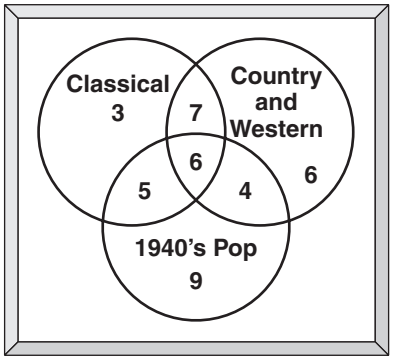
14. A department store employs 28 high school students, all juniors and seniors. Six of the 12 seniors are females and 12 of the juniors are males. One student employee is chosen at random. What is the probability of selecting a senior or a female?

15. A restaurant has 5 pieces of apple pie, 4 pieces of chocolate cream pie, and 3 pieces of blueberry pie. If Janine selects a piece of pie at random for dessert, what is the probability that she selects either apple or chocolate cream?

16. At a statewide meeting, there are 20 school superintendents, 13 principals, and 6 assistant principals. If one of these people is chosen at random, what is the probability that he or she is either a principal or an assistant principal?

17. An airline has one bank of 13 telephones at a reservations office. Of the 13 operators who work there, 8 take reservations for domestic flights and 5 take reservations for international flights. Seven of the operators taking domestic reservations and 3 of the operators taking international reservations are female. If an operator is chosen at random, what is the probability that the person chosen takes domestic reservations or is a male?

18. MUSIC Forty senior citizens were surveyed about their music preferences. The results are displayed in the Venn diagram. If a senior citizen from the survey group is selected at random, what is the probability that he or she likes only country and western music? What is the probability that he or she likes classical and/or country, but not 1940's pop?



NAME _____ DATE_____ PERIOD _____

12-5 Word Problem Practice

Adding Probabilities

1. PICK-UP When Tina's parents pick her up from school, there is a $\frac{1}{5}$ chance that she will be in the library, a $\frac{1}{2}$ chance that she will be on the playground, and a $\frac{3}{10}$ chance that she will be in her classroom. What is the probability that when Tina's parents pick her up, she is found in her classroom or on the playground?

2. TRAVEL John is randomly selected to be given a chance to win a new car. He must choose a red or yellow marble from a bag containing 1 red, 2 yellow, 10 green, and 12 blue marbles. What is the probability he will win the car?

3. DICE Alexis rolls two identical dice. What is the probability that the sum of the numbers rolled is odd? What is the probability that the sum of the numbers rolled is greater than 7? What is the probability that the sum of the numbers rolled is odd or greater than 7?

4. CLASSES At Jackson High School, 56 of the eleventh graders take physics and 70 of them take biology. There are 400 eleventh graders in total at the school. An eleventh grader is chosen at random from among all the eleventh graders at the high school. The probability that the selected student takes physics and biology is $\frac{11}{40}$. How many students at the high school take physics or biology?

PASSENGERS For Exercises 5 and 6, use the following information.

On an airplane flight, some passengers travel with carry-on luggage while others travel with a suitcase. Some passengers travel with carry-on luggage and a suitcase. Everyone travels with some form of luggage.

5. On one flight, there was no passenger with both carry-on luggage and a suitcase. On this flight are the events of picking a passenger with carry-on luggage and picking a passenger with a suitcase mutually exclusive?

6. On another flight, there are 120 passengers. Of those 120 passengers, 80 have carry-on luggage and 70 have a suitcase. What is the probability that a passenger has both carry-on luggage and a suitcase?

NAME _____ DATE_____ PERIOD _____

12-5 Enrichment

Probability and Tic-Tac-Toe

What would be the chances of winning at tic-tac-toe if it were turned into a game of pure chance? To find out, the nine cells of the tic-tac-toe board are numbered from 1 to 9 and nine chips (also numbered from 1 to 9) are put into a bag. Player A draws a chip at random and enters an *X* in the corresponding cell. Player B does the same and enters an *O*.

To solve the problem, assume that both players draw all their chips without looking and all *X* and *O* entries are made at the same time. There are four possible outcomes: a draw, A wins, B wins, and either A or B can win.

There are 16 arrangements that result in a draw. Reflections and rotations must be counted as shown below.

o x o	x o x	o o x
x o x 4	o o x 4	x x o 8
x o x	x x o	o x x

There are 36 arrangements in which either player may win because both players have winning triples.

x x x	x x x	x o x	x x x	x x x	x x o
o o o 4	x o x 4	x x x 4	x x o 8	o o o 8	x x x 8
x o x	o o o	o o o	o o o	x x o	o o o

In these 36 cases, A's chances of winning are $\frac{13}{40}$.

1. Find the 12 arrangements in which B wins and A cannot.

2. Below are 12 of the arrangements in which A wins and B cannot. Write the numbers to show the reflections and rotations for each arrangement. What is the total number?

o x o	x o x	x x x	x x x	x o o	x o o
x x x	o x o	x o o	o x o	x x x	x x o
o x o	x o x	x o o	o x o	o o x	o o x
x x o	x x x	x x x	x x x	x o o	x x o
o x x	o x o	x o o	x o o	x x x	o x o
o o x	o o x	o x o	o o x	o x o	x o x

3. There are $\frac{9!}{(5!4!)}$ different and equally probable distributions. Complete the chart to find the probability for a draw or for A or B to win.

Draw: $\frac{16}{126}$	= _____
A wins: _____	+ $\frac{13}{40}\left(\frac{36}{126}\right)$ = _____
B wins: _____	+ _____ = _____

NAME _____ DATE_____ PERIOD _____

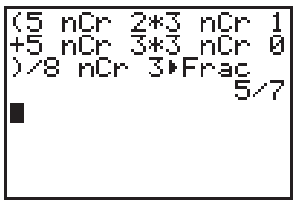
12-5 Graphing Calculator Activity

Probabilities

A graphing calculator can be used to perform calculations involving permutations, combinations, and probability.

Example 1 **There are 5 girls and 3 boys on a class committee. A subcommittee of 3 people is being chosen at random. What is the probability that the subcommittee will have at least 2 girls?**

P(at least 2 girls) = P(2 girls) + P(3 girls). Each probability is the product of the combinations of girls and boys divided by the combinations of all the students taken 3 at a time.

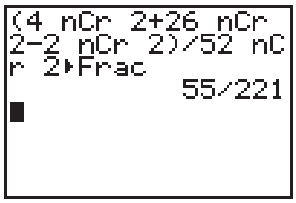


Keystrokes: (5 MATH ◀ 3 2 × 3 MATH ◀ 3 1 + 5 MATH ◀ 3 3 × 3 MATH ◀ 3 0) ÷ 8 MATH ◀ 3 3 MATH ENTER ENTER.

The probability that the subcommittee has at least 2 girls is $\frac{5}{7}$.

Example 2 **Two cards are randomly selected from a standard deck of cards. Find the probability that both cards are kings or that both cards are red.**

Since these events are mutually inclusive find the combinations of 4 kings taken 2 at a time plus 26 red cards taken 2 at a time minus 2 red kings taken 2 at a time divided by the combinations of 52 cards taken 2 at a time.



Keystrokes: (4 MATH ◀ 3 2 + 26 MATH ◀ 3 2 — 2 MATH ◀ 3 2) ÷ 52 MATH ◀ 3 2 MATH ENTER ENTER.

The probability of choosing 2 kings or two red cards is $\frac{55}{221}$.

Exercises

Find each probability.

1. There are 5 girls and 4 boys on the school publications committee. A group of 5 members is being chosen at random to attend a workshop on school newspapers. Find each probability.
a. at least 3 girls **b.** 4 girls or 4 boys **c.** at least 2 boys

2. Two cards are drawn from a standard deck of cards. Find each probability.
a. both queens or both black **b.** both kings or both aces **c.** both face cards or both black

3. Find the probability that a committee of 6 U.S. Representatives selected at random from 7 Democrats and 7 Republicans will have at least 3 Republicans on the committee.

4. Three CDs are randomly selected from a collection of 6 rock and 5 rap CDs. Find the probability that at least 2 are rock.

NAME _____ DATE_____ PERIOD _____

12-6 Lesson Reading Guide

Statistical Measures

Get Ready for the Lesson

Read the introduction to Lesson 12-6 in your textbook.

There is more than one way to give an "average" score for this test. Three measures of central tendency for these scores are 94, 76.5 and 73.9. Can you tell which of these is the mean, the median, and the mode without doing any calculations? Explain your answer.

Read the Lesson

1. Match each measure with one of the six descriptions of how to find measures of central tendency and variation.

a. median **b.** mode **c.** range

d. variance **e.** mean **f.** standard deviation

i. Find the most commonly occurring values or values in a set of data.

ii. Add the data and divide by the number of items.

iii. Find the mean of the squares of the differences between each value in the set of data and the mean.

iv. Find the difference between the largest and smallest values in the set of data.

v. Take the positive square root of the variance.

vi. If there is an odd number of items in a set of data, take the middle one. If there is an even number of items, add the two middle items and divide by 2.

Remember What You Learned

2. It is usually easier to remember a complicated procedure if you break it down into steps. Write the procedure for finding the standard deviation for a set of data in a series of brief, numbered steps.

NAME _____ DATE_____ PERIOD _____

12-6 Study Guide and Intervention

Statistical Measures

Measures of Central Tendency

	Use	When
Measures of Central Tendency	mean	the data are spread out and you want an average of values
	median	the data contain outliers
	mode	the data are tightly clustered around one or two values

Example **Find the mean, median, and mode of the following set of data: {42, 39, 35, 40, 38, 35, 45}.**

To find the mean, add the values and divide by the number of values.

$$\text{mean} = \frac{42 + 39 + 35 + 40 + 38 + 35 + 45}{7} \approx 39.14.$$

To find the median, arrange the values in ascending or descending order and choose the middle value. (If there is an even number of values, find the mean of the two middle values.) In this case, the median is 39.

To find the mode, take the most common value. In this case, the mode is 35.

Exercises

Find the mean, median, and mode of each set of data. Round to the nearest hundredth, if necessary.

1. {238, 261, 245, 249, 255, 262, 241, 245}

2. {9, 13, 8, 10, 11, 9, 12, 16, 10, 9}

3. {120, 108, 145, 129, 102, 132, 134, 118, 108, 142}

4. {68, 54, 73, 58, 63, 72, 65, 70, 61}

5. {34, 49, 42, 38, 40, 45, 34, 28, 43, 30}

6. The table at the right shows the populations of the six New England capitals. Which would be the most appropriate measure of central tendency to represent the data? Explain why and find that value.

Source: www.factfinder.census.gov

City	Population (rounded to the nearest 1000)
Augusta, ME	19,000
Boston, MA	589,000
Concord, NH	37,000
Hartford, CT	122,000
Montpelier, VT	8,000
Providence, RI	174,000

NAME _____ DATE_____ PERIOD _____

12-6 Study Guide and Intervention *(continued)*

Statistical Measures

Measures of Variation The *range* and the **standard deviation** measure how scattered a set of data is.

Standard Deviation	If a set of data consists of the n values $x_1, x_2, \ldots, x_n$ and has mean $\bar{x}$, then the standard deviation is given by $\sigma = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2}{n}}$.

The square of the standard deviation is called the **variance**.

Example **Find the variance and standard deviation of the data set {10, 9, 6, 9, 18, 4, 8, 20}.**

Step 1 Find the mean.

$$\bar{x} = \frac{10 + 9 + 6 + 9 + 18 + 4 + 8 + 20}{8} = 10.5$$

Step 2 Find the variance.

$$\sigma^2 = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2}{n}$$ Standard variance formula

$$= \frac{(10 - 10.5)^2 + (9 - 10.5)^2 + \ldots + (20 - 10.5)^2}{8}$$

$$= \frac{220}{8} \text{ or } 27.5$$

Step 3 Find the standard deviation.

$$\sigma = \sqrt{27.5}$$

$$\approx 5.2$$

The variance is 27.5 and the standard deviation is about 5.2.

Exercises

Find the variance and standard deviation of each set of data. Round to the nearest tenth.

1. {100, 89, 112, 104, 96, 108, 93}

2. {62, 54, 49, 62, 48, 53, 50}

3. {8, 9, 8, 8, 9, 7, 8, 9, 6}

4. {4.2, 5.0, 4.7, 4.5, 5.2, 4.8, 4.6, 5.1}

5. The table at the right lists the prices of ten brands of breakfast cereal. What is the standard deviation of the values to the nearest penny?

Price of 10 Brands of Breakfast Cereal	
$2.29	$3.19
$3.39	$2.79
$2.99	$3.09
$3.19	$2.59
$2.79	$3.29

NAME _____ DATE_____ PERIOD _____

12-6 Skills Practice

Statistical Measures

Find the variance and standard deviation of each set of data to the nearest tenth.

1. {32, 41, 35, 35, 46, 42}

2. {13, 62, 77, 24, 38, 19, 88}

3. {89, 99, 42, 16, 42, 71, 16}

4. {450, 400, 625, 225, 300, 750, 650, 625}

5. {17, 23, 65, 94, 33, 33, 33, 8, 57, 75, 44, 12, 11, 68, 39}

6. {7.2, 3.1, 3.8, 9.5, 8.3, 8.4}

7. {1.5, 2.5, 3.5, 4.5, 4.5, 5.5, 6.5, 7.5}

For Exercises 8 and 9, use the table that shows the profit in billions of dollars reported by U.S. manufacturers for the first quarter of the years from 1997 through 2001.

Year	1997	1998	1999	2000	2001
Seasonally-Adjusted Profit ($ billions)	$61.4	$75.6	$60.9	$78.5	$45.3

Source: U. S. Census Bureau

8. Find the mean and median of the data to the nearest tenth.

9. Which measure of central tendency best represents the data? Explain.

For Exercises 10 and 11, use the table that shows the percent of fourth grade students reading at or above the proficiency level in a nationally-administered reading assessment.

Year	1992	1994	1998	2000
Percent at or above proficiency level	29%	30%	31%	32%

Source: National Center for Education Statistics

10. Find the mean, median, and standard deviation of the data to the nearest tenth.

11. What do the statistics from Exercise 11 tell you about the data?

NAME _____ DATE_____ PERIOD _____

12-6 Practice

Statistical Measures

Find the variance and standard deviation of each set of data to the nearest tenth.

1. {47, 61, 93, 22, 82, 22, 37}

2. {10, 10, 54, 39, 96, 91, 91, 18}

3. {1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5}

4. {1100, 725, 850, 335, 700, 800, 950}

5. {3.4, 7.1, 8.5, 5.1, 4.7, 6.3, 9.9, 8.4, 3.6}

6. {2.8, 0.5, 1.9, 0.8, 1.9, 1.5, 3.3, 2.6, 0.7, 2.5}

7. HEALTH CARE Eight physicians with 15 patients on a hospital floor see these patients an average of 18 minutes a day. The 22 nurses on the same floor see the patients an average of 3 hours a day. As a hospital administrator, would you quote the mean, median, or mode as an indicator of the amount of daily medical attention the patients on this floor receive? Explain.

For Exercises 8–10, use the frequency table that shows the percent of public school teachers in the U. S. in 1999 who used computers or the Internet at school for various administrative and teaching activities.

Activity	Percent Using Computer or Internet
Create instructional materials	39
Administrative record keeping	34
Communicate with colleagues	23
Gather information for planning lessons	16
Multimedia classroom presentations	8
Access research and best practices for teaching	8
Communicate with parents or students	8
Access model lesson plans	6

Source: National Assessment of Educational Progress

8. Find the mean, median, and mode of the data.

9. Suppose you believe teachers use computers or the Internet too infrequently. Which measure would you quote as the "average?" Explain.

10. Suppose you believe teachers use computers or the Internet too often. Which measure would you quote as the "average?" Explain.

For Exercises 11 and 12, use the frequency table that shows the number of games played by 24 American League baseball players between opening day, 2001 and September 8, 2001.

No. of Games	Frequency
141	4
140	3
139	4
138	5
137	2
136	3
135	3

Source: Major League Baseball

11. Find the mean, median, mode, and standard deviation of the number of games played to the nearest tenth.

12. For how many players is the number of games within one standard deviation of the mean?

NAME _____ DATE_____ PERIOD _____

12-6 Word Problem Practice

Statistical Measures

1. SPORTS The table below shows the number of times some teams in the National Football League have won the Super Bowl.

NFL Team	Number of Super Bowl Victories
New England	3
Baltimore	2
Kansas City	1
St. Louis	1
Denver	2
Green Bay	1
Dallas	5
San Francisco	5
Oakland	2
Pittsburgh	5
Miami	2
Washington	3
NY Giants	2
NY Jets	1
Chicago	1

Source: www.pubquizhelp.34sp.com

Which statistical measure represents the team(s) with the least Super Bowl victories?

2. SALARIES The median salary in a small company is $10.20 per hour. What percentage of the employees at the company earns more than $10.20 per hour?

3. RANDOM GENERATORS Samuel has written a computer program to generate a random selection of the following two-digit numbers.

25, 67, 54, 99, 41, 87, 90, 18, 32

Find the mean, median, and mode of this data.

4. HEIGHTS The following table lists the heights of some of the great NBA players.

Player	Height (in inches)
Kareem Abdul-Jabbar	86
Larry Bird	81
Shaquille O'Neal	85
Wilt Chamberlain	85
Michael Jordan	78

Source: www.sidwell.edu

Find the mean and standard deviation of the data in the table. Round your answer to the nearest hundredth.

METEORS For Exercises 5-8, use the following information.

Arlene stayed up late one night to watch the Perseid meteor shower. She recorded the number of meteors she saw every ten minutes starting at 1 A.M. and going until 4 A.M. Her data are shown below.

8, 7, 8, 12, 17, 15, 22, 28, 29, 31, 28, 23, 29, 28, 25, 23, 15, 12

5. What is the mean of this data set?

6. What is the median of this data set?

7. What is the mode of this data set?

8. What is the standard deviation of this data set? Round your answer to the nearest hundredth.

NAME _____ DATE_____ PERIOD _____

12-6 Enrichment

Standard Deviation of Sample Data

A *population* is the set of all measurements of interest to an investigator. A *sample* is a subset of measurements selected from the population of interest. A *statistic* is any quantity whose value can be calculated from sample data. A common mistake is to use the terms *probability* and *statistics* interchangeably. Probabilities are used to make statements from a population to a sample, but statistics are calculated from a sample and are to make inferences about a population.

The *range* is a statistic calculated by taking the difference between the largest observation and the smallest observation. Range $= x_{\max} - x_{\min}$.

The *sample variance* is calculated using the formula: $s^2 = \frac{\sum_{i=1}^{n} (\bar{x} - x_i)^2}{n - 1}$ where $\bar{x}$ is the sample mean. Therefore, the *sample standard deviation* is the square root of the sample variance, $s = \sqrt{s^2}$.

To calculate the sample variance:

1. Calculate the sample mean. For example, suppose a sample contains the numbers {2, 5, 6, 9, 11}. The sample mean is $\bar{x} = \frac{2 + 5 + 6 + 9 + 11}{5} = 6.6$.

2. Next use the formula above to calculate the sample variance, in this case:

$$s^2 = \frac{(6.6 - 2)^2 + (6.6 - 5)^2 + (6.6 - 6)^2 + (6.6 - 9)^2 + (6.6 - 11)^2}{4} = 12.3.$$

3. Finally, the sample standard deviation is equal to 3.507 by taking the square root of 12.3.

Exercises

1. What are some differences in the formula for the sample variance compared to the formula for the population variance?

2. Given the random sample {5, 7, 1, 2, 4}, find the sample variance.

3. Calculate the sample standard deviation.

4. Calculate the range of the sample data {5, 7, 1, 2, 4}.

5. An approximation for the sample standard deviation is given by: $s \approx \frac{Range}{4}$. Compare this answer to your answer from 3.

NAME _____ DATE_____ PERIOD _____

12-7 Reading to Learn Mathematics

The Normal Distribution

Get Ready for the Lesson

Read the introduction to Lesson 12-7 in your textbook.

There were 66 players on the team and the mean height was approximately 74.1. About what fraction of the players' heights are between 72 and 75, inclusive?

Read the Lesson

1. Indicate whether each of the following statements is *true* or *false*.

a. In a continuous probability distribution, there is a finite number of possible outcomes.

b. Every normal distribution can be represented by a bell curve.

c. A distribution that is represented by a curve that is high at the left and has a tail to the right is negatively skewed.

d. A normal distribution is an example of a skewed distribution.

2. Ms. Rose gave the same quiz to her two geometry classes. She recorded the following scores.

First-period class:

Score	0	1	2	3	4	5	6	7	8	9	10
Frequency	1	0	1	0	3	4	5	7	4	3	2

Fifth-period class:

Score	0	1	2	3	4	5	6	7	8	9	10
Frequency	0	0	0	0	3	4	9	7	6	1	0

In each class, 30 students took the quiz. The mean score for each class was 6.4. Which set of scores has the greater standard deviation? (Answer this question without doing any calculations.) Explain your answer.

Remember What You Learned

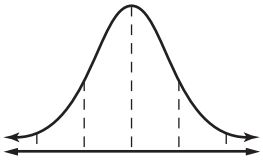
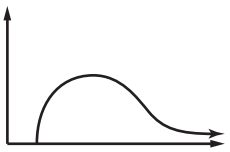
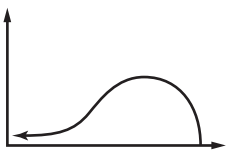
3. Many students have trouble remembering how to determine if a curve represents a distribution that is *positively skewed* or *negatively skewed*. What is an easy way to remember this?

NAME _____ DATE_____ PERIOD _____

12-7 Study Guide and Intervention

The Normal Distribution

Normal and Skewed Distributions A **continuous probability** distribution is represented by a curve.

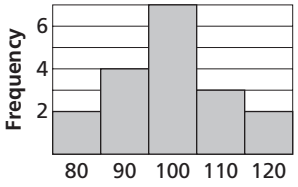
Types of Continuous Distributions	Normal	Positively Skewed	Negatively Skewed
			

Example **Determine whether the data below appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.**
{100, 120, 110, 100, 110, 80, 100, 90, 100, 120, 100, 90, 110, 100, 90, 80, 100, 90}

Make a frequency table for the data.

Value	80	90	100	110	120
Frequency	2	4	7	3	2

Then use the data to make a histogram.



Since the histogram is roughly symmetric, the data appear to be normally distributed.

Exercises

Determine whether the data in each table appear to be *positively skewed*, *negatively skewed*, or *normally distributed*. Make a histogram of the data.

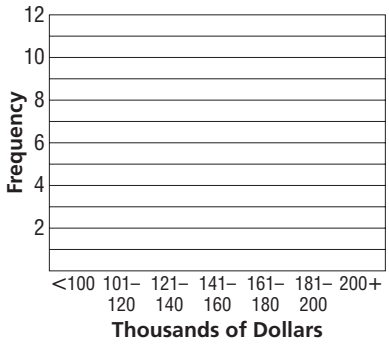
1. {27, 24, 29, 25, 27, 22, 24, 25, 29, 24, 25, 22, 27, 24, 22, 25, 24, 22}

2.

Shoe Size	4	5	6	7	8	9	10
No. of Students	1	2	4	8	5	1	2

3.

Housing Price	No. of Houses Sold
less than $100,000	0
$100,00–$120,000	1
$121,00–$140,000	3
$141,00–$160,000	7
$161,00–$180,000	8
$181,00–$200,000	6
over $200,000	12



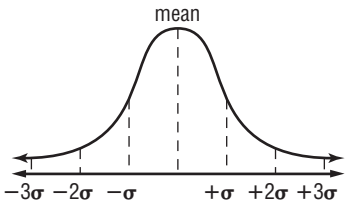
NAME _____ DATE_____ PERIOD _____

12-7 Study Guide and Intervention *(continued)*

The Normal Distribution

Use Normal Distributions

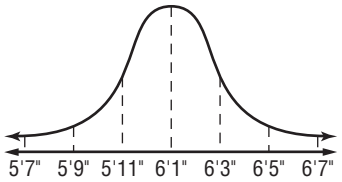
Normal Distribution



Normal distributions have these properties.
The graph is maximized at the mean.
The mean, median, and mode are about equal.
About 68% of the values are within one standard deviation of the mean.
About 95% of the values are within two standard deviations of the mean.
About 99% of the values are within three standard deviations of the mean.

Example **The heights of players in a basketball league are normally distributed with a mean of 6 feet 1 inch and a standard deviation of 2 inches.**

a. What is the probability that a player selected at random will be shorter than 5 feet 9 inches?



Draw a normal curve. Label the mean and the mean plus or minus multiples of the standard deviation.

The value of 5 feet 9 inches is 2 standard deviations below the mean, so approximately 2.5% of the players will be shorter than 5 feet 9 inches.

b. If there are 240 players in the league, about how many players are taller than 6 feet 3 inches?

The value of 6 feet 3 inches is one standard deviation above the mean. Approximately 16% of the players will be taller than this height.

$240 \times 0.16 \approx 38$

About 38 of the players are taller than 6 feet 3 inches.

Exercises

EGG PRODUCTION The number of eggs laid per year by a particular breed of chicken is normally distributed with a mean of 225 and a standard deviation of 10 eggs.

1. About what percent of the chickens will lay between 215 and 235 eggs per year?

2. In a flock of 400 chickens, about how many would you expect to lay more than 245 eggs per year?

MANUFACTURING The diameter of bolts produced by a manufacturing plant is normally distributed with a mean of 18 mm and a standard deviation of 0.2 mm.

3. What percent of bolts coming off of the assembly line have a diameter greater than 18.4 mm?

4. What percent have a diameter between 17.8 and 18.2 mm?

Lesson 12-7

NAME _____ DATE_____ PERIOD _____

12-7 Skills Practice

The Normal Distribution

Determine whether the data in each table appear to be *positively skewed*, *negatively skewed*, *or normally distributed*.

1.

Miles Run	Track Team Members
0–4	3
5–9	4
10–14	7
15–19	5
20–23	2

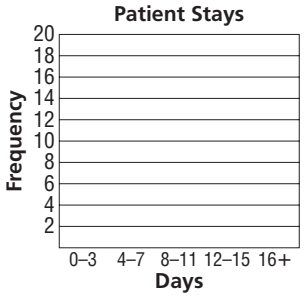
2.

Speeches Given	Political Candidates
0–5	1
6–11	2
12–17	3
18–23	8
24–29	8

For Exercises 3 and 4, use the frequency table that shows the average number of days patients spent on the surgical ward of a hospital last year.

Days	Number of Patients
0–3	5
4–7	18
8–11	11
12–15	9
16+	6

3. Make a histogram of the data.



4. Do the data appear to be *positively skewed*, *negatively skewed*, or *normally distributed*? Explain.

DELIVERY For Exercises 5–7, use the following information.

The time it takes a bicycle courier to deliver a parcel to his farthest customer is normally distributed with a mean of 40 minutes and a standard deviation of 4 minutes.

5. About what percent of the courier's trips to this customer take between 36 and 44 minutes?

6. About what percent of the courier's trips to this customer take between 40 and 48 minutes?

7. About what percent of the courier's trips to this customer take less than 32 minutes?

TESTING For Exercises 8–10, use the following information.

The average time it takes sophomores to complete a math test is normally distributed with a mean of 63.3 minutes and a standard deviation of 12.3 minutes.

8. About what percent of the sophomores take more than 75.6 minutes to complete the test?

9. About what percent of the sophomores take between 51 and 63.3 minutes?

10. About what percent of the sophomores take less than 63.3 minutes to complete the test?

NAME _____ DATE_____ PERIOD _____

12-7 Practice

The Normal Distribution

Determine whether the data in each table appear to be *positively skewed*, *negatively skewed*, *or normally distributed*.

1.

Time Spent at a Museum Exhibit	
Minutes	**Frequency**
0–25	27
26–50	46
51–75	89
75–100	57
100+	24

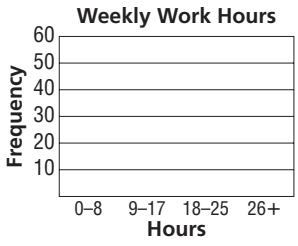
2.

Average Age of High School Principals	
Age in Years	**Number**
31–35	3
36–40	8
41–45	15
46–50	32
51–55	40
56–60	38
60+	4

For Exercises 3 and 4, use the frequency table that shows the number of hours worked per week by 100 high school seniors.

Hours	Number of Students
0–8	30
9–17	45
18–25	20
26+	5

3. Make a histogram of the data.



4. Do the data appear to be *positively skewed*, *negatively skewed*, or *normally distributed*? Explain.

TESTING For Exercises 5–10, use the following information.

The scores on a test administered to prospective employees are normally distributed with a mean of 100 and a standard deviation of 15.

5. About what percent of the scores are between 70 and 130?

6. About what percent of the scores are between 85 and 130?

7. About what percent of the scores are over 115?

8. About what percent of the scores are lower than 85 or higher than 115?

9. If 80 people take the test, how many would you expect to score higher than 130?

10. If 75 people take the test, how many would you expect to score lower than 85?

11. TEMPERATURE The daily July surface temperature of a lake at a resort has a mean of 82° and a standard deviation of 4.2°. If you prefer to swim when the temperature is at least 77.8°, about what percent of the days does the temperature meet your preference?

Lesson 12-7

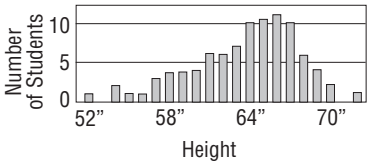
NAME _____ DATE_____ PERIOD _____

12-7 Word Problem Practice

The Normal Distribution

1. PARKING Over several years, Bertram conducted a study of how far into parking spaces people tend to park by measuring the distance from the end of a parking space to the front fender of a car parked in the space. He discovered that the distribution of the data closely approximated a normal distribution with mean 8.5 inches. He found that about 5% of cars parked more than 11.5 inches away from the end of the parking space. What percentage of cars would you expect parked less than 5.5 inches away from the end of the parking space?

2. HEIGHT Chandra's graph of the number of tenth grade students of different heights is shown below.



Is the data positively skewed, negatively skewed, or normally distributed?

3. OVENS An oven manufacturer tries to make the temperature setting on its ovens as accurate as possible. However, if one measures the actual temperatures in the ovens when the temperature setting is 350°F, they will differ slightly from 350°F. The set of actual temperatures for all the ovens is normally distributed around 350°F with a standard deviation of 0.5°F. About what percentage of ovens will be between 350°F and 351°F when their temperature setting is 350°F?

4. LIGHT BULBS The time that a certain brand of light bulb will last before burning out is normally distributed. About 2.5% of the bulbs last longer than 6800 hours and about 16% of the bulbs last longer than 6500 hours. How long does the average bulb last?

DOGS For Exercises 5-8, use the following information.

The weights of adult greyhound dogs are normally distributed. The mean weight is about 69 pounds and the standard deviation is about 10 pounds.

5. Approximately what percentage of adult greyhound dogs would you expect weigh between 59 and 79 pounds?

6. Approximately what percentage of adult greyhound dogs would you expect weigh more than 99 pounds?

7. Approximately what percentage of adult greyhound dogs would you expect weigh less than 49 pounds?

8. What would you expect an adult greyhound dog to weigh if it weighed less than 0.5% of an average adult greyhound?

NAME _____ DATE_____ PERIOD _____

12-7 Enrichment

Calculating Z-Scores

The normal distribution is the most important probability distribution. Many physical measurements have distributions approximately normal. Examples include height, weight, and measures of intelligence. More importantly, even if the individual variables are not normally distributed, sums and averages tend to still be normally distributed. Unfortunately, normal probability distribution functions are difficult to calculate. Fortunately, statisticians have compiled a table for a normal distribution with mean of zero and standard deviation of one. This is called the Standard Normal Distribution and is typically denoted by N(0, 1), where the N indicates a normal distribution which has mean, μ (mu) = 0, and standard deviation, σ (sigma) = 1.

Suppose the variable x is normally distributed with mean μ and standard deviation σ. In order to calculate probabilities of this normal distribution, we must standardize the variable x by an appropriate transformation. The letter Z denotes the transformed variable and is called the Z-score, which is a measure of relative standing. The following steps are needed to complete the transformation.

- If the mean and standard deviation are not given, then calculate the mean and standard deviation of the given (population) data.
- Define $Z = \frac{x - \mu}{\sigma}$.

Example Find the standard normal variable Z given and $\mu = 15$ and $\sigma = 3$.

Apply the transform to the variable X using the definition above, that is: $Z = \frac{X - 15}{3}$.

1. Suppose that the time, X, to complete an exam is normally distributed. The time, in minutes, of a class of 12 to complete the exam is given in the table. Transform X to a Z-score.

Student	1	2	3	4	5	6	7	8	9	10	11	12
Time	35	42	48	33	32	39	40	52	48	34	36	44

2. Suppose that a random variable X is normally distributed with $\mu = 20$ and $\sigma = 5$. Convert the following probability statements to the equivalent statements by standardizing X.

Example $P(X < 25) = P\left(\frac{X - 20}{5} < 25\right) = P(Z < 25)$

a. $P(X > 18)$

b. $P(17 < X < 23)$

c. $P(X < 19)$

Lesson 12-7

NAME _____ DATE_____ PERIOD _____

12-8 Lesson Reading Guide

Binomial Experiments

Get Ready for the Lesson

Read the introduction to Lesson 12-8 in your textbook.

Suppose you are taking a 50-question multiple-choice test in which there are 5 answer choices for each question. You are told that no points will be deducted for wrong answers. Should you guess the answers to the questions you do not know? Explain your reasoning.

Read the Lesson

1. Indicate whether each of the following is a *binomial experiment* or *not a binomial experiment*. If the experiment is not a binomial experiment, explain why.

a. A fair coin is tossed 10 times and "heads" or "tails" is recorded each time.

b. A pair of dice is thrown 5 times and the sum of the numbers that come up is recorded each time.

c. There are 5 red marbles and 6 blue marbles in a bag. One marble is drawn from the bag and its color recorded. The marble is not put back in the bag. A second marble is drawn and its color recorded.

d. There are 5 red marbles and 6 blue marbles in a bag. One marble is drawn from the bag and its color recorded. The marble is put back in the bag. A second marble is drawn and its color recorded.

2. Len randomly guesses the answers to all 6 multiple-choice questions on his chemistry test. Each question has 5 choices. Which of the following expressions gives the probability that he will get at least 4 of the answers correct?

A. $P(6, 4)\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^2 + P(6, 5)\left(\frac{1}{5}\right)^5\left(\frac{4}{5}\right)^1 + P(6, 6)\left(\frac{1}{5}\right)^6\left(\frac{4}{5}\right)^0$

B. $C(6, 4)\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^2 + C(6, 5)\left(\frac{1}{5}\right)^5\left(\frac{4}{5}\right)^1 + C(6, 6)\left(\frac{1}{5}\right)^6\left(\frac{4}{5}\right)^0$

C. $C(6, 4)\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^4 + C(6, 5)\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^5 + C(6, 6)\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^6$

Remember What You Learned

3. Some students have trouble remembering how to calculate binomial probabilities. What is an easy way to remember which numbers to put into an expression like $C(6, 4)\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^4$?

NAME _____ DATE_____ PERIOD _____

12-8 Study Guide and Intervention

Binomial Experiments

Binomial Expansions For situations with only 2 possible outcomes, you can use the Binomial Theorem to find probabilities. The coefficients of terms in a binomial expansion can be found by using combinations.

Example **What is the probability that 3 coins show heads and 3 show tails when 6 coins are tossed?**

There are 2 possible outcomes that are equally likely: heads (H) and tails (T). The tosses of 6 coins are independent events. When $(H + T)^6$ is expanded, the term containing H^3T^3, which represents 3 heads and 3 tails, is used to get the desired probability. By the Binomial Theorem the coefficient of H^3T^3 is $C(6, 3)$.

$$P(3 \text{ heads, } 3 \text{ tails}) = \frac{6!}{3!3!}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^3 \quad P(\text{H}) = \frac{1}{2} \text{ and } P(\text{T}) = \frac{1}{2}$$

$$= \frac{20}{64}$$

$$= \frac{5}{16}$$

The probability of getting 3 heads and 3 tails is $\frac{5}{16}$ or 0.3125.

Exercises

Find each probability if a coin is tossed 8 times.

1. P(exactly 5 heads)

2. P(exactly 2 heads)

3. P(even number of heads)

4. P(at least 6 heads)

Mike guesses on all 10 questions of a true-false test. If the answers true and false are evenly distributed, find each probability.

5. Mike gets exactly 8 correct answers.

6. Mike gets at most 3 correct answers.

7. A die is tossed 4 times. What is the probability of tossing exactly two sixes?

Lesson 12-8

NAME _____ DATE_____ PERIOD _____

12-8 Study Guide and Intervention *(continued)*

Binomial Experiments

Binomial Experiments

Binomial Experiments	A binomial experiment is possible if and only if all of these conditions occur. • There are exactly two outcomes for each trial. • There is a fixed number of trials. • The trials are independent. • The probabilities for each trial are the same.

Example **Suppose a coin is weighted so that the probability of getting heads in any one toss is 90%. What is the probability of getting exactly 7 heads in 8 tosses?**

The probability of getting heads is $\frac{9}{10}$, and the probability of getting tails is $\frac{1}{10}$. There are $C(8, 7)$ ways to choose the 7 heads.

$$P(7 \text{ heads}) = C(8, 7)\left(\frac{9}{10}\right)^7\left(\frac{1}{10}\right)^1$$

$$= 8 \cdot \frac{9^7}{10^8}$$

$$\approx 0.38$$

The probability of getting 7 heads in 8 tosses is about 38%.

Exercises

1. **BASKETBALL** For any one foul shot, Derek has a probability of 0.72 of getting the shot in the basket. As part of a practice drill, he shoots 8 shots from the foul line.
   - **a.** What is the probability that he gets in exactly 6 foul shots?
   - **b.** What is the probability that he gets in at least 6 foul shots?

2. **SCHOOL** A teacher is trying to decide whether to have 4 or 5 choices per question on her multiple choice test. She wants to prevent students who just guess from scoring well on the test.
   - **a.** On a 5-question multiple-choice test with 4 choices per question, what is the probability that a student can score at least 60% by guessing?
   - **b.** What is the probability that a student can score at least 60% by guessing on a test of the same length with 5 choices per question?

3. Julie rolls two dice and adds the two numbers.
   - **a.** What is the probability that the sum will be divisible by 3?
   - **b.** If she rolls the dice 5 times what is the chance that she will get exactly 3 sums that are divisible by 3?

4. **SKATING** During practice a skater falls 15% of the time when practicing a triple axel. During one practice session he attempts 20 triple axels.
   - **a.** What is the probability that he will fall only once?
   - **b.** What is the probability that he will fall 4 times?

NAME _____ DATE_____ PERIOD _____

12-8 Skills Practice

Binomial Experiments

Find each probability if a coin is tossed 4 times.

1. P(4 heads)

2. P(0 heads)

3. P(exactly 3 heads)

4. P(exactly 2 heads)

5. P(exactly 1 head)

6. P(at least 3 heads)

Find each probability if a die is rolled 3 times.

7. P(exactly one 2)

8. P(exactly two 2s)

9. P(exactly three 2s)

10. P(at most one 2)

A town that presents a fireworks display during its July 4 celebration found the probability that a family with two or more children will watch the fireworks is $\frac{3}{5}$. If 5 of these families are selected at random, find each probability.

11. P(exactly 3 families watch the fireworks)

12. P(exactly 2 families watch the fireworks)

13. P(exactly 5 families watch the fireworks)

14. P(no families watch the fireworks)

15. P(at least 4 families watch the fireworks)

16. P(at most 1 family watches the fireworks)

One section of a standardized English language test has 10 true/false questions. Find each probability when a student guesses at all ten questions.

17. P(exactly 8 correct)

18. P(exactly 2 correct)

19. P(exactly half correct)

20. P(all 10 correct)

21. P(0 correct)

22. P(at least 8 correct)

Lesson 12-8

NAME _____ DATE_____ PERIOD _____

12-8 Practice

Binomial Experiments

Find each probability if a coin is tossed 6 times.

1. P(exactly 3 tails)

2. P(exactly 5 tails)

3. P(0 tails)

4. P(at least 4 heads)

5. P(at least 4 tails)

6. P(at most 2 tails)

The probability of Chris making a free throw is $\frac{2}{3}$. If she shoots 5 times, find each probability.

7. P(all missed)

8. P(all made)

9. P(exactly 2 made)

10. P(exactly 1 missed)

11. P(at least 3 made)

12. P(at most 2 made)

When Tarin and Sam play a certain board game, the probability that Tarin will win a game is $\frac{3}{4}$. If they play 5 games, find each probability.

13. P(Sam wins only once)

14. P(Tarin wins exactly twice)

15. P(Sam wins exactly 3 games)

16. P(Sam wins at least 1 game)

17. P(Tarin wins at least 3 games)

18. P(Tarin wins at most 2 games)

19. SAFETY In August 2001, the American Automobile Association reported that 73% of Americans use seat belts. In a random selection of 10 Americans in 2001, what is the probability that exactly half of them use seat belts?

HEALTH For Exercises 20 and 21, use the following information.

In 2001, the American Heart Association reported that 50 percent of the Americans who receive heart transplants are ages 50–64 and 20 percent are ages 35–49.

20. In a randomly selected group of 10 heart transplant recipients, what is the probability that at least 8 of them are ages 50–64?

21. In a randomly selected group of 5 heart transplant recipients, what is the probability that 2 of them are ages 35–49?

NAME _____ DATE_____ PERIOD _____

12-8 Word Problem Practice

Binomial Experiments

1. GENETICS Dagmar is conducting a genetic experiment. Before she performs the experiment, she would like to compute theoretically probabilities for some of the outcomes. One of these computations involves expanding $(p + q)^4$. What is this expansion?

2. GAMES The probability that Kendra will win a card game is $\frac{2}{3}$. If Kendra plays 7 games what is the probability she wins exactly 4 games? Round your answer to the nearest thousandth.

3. DEFECTS An electronics parts manufacturer produces capacitors for electronic circuits. The probability that a capacitor comes out defective is 1 in 1,000. In a batch of 10,000 capacitors, write an expression for the probability that 10 of the capacitors are defective.

4. SUBWAYS Fiona uses the subway to commute to work. During the morning commute, the trains run frequently and there is a 1 in 8 chance that she will find a train waiting for her as soon as she gets to the platform. Over the course of a five-day work week, what is the probability that she found a train waiting for her at least twice? Round your answer to the nearest thousandth.

5. SOCCER The boys varsity soccer team at Lincoln High School has a 75% probability of winning each of their 17 games this season. What is the probability that the team will win at least 13 games this season? Round your answer to the nearest thousandth.

CHESS For Exercises 6-8, use the following information.

Gary and Howard play chess. Gary's chess rating is 2050 and Howard's chess rating is 1948. This means that whenever they play, Gary has a 64% chance of defeating Howard. One day, Gary and Howard play three games against each other. Round your answers to the nearest thousandth.

6. What is the probability that Gary will win all three of the matches?

7. What is the probability that Gary will win at least two of the three matches?

8. What is the probability that Gary will win only one of the matches?

Lesson 12-8

NAME _____ DATE_____ PERIOD _____

12-8 Enrichment

Multinomial Experiments

A multinomial is a generalization of a binomial. For example, $(a + b + c)^2$ is a multinomial. One way to determine the coefficients is by direct multiplication using the distributive property. Take each term in the first factor and multiply by each term (a, b, and c) in the second factor, then combine like terms. (Notice that the sum of the exponents is always equal to two.)

$$\begin{aligned}(a + b + c)^2 = (a + b + c)(a + b + c) &= a(a + b + c) + b(a + b + c) + c(a + b + c)\\ &= a^2 + ab + ac + ab + b^2 + bc + ac + bc + c^2\\ &= a^2 + b^2 + c^2 + 2ab + 2ac + 2bc\end{aligned}$$

Underlying this expansion is the notion of a *partition* into categories. The example partitions two 'items' among three categories. In this case the categories are the variables a, b, and c and the items are exponents. For example the partition, {1, 0, 1} represents the term ac in the expansion, which could also be written as $a^1b^0c^1$, whereas the partition {0, 2, 0} represents the b^2 term. The coefficients of each term can be computed by the formula:

$\frac{n!}{n_1! \cdot n_2! \cdot n_3! \cdots n_k!}$, where n is the exponent and $n_1 + n_2 + n_3 + \cdots + n_k = n$. Recall, $0! = 1$.

Term	Partition	Coefficient
a^2	{2,0,0}	$\frac{2!}{2! \cdot 0! \cdot 0!} = 1$
b^2	{0,2,0}	$\frac{2!}{0! \cdot 2! \cdot 0!} = 1$
c^2	{0,0,2}	$\frac{2!}{0! \cdot 0! \cdot 2!} = 1$
ab	{1,1,0}	$\frac{2!}{1! \cdot 1! \cdot 0!} = 2$
ac	{1,0,1}	$\frac{2!}{1! \cdot 0! \cdot 1!} = 2$
bc	{0,1,1}	$\frac{2!}{0! \cdot 1! \cdot 1!} = 2$

1. Determine the all the *partitions* of $(x + y + z)^3$.

2. Determine the coefficients in the expansion of $(x + y + z)^3$ associated with each partition.

3. How can you build and interpret a trinomial distribution?

NAME _____ DATE_____ PERIOD _____

12-9 Lesson Reading Guide

Sampling and Error

Get Ready for the Lesson

Read the introduction to Lesson 12-9 in your textbook.

Do you think the results of the survey show that more mothers spend $249 or less than $250–$349? If there is not enough information given to determine this, list at least two questions you would ask about the survey that would help you determine the significance of the survey.

Read the Lesson

1. Determine whether each situation would produce a random sample. Write *yes* or *no* and explain your answer.

a. asking all the customers at five restaurants on the same evening how many times a month they eat dinner in restaurants to determine how often the average American eats dinner in a restaurants

b. putting the names of all seniors at your high school in a hat and then drawing 20 names for a survey to find out where seniors would like to hold their prom

2. A survey determined that 58% of registered voters in the United States support increased federal spending for education. The margin of error for this survey is 4%. Explain in your own words what this tells you about the actual percentage of registered voters who support increased spending for education.

Remember What You Learned

3. The formula for margin of sampling error may be tricky to remember. A good way to start is to think about the variables that must be included in the formula. What are these variables, and what do they represent? What is an easy way to remember which variable goes in the denominator in the formula?

NAME _____ DATE_____ PERIOD _____

12-9 Study Guide and Intervention

Sampling and Error

Bias A sample of size n is random (or **unbiased**) when every possible sample of size n has an equal chance of being selected. If a sample is biased, then information obtained from it may not be reliable.

Example **To find out how people in the U.S. feel about mass transit, people at a commuter train station are asked their opinion. Does this situation represent a random sample?**

No; the sample includes only people who actually use a mass-transit facility. The sample does not include people who ride bikes, drive cars, or walk.

Exercises

Determine whether each situation would produce a random sample. Write *yes* or *no* and explain your answer.

1. asking people in Phoenix, Arizona, about rainfall to determine the average rainfall for the United States

2. obtaining the names of tree types in North America by surveying all of the U.S. National Forests

3. surveying every tenth person who enters the mall to find out about music preferences in that part of the country

4. interviewing country club members to determine the average number of televisions per household in the community

5. surveying all students whose ID numbers end in 4 about their grades and career counseling needs

6. surveying parents at a day care facility about their preferences for brands of baby food for a marketing campaign

7. asking people in a library about the number of magazines to which they subscribe in order to describe the reading habits of a town

NAME _____ DATE_____ PERIOD _____

12-9 Study Guide and Intervention *(continued)*

Sampling and Error

Margin of Error The **margin of sampling error** gives a limit on the difference between how a sample responds and how the total population would respond.

Margin of Error	If the percent of people in a sample responding in a certain way is p and the size of the sample is n, then 95% of the time, the percent of the population responding in that same way will be between $p - ME$ and $p + ME$, where $ME = 2\sqrt{\frac{p(1-p)}{n}}$.

Example 1 **In a survey of 4500 randomly selected voters, 62% favored candidate A. What is the margin of error?**

$ME = 2\sqrt{\frac{p(1-p)}{n}}$ Formula for margin of sampling error

$= 2\sqrt{\frac{0.62 \cdot (1 - 0.62)}{4500}}$ p = 62% or 0.62, n = 4500

≈ 0.01447 Use a calculator.

The margin of error is about 1%. This means that there is a 95% chance that the percent of voters favoring candidate A is between 62 − 1 or 61% and 62 + 1 or 63%.

Example 2 **The CD that 32% of teenagers surveyed plan to buy next is the latest from the popular new group BFA. If the margin of error of the survey is 2%, how many teenagers were surveyed?**

$ME = 2\sqrt{\frac{p(1-p)}{n}}$ Formula for margin of sampling error

$0.02 = 2\sqrt{\frac{0.32 \cdot (1 - 0.32)}{n}}$ ME = 0.02, p = 0.32

$0.01 = \sqrt{\frac{0.32(0.68)}{n}}$ Divide each side by 2.

$0.0001 = \frac{0.32(0.68)}{n}$ Square each side.

$n = \frac{0.32(0.68)}{0.0001}$ Multiply by n and divide by 0.0001

$n = 2176$

2176 teenagers were surveyed.

Exercises

Find the margin of sampling error to the nearest percent.

1. $p = 45\%, n = 350$ **2.** $p = 12\%, n = 1500$ **3.** $p = 86\%, n = 600$

4. A study of 50,000 drivers in Indiana, Illinois, and Ohio showed that 68% preferred a speed limit of 75 mph over 65 mph on highways and country roads. What was the margin of sampling error to the nearest tenth of a percent?

NAME _____ DATE_____ PERIOD _____

12-9 Skills Practice

Sampling and Error

Determine whether each situation would produce a random sample. Write *yes* or *no* and explain your answer.

1. calling households at 3:30 P.M. on Tuesday to determine a political candidate's support

2. polling customers as they exit a sporting goods store about their attitudes about exercise

3. recording the number of sit-ups performed by 15-year old girls in the high schools of a large school district to determine the fitness of all high-school girls in the district

4. selecting two of a city's 20 apartment buildings for a survey to determine the desire of apartment dwellers in the city to own a home

5. In a large school district, the superintendent of schools interviews two teachers at random from each school to determine whether teachers in the district think students are assigned too much or too little homework.

6. For seven consecutive days, one hour each in the morning, afternoon, and evening, every tenth customer who enters a mall is asked to choose her or his favorite store.

Find the margin of sampling error to the nearest percent.

7. $p = 85\%, n = 100$

8. $p = 78\%, n = 100$

9. $p = 15\%, n = 100$

10. $p = 37\%, n = 500$

11. $p = 12\%, n = 500$

12. $p = 93\%, n = 500$

13. $p = 23\%, n = 1000$

14. $p = 56\%, n = 1000$

15. HEALTH In a recent poll of cigarette smokers, 67% of those surveyed said they had tried to quit smoking within the last year. The margin of error was 3%. About how many people were surveyed?

NAME _____ DATE_____ PERIOD _____

12-9 Practice

Sampling and Error

Determine whether each situation would produce a random sample. Write *yes* or *no* and explain your answer.

1. calling every twentieth registered voter to determine whether people own or rent their homes in your community

2. predicting local election results by polling people in every twentieth residence in all the different neighborhoods of your community

3. to find out why not many students are using the library, a school's librarian gives a questionnaire to every tenth student entering the library

4. testing overall performance of tires on interstate highways only

5. selecting every 50th hamburger from a fast-food restaurant chain and determining its fat content to assess the fat content of hamburgers served in fast-food restaurant chains throughout the country

6. assigning all shift workers in a manufacturing plant a unique identification number, and then placing the numbers in a hat and drawing 30 at random to determine the annual average salary of the workers

Find the margin of sampling error to the nearest percent.

7. $p = 26\%, n = 100$ **8.** $p = 55\%, n = 100$ **9.** $p = 75\%, n = 500$

10. $p = 14\%, n = 500$ **11.** $p = 96\%, n = 1000$ **12.** $p = 21\%, n = 1000$

13. $p = 34\%, n = 1000$ **14.** $p = 49\%, n = 1500$ **15.** $p = 65\%, n = 1500$

16. COMPUTING According to a poll of 500 teenagers, 43% said that they use a personal computer at home. What is the margin of sampling error?

17. TRUST A survey of 605 people, ages 13–33, shows that 68% trust their parents more than their best friends to tell them the truth. What is the margin of sampling error?

18. PRODUCTIVITY A study by the University of Illinois in 1995 showed an increase in productivity by 10% of the employees who wore headsets and listened to music of their choice while they were working. The margin of sampling error for the study was about 7%. How many employees participated in the study?

NAME _____ DATE_____ PERIOD _____

12-9 Word Problem Practice

Sampling and Error

1. COMICS Isaac would like to know if people prefer reading comic books or novels. He decides to wait outside of a bookstore and ask people exiting whether they purchased comics or novels. Discuss whether this method of acquiring data would produce a biased or unbiased sample.

2. PARKING A town wants to find out if people are happy with a proposal to tear down a section of a park and replace it with a parking lot. The town council decides to conduct a random survey of the town's citizens. They send a person to the location in the park where the proposed parking lot will be and have that person ask all passersby whether they would like to see a parking lot built at the location. Discuss whether or not this would produce a random sample.

3. PROMS A poll asked 50 random seniors at a high school whether they would like to have the senior prom at a nearby hotel or at a local convention hall. Sixteen students responded that they would prefer the hotel. What is the margin of sampling error? Round your answer to the nearest percent.

4. AIRPORTS In a large city, a random survey found that 18% of the city's population want a new runway built at the city airport. The survey had a margin of error of 5%. About how many people were surveyed?

INTERNET USE For Exercises 5-7, use the following information.

Two surveys were conducted to find out if people think that Americans are becoming more knowledgeable about the Internet. One survey polled 500 people and found that 395 of them felt that Americans are becoming more Internet savvy. A second survey concluded that 79% of those polled think that Americans are becoming more Internet savvy with a margin of error of 2%.

5. What was the margin of error for the first survey? Round your answer to the nearest percent.

6. About how many people were polled in the second survey?

7. Based on the results of the second survey, between what two percentages would you estimate is the true percentage of people who think that Americans are more Internet savvy, with 95% confidence?

NAME _____ DATE_____ PERIOD _____

12-9 Enrichment

Sample Mean and Standard Error

The mean of a sample of size n is an estimate of the mean of the entire population under study. A different sample of size n probably will have a different mean, and a third sample yet another. For example, suppose the means for three different samples were 9, 5, and 11. What prediction would you make for the mean of a fourth sample? Clearly there is some uncertainty involved in the prediction. Therefore the sample mean must have a probability distribution associated with it. If the standard deviation of the population is known, σ (sigma), then the standard deviation of the sample mean, called the *standard error*, is given by: $SE = \frac{\sigma}{\sqrt{n}}$, where n is the sample size.

A 95% confidence interval for the population mean is given by:
$(\bar{x} - 2SE, \bar{x} + 2SE) = \left(\bar{x} - 2\frac{\sigma}{\sqrt{n}}, \bar{x} + 2\frac{\sigma}{\sqrt{n}}\right)$. That is, there is a 95% chance that the *true* population mean will be contained in this interval.

Suppose a high school football team has 50 players. The coach wants to know what the average amount a player can bench press. The only exact way to calculate this average would be to measure, in the weight room, each player on the team. Instead the coach randomly samples 10 players and computes the average of these 10 to estimate the mean bench press weight for the entire team. Suppose the coach knew that $\sigma = 41.6$. (This data is unknown to the coach but presented here for the purpose of the exercises.)

185	185	200	205	215
235	240	200	205	180
175	190	195	215	205
250	275	250	230	250
205	260	190	210	230
180	175	165	165	195
290	300	305	315	210
315	210	230	205	200
150	155	145	200	215
160	190	195	185	180

Exercises

1. Calculate the standard error of the sample mean.

2. Calculate a 95% confidence interval for the population mean.

3. How can the coach make more accurate estimations?

4. Calculate the sample mean and standard error for a sample of size 20.

NAME _____ DATE_____ PERIOD _____

12 Student Recording Sheet

Use this recording sheet with pages 746–747 of the Student Edition.

Read each question. Then fill in the correct answer.

1. Ⓐ Ⓑ Ⓒ Ⓓ

2. Ⓕ Ⓖ Ⓗ Ⓙ

3. Record your answer and fill in the bubbles in the grid below. Be sure to use the correct place value.

				.			
⓪	⓪	⓪	⓪		⓪	⓪	⓪
①	①	①	①		①	①	①
②	②	②	②		②	②	②
③	③	③	③		③	③	③
④	④	④	④		④	④	④
⑤	⑤	⑤	⑤		⑤	⑤	⑤
⑥	⑥	⑥	⑥		⑥	⑥	⑥
⑦	⑦	⑦	⑦		⑦	⑦	⑦
⑧	⑧	⑧	⑧		⑧	⑧	⑧
⑨	⑨	⑨	⑨		⑨	⑨	⑨

4. Ⓐ Ⓑ Ⓒ Ⓓ

5. Ⓕ Ⓖ Ⓗ Ⓙ

6. Ⓐ Ⓑ Ⓒ Ⓓ

7. Ⓕ Ⓖ Ⓗ Ⓙ

8. Ⓐ Ⓑ Ⓒ Ⓓ

9. Ⓕ Ⓖ Ⓗ Ⓙ

10. Ⓐ Ⓑ Ⓒ Ⓓ

11. Record your answer and fill in the bubbles in the grid below. Be sure to use the correct place value.

				.			
⓪	⓪	⓪	⓪		⓪	⓪	⓪
①	①	①	①		①	①	①
②	②	②	②		②	②	②
③	③	③	③		③	③	③
④	④	④	④		④	④	④
⑤	⑤	⑤	⑤		⑤	⑤	⑤
⑥	⑥	⑥	⑥		⑥	⑥	⑥
⑦	⑦	⑦	⑦		⑦	⑦	⑦
⑧	⑧	⑧	⑧		⑧	⑧	⑧
⑨	⑨	⑨	⑨		⑨	⑨	⑨

Pre-AP

Record your answers for Question 12 on the back of this paper.

Assessment

NAME _____ DATE_____ PERIOD _____

12 Rubric for Scoring Pre-AP

(Use to score the Pre-AP question on page 747 of the Student Edition.)

General Scoring Guidelines

- If a student gives only a correct numerical answer to a problem but does not show how he or she arrived at the answer, the student will be awarded only 1 credit. All extended response questions require the student to show work.
- A fully correct answer for a multiple-part question requires correct responses for all parts of the question. For example, if a question has three parts, the correct response to one or two parts of the question that required work to be shown is *not* considered a fully correct response.
- Students who use trial and error to solve a problem must show their method. Merely showing that the answer checks or is correct is not considered a complete response for full credit.

Exercise 12 Rubric

Score	Specific Criteria
4	A correct solution that is supported by well-developed, accurate explanations. Naomi can make $(6 \cdot 3 \cdot 4) = 72$ different combinations of shirt, pants, and ties. Naomi can make $(6 \cdot 3) = 18$ different shirt and pants combinations. The probability that Naomi wears a red shirt the first two days is $\left(\frac{1}{3} \cdot \frac{1}{5}\right) = \frac{1}{15}$.
3	A generally correct solution, but may contain minor flaws in reasoning or computation.
2	A partially correct interpretation and/or solution to the problem.
1	A correct solution with no supporting evidence or explanation.
0	An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given.

NAME _____ DATE _____ PERIOD _____

12 Chapter 12 Quiz 1

(Lessons 12–1 through 12–3)

SCORE _____

1. MULTIPLE CHOICE Lisa selects a car from 4 models. Each model comes in 5 colors. How many different ways can she select a car?

A. 24 **B.** 16 **C.** 9 **D.** 20 **1.** _____

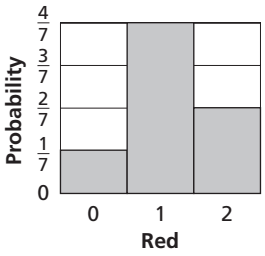
2. How many four-digit codes are possible if no digit may be used more than once? **2.** _____

3. A group of 3 women and 1 man is chosen from 7 women and 5 men. Does this involve a *permutation* or a *combination*? Find the number of different groups that can be formed. **3.** _____

4. Cards are numbered 1 through 20. Find the probability that a card drawn at random will contain a number greater than 11. **4.** _____

5. Two marbles are chosen at random from a bag containing 4 red and 3 blue marbles. The table and relative-frequency histogram show the distribution of the number of red marbles chosen. Find $P(R = 2)$. **5.** _____

R = Red	0	1	2
Probability	$\frac{1}{7}$	$\frac{4}{7}$	$\frac{2}{7}$



NAME _____ DATE _____ PERIOD _____

12 Chapter 12 Quiz 2

(Lessons 12–4 and 12–5)

SCORE _____

1. A pair of dice is thrown. What is the probability that both dice show a number less than 5? **1.** _____

For Questions 2 and 3, consider a bag that contains 8 red marbles, 5 white marbles, and 2 blue marbles.

2. If 3 marbles are selected in succession with replacement, what is the probability that the marbles are white, blue, and red in that order? **2.** _____

3. If 3 marbles are selected in succession without replacement, what is the probability that the marbles are white, blue, and red in that order? **3.** _____

4. Janet has 3 dimes and 6 nickels in her pocket. She selects 3 coins without replacement. What is the probability that she selects all dimes or all nickels? **4.** _____

5. A card is drawn from a standard deck of 52 playing cards. What is the probability that a heart or face card is drawn? (*Hint*: A face card is a jack, queen, or king.) **5.** _____

Assessment

NAME _____ DATE_____ PERIOD ____

12 Chapter 12 Quiz 3

(Lessons 12–6 and 12–7)

SCORE _____

1. Find the variance of the data set {13, 16, 17, 18, 16, 12, 14, 12}. Round to the nearest hundredth, if necessary. **1.** _____

2. Find the standard deviation for the data in Question 1. Round to the nearest hundredth, if necessary. **2.** _____

3. Determine if the data in the table appear to be *positively or negatively skewed* or *normally distributed.* **3.** _____

Family Income and Benefits in 2000	
Income and Benefits	**Number of Families (in millions)**
less than $50,000	35.8
$50,000–$99,999	24.3
$100,000–$149,999	6.9
$150,000–$199,999	2.0
$200,000 or more	1.9

The times a group of high school students wake up on weekday mornings was found to be normally distributed. The mean wake-up time was 6:45 A.M. and the times had a standard deviation of 15 minutes.

4. What percent of the students would you expect to wake up between 6:30 A.M. and 7:00 A.M.? **4.** _____

5. If 400 students were surveyed, how many would you expect to wake up between 6:00 A.M. and 7:30 A.M.? **5.** _____

NAME _____ DATE_____ PERIOD ____

12 Chapter 12 Quiz 4

(Lessons 12–8 and 12–9)

SCORE _____

For Questions 1 and 2, find each probability if a die is rolled 3 times. **1.** _____

1. P(exactly two 4s)

2. P(at most two 4s) **2.** _____

3. A batter's probability of getting a hit is $\frac{1}{3}$. In his next 5 times at bat, what is the probability that he will get at least 4 hits? **3.** _____

4. Determine whether the situation would produce a random sample and explain your answer: *surveying students on the basketball team to determine the favorite sport of students in your school.* **4.** _____

5. In a survey of 50 people, 80% read a newspaper at least once per week. Find the margin of sampling error. **5.** _____

NAME _____ DATE _____ PERIOD _____

12 Chapter 12 Mid-Chapter Test

(Lessons 12–1 through 12–5)

SCORE _____

Part I ***Write the letter for the correct answer in the blank at the right of each question.***

1. A company manufactures bicycles in 8 different styles. Each style comes in 7 different colors. How many different bicycles does the company make?

A. 64 **B.** 49 **C.** 56 **D.** 15 **1.** _____

2. How many ways can 6 children form a line to use the drinking fountain?

F. 120 **G.** 720 **H.** 36 **J.** 30 **2.** _____

3. Find $P(9, 4)$.

A. 126 **B.** 15,120 **C.** 36 **D.** 3024 **3.** _____

4. Find $C(10, 8)$.

F. 1,814,400 **G.** 80 **H.** 90 **J.** 45 **4.** _____

5. Two letters are selected at random from the word *space*. What is the probability of selecting 2 vowels?

A. $\frac{1}{10}$ **B.** $\frac{2}{25}$ **C.** $\frac{2}{5}$ **D.** $\frac{1}{20}$ **5.** _____

6. A die is rolled. Find $P(1 \text{ or } 5)$.

F. $\frac{1}{30}$ **G.** $\frac{1}{3}$ **H.** $\frac{1}{36}$ **J.** $\frac{5}{9}$ **6.** _____

Part II

7. A jar contains 7 red, 8 blue, and 4 green marbles. What is the probability of choosing 3 blue marbles in a row, if no replacement occurs? **7.** _____

8. A stained glass window has 25 blue pieces and 20 red pieces. If 2 pieces are selected at random, what is $P(2 \text{ red or } 2 \text{ blue})$? **8.** _____

9. SOCCER On the all-state soccer team, 5 of the 8 players from the North Region are seniors, and 8 of the 12 players from the South Region are seniors. What is the probability that a randomly-selected student is a senior or is a student from the North Region? **9.** _____

10. How many different arrangements of three folders can be made if you have one green, one red, one blue, and one black folder? **10.** _____

11. A bag contains 6 red dice, 3 green dice, and 10 blue dice. A die is selected at random. Find the probability of selecting a red die or a blue die. **11.** _____

12. How many different groups of 3 students can be formed if there are 20 students in the class? **12.** _____

NAME _____ DATE_____ PERIOD _____

12 Chapter 12 Vocabulary Test

SCORE _____

binomial experiment	measure of variation	random	unbiased sample
combination	mutually exclusive events	random variable	uniform distribution
compound event	normal distribution	relative-frequency histogram	univariate data
dependent events	outcome	sample space	variance
event	permutation	simple event	
inclusive events	probability	standard deviation	
independent events	probability distribution		

Write whether each sentence is *true* or *false*. If false, replace the underlined word or words to make a true sentence.

1. A selection of objects in which order is not important is called a permutation. **1.** _____

2. The graph of a normal distribution is a bell curve. **2.** _____

3. Range, variance, and probability are measures of the spread of a set of data. **3.** _____

4. Data with one variable are called univariate data. **4.** _____

5. Tossing a coin ten times is an example of a linear permutation. **5.** _____

6. A relative-frequency distribution is a graph of a probability distribution. **6.** _____

7. Two or more choices for which the result of one choice does not affect the result of another are called independent events. **7.** _____

8. Events that consist of two or more simple events are called dependent events. **8.** _____

9. The square root of the variance is called the outcome. **9.** _____

10. A sample in which every possible sample has an equal chance of being selected is a(n) unbiased sample. **10.** _____

Define each term in your own words.

11. mutually exclusive events

12. sample space

NAME _____ DATE _____ PERIOD _____

12 Chapter 12 Test, Form 1

SCORE _____

Write the letter for the correct answer in the blank at the right of each question.

1. Carl purchased four new shirts and three new pairs of pants. How many new outfits can he make with these items?

A. 12 **B.** 7 **C.** 9 **D.** 81 **1.** _____

2. TRIATHALON During training, a triathlete works on biking, swimming, and running times. How many ways can a triathlete choose the order of these activities in a training session?

F. 4 **G.** 9 **H.** 5 **J.** 6 **2.** _____

3. Evaluate $P(7, 2)$.

A. 49 **B.** 21 **C.** 42 **D.** 14 **3.** _____

4. Evaluate $C(6, 2)$.

F. 30 **G.** 15 **H.** 12 **J.** 36 **4.** _____

5. Two letters are selected at random from the word *should*. Find $P(2 \text{ consonants})$.

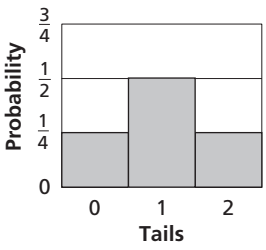
A. $\frac{1}{15}$ **B.** $\frac{1}{3}$ **C.** $\frac{4}{9}$ **D.** $\frac{2}{5}$ **5.** _____

6. The table and relative-frequency histogram show the distribution of the number of tails when 2 coins are tossed. Find $P(T = 2 \text{ tails})$.

F. $\frac{1}{4}$ **H.** 1

G. $\frac{1}{2}$ **J.** 0

T = Tails	0	1	2
Probability	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$



6. _____

7. A blue die and a red die are tossed. What is the probability that a 6 will appear on both dice?

A. $\frac{1}{18}$ **B.** $\frac{1}{36}$ **C.** $\frac{1}{2}$ **D.** $\frac{1}{11}$ **7.** _____

8. A jar contains 10 purple marbles and 2 red marbles. If two marbles are chosen at random with no replacement, what is the probability that 2 purple marbles are chosen?

F. $\frac{25}{36}$ **G.** $\frac{5}{6}$ **H.** $\frac{15}{22}$ **J.** $\frac{1}{5}$ **8.** _____

9. A bag contains 6 cherry, 8 strawberry, and 9 grape-flavored candies. What is the probability of selecting a cherry or a grape flavored candy?

A. $\frac{15}{23}$ **B.** $\frac{14}{23}$ **C.** $\frac{17}{23}$ **D.** $\frac{54}{529}$ **9.** _____

10. A die is rolled. What is the probability of rolling a 6 or a number greater than 4?

F. $\frac{2}{3}$ **G.** $\frac{1}{2}$ **H.** $\frac{1}{6}$ **J.** $\frac{1}{3}$ **10.** _____

11. A coin is tossed 5 times. Find $P(5 \text{ tails})$.

A. $\frac{1}{5}$ **B.** $\frac{1}{10}$ **C.** $\frac{1}{16}$ **D.** $\frac{1}{32}$ **11.** _____

12 Chapter 12 Test, Form 1 *(continued)*

12. Which measure of central tendency best represents a data set with outliers?

F. mode **G.** mean **H.** median **J.** variance **12.** _____

For Questions 13–15, use the data set {10, 12, 12, 14, 22}.

13. Find the mean.

A. 17.5 **B.** 14 **C.** 70 **D.** 13 **13.** _____

14. Find the variance. Round to the nearest tenth, if necessary.

F. 17.6 **G.** 88 **H.** 4.2 **J.** 4 **14.** _____

15. Find the standard deviation. Round to the nearest tenth, if necessary.

A. 17.6 **B.** 14.6 **C.** 4.2 **D.** 14 **15.** _____

16. Classify the data in the table.

F. positively skewed

G. negatively skewed

H. normally distributed

J. discrete distribution

Amount Spent on Lunch	
Less than $4.00	18%
$4.00–$7.99	47%
$8.00–$11.99	16%
$12.00–$15.99	11%
$16.00 or more	8%

16. _____

17. CAR SALES The mean stay of a car on a lot before being sold is 21 days, with a standard deviation of 3 days. The lengths of stay are normally distributed. What percent of the cars are sold after having been on the lot between 18 and 24 days?

A. 95% **B.** 34% **C.** 68% **D.** 5% **17.** _____

18. The probability that a certain team will win a baseball game is $\frac{1}{3}$. In a 5-game series, what is the probability that the team will win all five games?

F. $\frac{1}{15}$ **G.** $\frac{1}{243}$ **H.** $\frac{1}{3}$ **J.** $\frac{5}{243}$ **18.** _____

19. COMMUTERS Which group should be surveyed to determine how people commute to work in order to produce a random sample?

A. students in your school

B. people passing through a toll booth on a given day

C. people in your state whose last name begins with S

D. people whose annual income is greater than $1,000,000 **19.** _____

20. Find the margin of sampling error when $p = 45\%$ and $n = 100$ if $ME = 2\sqrt{\frac{p(1-p)}{n}}$.

F. 9% **G.** 10% **H.** 5% **J.** 1% **20.** _____

Bonus If f represents the probability of rolling a 5 and n represents the probability of rolling any other number, which term of $(f + n)^4 = f^4 + 4f^3n + 6f^2n^2 + 4fn^3 + n^4$ represents the probability of rolling exactly three 5s in 4 rolls of a die? Find the probability. **B:** _____

NAME _____ DATE_____ PERIOD _____

12 Chapter 12 Test, Form 2A

SCORE _____

Write the letter for the correct answer in the blank at the right of each question.

1. LICENSE PLATES A license plate has one letter (not I or O) followed by five digits. How many different plates are possible?

A. 1200 **B.** 2,400,000 **C.** 725,760 **D.** 100,000 **1.** _____

2. How many 3-letter identification codes are possible if no letter is repeated?

F. 17,576 **G.** 2600 **H.** 78 **J.** 15,600 **2.** _____

3. Evaluate $P(10, 4)$.

A. 5040 **B.** 151,200 **C.** 30,240 **D.** 210 **3.** _____

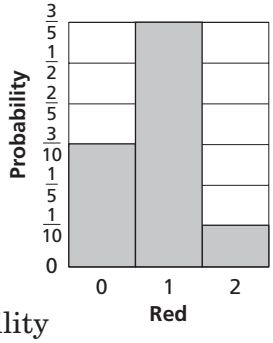
4. A group has 6 men and 5 women. How many ways can a committee of 3 men and 2 women be formed?

F. 200 **G.** 150 **H.** 7200 **J.** 2400 **4.** _____

5. Two letters are selected at random from the word *student*. Find P(2 vowels).

A. $\frac{2}{21}$ **B.** $\frac{1}{21}$ **C.** $\frac{10}{21}$ **D.** $\frac{2}{49}$ **5.** _____

6. Two marbles are chosen at random from a bag containing 3 blue and 2 red marbles. The relative-frequency histogram shows the distribution of the number of red marbles chosen. Find P(2 red).



F. $\frac{1}{10}$ **H.** $\frac{3}{5}$

G. $\frac{1}{5}$ **J.** $\frac{3}{10}$ **6.** _____

7. A red die and a blue die are tossed. What is the probability that the red die shows a 5 and the blue die shows an even number?

A. $\frac{1}{36}$ **B.** $\frac{1}{18}$ **C.** $\frac{1}{12}$ **D.** $\frac{2}{3}$ **7.** _____

8. Tickets are numbered 1 to 50 and are placed in a box. Three tickets are drawn at random without replacement. What is the probability that the numbers are all greater than 35?

F. $\frac{27}{1000}$ **G.** $\frac{13}{560}$ **H.** $\frac{3}{10}$ **J.** $\frac{1}{7840}$ **8.** _____

9. From 4 yellow and 9 blue marbles, 3 are selected. What is the probability that all 3 are yellow or all 3 are blue?

A. $\frac{4}{143}$ **B.** $\frac{4}{13}$ **C.** $\frac{42}{143}$ **D.** $\frac{84}{143}$ **9.** _____

10. A card is drawn from a deck of cards. What is the probability of drawing a club or a face card? (*Hint*: A face card is a jack, queen, or king.)

F. $\frac{25}{52}$ **G.** $\frac{3}{13}$ **H.** $\frac{11}{26}$ **J.** $\frac{7}{13}$ **10.** _____

11. A coin is tossed 5 times. Find P(at least 3 tails).

A. $\frac{3}{16}$ **B.** $\frac{1}{2}$ **C.** $\frac{5}{16}$ **D.** $\frac{3}{5}$ **11.** _____

12 Chapter 12 Test, Form 2A *(continued)*

12. How many different arrangements of the letters of the word *radar* are possible?

F. 120 **G.** 60 **H.** 30 **J.** 480 **12.** _____

TEMPERATURES For Questions 13–15, use the data in the table. Round to the nearest tenth, if necessary.

Record Low Temperatures in Honolulu, HI (°F)											
Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
52	53	55	56	60	65	66	67	66	61	57	54

Source: www.weather.com

13. Which measure of central tendency is *not* a good representation of the data?

A. mean **B.** mode **C.** median **D.** middle **13.** _____

14. Find the variance of the temperatures.

F. 28.4 **G.** 5.3 **H.** 59.3 **J.** 340.7 **14.** _____

15. Find the standard deviation of the temperatures.

A. 52°F **B.** 5.3°F **C.** 5.6°F **D.** 28.4°F **15.** _____

16. Classify the data in the table.

F. positively skewed

G. negatively skewed

H. normally distributed

J. discrete distribution **16.** _____

Age of Population of Iowa in 2000	
Age	Number of People
0–24	978,875
25–44	795,499
45–64	644,861
65–84	357,074
Over 84	45,848

17. POTTERY The diameters of pottery bowls are normally distributed. The mean of the diameters is 22 cm and the standard deviation is 2 cm. What percent of the bowls have diameters between 18 and 26 cm?

A. 13.5% **B.** 34% **C.** 68% **D.** 95% **17.** _____

18. In a local car lot, $\frac{1}{6}$ of the cars have standard transmissions. Find the probability that 3 of 4 randomly-selected cars have standard transmissions.

F. $\frac{125}{324}$ **G.** $\frac{5}{9}$ **H.** $\frac{5}{324}$ **J.** $\frac{5}{1296}$ **18.** _____

19. A school librarian wants to determine the reading interests of students. A survey of which group would produce a random sample?

A. every third student leaving the library on a given day

B. students on the football team

C. every fifth person entering the school in the morning

D. seniors planning to attend college **19.** _____

20. HOMEWORK In a survey of 320 students, 32% spent at least 1 hour per night on homework. Find the margin of sampling error.

F. 5% **G.** 21% **H.** 3% **J.** 10% **20.** _____

Bonus Write a data set having 7 values that has a median of 24 and a mean of 20. **B:** _____

NAME _____ DATE_____ PERIOD _____

12 Chapter 12 Test, Form 2B

SCORE _____

Write the letter for the correct answer in the blank at the right of each question.

1. An ice cream store has 31 flavors of ice cream and 10 toppings. A regular sundae has one flavor of ice cream, one topping, and comes with or without whipped cream. How many different ice cream sundaes can be ordered?

A. 310 **B.** 372 **C.** 620 **D.** 82 **1.** _____

2. How many 5-digit codes are possible if 0 cannot be used and no digit can be repeated?

F. 15,120 **G.** 45 **H.** 30,240 **J.** 59,049 **2.** _____

3. A clown has 7 balloons, each a different color. There are 5 children. How many ways can the clown give each child a balloon?

A. 21 **B.** 5040 **C.** 42 **D.** 2520 **3.** _____

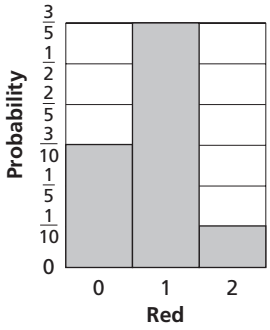
4. Evaluate $C(13, 9)$.

F. 17,160 **G.** 715 **H.** 259,459,200 **J.** 117 **4.** _____

5. Two letters are chosen at random from the word *question*. Find $P(2 \text{ consonants})$.

A. $\frac{2}{7}$ **B.** $\frac{3}{7}$ **C.** $\frac{3}{28}$ **D.** $\frac{3}{14}$ **5.** _____

6. Two marbles are chosen at random from a bag containing 3 blue and 2 red marbles. The relative-frequency histogram shows the distribution of the number of red marbles chosen. Find $P(0 \text{ red})$.



F. $\frac{1}{10}$ **G.** $\frac{1}{5}$ **H.** $\frac{3}{10}$ **J.** $\frac{3}{5}$ **6.** _____

7. A red die and a blue die are tossed. What is the probability that the red die shows a 3 and the blue die shows a number greater than 3?

A. $\frac{1}{10}$ **B.** $\frac{1}{5}$ **C.** $\frac{1}{12}$ **D.** $\frac{3}{5}$ **7.** _____

8. Tickets are numbered 1 to 50 and placed in a box. Three tickets are drawn at random without replacement. What is the probability that their numbers are all greater than 25?

F. $\frac{1}{8}$ **G.** $\frac{23}{196}$ **H.** $\frac{69}{625}$ **J.** $\frac{1}{2}$ **8.** _____

9. From 4 yellow and 8 blue marbles, 3 are selected. What is the probability that all three are yellow or all three are blue?

A. $\frac{3}{11}$ **B.** $\frac{1}{55}$ **C.** $\frac{14}{55}$ **D.** $\frac{3}{220}$ **9.** _____

10. A card is drawn from a standard deck of cards. What is $P(\text{heart or a 6})$?

F. $\frac{9}{26}$ **G.** $\frac{17}{52}$ **H.** $\frac{1}{4}$ **J.** $\frac{4}{13}$ **10.** _____

11. A coin is tossed 5 times. Find $P(\text{at most 4 tails})$.

A. $\frac{3}{16}$ **B.** $\frac{13}{16}$ **C.** $\frac{1}{32}$ **D.** $\frac{31}{32}$ **11.** _____

12 Chapter 12 Test, Form 2B *(continued)*

12. How many different arrangements of the letters of the word *doodle* are possible?

F. 180 **G.** 720 **H.** 15 **J.** 90 **12.** _____

TEMPERATURES For Questions 13–15, use the data in the table. Round to the nearest tenth, if necessary.

Record High Temperatures in Anchorage, Alaska (°F)											
Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
50	48	51	65	77	85	82	82	73	61	53	48

Source: www.weather.com

13. Which measure of central tendency is *not* a good representation of the data?

A. middle **B.** median **C.** mode **D.** mean **13.** _____

14. Find the variance of the temperatures.

F. 4366.2 **G.** 64.6 **H.** 2342.9 **J.** 195.2 **14.** _____

15. Find the standard deviation of the temperatures.

A. 14.6°F **B.** 14.0°F **C.** 63.0°F **D.** 64.6°F **15.** _____

16. Classify the data in the table.

F. positively skewed

G. negatively skewed

H. normally distributed

J. discrete distribution **16.** _____

Age of Population of Rhode Island in 2000	
Age	**Number of People**
0–14	206,423
15–34	265,778
35–54	308,946
55–74	159,092
over 74	69,264

Source: Census 2000

17. For 2000 patients, blood-clotting time was normally distributed with a mean of 8 seconds and a standard deviation of 3 seconds. What percent had blood-clotting times between 5 and 11 seconds?

A. 68% **B.** 34% **C.** 49.5% **D.** 47.5% **17.** _____

18. During a sale, $\frac{1}{6}$ of the CD prices are reduced. Find the probability that 2 of 4 randomly-selected CDs have reduced prices.

F. $\frac{5}{36}$ **G.** $\frac{25}{1296}$ **H.** $\frac{25}{216}$ **J.** $\frac{5}{216}$ **18.** _____

19. A music teacher wants to determine the music preferences of students. A survey of which group would produce a random sample?

A. students in the school band

B. students attending the annual jazz concert

C. students in every odd-numbered homeroom

D. every other player on the baseball roster **19.** _____

20. ELECTIONS In an election poll, 56% of 400 voters chose a certain candidate. Find the margin of sampling error.

F. 5% **G.** 2% **H.** 4% **J.** 7% **20.** _____

Bonus Write a data set having 7 data values that has a median of 20 and a mean of 24. **B:** _____

NAME _____ DATE _____ PERIOD _____

12 Chapter 12 Test, Form 2C

SCORE _____

1. **BELTS** A clothing store sells belts in 3 colors, 4 designs, and 6 sizes. How many different belts are available? 1. _____

2. Five children stand in a line to play a game. How many different ways can the children be arranged? 2. _____

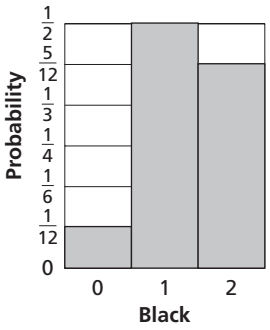
3. **CROSS-COUNTRY** Twelve runners are in a cross-country race. How many different ways can they finish first, second, and third? 3. _____

4. Five cheerleaders will be chosen from a group of 15 students. How many different cheerleading squads can be formed? 4. _____

5. Three letters are selected at random from the word *practice*. Find P(3 consonants). 5. _____

6. Two socks are chosen at random from a drawer containing 6 black and 3 blue socks. The table and relative-frequency histogram show the distribution of the number of black socks chosen. Find $P(B = 2)$. 6. _____

B = Black	0	1	2
Probability	$\frac{1}{12}$	$\frac{1}{2}$	$\frac{5}{12}$



7. A die is rolled three times. What is P(no 5s)? 7. _____

8. Two cards are drawn from a standard deck of 52 cards without replacement. Find the probability that the first card is an ace and the second is a 2. 8. _____

9. From a group of 6 men and 8 women, a committee of 3 is selected. Find the probability that all 3 are men or all 3 are women. 9. _____

10. Each of the numbers 1 to 25 is written on a card and placed in a bag. If one card is drawn at random, what is the probability that it is a multiple of 4 or a multiple of 5? 10. _____

11. Seven coins are tossed. Find P(at least 6 tails). 11. _____

12. If the probability of rain in a certain city is $\frac{1}{8}$ on any given day, find the probability that rain will fall on exactly one day of a three-day visit to the city. 12. _____

13. How many different arrangements of the letters in the word ILLINOIS are possible? 13. _____

Assessment

NAME _____ DATE_____ PERIOD _____

12 Chapter 12 Test, Form 2C *(continued)*

TEMPERATURES For Questions 14–16, use the data in the table. Round to the nearest tenth, if necessary.

Record High Temperatures in Memphis, TN (°F)											
Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
78	81	85	94	99	104	108	105	103	95	85	81

Source: www.weather.com

14. If you were a member of the Chamber of Commerce, which measure of central tendency would you use to convince someone that Memphis has a comfortable climate? Explain. **14.** _____

15. Find the variance of the temperatures. **15.** _____

16. Find the standard deviation of the temperatures. **16.** _____

17. EDUCATION Determine whether the data in the table is *positively skewed, negatively skewed,* or *normally distributed.* **17.** _____

Educational Attainment in Georgia for persons over 25 years of age, as of 2000	
Less than 9th grade	484,000
9th to 12th grade, no diploma	686,000
High school graduate	1,193,000
Some college or associate degree	884,000
Bachelor's degree	520,000
Graduate or professional degree	258,000

Source: www.census.gov

18. COLLEGE ENTRANCE EXAM The scores on a standardized college entrance examination are found to be normally distributed. The mean is 85 and the standard deviation is 11. What percent scored between 85 and 107? **18.** _____

19. Determine whether the situation would produce a random sample and explain your answer: *surveying your class to determine the most-admired person in the United States by people your age.* **19.** _____

20. In a sample of 120 small business owners, 64% said they preferred a certain company for office supplies. Find the margin of sampling error. **20.** _____

Bonus Student test grades were normally distributed, and grades between 62 and 86 were within three standard deviations of the mean. Find the mean and standard deviation of the set of grades. **B:** _____

NAME _____ DATE_____ PERIOD _____

12 Chapter 12 Test, Form 2D

SCORE _____

1. A store sells T-shirts in 7 colors, 5 designs, and 3 sizes. How many different T-shirts are available? **1.** _____

2. Marva needs to mow the lawn, pay her bills, walk the dog, and return a phone call. How many ways can she choose to order her tasks? **2.** _____

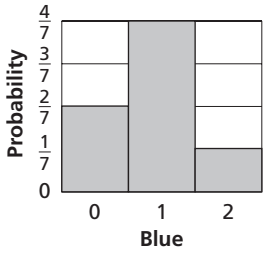
3. How many different arrangements of three coins can be made if you have a penny, a nickel, a dime, a quarter, and a silver dollar? **3.** _____

4. How many different 5-player basketball teams can be formed from a group of 12 people? **4.** _____

5. Three letters are selected at random from the word *distance*. Find P(3 vowels). **5.** _____

6. Two socks are chosen at random from a drawer containing 4 black and 3 blue socks. The table and relative-frequency histogram show the distribution of the number of blue socks chosen. Find $P(B = 1)$. **6.** _____

B = Blue	0	1	2
Probability	$\frac{2}{7}$	$\frac{4}{7}$	$\frac{1}{7}$



7. A die is rolled three times. What is P(three 5s)? **7.** _____

8. Two cards are drawn from a standard deck of 52 cards without replacement. Find the probability that both cards are aces. **8.** _____

9. From a group of 7 men and 5 women, a 4-person committee is chosen. What is the probability that all 4 are men or all 4 are women? **9.** _____

10. Each of the numbers 1 to 20 is written on a card and placed in a bag. If one card is drawn at random, what is the probability that it is a multiple of 3 or a multiple of 5? **10.** _____

11. Eight coins are tossed. Find P(at least 7 heads). **11.** _____

12. If the probability of rain in a certain city is $\frac{2}{5}$ on any given day, find the probability that rain will fall on exactly one day of a three-day visit to the city. **12.** _____

NAME _____ DATE_____ PERIOD _____

12 Chapter 12 Test, Form 2D *(continued)*

13. How many different arrangements of the letters in the word INDIANA are possible? **13.** _____

14. The sales prices of several cars on a used car lot are \$18,900; \$20,500; \$29,900; \$19,800; and \$21,750. Which measure of central tendency best represents the data? Explain. **14.** _____

For Questions 15 and 16, use the data in the table that shows average precipitation in Grand Junction, Colorado. Round to the nearest hundredth, if necessary.

Average Precipitation in Grand Junction, CO (in.)											
Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
0.6	0.5	0.9	0.8	0.9	0.5	0.7	0.8	0.8	1.0	0.7	0.6

Source: www.weather.com

15. Find the variance of the data. **15.** _____

16. Find the standard deviation of the data. **16.** _____

17. Determine whether the data in the table is *positively skewed, negatively skewed,* or *normally distributed.* **17.** _____

Age of Population of Florida in 2000	
Age	**Number of People**
0–24	4,870,160
25–44	4,442,638
45–64	3,581,676
65–84	2,449,573
Over 84	269,388

Source: Census 2000

18. COLLEGE ENTRANCE EXAM The scores on a standardized college entrance examination are found to be normally distributed. The mean is 78 and the standard deviation is 13. What percent scored between 52 and 78? **18.** _____

19. Determine whether the situation would produce a random sample and explain your answer: *surveying persons with library cards to determine if a city should raise taxes to pay for a new library.* **19.** _____

20. In a survey of 60 customers in a supermarket, 40% expect to use the express line. What is the margin of sampling error? **20.** _____

Bonus Student test grades were normally distributed, and grades between 68 and 86 were within three standard deviations of the mean. Find the mean and standard deviation of the set of grades. **B:** _____

NAME _____ DATE _____ PERIOD _____

12 Chapter 12 Test, Form 3

SCORE _____

Assessment

1. Each day, Jonathan chooses one of six routes to work. How many different ways can Jonathan get to work over a five-day period? **1.** _____

2. How many different ways can 9 entertainers appear on an awards show if the guest of honor must appear first or last? **2.** _____

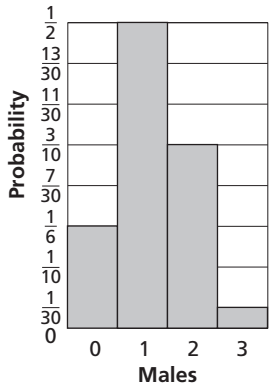
3. How many ways can you select 4 pizza toppings from a total of 8 toppings? Is this a permutation or a combination? Explain. **3.** _____

4. Evaluate $C(13, 5) \cdot C(9, 4)$. **4.** _____

5. A coin purse contains 4 pennies, 5 nickels, and 8 dimes. Three coins are selected at random. Find the probability of selecting one coin of each type. **5.** _____

6. Three students are selected at random from a group of 4 males and 6 females. The table and relative-frequency histogram show the distribution of the number of males chosen. Find P(two females). **6.** _____

M = Male	0	1	2	3
Probability	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$



7. A die is rolled four times. Find P(four of the same number). **7.** _____

8. Four cards are drawn from a standard deck of 52 cards without replacement. Find the probability that the first card is a heart, the second is a club, and the third and fourth are diamonds. **8.** _____

9. From a group of 8 men and 10 women, a committee of 5 is to be selected at random. Find P(at least 3 men). **9.** _____

10. Two cards are drawn from a standard deck of cards. Find P(both black or both 9s). **10.** _____

11. How many ways can 4 basketball shoes, 2 tennis shoes, and 5 running shoes be arranged on a shelf if the shoes are grouped according to type? **11.** _____

NAME _____ DATE_____ PERIOD _____

12 Chapter 12 Test, Form 3 *(continued)*

TAXES For Questions 12–15, use the data in the table that shows the per capita taxes, in dollars, in the 10 states listed. Round to the nearest cent, if necessary.

State	Taxes	State	Taxes
Arizona	1489	Iowa	1678
California	2073	Maine	1905
Colorado	1483	Maryland	1790
Delaware	2665	Michigan	2161
Illinois	1641	Missouri	1512

Source: *World Almanac*

12. Which measure of central tendency might a realtor in Maryland use to convince a client that the per capita taxes were reasonable? Explain. **12.** _____

13. Find the variance of the taxes. **13.** _____

14. Find the standard deviation of the taxes. **14.** _____

15. Determine whether the data in the table is *positively skewed, negatively skewed,* or *normally distributed.* **15.** _____

16. IQ TESTS Scores on an IQ test are normally distributed. The mean is 100 and the standard deviation is 15. If 6000 people took the test, how many of them scored between 85 and 130? **16.** _____

17. Find P(at least four 4s) if a die is rolled 6 times. **17.** _____

18. In a certain city in June, the probability that the temperature will rise above 80°F is 0.7. For the first 8 days, what is P(temperature will rise above 80°F exactly 3 times)? Round to the nearest hundredth. **18.** _____

19. Determine whether the situation would produce a random sample and explain your answer: *surveying town residents whose license number ends in 5 to determine whether to increase taxes to pay for road repair.* **19.** _____

20. PETS In a survey of pet owners, 68% preferred dogs to any other kind of pet. The margin of sampling error was 5%. How many people were surveyed? **20.** _____

Bonus 20% of the students in a high school were surveyed to determine their favorite pizza topping. If 43% of those surveyed responded "pepperoni," and the margin of sampling error was 6.2%, how many students attend the high school? **B:** _____

NAME _____ DATE_____ PERIOD _____

12 Chapter 12 Extended-Response Test

SCORE _____

Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solutions in more than one way or investigate beyond the requirements of the problem.

1. Kathy, Alma, and Steven are working on a group quiz. One question is as follows.

Two dice are rolled. Find the probability that the first die is a 5 or a 6, and the second die is an even number.

All three students agree to let A represent rolling a 5 or a 6, and B represent rolling an even number. But Kathy argues that the solution is $P(A) + P(B) = \frac{2}{6} + \frac{3}{6} = \frac{5}{6}$, Alma feels certain that the solution should be $P(A) \cdot P(B) = \frac{2}{6} \cdot \frac{3}{6} = \frac{6}{36} = \frac{1}{6}$, and Steven is convinced that the correct solution is $P(A) + P(B) - P(A \text{ and } B) = \frac{2}{6} + \frac{3}{6} - \frac{1}{6} = \frac{4}{6} - \frac{2}{3}$.

a. Which student, if any, is correct? Explain your reasoning.

b. For one of the incorrect solutions above, write a probability problem for which that solution would be correct.

2. a. One day, your math teacher, Mr. Butler, looks at your exam scores and informs you that your score distribution is negatively skewed. How do you feel about this news? Explain your reasoning.

b. The next day, Mr. Butler announces that the class scores on the last exam were normally distributed, that scores between 56 and 98 fell within three standard deviations of the mean, and that students whose scores fell within one standard deviation of the mean would earn a grade of C on the exam. Explain how to estimate the mean score, the standard deviation of the class scores, and the range of grades for which a student would earn a grade of C. Determine the indicated values.

3. Greg and Jacqui are planning a dinner party for 6 guests. After dinner, they plan to separate into two teams to play charades.

a. Explain how you could determine the number of different possible arrangements of guests and hosts into two teams. Include in your explanation whether the formula $P(n, r) = \frac{n!}{(n-r)!}$ or the formula $C(n, r) = \frac{n!}{(n-r)!r!}$ would be helpful in determining the number of arrangements. Explain your reasoning and determine the number of arrangements that are possible.

b. Would the number of possible arrangements change if Greg and Jacqui decided that they should be on different teams? If so, how many arrangements would be possible under those conditions? Explain your reasoning.

Assessment

NAME _____ DATE_____ PERIOD _____

12 Standardized Test Practice

(Chapters 1–12)

SCORE _____

Part 1: Multiple Choice

Instructions: Fill in the appropriate circle for the best answer.

1. What is the seventh term in the sequence $-\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \frac{1}{16}, \ldots$?

A $\frac{1}{64}$ **B** $-\frac{1}{64}$ **C** $\frac{1}{128}$ **D** $-\frac{1}{128}$ **1.** Ⓐ Ⓑ Ⓒ Ⓓ

2. In the correctly completed addition problem shown, ▲ and ∗ are nonzero digits. What number does ∗ represent?

$$\begin{array}{r} ▲\ 5 \\ *\ ▲ \\ 8\ 1 \\ +\ 1\ ▲ \\ \hline 1\ 6\ 0 \end{array}$$

F 5 **H** 4
G 7 **J** 2 **2.** Ⓕ Ⓖ Ⓗ Ⓙ

3. The volume of a rectangular box is 405. The length, width, and height of the box are in the ratio 5 : 3 : 1. What is the total surface area of the box?

A 414 **B** 27 **C** 54 **D** 324 **3.** Ⓐ Ⓑ Ⓒ Ⓓ

4. If $3^{4x+2} = 9^5$, what is the value of x?

F $\frac{3}{4}$ **G** 2 **H** 3 **J** $\frac{1}{2}$ **4.** Ⓕ Ⓖ Ⓗ Ⓙ

5. David is twice as old as his sister, Jennifer. Three years ago, David was three times as old as Jennifer. How old is David now?

A 12 **B** 9 **C** 6 **D** 3 **5.** Ⓐ Ⓑ Ⓒ Ⓓ

6. If $a < 0$ and $b > 0$, which of the following statements must be true?

I. $ab < 0$ **II.** $b - a > 0$ **III.** $ac < bc$

F I, II, and III **H** I and II only
G I only **J** II and III only **6.** Ⓕ Ⓖ Ⓗ Ⓙ

7. If $(x - y)^2 = 200$ and $x^2 + y^2 = 50$, what is the value of xy?

A -75 **B** 75 **C** 150 **D** -150 **7.** Ⓐ Ⓑ Ⓒ Ⓓ

8. What is 25% of 20% of $\frac{3}{4}$?

F 0.375 **G** 3.75 **H** 0.00375 **J** 0.0375 **8.** Ⓕ Ⓖ Ⓗ Ⓙ

9. What is the sum of all composite numbers between 1 and 15?

A 120 **B** 59 **C** 78 **D** 63 **9.** Ⓐ Ⓑ Ⓒ Ⓓ

10. If $(x + y)^2 = 8$ and $(x - y)^2 = 4$, what is xy?

F 0 **G** 1 **H** 2 **J** 3 **10.** Ⓕ Ⓖ Ⓗ Ⓙ

12 Standardized Test Practice *(continued)*

11. Alejandra has been saving to purchase a VCR. The model she wants is priced at \$180, on which she will be required to pay 5% sales tax. She has already saved \$53. If Alejandra earns \$8.50 per hour after all payroll deductions have been made, for how many hours will she need to work in order to have enough money to purchase the VCR?

A 15 **B** 16 **C** 29 **D** 160 **11.** Ⓐ Ⓑ Ⓒ Ⓓ

12. If $4^{3x-2} = 256$, what is the value of 3^{2x+1}?

F 2 **G** 243 **H** 22 **J** 45 **12.** Ⓕ Ⓖ Ⓗ Ⓙ

13. What is the graph of $4x^2 = 9 - 4y^2$?

A circle **B** ellipse **C** hyperbola **D** parabola **13.** Ⓐ Ⓑ Ⓒ Ⓓ

14. Write $0.\overline{627}$ as a fraction.

F $\frac{5}{8}$ **G** $\frac{209}{333}$ **H** $\frac{6}{27}$ **J** $\frac{627}{1000}$ **14.** Ⓕ Ⓖ Ⓗ Ⓙ

15. How many different ways can the letters of the word *PERMUTATION* be arranged?

A 1 **B** 3,628,800 **C** 19,958,400 **D** 39,916,800 **15.** Ⓐ Ⓑ Ⓒ Ⓓ

16. Three cards are drawn from a standard deck of cards without replacement. Find the probability of drawing a king, a queen, and another king in that order.

F $\frac{1}{2197}$ **G** $\frac{2}{5525}$ **H** $\frac{3}{8788}$ **J** $\frac{12}{5525}$ **16.** Ⓕ Ⓖ Ⓗ Ⓙ

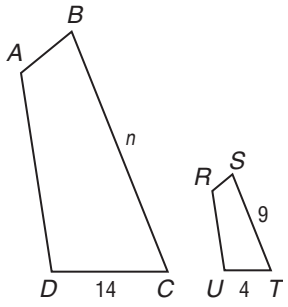
17. The scores on an algebra test are found to be normally distributed. The mean is 72 and the standard deviation is 8. What percent scored between 72 and 88?

A 13.5% **B** 68% **C** 47.5% **D** 95% **17.** Ⓐ Ⓑ Ⓒ Ⓓ

Part 2: Griddable

Instructions: Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate circle that corresponds to that entry.

18. In the figure at the right, quadrilaterals *ABCD* and *RSTU* are similar. What is the value of n?



				.			
⓪	⓪	⓪	⓪		⓪	⓪	⓪
①	①	①	①		①	①	①
②	②	②	②		②	②	②
③	③	③	③		③	③	③
④	④	④	④		④	④	④
⑤	⑤	⑤	⑤		⑤	⑤	⑤
⑥	⑥	⑥	⑥		⑥	⑥	⑥
⑦	⑦	⑦	⑦		⑦	⑦	⑦
⑧	⑧	⑧	⑧		⑧	⑧	⑧
⑨	⑨	⑨	⑨		⑨	⑨	⑨

19. If the average of a and b is 87, the average of a and c is 73, and the average of b and c is 50, what is the average of a, b, and c?

				.			
⓪	⓪	⓪	⓪		⓪	⓪	⓪
①	①	①	①		①	①	①
②	②	②	②		②	②	②
③	③	③	③		③	③	③
④	④	④	④		④	④	④
⑤	⑤	⑤	⑤		⑤	⑤	⑤
⑥	⑥	⑥	⑥		⑥	⑥	⑥
⑦	⑦	⑦	⑦		⑦	⑦	⑦
⑧	⑧	⑧	⑧		⑧	⑧	⑧
⑨	⑨	⑨	⑨		⑨	⑨	⑨

NAME _____ DATE_____ PERIOD _____

12 Standardized Test Practice *(continued)*

Part 3: Short Answer

Instructions: Write your answers in the space provided.

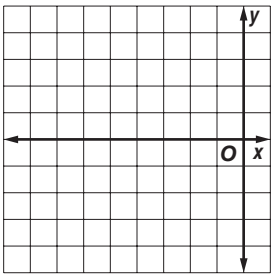
20. The vertices of RST are $R(-1, -3)$, $S(2, 4)$, and $T(-4, 3)$. The triangle is reflected over the line $y = x$. Find the coordinates of R', S', and T'. **20.** _____

21. Simplify $(x^2 + 3) - (4x^2 - 5x - 9)$. **21.** _____

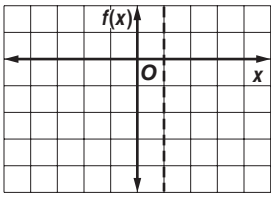
22. Determine whether $f(x) = 5x - 8$ and $g(x) = x + \frac{8}{5}$ are inverse functions. **22.** _____

For Questions 23 and 24, graph the function or equation.

23. $(x + 5)^2 + y^2 = 9$ **23.**



24. $f(x) = -\frac{2}{(x - 1)^2}$ **24.**



25. Solve $\frac{w}{w - 3} + w = \frac{3}{w - 3}$. Check your solution(s). **25.** _____

26. Write the equation $\log_{1000} \frac{1}{10} = -\frac{1}{3}$ in exponential form. **26.** _____

27. A savings account deposit of \$500 is to earn 5.7% interest. After how many years will the investment be worth \$750? Use $y = a(1 + r)^t$ and round to the nearest tenth. **27.** _____

28. Find the three arithmetic means between 5 and -7. **28.** _____

29. Find four geometric means between 27 and $\frac{1}{9}$. **29.** _____

30. The number tiles 11, 12, 13, 14, and 15 are placed in a bag. Two tiles are drawn at random.

a. Find P(13, then 14) if replacement occurs. **30a.** _____

b. Find P(2 odd numbers) if no replacement occurs. **30b.** _____

c. Find P(2 composite numbers) if replacement occurs. **30c.** _____

NAME _____ DATE_____ PERIOD _____

12 Unit 4 Test

(Chapters 11 and 12)

SCORE _____

1. Find the next four terms of the arithmetic sequence 4, 10, 16, … . **1.** _____

2. Find the three arithmetic means between 21 and 13. **2.** _____

3. Find S_n for the arithmetic series in which $a_1 = -11$, $a_n = 13$, and $n = 7$. **3.** _____

4. Find the next two terms of the geometric sequence 6250, 5000, 4000, … . **4.** _____

5. Find four geometric means between 4096 and 972. **5.** _____

6. Find the sum of a geometric series for which $a_1 = 1$, $r = 2$, and $n = 6$. **6.** _____

7. Find a_1 in a geometric series for which $S_n = 189$, $r = \frac{1}{2}$, and $a_n = 3$. **7.** _____

8. Find the sum of the infinite geometric series $36 + 24 + 16 + \ldots$, if it exists. **8.** _____

9. Write $0.\overline{735}$ as a fraction. **9.** _____

10. Find the first five terms of the sequence for which $a_1 = 5$ and $a_{n+1} = 3a_n + 1$. **10.** _____

11. Find the first three iterates x_1, x_2, x_3 of $f(x) = 2x - 5$ for an initial value of $x_0 = 3$. **11.** _____

12. Use the Binomial Theorem to find the fifth term in the expansion of $(2x + 3y)^5$. **12.** _____

13. Prove that the statement $\frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3} + \ldots + \frac{1}{5^n} = \frac{1}{4}\left(1 - \frac{1}{5^n}\right)$ is true for all positive integers n. Write your proof on a separate piece of paper. **13.** _____

14. Find a counterexample to the statement $4^n + 1$ is divisible by 5. **14.** _____

15. A scout troop will prepare trail mix for their next hike. They have decided to mix one type of nut, one type of dried fruit, and one type of granola. The local store carries 8 types of nuts, 6 types of dried fruit, and 5 types of granola. How many different trail mixes are possible? **15.** _____

16. Students are given a list of ten vocabulary words to learn. In how many ways could four of the words be listed on a test? **16.** _____

Assessment

NAME _____ DATE_____ PERIOD _____

12 Unit 4 Test *(continued)*

(Chapters 11 and 12)

17. Evaluate $C(12, 10)$. **17.** _____

18. Evaluate $P(12, 4)$. **18.** _____

19. A red die and a blue die are tossed. What is the probability that the red die shows an odd number and the blue die shows a 1 or 2? **19.** _____

20. From a group of 6 men and 4 women, a committee of 3 is to be selected at random. Find P(at least 2 women). **20.** _____

21. One card is drawn from a standard deck of cards. Find the probability that a king or a red card is drawn. **21.** _____

For Questions 22 and 23, use the data in the table that shows the number of public secondary schools in eight eastern states in the fall of 1998.

State	Number of Public Secondary Schools
Florida	456
Georgia	306
Maine	160
Massachusetts	363
North Carolina	376
New York	935
Rhode Island	54
Virginia	349

Source: *World Almanac*

22. Find the mean, median, mode, and standard deviation of the data. Round to the nearest hundredth, if necessary. **22.** _____

23. Determine whether the data in the table appear to be *positively skewed*, *negatively skewed*, or *normally distributed*. **23.** _____

24. The time a group of high school students arrive home from school each day was found to be normally distributed. The mean time was 3:15 P.M. and the times had a standard deviation of 15 minutes. What is the probability that a student chosen at random arrives home from school before 2:30 P.M.? **24.** _____

25. During a clothing sale, $\frac{1}{4}$ of the store merchandise is reduced in price. Find the probability that 3 of 5 randomly-selected shirts have reduced prices. **25.** _____

26. Determine whether the situation would produce a random sample and explain your answer: *surveying people at a concert to determine their favorite local radio station*. **26.** _____

NAME _____ DATE _____ PERIOD ____

12 Anticipation Guide

Probability and Statistics

Step 1 ***Before you begin Chapter 12***

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

STEP 1 A, D, or NS	Statement	STEP 2 A or D
	1. A sample space is a partial list of possible outcomes of an experiment.	D
	2. Two events are called independent if choosing one does not affect choosing the other.	A
	3. According to the Fundamental Counting Principle, if one event can occur in 6 ways and another event can occur in 3 ways, then the events together can occur in 6 + 3 or 9 ways.	D
	4. Since order is not important in a combination, an outcome ab is the same as an outcome ba.	A
	5. The odds of an event occurring can be expressed as a ratio of the number of successes to the total number of outcomes.	D
	6. If two events are dependent, then the probability of both events occurring is the product of the probabilities of each event.	D
	7. Two events are *mutually exclusive* if they cannot occur at the same time.	A
	8. If a set of data contains outliers, the median would be a good choice to represent the set.	A
	9. Measures of variation are the differences between consecutive values in the set.	D
	10. The curve representing a normal distribution is symmetric.	A
	11. The Binomial Theorem can be used to find probabilities only when there are two possible outcomes.	A
	12. Asking people in a music store how many hours they spend listening to music to determine how many hours people in the city listen to music is an example of an unbiased survey.	D

Step 2 ***After you complete Chapter 12***

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

NAME _____ DATE _____ PERIOD ____

12-1 Lesson Reading Guide

The Counting Principle

Get Ready for the Lesson

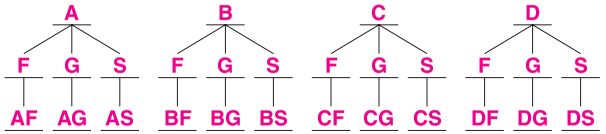
Read the introduction to Lesson 12-1 in your textbook.

Assume that all Florida license plates have three letters followed by three digits, and that there are no rules against using the same letter or number more than once. How many choices are there for each letter? for each digit? **26; 10**

Read the Lesson

1. Shamim is signing up for her classes. Most of her classes are required, but she has two electives. For her arts class, she can chose between Art, Band, Chorus, or Drama. For her language class, she can choose between French, German, and Spanish.

a. To organize her choices, Shamim decides to make a tree diagram. Let A, B, C, and D represent Art, Band, Chorus, and Drama, and F, G, and S represent French, German, and Spanish. Complete the following diagram.



b. How could Shamim have found the number of possible combinations without making a tree diagram? **Sample answer: Multiply the number of choices for her arts class by the number of choices for her language class: $4 \times 3 = 12$.**

2. A jar contains 6 red marbles, 4 blue marbles, and 3 yellow marbles. Indicate whether the events described are *dependent* or *independent*.

a. A marble is drawn out of the jar and is not replaced. A second marble is drawn. **dependent**

b. A marble is drawn out of the jar and is put back in. The jar is shaken. A second marble is drawn. **independent**

Remember What You Learned

3. One definition of *independent* is "not determined or influenced by someone or something else." How can this definition help you remember the difference between *independent* and *dependent* events? **Sample answer: If the outcome of one event does *not* affect or influence the outcome of another, the events are independent. If the outcome of one event *does* affect or influence the outcome of another, the events are dependent.**

NAME _____ DATE_____ PERIOD ____

12-1 Study Guide and Intervention

The Counting Principle

Independent Events If the outcome of one event does not affect the outcome of another event and vice versa, the events are called **independent events**.

Fundamental Counting Principle	If event M can occur in m ways and is followed by event N that can occur in n ways, then the event M followed by the event N can occur in $m \cdot n$ ways.

Example **FOOD For the Breakfast Special at the Country Pantry, customers can choose their eggs scrambled, fried, or poached, whole wheat or white toast, and either orange, apple, tomato, or grapefruit juice. How many different Breakfast Specials can a customer order?**

A customer's choice of eggs does not affect his or her choice of toast or juice, so the events are independent. There are 3 ways to choose eggs, 2 ways to choose toast, and 4 ways to choose juice. By the Fundamental Counting Principle, there are $3 \cdot 2 \cdot 4$ or 24 ways to choose the Breakfast Special.

Exercises

Solve each problem.

1. The Palace of Pizza offers small, medium, or large pizzas with 14 different toppings available. How many different one-topping pizzas do they serve? 42

2. The letters A, B, C, and D are used to form four-letter passwords for entering a computer file. How many passwords are possible if letters can be repeated? 256

3. A restaurant serves 5 main dishes, 3 salads, and 4 desserts. How many different meals could be ordered if each has a main dish, a salad, and a dessert? 60

4. Marissa brought 8 T-shirts and 6 pairs of shorts to summer camp. How many different outfits consisting of a T-shirt and a pair of shorts does she have? 48

5. There are 6 different packages available for school pictures. The studio offers 5 different backgrounds and 2 different finishes. How many different options are available? 60

6. How many 5-digit even numbers can be formed using the digits 4, 6, 7, 2, 8 if digits can be repeated? 2500

7. How many license plate numbers consisting of three letters followed by three numbers are possible when repetition is allowed? 17,576,000

8. How many 4-digit positive even integers are there? 4500

NAME _____ DATE_____ PERIOD ____

12-1 Study Guide and Intervention *(continued)*

The Counting Principle

Dependent Events If the outcome of an event *does* affect the outcome of another event, the two events are said to be **dependent**. The Fundamental Counting Principle still applies.

Example **ENTERTAINMENT The guests at a sleepover brought 8 videos. They decided they would only watch 3 videos. How many orders of 3 different videos are possible?**

After the group chooses to watch a video, they will not choose to watch it again, so the choices of videos are dependent events.

There are 8 choices for the first video. That leaves 7 choices for the second. After they choose the first 2 videos, there are 6 remaining choices. Thus by the Fundamental Counting Principle, there are $8 \cdot 7 \cdot 6$ or 336 orders of 3 different videos.

Exercises

Solve each problem.

1. Three students are scheduled to give oral reports on Monday. In how many ways can their presentations be ordered? 6

2. In how many ways can the first five letters of the alphabet be arranged if each letter is used only once? 120

3. In how many different ways can 4 different books be arranged on the shelf? 24

4. How many license plates consisting of three letters followed by three numbers are possible when no repetition is allowed? 11,232,000

5. Sixteen teams are competing in a soccer match. Gold, silver, and bronze medals will be awarded to the top three finishers. In how many ways can the medals be awarded? 3360

6. In a word-building game each player picks 7 letter tiles. If Julio's letters are all different, how many 3-letter combinations can he make out of his 7 letters? 210

7. The editor has accepted 6 articles for the news letter. In how many ways can the 6 articles be ordered? 720

8. There are 10 one-hour workshops scheduled for the open house at the greenhouse. There is only one conference room available. In how many ways can the workshops be ordered? 3,628,800

9. The top 5 runners at the cross-country meet will receive trophies. If there are 22 runners in the race, in how many ways can the trophies be awarded? 3,160,080

NAME _____ DATE_____ PERIOD ____

12-1 Skills Practice

The Counting Principle

State whether the events are *independent* or *dependent*.

1. finishing in first, second, or third place in a ten-person race **dependent**
2. choosing a pizza size and a topping for the pizza **independent**
3. Seventy-five raffle tickets are placed in a jar. Three tickets are then selected, one after the other, without replacing a ticket after it is chosen. **dependent**
4. The 232 members of the freshman class all vote by secret ballot for the class representative to the Student Senate. **independent**

Solve each problem.

5. A surveying firm plans to buy a color printer for printing its maps. It has narrowed its choice to one of three models. Each of the models is available with either 32 megabytes of random access memory (RAM), 64 megabytes of RAM, or 128 megabytes of RAM. From how many combinations of models and RAM does the firm have to choose? **9**
6. How many arrangements of three letters can be formed from the letters of the word *MATH* if any letter will not be used more than once? **24**
7. Allan is playing the role of Oliver in his school's production of *Oliver Twist*. The wardrobe crew has presented Allan with 5 pairs of pants and 4 shirts that he can wear. How many possible costumes consisting of a pair of pants and a shirt does Allan have to choose from? **20**
8. The 10-member steering committee that is preparing a study of the public transportation needs of its town will select a chairperson, vice-chairperson, and secretary from the committee. No person can serve in more than one position. In how many ways can the three positions be filled? **720**
9. Jeanine has decided to buy a pickup truck. Her choices include either a V-6 engine or a V-8 engine, a standard cab or an extended cab, and 2-wheel drive or 4-wheel drive. How many possible models does she have to choose from? **8**
10. A mail-order company that sells gardening tools offers rakes in two different lengths. Customers can also choose either a wooden, plastic, or fiberglass handle for the rake. How many different kinds of rakes can a customer buy? **6**
11. A Mexican restaurant offers chicken, beef, or vegetarian fajitas wrapped with either corn or flour tortillas, and topped with either mild, medium, or hot salsa. How many different choices of fajitas does a customer have? **18**

NAME _____ DATE_____ PERIOD ____

12-1 Practice

The Counting Principle

State whether the events are *independent* or *dependent*.

1. choosing an ice cream flavor and choosing a topping for the ice cream **independent**
2. choosing an offensive player of the game and a defensive player of the game in a professional football game **independent**
3. From 15 entries in an art contest, a camp counselor chooses first, second, and third place winners. **dependent**
4. Jillian is selecting two more courses for her block schedule next semester. She must select one of three morning history classes and one of two afternoon math classes. **independent**

Solve each problem.

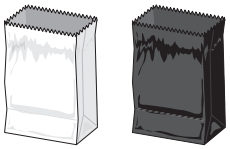
5. A briefcase lock has 3 rotating cylinders, each containing 10 digits. How many numerical codes are possible? **1000**
6. A golf club manufacturer makes irons with 7 different shaft lengths, 3 different grips, 5 different lies, and 2 different club head materials. How many different combinations are offered? **210**
7. There are five different routes that a commuter can take from her home to the office. In how many ways can she make a round trip if she uses a different route coming than going? **20**
8. In how many ways can the four call letters of a radio station be arranged if the first letter must be W or K and no letters repeat? **27,600**
9. How many 7-digit phone numbers can be formed if the first digit cannot be 0 or 1, and any digit can be repeated? **8,000,000**
10. How many 7-digit phone numbers can be formed if the first digit cannot be 0, and any digit can be repeated? **9,000,000**
11. How many 7-digit phone numbers can be formed if the first digit cannot be 0 or 1, and if no digit can be repeated? **483,840**
12. How many 7-digit phone numbers can be formed if the first digit cannot be 0, and if no digit can be repeated? **544,320**
13. How many 6-character passwords can be formed if the first character is a digit and the remaining 5 characters are letters that can be repeated? **118,813,760**
14. How many 6-character passwords can be formed if the first and last characters are digits and the remaining characters are letters? Assume that any character can be repeated. **45,697,600**

NAME _____ DATE_____ PERIOD ____

12-1 Word Problem Practice

The Counting Principle

1. **CANDY** Amy, Bruce, and Carol can choose one piece of candy from either a white or black bag. The white bag contains various chocolates. The black bag contains small bags of jelly beans. Amy picks from the white bag and Bruce and Carol both pick from the black bag. Describe whether each of the picks is related as dependent or independent events.



Amy's pick is independent of each of Bruce and Carol's picks; Bruce and Carol's picks are examples of dependent events.

2. **PHOTOS** Morgan has three pictures that she would like to display side by side.

In how many different ways can the pictures be displayed?
6

3. **COMBINATION LOCKS** Eric uses a combination lock for his locker. The lock uses a three number secret code. Each number ranges from 1 to 35, inclusive. How many different combinations are possible with Eric's lock?
42,875

4. **TRUE OR FALSE** Faith is preparing a true or false quiz for her biology class. How many different answer keys can there be for a 10 question true or false quiz?
1024

WEBSITES For Exercises 5-8, use the following information.

Greg is registering to use a website. The website requires him to choose an 8 character alphanumeric password that is not case-sensitive. In other words, for each character, he can choose one of the 26 letters A through Z or one of the 10 digits 0 through 9.

5. How many different passwords are possible?
2,821,109,907,456

6. Greg decides to use a password that does not contain any repeated characters. How many different passwords are possible with this constraint?
1,220,096,908,800

7. Suppose Greg chooses to use only letters with possible repeats. How many different passwords would be possible?
208,827,064,576

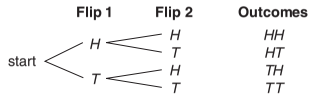
8. If Greg's password begins with his first name and ends with his birth month and date, how many possibilities would need to be checked to find his password?
372

NAME _____ DATE_____ PERIOD ____

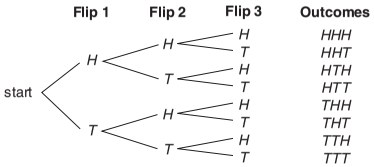
12-1 Enrichment

Tree Diagrams and the Power Rule

If you flip a coin once, there are two possible outcomes: heads showing (H) or tails showing (T). The tree diagram to the right shows the four (2^2) possible outcomes if you flip a coin twice.

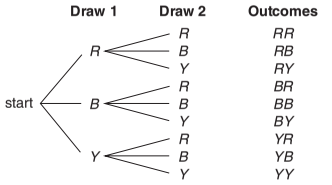


Example 1 **Draw a tree diagram to show all the possible outcomes for flipping a coin three times. List the outcomes.**



There are eight (2^3) possible outcomes. With each extra flip, the number of outcomes doubles. With 4 flips, there would be sixteen (2^4) outcomes.

Example 2 **In a cup there are a red, a blue, and a yellow marble. How many possible outcomes are there if you draw one marble at random, replace it, and then draw another?**



There are nine (3^2) possible outcomes.

The Power Rule for the number of outcomes states that if an experiment is repeated n times, and if there are b possible outcomes each time, there are b^n total possible outcomes.

Find the total number of possible outcomes for each experiment. Use tree diagrams to help you.

1. flipping a coin 5 times **2^5**
2. doing the marble experiment 6 times **3^6**
3. flipping a coin 8 times **2^8**
4. rolling a 6-sided die 2 times **6^2**
5. rolling a 6-sided die 3 times **6^3**
6. rolling a 4-sided die 2 times **4^2**
7. rolling a 4-sided die 3 times **4^3**
8. rolling a 12-sided die 2 times **12^2**

NAME _____ DATE_____ PERIOD ____

12-2 Lesson Reading Guide

Permutations and Combinations

Get Ready for the Lesson

Read the introduction to Lesson 12-2 in your textbook.

Suppose that 20 students enter a math contest. In how many ways can first, second, and third places be awarded? (Write your answer as a product. Do not calculate the product.)
20 · 19 · 18

Read the Lesson

1. Indicate whether each situation involves a *permutation* or a *combination*.

   a. choosing five students from a class to work on a special project **combination**

   b. arranging five pictures in a row on a wall **permutation**

   c. drawing a hand of 13 cards from a 52-card deck **combination**

   d. arranging the letters of the word *algebra* **permutation**

2. Write an expression that can be used to calculate each of the following.

   a. number of combinations of n distinct objects taken r at a time $\frac{n!}{(n-r)!r!}$

   b. number of permutations of n objects of which p are alike and q are alike $\frac{n!}{p!q!}$

   c. number of permutations of n distinct objects taken r at a time $\frac{n!}{(n-r)!}$

3. Five cards are drawn from a standard deck of cards. Suppose you are asked to determine how many possible hands consist of one heart, two diamonds, and two spades.

   a. Which of the following would you use to solve this problem: *Fundamental Counting Principle*, *permutations*, or *combinations*? (More than one of these may apply.)

   Fundamental Counting Principle, combinations

   b. Write an expression that involves the notation $P(n, r)$ and/or $C(n, r)$ that you would use to solve this problem. (Do not do any calculations.)

   $C(13, 1) \cdot C(13, 2) \cdot C(13, 2)$

Remember What You Learned

4. Many students have trouble knowing when to use permutations and when to use combinations to solve counting problems. How can the idea of *order* help you to remember the difference between permutations and combinations?

   Sample answer: A permutation is an arrangement of objects in which order is important. A combination is a selection of objects in which order is *not* important.

NAME _____ DATE_____ PERIOD ____

12-2 Study Guide and Intervention

Permutations and Combinations

Permutations When a group of objects or people are arranged in a certain order, the arrangement is called a **permutation**.

Permutations	The number of permutations of n distinct objects taken r at a time is given by $P(n, r) = \frac{n!}{(n-r)!}$.
Permutations with Repetitions	The number of permutations of n objects of which p are alike and q are alike is $\frac{n!}{p!q!}$.

The rule for permutations with repetitions can be extended to any number of objects that are repeated.

Example **From a list of 20 books, each student must choose 4 books for book reports. The first report is a traditional book report, the second a poster, the third a newspaper interview with one of the characters, and the fourth a timeline of the plot. How many different orderings of books can be chosen?**

Since each book report has a different format, order is important. You must find the number of permutations of 20 objects taken 4 at a time.

$$P(n, r) = \frac{n!}{(n-r)!} \quad \text{Permutation formula}$$

$$P(20, 4) = \frac{20!}{(20-4)!} \quad n = 20, r = 4$$

$$= \frac{20!}{16!} \quad \text{Simplify.}$$

$$= \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot \overset{1}{\cancel{16}} \cdot \overset{1}{\cancel{15}} \cdot \ldots \cdot \overset{1}{\cancel{1}}}{\underset{1}{\cancel{16}} \cdot \underset{1}{\cancel{15}} \cdot \ldots \cdot \underset{1}{\cancel{1}}} \quad \text{Divide by common factors.}$$

$$= 116{,}280$$

Books for the book reports can be chosen 116,280 ways.

Exercises

Evaluate each expression.

1. $P(6, 3)$ **120**
2. $P(8, 5)$ **6720**
3. $P(9, 4)$ **3024**
4. $P(11, 6)$ **332,640**

How many different ways can the letters of each word be arranged?

5. MOM **3**
6. MONDAY **720**
7. STEREO **360**

8. **SCHOOL** The high school chorus has been practicing 12 songs, but there is time for only 5 of them at the spring concert. How may different orderings of 5 songs are possible?
95,040

NAME _____ DATE _____ PERIOD ____

12-2 Study Guide and Intervention *(continued)*

Permutations and Combinations

Combinations An arrangement or selection of objects in which order is *not* important is called a combination.

Combinations	The number of combinations of n distinct objects taken r at a time is given by $C(n, r) = \frac{n!}{(n-r)!r!}$.

Example 1 **SCHOOL How many groups of 4 students can be selected from a class of 20?**

Since the order of choosing the students is not important, you must find the number of combinations of 20 students taken 4 at a time.

$C(n, r) = \frac{n!}{(n-r)!r!}$ Combination formula

$C(20, 4) = \frac{20!}{(20-4)!4!}$ $n = 20, r = 4$

$= \frac{20!}{16!4!}$ or 4845

There are 4845 possible ways to choose 4 students.

Example 2 **In how many ways can you choose 1 vowel and 2 consonants from a set of 26 letter tiles? (Assume there are 5 vowels and 21 consonants.)**

By the Fundamental Counting Principle, you can multiply the number of ways to select one vowel and the number of ways to select 2 consonants. Only the letters chosen matter, not the order in which they were chosen, so use combinations.

$C(5, 1)$ One of 5 vowels are drawn.

$C(21, 2)$ Two of 21 consonants are drawn.

$C(5, 1) \cdot C(21, 2) = \frac{5!}{(5-1)!1!} \cdot \frac{21!}{(21-2)!2!}$ Combination formula

$= \frac{5!}{4!} \cdot \frac{21!}{19!2!}$ Subtract.

$= 5 \cdot 210$ or 1050 Simplify.

There are 1050 combinations of 1 vowel and 2 consonants.

Exercises

Evaluate each expression.

1. $C(5, 3)$ 10
2. $C(7, 4)$ 35
3. $C(15, 7)$ 6435
4. $C(10, 5)$ 252

5. **PLAYING CARDS** From a standard deck of 52 cards, in how many ways can 5 cards be drawn? 2,598,960

6. **HOCKEY** How many hockey teams of 6 players can be formed from 14 players without regard to position played? 3003

7. **COMMITTEES** From a group of 10 men and 12 women, how many committees of 5 men and 6 women can be formed? 232,848

NAME _____ DATE _____ PERIOD ____

12-2 Skills Practice

Permutations and Combinations

Evaluate each expression.

1. $P(6, 3)$ 120
2. $P(8, 2)$ 56
3. $P(2, 1)$ 2
4. $P(3, 2)$ 6
5. $P(10, 4)$ 5040
6. $P(5, 5)$ 120
7. $C(2, 2)$ 1
8. $C(5, 3)$ 10
9. $C(4, 1)$ 4
10. $C(8, 7)$ 8
11. $C(3, 2)$ 3
12. $C(7, 4)$ 35

Determine whether each situation involves a *permutation* or a *combination*. Then find the number of possibilities.

13. seating 8 students in 8 seats in the front row of the school auditorium
permutation; 40,320

14. introducing the 5 starting players on the Woodsville High School basketball team at the beginning of the next basketball game
permutation; 120

15. checking out 3 library books from a list of 8 books for a research paper
combination; 56

16. choosing 2 movies to rent from 5 movies
combination; 10

17. the first-, second-, and third-place finishers in a race with 10 contestants
permutation; 720

18. electing 4 candidates to a municipal planning board from a field of 7 candidates
combination; 35

19. choosing 2 vegetables from a menu that offers 6 vegetable choices
combination; 15

20. an arrangement of the letters in the word *rhombus*
permutation; 5040

21. selecting 2 of 8 choices of orange juice at a store
combination; 28

22. placing a red rose bush, a yellow rose bush, a white rose bush, and a pink rose bush in a row in a planter permutation; 24

23. selecting 2 of 9 kittens at an animal rescue shelter
combination; 36

24. an arrangement of the letters in the word *isosceles*
permutation; 30,240

NAME _____ DATE_____ PERIOD ____

12-2 Practice

Permutations and Combinations

Evaluate each expression.

1. $P(8, 6)$ **20,160**
2. $P(9, 7)$ **181,440**
3. $P(3, 3)$ **6**
4. $P(4, 3)$ **24**
5. $P(4, 1)$ **4**
6. $P(7, 2)$ **42**
7. $C(8, 2)$ **28**
8. $C(11, 3)$ **165**
9. $C(20, 18)$ **190**
10. $C(9, 9)$ **1**
11. $C(3, 1)$ **3**
12. $C(9, 3) \cdot C(6, 2)$ **1260**

Determine whether each situation involves a *permutation* or a *combination*. Then find the number of possibilities.

13. selecting a 4-person bobsled team from a group of 9 athletes
combination; 126

14. an arrangement of the letters in the word *Canada*
permutation; 120

15. arranging 4 charms on a bracelet that has a clasp, a front, and a back
permutation; 24

16. selecting 3 desserts from 10 desserts that are displayed on a dessert cart in a restaurant
combination; 120

17. an arrangement of the letters in the word *annually*
permutation; 5040

18. forming a 2-person sales team from a group of 12 salespeople
combination; 66

19. making 5-sided polygons by choosing any 5 of 11 points located on a circle to be the vertices
combination; 462

20. seating 5 men and 5 women alternately in a row, beginning with a woman
permutation; 14,400

21. **STUDENT GROUPS** Farmington High is planning its academic festival. All math classes will send 2 representatives to compete in the math bowl. How many different groups of students can be chosen from a class of 16 students? **120**

22. **PHOTOGRAPHY** A photographer is taking pictures of a bride and groom and their 6 attendants. If she takes photographs of 3 people in a group, how many different groups can she photograph? **56**

23. **AIRLINES** An airline is hiring 5 flight attendants. If 8 people apply for the job, how many different groups of 5 attendants can the airline hire? **56**

24. **SUBSCRIPTIONS** A school librarian would like to buy subscriptions to 7 new magazines. Her budget, however, will allow her to buy only 4 new subscriptions. How many different groups of 4 magazines can she choose from the 7 magazines? **35**

NAME _____ DATE_____ PERIOD ____

12-2 Word Problem Practice

Permutations and Combinations

1. **WAITING IN LINE** When the 12 students in Mr. Jaybird's class go to lunch, they form a single file line. Does forming a line involve a permutation or a combination of the students?
A permutation

2. **ART** Isabel needs to select three different colors of construction paper to make a flag for a school project. She can choose from a selection of 15 different colors. In how many ways can she pick her colors?
455

3. **SUDOKU** A popular game called "Sudoku" involves square arrays of numbers. In a game of Sudoku, every entry is an integer between 1 and 9, inclusive. No number appears twice in any row or column.

7	1	8	6	9	4	2	3	5
9	2	5	7	3	1	6	4	8
4	6	3	8	5	2	7	9	1
5	9	2	1	7	3	4	8	6
8	3	1	4	6	5	9	2	7
9	7	4	2	8	9	5	1	3
3	4	9	5	1	7	8	6	2
2	8	7	3	4	6	1	5	9
1	5	6	9	2	8	3	7	4

For a game of Sudoku, how many different possibilities are there for the first row of numbers?
362,880

4. **NAMES** Hannah is curious to know how many different 6 letter sequences she can make using each of the letters of her name exactly once. For example, "HANNAH," "AAHHNN," and "NAHNAH" are all possible sequences. How many total sequences are possible?
90

METEORITES For Exercises 5 and 6, use the following information.

Over the course of several years, Kendra managed to collect 7 meteorites. Each one is unique.

5. For a school science fair, Kendra displays her meteorites in a row. How many ways are there to order the meteorites?
5040

6. She decides to trade three of her meteorites for a telescope after the fair. How many ways can she pick out 3 meteorites from her collection?
35

Answers

NAME _____ DATE _____ PERIOD ____

12-2 Enrichment

Combinations and Pascal's Triangle

Pascal's triangle is a special array of numbers invented by Blaise Pascal (1623–1662). The values in Pascal's triangle can be found using the combinations shown below.

1. Evaluate the expression in each cell of the triangle.

C(1,0) = 1	C(1,1) = 1				
C(2,0) = 1	C(2,1) = 2	C(2,2) = 1			
C(3,0) = 1	C(3,1) = 3	C(3,2) = 3	C(3,3) = 1		
C(4,0) = 1	C(4,1) = 4	C(4,2) = 6	C(4,3) = 4	C(4,4) = 1	
C(5,0) = 1	C(5,1) = 5	C(5,2) = 10	C(5,3) = 10	C(5,4) = 5	C(5,5) = 1

2. The pattern shows the relationship between $C(n, r)$ and Pascal's triangle. In general, it is true that $C(n, r) + C(n, r + 1) = C(n + 1, r + 1)$. Complete the proof of this property. In each step, the denominator has been given.

$$C(n, r) + C(n, r + 1) = \frac{n!}{r!(n - r)!} + \frac{n!}{(r + 1)!(n - r - 1)!}$$

$$= \frac{n!(r + 1)}{r!(n - r)!(r + 1)} + \frac{n!(n - r)}{(r + 1)!(n - r - 1)!(n - r)}$$

$$= \frac{n!(r + 1)}{(r + 1)!(n - r)!} + \frac{n!(n - r)}{(r + 1)!(n - r)!}$$

$$= \frac{n!(r + 1 + n - r)}{(r + 1)!(n - r)!}$$

$$= \frac{n!(n + 1)}{(r + 1)!(n - r)!}$$

$$= \frac{(n + 1)!}{(r + 1)!(n - r)!}$$

$$= \frac{(n + 1)!}{(r + 1)![(n + 1) - (r + 1)]!}$$

$$= C(n + 1, r + 1)$$

NAME _____ DATE _____ PERIOD ____

12-2 Spreadsheet Activity

Permutations and Combinations

You have learned the formulas for the number of permutations of n objects taken r at a time, $P(n, r)$, and the number of combinations of n objects taken r at a time, $C(n, r)$. You are going to set up a spreadsheet like the one shown below to perform analyses of these functions.

◇	A	B	C	D	E	F	G
1	*n*	5	5	5	5	5	5
2	*r*	0	1	2	3	4	5
3	*P (n , r)*	1	5	20	60	120	120
4	*C (n , r)*	1	5	10	10	5	1

Sheet 1 Sheet 2 Sheet 3

In the spreadsheet, the values in row 1 represent n, the values in row 2 represent r, and the formulas for $P(n, r)$ and $C(n, r)$ are in rows 3 and 4, respectively.

The formula to calculate $P(n, r)$ is =FACT(B1)/FACT(B1-B2).

FACT is a special function from the function list and should not be entered from the letters on the keyboard. Enter the formula in B3. Then drag the cursor across the row to copy the formula into cells C3 through G3.

The formula for $C(n, r)$ is =FACT(B1)/(FACT(B1−B2)*FACT(B2)) and should be entered in cell B4. Copy the formula into cells C4 through G4.

Exercises

1. Compare the values of $P(n, r)$ and $C(n, r)$ for $n = 5$ and $r = 0$ through 5, as well as for two other choices of n and r. **Most of the values of $P(n, r)$ are much larger than the corresponding values of $C(n, r)$. The values of $P(n, r)$ tend to increase, while the values of $C(n, r)$ tend to increase and then decrease.**

2. Several identities hold for $P(n, r)$ and $C(n, r)$. Use the spreadsheet to verify the following identities by finding three examples of each. **2a-2c. See students' work.**

a. $P(n, n) = P(n, n - 1)$

b. $C(n + 1, r) = C(n, r - 1) + C(n, r)$

c. $C(n, 0) + C(n, 1) + C(n, 2) + \ldots + C(n, n) = 2^n$

NAME _____ DATE_____ PERIOD ____

12-3 Lesson Reading Guide

Probability

Get Ready for the Lesson

Read the introduction to Lesson 12-3 in your textbook.

What is the probability that a person will *not* be struck by lightning in a given year?
749,999/750,000

Read the Lesson

1. Indicate whether each of the following statements is *true* or *false*.

a. If an event can never occur, its probability is a negative number. false

b. If an event is certain to happen, its probability is 1. true

c. If an event can succeed in s ways and fail in f ways, then the probability of success is $\frac{s}{f}$. false

d. If an event can succeed in s ways and fail in f ways, then the odds against the event are $s:f$. false

e. A probability distribution is a function in which the domain is the sample space of an experiment. true

2. A weather forecast says that the chance of rain tomorrow is 40%.

a. Write the probability that it will rain tomorrow as a fraction in lowest terms. $\frac{2}{5}$

b. Write the probability that it will not rain tomorrow as a fraction in lowest terms. $\frac{3}{5}$

c. What are the odds in favor of rain? 2:3

d. What are the odds against rain? 3:2

3. Refer to the table in Example 4 on page 646 in your textbook.

a. What other sum has the same probability as a sum of 11? 3

b. What are the odds of rolling a sum of 8? 5:31

c. What are the odds against rolling a sum of 9? 8:1

Remember What You Learned

4. A good way to remember something is to explain it to someone else. Suppose that your friend Roberto is having trouble remembering the difference between probability and odds. What would you tell him to help him remember this easily?

Sample answer: Probability gives the ratio of successes to the total number of outcomes, while odds gives the ratio of successes to failures.

NAME _____ DATE_____ PERIOD ____

12-3 Study Guide and Intervention

Probability

Probability and Odds In probability, a desired outcome is called a **success**; any other outcome is called a **failure**.

Probability of Success and Failure	If an event can succeed in s ways and fail in f ways, then the probabilities of success, $P(S)$, and of failure, $P(F)$, are as follows. $P(S) = \frac{s}{s+f}$ and $P(F) = \frac{f}{s+f}$.
Definition of Odds	If an event can succeed in s ways and fail in f ways, then the odds of success and of failure are as follows. Odds of success = $s:f$ Odds of failure = $f:s$

Example 1 **When 3 coins are tossed, what is the probability that at least 2 are heads?**

You can use a tree diagram to find the sample space.

First Coin	Second Coin	Third Coin	Possible Outcomes
H	H	H	HHH
		T	HHT
	T	H	HTH
		T	HTT
T	H	H	THH
		T	THT
	T	H	TTH
		T	TTT

Of the 8 possible outcomes, 4 have at least 2 heads. So the probability of tossing at least 2 heads is $\frac{4}{8}$ or $\frac{1}{2}$.

Example 2 **What is the probability of picking 4 fiction books and 2 biographies from a best-seller list that consists of 12 fiction books and 6 biographies?**

By the Fundamental Counting Principle, the number of successes is $C(12, 4) \cdot C(6, 2)$. The total number of selections, $s + f$, of 6 books is $C(18, 6)$.

$P(4 \text{ fiction, } 2 \text{ biography}) = \frac{C(12, 4) \cdot C(6, 2)}{C(18, 6)}$ or about 0.40

The probability of selecting 4 fiction books and 2 biographies is about 40%.

Exercises

Find the odds of an event occurring, given the probability of the event.

1. $\frac{3}{7}$ 3:4 **2.** $\frac{4}{5}$ 4:1 **3.** $\frac{2}{13}$ 2:11 **4.** $\frac{1}{15}$ 1:14

Find the probability of an event occurring, given the odds of the event.

5. 10:1 $\frac{10}{11}$ **6.** 2:5 $\frac{2}{7}$ **7.** 4:9 $\frac{4}{13}$ **8.** 8:3 $\frac{8}{11}$

One bag of candy contains 15 red candies, 10 yellow candies, and 6 green candies. Find the probability of each selection.

9. picking a red candy $\frac{15}{31}$

10. not picking a yellow candy $\frac{21}{31}$

11. picking a green candy $\frac{6}{31}$

12. not picking a red candy $\frac{16}{31}$

NAME _____ DATE_____ PERIOD ____

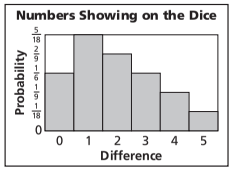
12-3 Study Guide and Intervention *(continued)*

Probability

Probability Distributions A **random variable** is a variable whose value is the numerical outcome of a random event. A **probability distribution** for a particular random variable is a function that maps the sample space to the probabilities of the outcomes in the sample space.

Example **Suppose two dice are rolled. The table and the relative-frequency histogram show the distribution of the absolute value of the difference of the numbers rolled. Use the graph to determine which outcome is the most likely. What is its probability?**

Difference	0	1	2	3	4	5
Probability	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{2}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{18}$



The greatest probability in the graph is $\frac{5}{18}$.
The most likely outcome is a difference of 1 and its probability is $\frac{5}{18}$.

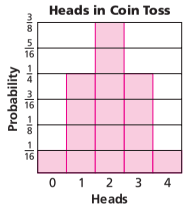
Exercises

Four coins are tossed.

1. Complete the table below to show the probability distribution of the number of heads.

Number of Heads	0	1	2	3	4
Probability	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

2. Make relative-frequency distribution of the data.



NAME _____ DATE_____ PERIOD ____

12-3 Skills Practice

Probability

Ahmed is posting 2 photographs on his website. He has narrowed his choices to 4 landscape photographs and 3 portraits. If he chooses the two photographs at random, find the probability of each selection.

1. P(2 portrait) $\frac{1}{7}$
2. P(2 landscape) $\frac{2}{7}$
3. P(1 of each) $\frac{4}{7}$

The Carubas have a collection of 28 video movies, including 12 westerns and 16 science fiction. Elise selects 3 of the movies at random to bring to a sleep-over at her friend's house. Find the probability of each selection.

4. P(3 westerns) $\frac{55}{819}$
5. P(3 science fiction) $\frac{20}{117}$
6. P(1 western and 2 science fiction) $\frac{40}{91}$
7. P(2 westerns and 1 science fiction) $\frac{88}{273}$
8. P(3 comedy) 0
9. P(2 science fiction and 2 westerns) 0

For Exercises 10–13, use the chart that shows the class and gender statistics for the students taking an Algebra 1 or Algebra 2 class at La Mesa High School.

Class/Gender	Number
Freshman/Male	95
Freshman/Female	101
Sophomore/Male	154
Sophomore/Female	145
Junior/Male	100
Junior/Female	102

If a student taking Algebra 1 or Algebra 2 is selected at random, find each probability. Express as decimals rounded to the nearest thousandth.

10. P(sophomore/female) 0.208
11. P(junior/male) 0.143
12. P(freshman/male) 0.136
13. P(freshman/female) 0.145

Find the odds of an event occurring, given the probability of the event.

14. $\frac{5}{8}$ 5:3
15. $\frac{2}{7}$ 2:5
16. $\frac{3}{5}$ 3:2
17. $\frac{1}{10}$ 1:9
18. $\frac{5}{6}$ 5:1
19. $\frac{5}{12}$ 5:7

Find the probability of an event occurring, given the odds of the event.

20. 2:1 $\frac{2}{3}$
21. 8:9 $\frac{8}{17}$
22. 4:1 $\frac{4}{5}$
23. 1:9 $\frac{1}{10}$
24. 2:7 $\frac{2}{9}$
25. 5:9 $\frac{5}{14}$

NAME _____ DATE _____ PERIOD ____

12-3 Practice

Probability

A bag contains 1 green, 4 red, and 5 yellow balls. Two balls are selected at random. Find the probability of each selection.

1. P(2 red) $\frac{2}{15}$
2. P(1 red and 1 yellow) $\frac{4}{9}$
3. P(1 green and 1 yellow) $\frac{1}{9}$
4. P(2 green) 0
5. P(2 red and 1 yellow) 0
6. P(1 red and 1 green) $\frac{4}{45}$

A bank contains 3 pennies, 8 nickels, 4 dimes, and 10 quarters. Two coins are selected at random. Find the probability of each selection.

7. P(2 pennies) $\frac{1}{100}$
8. P(2 dimes) $\frac{1}{50}$
9. P(1 nickel and 1 dime) $\frac{8}{75}$
10. P(1 quarter and 1 penny) $\frac{1}{10}$
11. P(1 quarter and 1 nickel) $\frac{4}{15}$
12. P(2 dimes and 1 quarter) 0

Henrico visits a home decorating store to choose wallpapers for his new house. The store has 28 books of wallpaper samples, including 10 books of WallPride samples and 18 books of Deluxe Wall Coverings samples. The store will allow Henrico to bring 4 books home for a few days so he can decide which wallpapers he wants to buy. If Henrico randomly chooses 4 books to bring home, find the probability of each selection.

13. P(4 WallPride) $\frac{2}{195}$
14. P(2 WallPride and 2 Deluxe) $\frac{153}{455}$
15. P(1 WallPride and 3 Deluxe) $\frac{544}{1365}$
16. P(3 WallPride and 1 Deluxe) $\frac{48}{455}$

For Exercises 17–20, use the table that shows the range of verbal SAT scores for freshmen at a small liberal arts college. If a freshman student is chosen at random, find each probability. Express as decimals rounded to the nearest thousandth.

Range	400–449	450–499	500–549	550–559	600–649	650+
Number of Students	129	275	438	602	620	412

17. P(400–449) 0.052
18. P(550–559) 0.243
19. P(at least 650) 0.166

Find the odds of an event occurring, given the probability of the event.

20. $\frac{4}{11}$ 4:7
21. $\frac{12}{13}$ 12:1
22. $\frac{5}{99}$ 5:94
23. $\frac{1}{1000}$ 1:999
24. $\frac{5}{16}$ 5:11
25. $\frac{3}{95}$ 3:92
26. $\frac{9}{70}$ 9:61
27. $\frac{8}{15}$ 8:7

Find the probability of an event occurring, given the odds of the event.

28. 2:23 $\frac{2}{25}$
29. 2:5 $\frac{2}{7}$
30. 15:1 $\frac{15}{16}$
31. 9:7 $\frac{9}{16}$
32. 11:14 $\frac{11}{25}$
33. 1000:1 $\frac{1000}{1001}$
34. 12:17 $\frac{12}{29}$
35. 8:13 $\frac{8}{21}$

NAME _____ DATE _____ PERIOD ____

12-3 Word Problem Practice

Probability

1. **ART** The letters "A", "R", and "T" are written on three different pieces of paper. The pieces of paper are then put in a bag and mixed up. Logan picks each letter without looking and places them side by side. What is the probability that the letters spell "ART"?
$\frac{1}{6}$

2. **AGE** There are 24 students in Miss Mason's third grade class, all born on different days. Eleven students are boys. In the morning, the classroom is empty. One student arrives followed by another. What is the probability that when the first two students arrive, one is a boy and the other a girl?
about 26%

3. **DICE** Jamal rolls two six-sided dice, one after the other. What is the probability that the second die shows a number larger than the first die?
$\frac{5}{12}$

4. **LANGUAGES** Noah cannot decide whether to learn French, German, Italian, Russian, or Chinese. He assigns each language a different number from 0 to 4. He then takes four fair coins and flips them. He decided to take the language corresponding to the number of coins that come up heads. Does Noah's method for choosing a language give each language the same chance of being chosen? Explain.
No. For example, the probability of getting 0 is 1 in 16 but the probability of getting 1 is 1 in 4.

ICE CREAM For Exercises 5-7, use the following information.

A survey of the students in Mr. Orr's fifth grade class asked each student to name their favorite flavor of ice cream. The results are shown in the table below.

Flavor	Number of Students
Vanilla	10
Chocolate	9
Butternut	5
Strawberry	4
Banana	1
Coffee	1

5. A student from Mr. Orr's class is selected at random. What is the probability that the student's favorite flavor of ice cream is chocolate?
$\frac{3}{10}$

6. A student from Mr. Orr's class is selected at random. What is the probability that the student's favorite flavor of ice cream is banana?
$\frac{1}{30}$

7. A student from Mr. Orr's class is selected at random. Is it more likely that the student prefers either butternut or strawberry or that the student prefers either chocolate or banana?
Chocolate or banana is more likely.

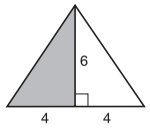
Answers

NAME _____ DATE _____ PERIOD ____

12-3 Enrichment

Geometric Probability

If a dart, thrown at random, hits the triangular board shown at the right, what is the chance that it will hit the shaded region? This chance, also called a probability, can be determined by comparing the area of the shaded region to the area of the board. This ratio indicates what fraction of the tosses should hit in the shaded region.



$$\frac{\text{area of shaded region}}{\text{area of triangular board}} = \frac{\frac{1}{2}(4)(6)}{\frac{1}{2}(8)(6)}$$

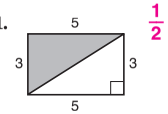
$$= \frac{12}{24} \text{ or } \frac{1}{2}$$

In general, if S is a subregion of some region R, then the probability, $P(S)$, that a point, chosen at random, belongs to subregion S is given by the following.

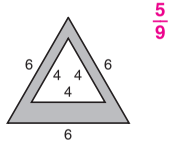
$$P(S) = \frac{\text{area of subregion } S}{\text{are of region } R}$$

Find the probability that a point, chosen at random, belongs to the shaded subregions of the following regions.

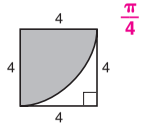
1. $\frac{1}{2}$



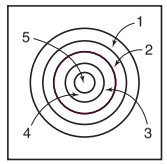
2. $\frac{5}{9}$



3. $\frac{\pi}{4}$



The dart board shown at the right has 5 concentric circles whose centers are also the center of the square board. Each side of the board is 38 cm, and the radii of the circles are 2 cm, 5 cm, 8 cm, 11 cm, and 14 cm. A dart hitting within one of the circular regions scores the number of points indicated on the board, while a hit anywhere else scores 0 points. If a dart, thrown at random, hits the board, find the probability of scoring the indicated number of points.



4. 0 points
$\frac{361 - 49\pi}{361}$

5. 1 point
$\frac{75\pi}{1444}$

6. 2 points
$\frac{57\pi}{1444}$

7. 3 points
$\frac{39\pi}{1444}$

8. 4 points
$\frac{21\pi}{1444}$

9. 5 points
$\frac{\pi}{361}$

NAME _____ DATE _____ PERIOD ____

12-4 Lesson Reading Guide

Multiplying Probabilities

Get Ready for the Lesson

Read the introduction to Lesson 12-4 in your textbook.

Write the probability that Yao Ming made a field goal shot during the 2004–05 season as a fraction in lowest terms. (Your answer should not include a decimal.) $\frac{276}{5}$

Read the Lesson

1. A bag contains 4 yellow balls, 5 red balls, 1 white ball, and 2 black balls. *A* ball is drawn from the bag and is not replaced. *A* second ball is drawn.

a. Let Y be the event "first ball is yellow" and B be the event "second ball is black." Are these events *independent* or *dependent*? dependent

b. Tell which formula you would use to find the probability that the first ball is yellow and the second ball is black. C

A. $P(Y \text{ and } B) = \frac{P(Y)}{P(Y) + P(B)}$

B. $P(Y \text{ and } B) = P(Y) \cdot P(B)$

C. $P(Y \text{ and } B) = P(Y) \cdot P(B \text{ following } Y)$

c. Which equation shows the correct calculation of this probability? B

A. $\frac{1}{3} + \frac{2}{11} = \frac{17}{33}$

B. $\frac{1}{3} \cdot \frac{2}{11} = \frac{2}{33}$

C. $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$

D. $\frac{1}{3} \cdot \frac{1}{6} = \frac{1}{18}$

d. Which equation shows the correct calculation of the probability that if three balls are drawn in succession without replacement, all three will be red? B

A. $\frac{5}{12} \cdot \frac{5}{12} \cdot \frac{5}{12} = \frac{125}{1728}$

B. $\frac{5}{12} \cdot \frac{4}{11} \cdot \frac{3}{10} = \frac{1}{22}$

C. $\frac{5}{12} + \frac{4}{11} + \frac{3}{10} = \frac{713}{660}$

Remember What You Learned

2. Some students have trouble remembering a lot of formulas, so they try to keep the number of formulas they have to know to a minimum. Can you learn just one formula that will allow you to find probabilities for both independent and dependent events? Explain your reasoning. Sample answer: Just remember the formula for dependent events: $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$. When the events are independent, $P(B \text{ following } A) = P(B)$, so the formula for dependent events simplifies to $P(A \text{ and } B) = P(A) \cdot P(B)$, which is the correct formula for independent events.

NAME _____ DATE_____ PERIOD ____

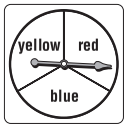
12-4 Study Guide and Intervention

Multiplying Probabilities

Probability of Independent Events

Probability of Two Independent Events	If two events, A and B, are independent, then the probability of both occurring is $P(A \text{ and } B) = P(A) \cdot P(B)$.

Example **In a board game each player has 3 different-colored markers. To move around the board the player first spins a spinner to determine which piece can be moved. He or she then rolls a die to determine how many spaces that colored piece should move. On a given turn what is the probability that a player will be able to move the yellow piece more than 2 spaces?**



Let A be the event that the spinner lands on yellow, and let B be the event that the die shows a number greater than 2. The probability of A is $\frac{1}{3}$, and the probability of B is $\frac{2}{3}$.

$P(A \text{ and } B) = P(A) \cdot P(B)$ Probability of independent events

$= \frac{1}{3} \cdot \frac{2}{3}$ or $\frac{2}{9}$ Substitute and multiply.

The probability that the player can move the yellow piece more than 2 spaces is $\frac{2}{9}$.

Exercises

A die is rolled 3 times. Find the probability of each event.

1. a 1 is rolled, then a 2, then a 3 $\frac{1}{216}$
2. a 1 or a 2 is rolled, then a 3, then a 5 or a 6 $\frac{1}{54}$
3. 2 odd numbers are rolled, then a 6 $\frac{1}{24}$
4. a number less than 3 is rolled, then a 3, then a number greater than 3 $\frac{1}{36}$
5. A box contains 5 triangles, 6 circles, and 4 squares. If a figure is removed, replaced, and a second figure is picked, what is the probability that a triangle and then a circle will be picked? $\frac{2}{15}$ or about 0.13
6. A bag contains 5 red marbles and 4 white marbles. A marble is selected from the bag, then replaced, and a second selection is made. What is the probability of selecting 2 red marbles? $\frac{25}{81}$ or about 0.31
7. A jar contains 7 lemon jawbreakers, 3 cherry jawbreakers, and 8 rainbow jawbreakers. What is the probability of selecting 2 lemon jawbreakers in succession providing the jawbreaker drawn first is then replaced before the second is drawn? $\frac{49}{324}$ or about 0.15

NAME _____ DATE_____ PERIOD ____

12-4 Study Guide and Intervention *(continued)*

Multiplying Probabilities

Probability of Dependent Events

Probability of Two Dependent Events	If two events, A and B, are dependent, then the probability of both events occurring is $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$.

Example 1 **There are 7 dimes and 9 pennies in a wallet. Suppose two coins are to be selected at random, without replacing the first one. Find the probability of picking a penny and then a dime.**

Because the coin is not replaced, the events are dependent.

Thus, $P(A \text{ and } B) = P(\text{A}) \cdot P(B \text{ following } A)$.

$P(\text{penny, then dime}) = P(\text{penny}) \cdot P(\text{dime following penny})$

$$\frac{9}{16} \cdot \frac{7}{15} = \frac{21}{80}$$

The probability is $\frac{21}{80}$ or about 0.26

Example 2 **What is the probability of drawing, without replacement, 3 hearts, then a spade from a standard deck of cards?**

Since the cards are not replaced, the events are dependent. Let H represent a heart and S represent a spade.

$P(\text{H, H, H, S}) = P(\text{H}) \cdot P(\text{H following H}) \cdot P(\text{H following 2 Hs}) \cdot P(\text{S following 3 Hs})$

$$= \frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} \cdot \frac{13}{49} \text{ or about } 0.003$$

The probability is about 0.003 of drawing 3 hearts, then a spade.

Exercises

Find each probability.

1. The cup on Sophie's desk holds 4 red pens and 7 black pens. What is the probability of her selecting first a black pen, then a red one? $\frac{14}{55}$ or about 0.25
2. What is the probability of drawing two cards showing odd numbers from a set of cards that show the first 20 counting numbers if the first card is not replaced before the second is chosen? $\frac{9}{38}$ or about 0.24
3. There are 3 quarters, 4 dimes, and 7 nickels in a change purse. Suppose 3 coins are selected without replacement. What is the probability of selecting a quarter, then a dime, and then a nickel? $\frac{1}{26}$ or about 0.04
4. A basket contains 4 plums, 6 peaches, and 5 oranges. What is the probability of picking 2 oranges, then a peach if 3 pieces of fruit are selected at random? $\frac{4}{91}$ or about 0.04
5. A photographer has taken 8 black and white photographs and 10 color photographs for a brochure. If 4 photographs are selected at random, what is the probability of picking first 2 black and white photographs, then 2 color photographs? $\frac{7}{102}$ or about 0.07

Answers

NAME _____ DATE _____ PERIOD ____

12-4 Skills Practice

Multiplying Probabilities

A die is rolled twice. Find each probability.

1. P(5, then 6) $\frac{1}{36}$
2. P(no 2s) $\frac{25}{36}$
3. P(two 1s) $\frac{1}{36}$
4. P(any number, then not 5) $\frac{5}{6}$
5. P(4, then not 6) $\frac{5}{36}$
6. P(not 1, then not 2) $\frac{25}{36}$

A board game uses a set of 6 different cards. Each card displays one of the following figures: a star, a square, a circle, a diamond, a rectangle, or a pentagon. The cards are placed face down, and a player chooses two cards. Find each probability.

7. P(circle, then star), if no replacement occurs $\frac{1}{30}$
8. P(diamond, then square), if replacement occurs $\frac{1}{36}$
9. P(2 polygons), if replacement occurs $\frac{25}{36}$
10. P(2 polygons), if no replacement occurs $\frac{2}{3}$
11. P(circle, then hexagon), if no replacement occurs 0

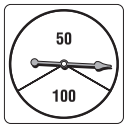
Determine whether the events are *independent* or *dependent*. Then find each probability.

12. A mixed box of herbal teabags contains 2 lemon teabags, 3 orange-mango teabags, 3 chamomile teabags, and 1 apricot-ginger teabag. Kevin chooses 2 teabags at random to bring to work with him. What is the probability that he first chooses a lemon teabag and then a chamomile teabag? dependent; $\frac{1}{12}$

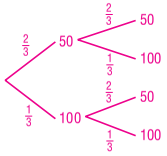
13. The chart shows the selection of olive oils that Hasha finds in a specialty foods catalog. If she randomly selects one type of oil, then randomly selects another, different oil, what is the probability that both selections are domestic, first cold pressed oils? dependent; $\frac{21}{820}$

Type of Oil	Domestic	Imported
Pure	2	5
Cold Pressed	4	8
First Cold Pressed	7	15

For Exercises 14 and 15, two thirds of the area of the spinner earns you 50 points. Suppose you spin the spinner twice.



14. Sketch a tree diagram showing all of the possibilities. Use it to find the probability of spinning 50 points, then 100 points. $\frac{2}{9}$



15. What is the probability that you get 100 points on each spin? $\frac{1}{9}$

NAME _____ DATE _____ PERIOD ____

12-4 Practice

Multiplying Probabilities

A die is rolled three times. Find each probability.

1. P(three 4s) $\frac{1}{216}$
2. P(no 4s) $\frac{125}{216}$
3. P(2, then 3, then 1) $\frac{1}{216}$
4. P(three different even numbers) $\frac{1}{36}$
5. P(any number, then 5, then 5) $\frac{1}{36}$
6. P(even number, then odd number, then 1) $\frac{1}{24}$

There are 3 nickels, 2 dimes, and 5 quarters in a purse. Three coins are selected in succession at random. Find the probability.

7. P(nickel, then dime, then quarter), if no replacement occurs $\frac{1}{24}$
8. P(nickel, then dime, then quarter), if replacement occurs $\frac{3}{100}$
9. P(2 nickels, then 1 quarter), if no replacement occurs $\frac{1}{24}$
10. P(3 dimes), if replacement occurs $\frac{1}{125}$
11. P(3 dimes), if no replacement occurs 0

For Exercises 12 and 13, determine whether the events are *independent* or *dependent*. Then find each probability.

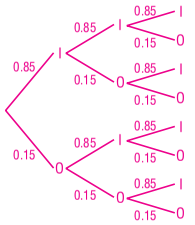
12. Serena is creating a painting. She wants to use 2 more colors. She chooses randomly from 6 shades of red, 10 shades of green, 4 shades of yellow, 4 shades of purple, and 6 shades of blue. What is the probability that she chooses 2 shades of green? dependent; $\frac{3}{29}$

13. Kershel's mother is shopping at a bakery. The owner offers Kershel a cookie from a jar containing 22 chocolate chip cookies, 18 sugar cookies, and 15 oatmeal cookies. Without looking, Kershel selects one, drops it back in, and then randomly selects another. What is the probability that neither selection was a chocolate chip cookie? independent; $\frac{9}{25}$

14. **METEOROLOGY** The Fadeeva's are planning a 3-day vacation to the mountains. A long-range forecast reports that the probability of rain each day is 10%. Assuming that the daily probabilities of rain are independent, what is the probability that there is no rain on the first two days, but that it rains on the third day? $\frac{81}{1000}$

RANDOM NUMBERS For Exercises 15 and 16, use the following information.

Anita has a list of 20 jobs around the house to do, and plans to do 3 of them today. She assigns each job a number from 1 to 20, and sets her calculator to generate random numbers from 1 to 20, which can reoccur. Of the jobs, 3 are outside, and the rest are inside.



15. Sketch a tree diagram showing all of the possibilities that the first three numbers generated correspond to inside jobs or outside jobs. Use it to find the probability that the first two numbers correspond to inside jobs, and the third to an outside job. 0.108375

16. What is the probability that the number generated corresponds to an outside job three times in a row? 0.003375

NAME _____ DATE_____ PERIOD ____

12-4 Word Problem Practice

Multiplying Probabilities

1. **BUSSING** Portia and Quinton use the same bus stop when they go to work. They arrive at the bus stop independently of each other. The probability that Portia catches the 7:45 A.M. bus is $\frac{3}{5}$. The probability that Quinton catches the 7:45 A.M. bus is $\frac{1}{2}$. What is the probability that they both catch the 7:45 A.M. bus on the same day?
$\frac{3}{10}$

2. **GOODY BAGS** Ryan and Sophia are given goody bags with identical contents. The probability of reaching into either of these goody bags and pulling out a stick of chewing gum is $\frac{1}{10}$. Ryan and Sophia each reach into their own goody bag and randomly pull something out. What is the probability that they both pulled out a stick of chewing gum?
$\frac{1}{100}$

3. **PENCILS** A box of pencils contains 11 type 2 pencils and 5 type 3 pencils. Tara picks out a pencil from the box without looking and keeps it. Then, Upton picks out a pencil from the box without looking. What is the probability that Tara picks a type 2 pencil and Upton picks a type 3 pencil?
$\frac{11}{48}$

4. **GUESSING GAMES** Valerie is playing a guessing game. Four cards are placed face down before her. The hidden side of each card shows either the word "LOSE" or "WIN". Only one card is labeled "WIN". Valerie is given two chances to find the card labeled "WIN".

What is the probability that she does not pick the "win" card on her first try but does find it with her second?
$\frac{1}{4}$

WALLETS For Exercises 5 and 6, use the following information.

Wayne has 1 ten-dollar bill, 2 five-dollar bills, and 5 one-dollar bills in his wallet.

5. Wayne randomly chooses a bill from his wallet, puts it back, then picks another bill, and puts that one back too. What is the probability that both were five-dollar bills?
$\frac{1}{16}$

6. Wayne randomly pulls out a bill from his wallet, and then, without putting it back, randomly pulls a second bill from his wallet. He then puts both bills back into the wallet. What is the probability that both of the bills pulled out were five-dollar bills?
$\frac{1}{28}$

NAME _____ DATE_____ PERIOD ____

12-4 Enrichment

Conditional Probability

Suppose a pair of dice is thrown. It is known that the sum is greater than seven. Find the probability that the dice match.

The probability of an event given the occurrence of another event is called *conditional probability*. The conditional probability of event A, the dice match, given event B, their sum is greater than seven, is denoted $P(A/B)$.

There are 15 sums greater than seven and there are 36 possible pairs altogether.
$P(B) = \frac{15}{36}$

There are three matching pairs greater than seven.
$P(A \text{ and } B) = \frac{3}{36}$

$P(A/B) = \frac{P(A \text{ and } B)}{P(B)}$

$P(A/B) = \frac{\frac{3}{36}}{\frac{15}{36}}$ or $\frac{1}{5}$

The conditional probability is $\frac{1}{5}$.

A card is drawn from a standard deck of 52 and is found to be red. Given that event, find each of the following probabilities.

1. P(heart) $\frac{1}{2}$
2. P(ace) $\frac{1}{13}$
3. P(face card) $\frac{3}{13}$
4. P(jack or ten) $\frac{2}{13}$
5. P(six of spades) 0
6. P(six of hearts) $\frac{1}{26}$

A sports survey taken at Stirers High School shows that 48% of the respondents liked soccer, 66% liked basketball, and 38% liked hockey. Also, 30% liked soccer and basketball, 22% liked basketball and hockey and 28% liked soccer and hockey. Finally, 12% liked all three sports. Find each of the following probabilities.

7. The probability Meg likes soccer if she likes basketball. $\frac{30}{66}$ or $\frac{5}{11}$
8. The probability Biff likes basketball if he likes soccer. $\frac{30}{48}$ or $\frac{5}{8}$
9. The probability Muffy likes hockey if she likes basketball. $\frac{22}{66}$ or $\frac{1}{3}$
10. The probability Greg likes hockey and basketball if he likes soccer. $\frac{12}{48}$ or $\frac{1}{4}$

Answers

NAME _____ DATE _____ PERIOD ____

12-5 Lesson Reading Guide

Adding Probabilities

Get Ready for the Lesson

Read the introduction to Lesson 12-5 in your textbook.

Why do the percentages shown on the bar graph add up to more than 100%? **Sample answer: Many teens do one or more of the listed online activities.**

Read the Lesson

1. Indicate whether the events in each pair are *inclusive* or *mutually exclusive*.

a. Q: drawing a queen from a standard deck of cards
D: drawing a diamond from a standard deck of cards **inclusive**

b. J: drawing a jack from a standard deck of cards
K: drawing a king from a standard deck of cards **mutually exclusive**

2. Marla took a quiz on this lesson that contained the following problem.
Each of the integers from 1 through 25 is written on a slip of paper and placed in an envelope. If one slip is drawn at random, what is the probability that it is odd or a multiple of 5?
Here is Marla's work.

$$P(\text{odd}) = \frac{13}{25} \qquad P(\text{multiple of 5}) = \frac{5}{25} \text{ or } \frac{1}{5}$$

$$P(\text{odd or multiple of 5}) = P(\text{odd}) + P(\text{multiple of 5})$$
$$= \frac{13}{25} + \frac{5}{25} = \frac{18}{25}$$

a. Why is Marla's work incorrect? **Sample answer: Marla used the formula for mutually exclusive events, but the events are inclusive. She should use the formula for inclusive events so that the odd multiples of 5 will not be counted twice.**

b. Show the corrected work.

$$P(\text{odd or multiple of 5}) = P(\text{odd}) + P(\text{multiple of 5}) - P(\text{odd multiple of 5})$$
$$= \frac{13}{25} + \frac{5}{25} - \frac{3}{25} = \frac{15}{25} = \frac{3}{5}$$

Remember What You Learned

3. Some students have trouble remembering a lot of formulas, so they try to keep the number of formulas they have to know to a minimum. Can you learn just one formula that will allow you to find probabilities for both mutually exclusive and inclusive events? Explain your reasoning. **Sample answer: Just remember the formula for inclusive events: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. When the events are mutually exclusive, $P(A \text{ and } B) = 0$, so the formula for inclusive events simplifies to $P(A \text{ and } B) = P(A) + P(B)$, which is the correct formula for mutually exclusive events.**

NAME _____ DATE _____ PERIOD ____

12-5 Study Guide and Intervention

Adding Probabilities

Mutually Exclusive Events Events that cannot occur at the same time are called mutually exclusive events.

Probability of Mutually Exclusive Events	If two events, A and B, are mutually exclusive, then $P(A \text{ or } B) = P(A) + P(B)$.

This formula can be extended to any number of mutually exclusive events.

Example 1 **To choose an afternoon activity, summer campers pull slips of paper out of a hat. Today there are 25 slips for a nature walk, 35 slips for swimming, and 30 slips for arts and crafts. What is the probability that a camper will pull a slip for a nature walk or for swimming?**

These are mutually exclusive events. Note that there is a total of 90 slips.

$$P(\text{nature walk or swimming}) = P(\text{nature walk}) + P(\text{swimming})$$
$$= \frac{25}{90} + \frac{35}{90} \text{ or } \frac{2}{3}$$

The probability of a camper's pulling out a slip for a nature walk or for swimming is $\frac{2}{3}$.

Example 2 **By the time one tent of 6 campers gets to the front of the line, there are only 10 nature walk slips and 15 swimming slips left. What is the probability that more than 4 of the 6 campers will choose a swimming slip?**

$$P(\text{more than 4 swimmers}) = P(5 \text{ swimmers}) + P(6 \text{ swimmers})$$
$$= \frac{C(10, 1) \cdot C(15, 5)}{C(25, 6)} + \frac{C(10, 0) \cdot C(15, 6)}{C(25, 6)}$$
$$\approx 0.2$$

The probability of more than 4 of the campers swimming is about 0.2.

Exercises

Find each probability.

1. A bag contains 45 dyed eggs: 15 yellow, 12 green, and 18 red. What is the probability of selecting a green or a red egg? $\frac{2}{3}$

2. The letters from the words LOVE and LIVE are placed on cards and put in a box. What is the probability of selecting an L or an O from the box? $\frac{3}{8}$

3. A pair of dice is rolled, and the two numbers are added. What is the probability that the sum is either a 5 or a 7? $\frac{5}{18}$ **or about 0.28**

4. A bowl has 10 whole wheat crackers, 16 sesame crackers, and 14 rye crisps. If a person picks a cracker at random, what is the probability of picking either a sesame cracker or a rye crisp? $\frac{3}{4}$

5. An art box contains 12 colored pencils and 20 pastels. If 5 drawing implements are chosen at random, what is the probability that at least 4 of them are pastels? **about 0.37**

NAME _____ DATE_____ PERIOD ____

12-5 Study Guide and Intervention *(continued)*

Adding Probabilities

Inclusive Events

Probability of Inclusive Events	If two events, A and B, are inclusive, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

Example **What is the probability of drawing a face card or a black card from a standard deck of cards?**

The two events are inclusive, since a card can be both a face card and a black card.

$P(\text{face card or black card}) = P(\text{face card}) + P(\text{black card}) - P(\text{black face card})$

$= \frac{3}{13} + \frac{1}{2} - \frac{3}{26}$

$= \frac{8}{13}$ or about 0.62

The probability of drawing either a face card or a black card is about 0.62

Exercises

Find each probability.

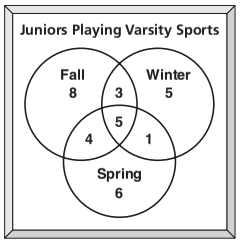
1. What is the probability of drawing a red card or an ace from a standard deck of cards?
$\frac{7}{13}$ or about 0.54

2. Three cards are selected from a standard deck of 52 cards. What is the probability of selecting a king, a queen, or a red card?
$\frac{15}{26}$ or about 0.58

3. The letters of the alphabet are placed in a bag. What is the probability of selecting a vowel or one of the letters from the word QUIZ?
$\frac{7}{26}$ or about 0.27

4. A pair of dice is rolled. What is the probability that the sum is odd or a multiple of 3?
$\frac{2}{3}$ or about 0.67

5. The Venn diagram at the right shows the number of juniors on varsity sports teams at Elmwood High School. Some athletes are on varsity teams for one season only, some athletes for two seasons, and some for all three seasons. If a varsity athlete is chosen at random from the junior class, what is the probability that he or she plays a fall or winter sport? $\frac{13}{16}$



NAME _____ DATE_____ PERIOD ____

12-5 Skills Practice

Adding Probabilities

Eli has 10 baseball cards of 10 different players in his pocket. Three players are pitchers, 5 are outfielders, and 2 are catchers. If Eli randomly selects a card to trade, find each probability.

1. P(pitcher or outfielder) $\frac{4}{5}$
2. P(pitcher or catcher) $\frac{1}{2}$
3. P(outfielder or catcher) $\frac{7}{10}$

A die is rolled. Find each probability.

4. P(5 or 6) $\frac{1}{3}$
5. P(at least a 3) $\frac{2}{3}$
6. P(less than 4) $\frac{1}{2}$

Determine whether the events are *mutually exclusive* or *inclusive*. Then find the probability.

7. A die is rolled. What is the probability of rolling a 3 or a 4? mutually exclusive; $\frac{1}{3}$
8. A die is rolled. What is the probability of rolling an even number or a 4? inclusive; $\frac{1}{2}$
9. A card is drawn from a standard deck of cards. What is the probability of drawing a king or a queen? mutually exclusive; $\frac{2}{13}$
10. A card is drawn from a standard deck of cards. What is the probability of drawing a jack or a heart? inclusive; $\frac{4}{13}$
11. The sophomore class is selling Mother's Day plants to raise money. Susan's prize for being the top seller of plants is a choice of a book, a CD, or a video. She can choose from 6 books, 3 CDs, and 5 videos. What is the probability that Susan selects a book or a CD? mutually exclusive; $\frac{9}{14}$

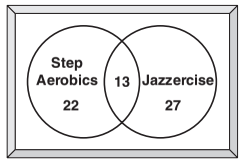
A spinner numbered 1–10 is spun. Find each probability.

12. P(less than 5 or even) $\frac{7}{10}$
13. P(even or odd) 1
14. P(prime or even) $\frac{4}{5}$

Two cards are drawn from a standard deck of cards. Find each probability.

15. P(both red or both black) $\frac{25}{51}$
16. P(both aces or both red) $\frac{55}{221}$
17. P(both 2s or both less than 5) $\frac{11}{221}$
18. P(both black or both less than 5) $\frac{188}{663}$

For Exercises 19 and 20, use the Venn diagram that shows the number of participants in two different kinds of aerobic exercise classes that are offered at a health club. Determine each probability if a person is selected at random from the participants.



19. P(step aerobics or jazzercise, but not both) $\frac{49}{62}$
20. P(step aerobics and jazzercise) $\frac{13}{62}$

Answers

NAME _____ DATE _____ PERIOD ____

12-5 Practice

Adding Probabilities

An urn contains 7 white marbles and 5 blue marbles. Four marbles are selected without replacement. Find each probability.

1. P(4 white or 4 blue) $\frac{8}{99}$
2. P(exactly 3 white) $\frac{35}{99}$
3. P(at least 3 white) $\frac{14}{33}$
4. P(fewer than 3 white) $\frac{19}{33}$
5. P(3 white or 3 blue) $\frac{49}{99}$
6. P(no white or no blue) $\frac{8}{99}$

Jason and Maria are playing a board game in which three dice are tossed to determine a player's move. Find each probability.

7. P(two 5s) $\frac{5}{72}$
8. P(three 5s) $\frac{1}{216}$
9. P(at least two 5s) $\frac{2}{27}$
10. P(no 5s) $\frac{125}{216}$
11. P(one 5) $\frac{25}{72}$
12. P(one 5 or two 5s) $\frac{5}{12}$

Determine whether the events are *mutually exclusive* or *inclusive*. Then find the probability.

13. A clerk chooses 4 CD players at random for floor displays from a shipment of 24 CD players. If 15 of the players have a blue case and the rest have a red case, what is the probability of choosing 4 players with a blue case or 4 players with a red case? mutual. exclus.; $\frac{71}{506}$

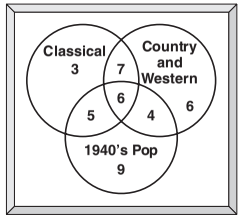
14. A department store employs 28 high school students, all juniors and seniors. Six of the 12 seniors are females and 12 of the juniors are males. One student employee is chosen at random. What is the probability of selecting a senior or a female? inclusive; $\frac{4}{7}$

15. A restaurant has 5 pieces of apple pie, 4 pieces of chocolate cream pie, and 3 pieces of blueberry pie. If Janine selects a piece of pie at random for dessert, what is the probability that she selects either apple or chocolate cream? mutually exclusive; $\frac{3}{4}$

16. At a statewide meeting, there are 20 school superintendents, 13 principals, and 6 assistant principals. If one of these people is chosen at random, what is the probability that he or she is either a principal or an assistant principal? mutually exclusive; $\frac{19}{39}$

17. An airline has one bank of 13 telephones at a reservations office. Of the 13 operators who work there, 8 take reservations for domestic flights and 5 take reservations for international flights. Seven of the operators taking domestic reservations and 3 of the operators taking international reservations are female. If an operator is chosen at random, what is the probability that the person chosen takes domestic reservations or is a male? inclusive; $\frac{10}{13}$

18. **MUSIC** Forty senior citizens were surveyed about their music preferences. The results are displayed in the Venn diagram. If a senior citizen from the survey group is selected at random, what is the probability that he or she likes only country and western music? What is the probability that he or she likes classical and/or country, but not 1940's pop?
$\frac{3}{20}$; $\frac{2}{5}$



NAME _____ DATE _____ PERIOD ____

12-5 Word Problem Practice

Adding Probabilities

1. **PICK-UP** When Tina's parents pick her up from school, there is a $\frac{1}{5}$ chance that she will be in the library, a $\frac{1}{2}$ chance that she will be on the playground, and a $\frac{3}{10}$ chance that she will be in her classroom. What is the probability that when Tina's parents pick her up, she is found in her classroom or on the playground?
$\frac{4}{5}$

2. **TRAVEL** John is randomly selected to be given a chance to win a new car. He must choose a red or yellow marble from a bag containing 1 red, 2 yellow, 10 green, and 12 blue marbles. What is the probability he will win the car?
$\frac{3}{25}$

3. **DICE** Alexis rolls two identical dice. What is the probability that the sum of the numbers rolled is odd? What is the probability that the sum of the numbers rolled is greater than 7? What is the probability that the sum of the numbers rolled is odd or greater than 7?
$P(\text{odd}) = \frac{1}{2}$; $P(>7) = \frac{5}{12}$;
$P(\text{odd or } >7) = \frac{3}{4}$

4. **CLASSES** At Jackson High School, 56 of the eleventh graders take physics and 70 of them take biology. There are 400 eleventh graders in total at the school. An eleventh grader is chosen at random from among all the eleventh graders at the high school. The probability that the selected student takes physics and biology is $\frac{11}{40}$. How many students at the high school take physics or biology?
16

PASSENGERS For Exercises 5 and 6, use the following information.

On an airplane flight, some passengers travel with carry-on luggage while others travel with a suitcase. Some passengers travel with carry-on luggage and a suitcase. Everyone travels with some form of luggage.

5. On one flight, there was no passenger with both carry-on luggage and a suitcase. On this flight are the events of picking a passenger with carry-on luggage and picking a passenger with a suitcase mutually exclusive?
Yes

6. On another flight, there are 120 passengers. Of those 120 passengers, 80 have carry-on luggage and 70 have a suitcase. What is the probability that a passenger has both carry-on luggage and a suitcase?
$\frac{1}{4}$

NAME _____ DATE _____ PERIOD ____

12-5 Enrichment

Probability and Tic-Tac-Toe

What would be the chances of winning at tic-tac-toe if it were turned into a game of pure chance? To find out, the nine cells of the tic-tac-toe board are numbered from 1 to 9 and nine chips (also numbered from 1 to 9) are put into a bag. Player A draws a chip at random and enters an *X* in the corresponding cell. Player B does the same and enters an *O*.

To solve the problem, assume that both players draw all their chips without looking and all *X* and *O* entries are made at the same time. There are four possible outcomes: a draw, A wins, B wins, and either A or B can win.

There are 16 arrangements that result in a draw. Reflections and rotations must be counted as shown below.

o x o x o x 4 x o x	x o x o o x 4 x x o	o o x x x o 8 o x x

There are 36 arrangements in which either player may win because both players have winning triples.

x x x o o o 4 x o x	x x x x o x 4 o o o	x o x x x x 4 o o o	x x x x x o 8 o o o	x x x o o o 8 x x o	x x o x x x 8 o o o

In these 36 cases, A's chances of winning are $\frac{13}{40}$.

1. Find the 12 arrangements in which B wins and A cannot.

o o x x o x 8 x x o	o x o x o x 4 x x o

2. Below are 12 of the arrangements in which A wins and B cannot. Write the numbers to show the reflections and rotations for each arrangement. What is the total number? 62

o x o x x x 1 o x o	x o x o x o 1 x o x	x x x x o o 4 x o o	x x x o x o 4 o x o	x o o x x x 4 o o x	x o o x x o 4 o o x
x x o o x x 4 o o x	x x x o x o 8 o o x	x x x x o o 8 o x o	x x x x o o 8 o o x	x o o x x x 8 o x o	x x o o x o 8 x o x

3. There are $\frac{9!}{(5!4!)}$ different and equally probable distributions. Complete the chart to find the probability for a draw or for A or B to win.

Draw: $\frac{16}{126}$	$= \frac{8}{63}$
A wins: $\frac{62}{126} + \frac{13}{40}\left(\frac{36}{126}\right)$	$= \frac{737}{1260}$
B wins: $\frac{12}{126} + \frac{27}{40}\left(\frac{36}{126}\right)$	$= \frac{121}{420}$

NAME _____ DATE _____ PERIOD ____

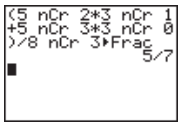
12-5 Graphing Calculator Activity

Probabilities

A graphing calculator can be used to perform calculations involving permutations, combinations, and probability.

Example 1 **There are 5 girls and 3 boys on a class committee. A subcommittee of 3 people is being chosen at random. What is the probability that the subcommittee will have at least 2 girls?**

P(at least 2 girls) = P(2 girls) + P(3 girls). Each probability is the product of the combinations of girls and boys divided by the combinations of all the students taken 3 at a time.

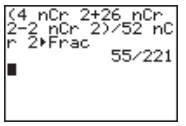


Keystrokes: (5 MATH ◀ 3 2 × 3 MATH ◀ 3 1 + 5 MATH ◀ 3 3 × 3 MATH ◀ 3 0) ÷ 8 MATH ◀ 3 3 MATH ENTER ENTER.

The probability that the subcommittee has at least 2 girls is $\frac{5}{7}$.

Example 2 **Two cards are randomly selected from a standard deck of cards. Find the probability that both cards are kings or that both cards are red.**

Since these events are mutually inclusive find the combinations of 4 kings taken 2 at a time plus 26 red cards taken 2 at a time minus 2 red kings taken 2 at a time divided by the combinations of 52 cards taken 2 at a time.



Keystrokes: (4 MATH ◀ 3 2 + 26 MATH ◀ 3 2 − 2 MATH ◀ 3 2) ÷ 52 MATH ◀ 3 2 MATH ENTER ENTER.

The probability of choosing 2 kings or two red cards is $\frac{55}{221}$.

Exercises

Find each probability.

1. There are 5 girls and 4 boys on the school publications committee. A group of 5 members is being chosen at random to attend a workshop on school newspapers. Find each probability.

a. at least 3 girls $\frac{10}{21}$ **b.** 4 girls or 4 boys $\frac{25}{126}$ **c.** at least 2 boys $\frac{10}{21}$

2. Two cards are drawn from a standard deck of cards. Find each probability.

a. both queens or both black $\frac{55}{221}$ **b.** both kings or both aces $\frac{2}{221}$ **c.** both face cards or both black $\frac{188}{663}$

3. Find the probability that a committee of 6 U.S. Representatives selected at random from 7 Democrats and 7 Republicans will have at least 3 Republicans on the committee. $\frac{302}{409}$

4. Three CDs are randomly selected from a collection of 6 rock and 5 rap CDs. Find the probability that at least 2 are rock. $\frac{19}{33}$

Answers

NAME _____ DATE_____ PERIOD ____

12-6 Lesson Reading Guide

Statistical Measures

Get Ready for the Lesson

Read the introduction to Lesson 12-6 in your textbook.

There is more than one way to give an "average" score for this test. Three measures of central tendency for these scores are 94, 76.5 and 73.9. Can you tell which of these is the mean, the median, and the mode without doing any calculations? Explain your answer.

Sample answer: Yes. The mode must be one of the scores, so it must be an integer. The median must be either one of the scores or halfway between two of the scores, so it must be an integer or a decimal ending with .5. Therefore, 94 is the mode, 76.5 is the median, and 73.9 is the mean.

Read the Lesson

1. Match each measure with one of the six descriptions of how to find measures of central tendency and variation.

a. median **vi** **b.** mode **i** **c.** range **iv**

d. variance **iii** **e.** mean **ii** **f.** standard deviation **v**

i. Find the most commonly occurring values or values in a set of data.

ii. Add the data and divide by the number of items.

iii. Find the mean of the squares of the differences between each value in the set of data and the mean.

iv. Find the difference between the largest and smallest values in the set of data.

v. Take the positive square root of the variance.

vi. If there is an odd number of items in a set of data, take the middle one. If there is an even number of items, add the two middle items and divide by 2.

Remember What You Learned

2. It is usually easier to remember a complicated procedure if you break it down into steps. Write the procedure for finding the standard deviation for a set of data in a series of brief, numbered steps.

Sample answer:
1. Find the mean.
2. Find the difference between each value and the mean.
3. Square each difference.
4. Find the mean of the squares.
5. Take the positive square root.

NAME _____ DATE_____ PERIOD ____

12-6 Study Guide and Intervention

Statistical Measures

Measures of Central Tendency

	Use	When
Measures of Central Tendency	mean	the data are spread out and you want an average of values
	median	the data contain outliers
	mode	the data are tightly clustered around one or two values

Example **Find the mean, median, and mode of the following set of data: {42, 39, 35, 40, 38, 35, 45}.**

To find the mean, add the values and divide by the number of values.

$$\text{mean} = \frac{42 + 39 + 35 + 40 + 38 + 35 + 45}{7} \approx 39.14.$$

To find the median, arrange the values in ascending or descending order and choose the middle value. (If there is an even number of values, find the mean of the two middle values.) In this case, the median is 39.

To find the mode, take the most common value. In this case, the mode is 35.

Exercises

Find the mean, median, and mode of each set of data. Round to the nearest hundredth, if necessary.

1. {238, 261, 245, 249, 255, 262, 241, 245} **249.5; 247; 245**

2. {9, 13, 8, 10, 11, 9, 12, 16, 10, 9} **10.7; 10; 9**

3. {120, 108, 145, 129, 102, 132, 134, 118, 108, 142} **123.8; 124.5; 108**

4. {68, 54, 73, 58, 63, 72, 65, 70, 61} **64.89; 65; no mode**

5. {34, 49, 42, 38, 40, 45, 34, 28, 43, 30} **38.3; 39; 34**

6. The table at the right shows the populations of the six New England capitals. Which would be the most appropriate measure of central tendency to represent the data? Explain why and find that value.

Source: www.factfinder.census.gov **There is no mode. The population of Boston is an outlier and would raise the mean too high. The median, 79,500, would be the best choice.**

City	Population (rounded to the nearest 1000)
Augusta, ME	19,000
Boston, MA	589,000
Concord, NH	37,000
Hartford, CT	122,000
Montpelier, VT	8,000
Providence, RI	174,000

NAME _____ DATE_____ PERIOD ____

12-6 Study Guide and Intervention *(continued)*

Statistical Measures

Measures of Variation The *range* and the **standard deviation** measure how scattered a set of data is.

Standard Deviation	If a set of data consists of the n values $x_1, x_2, \ldots, x_n$ and has mean $\bar{x}$, then the standard deviation is given by $\sigma = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2}{n}}$.

The square of the standard deviation is called the **variance**.

Example **Find the variance and standard deviation of the data set {10, 9, 6, 9, 18, 4, 8, 20}.**

Step 1 Find the mean.

$$\bar{x} = \frac{10 + 9 + 6 + 9 + 18 + 4 + 8 + 20}{8} = 10.5$$

Step 2 Find the variance.

$$\sigma^2 = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2}{n}$$ Standard variance formula

$$= \frac{(10 - 10.5)^2 + (9 - 10.5)^2 + \ldots + (20 - 10.5)^2}{8}$$

$$= \frac{220}{8} \text{ or } 27.5$$

Step 3 Find the standard deviation.

$$\sigma = \sqrt{27.5}$$

$$\approx 5.2$$

The variance is 27.5 and the standard deviation is about 5.2.

Exercises

Find the variance and standard deviation of each set of data. Round to the nearest tenth.

1. {100, 89, 112, 104, 96, 108, 93}
   58.5; 7.6
2. {62, 54, 49, 62, 48, 53, 50}
   29.4; 5.4
3. {8, 9, 8, 8, 9, 7, 8, 9, 6}
   0.9; 0.9
4. {4.2, 5.0, 4.7, 4.5, 5.2, 4.8, 4.6, 5.1}
   0.1; 0.3
5. The table at the right lists the prices of ten brands of breakfast cereal. What is the standard deviation of the values to the nearest penny? $0.33

Price of 10 Brands of Breakfast Cereal	
$2.29	$3.19
$3.39	$2.79
$2.99	$3.09
$3.19	$2.59
$2.79	$3.29

NAME _____ DATE_____ PERIOD ____

12-6 Skills Practice

Statistical Measures

Find the variance and standard deviation of each set of data to the nearest tenth.

1. {32, 41, 35, 35, 46, 42} 23.6, 4.9
2. {13, 62, 77, 24, 38, 19, 88} 763.8, 27.6
3. {89, 99, 42, 16, 42, 71, 16} 959.1, 31.0
4. {450, 400, 625, 225, 300, 750, 650, 625} 30,537.1; 174.7
5. {17, 23, 65, 94, 33, 33, 33, 8, 57, 75, 44, 12, 11, 68, 39} 630.7, 25.1
6. {7.2, 3.1, 3.8, 9.5, 8.3, 8.4} 5.8, 2.4
7. {1.5, 2.5, 3.5, 4.5, 4.5, 5.5, 6.5, 7.5} 3.5, 1.9

For Exercises 8 and 9, use the table that shows the profit in billions of dollars reported by U.S. manufacturers for the first quarter of the years from 1997 through 2001.

Year	1997	1998	1999	2000	2001
Seasonally-Adjusted Profit ($ billions)	$61.4	$75.6	$60.9	$78.5	$45.3

Source: U. S. Census Bureau

8. Find the mean and median of the data to the nearest tenth. $64.3 billion, $61.4 billion
9. Which measure of central tendency best represents the data? Explain.
   The median is more representative because the value 45.3 is not close to the other data points, and it lowers the mean.

For Exercises 10 and 11, use the table that shows the percent of fourth grade students reading at or above the proficiency level in a nationally-administered reading assessment.

Year	1992	1994	1998	2000
Percent at or above proficiency level	29%	30%	31%	32%

Source: National Center for Education Statistics

10. Find the mean, median, and standard deviation of the data to the nearest tenth.
    30.5%, 30.5%, 1.1%
11. What do the statistics from Exercise 11 tell you about the data?
    Sample answer: Since the median and mean are equal and the standard deviation is small, the percent of students reading at or above the proficiency level has not varied much from 1992 to 2000.

NAME _____ DATE _____ PERIOD ____

12-6 Practice

Statistical Measures

Find the variance and standard deviation of each set of data to the nearest tenth.

1. {47, 61, 93, 22, 82, 22, 37}
   673.1, 25.9
2. {10, 10, 54, 39, 96, 91, 91, 18}
   1228.6, 35.1
3. {1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5}
   1.6, 1.2
4. {1100, 725, 850, 335, 700, 800, 950}
   49,150.0; 221.7
5. {3.4, 7.1, 8.5, 5.1, 4.7, 6.3, 9.9, 8.4, 3.6}
   4.7, 2.2
6. {2.8, 0.5, 1.9, 0.8, 1.9, 1.5, 3.3, 2.6, 0.7, 2.5}
   0.8, 0.9

7. **HEALTH CARE** Eight physicians with 15 patients on a hospital floor see these patients an average of 18 minutes a day. The 22 nurses on the same floor see the patients an average of 3 hours a day. As a hospital administrator, would you quote the mean, median, or mode as an indicator of the amount of daily medical attention the patients on this floor receive? Explain. Either the median or the mode; they are equal and higher than the mean, which is lowered by the smaller amount of time the physicians spend with the patients.

For Exercises 8–10, use the frequency table that shows the percent of public school teachers in the U. S. in 1999 who used computers or the Internet at school for various administrative and teaching activities.

Activity	Percent Using Computer or Internet
Create instructional materials	39
Administrative record keeping	34
Communicate with colleagues	23
Gather information for planning lessons	16
Multimedia classroom presentations	8
Access research and best practices for teaching	8
Communicate with parents or students	8
Access model lesson plans	6

Source: National Assessment of Educational Progress

8. Find the mean, median, and mode of the data. 17.75%, 12%, 8%
9. Suppose you believe teachers use computers or the Internet too infrequently. Which measure would you quote as the "average?" Explain. Mode; it is lowest.
10. Suppose you believe teachers use computers or the Internet too often. Which measure would you quote as the "average?" Explain. Mean; it is highest.

For Exercises 11 and 12, use the frequency table that shows the number of games played by 24 American League baseball players between opening day, 2001 and September 8, 2001.

No. of Games	Frequency
141	4
140	3
139	4
138	5
137	2
136	3
135	3

Source: Major League Baseball

11. Find the mean, median, mode, and standard deviation of the number of games played to the nearest tenth.
    138.2, 138; 138, 2.0
12. For how many players is the number of games within one standard deviation of the mean? 14

NAME _____ DATE _____ PERIOD ____

12-6 Word Problem Practice

Statistical Measures

1. **SPORTS** The table below shows the number of times some teams in the National Football League have won the Super Bowl.

NFL Team	Number of Super Bowl Victories
New England	3
Baltimore	2
Kansas City	1
St. Louis	1
Denver	2
Green Bay	1
Dallas	5
San Francisco	5
Oakland	2
Pittsburgh	5
Miami	2
Washington	3
NY Giants	2
NY Jets	1
Chicago	1

Source: www.pubquizhelp.34sp.com

Which statistical measure represents the team(s) with the least Super Bowl victories? the mode

2. **SALARIES** The median salary in a small company is $10.20 per hour. What percentage of the employees at the company earns more than $10.20 per hour? 50%
3. **RANDOM GENERATORS** Samuel has written a computer program to generate a random selection of the following two-digit numbers.

   25, 67, 54, 99, 41, 87, 90, 18, 32

   Find the mean, median, and mode of this data. 57; 54; none
4. **HEIGHTS** The following table lists the heights of some of the great NBA players.

Player	Height (in inches)
Kareem Abdul-Jabbar	86
Larry Bird	81
Shaquille O'Neal	85
Wilt Chamberlain	85
Michael Jordan	78

Source: www.sidwell.edu

Find the mean and standard deviation of the data in the table. Round your answer to the nearest hundredth.
83; 3.0

METEORS For Exercises 5-8, use the following information.

Arlene stayed up late one night to watch the Perseid meteor shower. She recorded the number of meteors she saw every ten minutes starting at 1 A.M. and going until 4 A.M. Her data are shown below.

8, 7, 8, 12, 17, 15, 22, 28, 29, 31, 28, 23, 29, 28, 25, 23, 15, 12

5. What is the mean of this data set?
   20
6. What is the median of this data set?
   22.5
7. What is the mode of this data set?
   28
8. What is the standard deviation of this data set? Round your answer to the nearest hundredth.
   8.05

NAME _____ DATE_____ PERIOD ____

12-6 Enrichment

Standard Deviation of Sample Data

A *population* is the set of all measurements of interest to an investigator. A *sample* is a subset of measurements selected from the population of interest. A *statistic* is any quantity whose value can be calculated from sample data. A common mistake is to use the terms *probability* and *statistics* interchangeably. Probabilities are used to make statements from a population to a sample, but statistics are calculated from a sample and are to make inferences about a population.

The *range* is a statistic calculated by taking the difference between the largest observation and the smallest observation. Range $= x_{\max} - x_{\min}$.

The *sample variance* is calculated using the formula: $s^2 = \frac{\sum_{i=1}^{n} (\bar{x} - x_i)^2}{n - 1}$ where $\bar{x}$ is the sample mean. Therefore, the *sample standard deviation* is the square root of the sample variance, $s = \sqrt{s^2}$.

To calculate the sample variance:

1. Calculate the sample mean. For example, suppose a sample contains the numbers {2, 5, 6, 9, 11}. The sample mean is $\bar{x} = \frac{2 + 5 + 6 + 9 + 11}{5} = 6.6$.
2. Next use the formula above to calculate the sample variance, in this case: $s^2 = \frac{(6.6 - 2)^2 + (6.6 - 5)^2 + (6.6 - 6)^2 + (6.6 - 9)^2 + (6.6 - 11)^2}{4} = 12.3$.
3. Finally, the sample standard deviation is equal to 3.507 by taking the square root of 12.3.

Exercises

1. What are some differences in the formula for the sample variance compared to the formula for the population variance? **It uses the sample mean instead of the population mean, and since the sample mean is an estimator for the population mean, the denominator is $n - 1$ instead of n.**
2. Given the random sample {5, 7, 1, 2, 4}, find the sample variance. **5.70**
3. Calculate the sample standard deviation. **2.387**
4. Calculate the range of the sample data {5, 7, 1, 2, 4}. **6**
5. An approximation for the sample standard deviation is given by: $s \approx \frac{Range}{4}$. Compare this answer to your answer from 3. **6**

NAME _____ DATE_____ PERIOD ____

12-7 Reading to Learn Mathematics

The Normal Distribution

Get Ready for the Lesson

Read the introduction to Lesson 12-7 in your textbook.

There were 66 players on the team and the mean height was approximately 74.1. About what fraction of the players' heights are between 72 and 75, inclusive?

Sample answer: about $\frac{1}{2}$

Read the Lesson

1. Indicate whether each of the following statements is *true* or *false*.
   a. In a continuous probability distribution, there is a finite number of possible outcomes. **false**
   b. Every normal distribution can be represented by a bell curve. **true**
   c. A distribution that is represented by a curve that is high at the left and has a tail to the right is negatively skewed. **false**
   d. A normal distribution is an example of a skewed distribution. **false**
2. Ms. Rose gave the same quiz to her two geometry classes. She recorded the following scores.

First-period class:

Score	0	1	2	3	4	5	6	7	8	9	10
Frequency	1	0	1	0	3	4	5	7	4	3	2

Fifth-period class:

Score	0	1	2	3	4	5	6	7	8	9	10
Frequency	0	0	0	0	3	4	9	7	6	1	0

In each class, 30 students took the quiz. The mean score for each class was 6.4. Which set of scores has the greater standard deviation? (Answer this question without doing any calculations.) Explain your answer.

First period class; sample answer: The scores are more spread out from the mean than for the fifth period class.

Remember What You Learned

3. Many students have trouble remembering how to determine if a curve represents a distribution that is *positively skewed* or *negatively skewed*. What is an easy way to remember this?

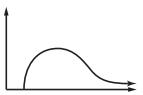
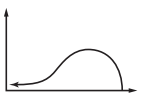
Sample answer: Follow the tail! If the tail is on the right (positive direction), the distribution is positively skewed. If the tail is on the left (negative direction), the distribution is negatively skewed.

NAME _____ DATE _____ PERIOD ____

12-7 Study Guide and Intervention

The Normal Distribution

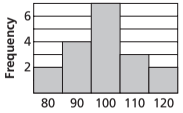
Normal and Skewed Distributions A **continuous probability** distribution is represented by a curve.

Types of Continuous Distributions	Normal	Positively Skewed	Negatively Skewed
			

Example **Determine whether the data below appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.**
{100, 120, 110, 100, 110, 80, 100, 90, 100, 120, 100, 90, 110, 100, 90, 80, 100, 90}

Make a frequency table for the data.

Value	80	90	100	110	120
Frequency	2	4	7	3	2



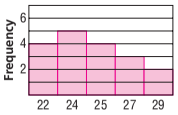
Then use the data to make a histogram.

Since the histogram is roughly symmetric, the data appear to be normally distributed.

Exercises

Determine whether the data in each table appear to be *positively skewed*, *negatively skewed*, or *normally distributed*. Make a histogram of the data.

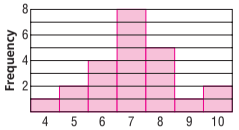
1. {27, 24, 29, 25, 27, 22, 24, 25, 29, 24, 25, 22, 27, 24, 22, 25, 24, 22}
   positively skewed



2.

Shoe Size	4	5	6	7	8	9	10
No. of Students	1	2	4	8	5	1	2

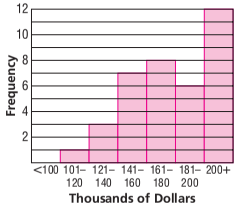
normally distributed



3.

Housing Price	No. of Houses Sold
less than $100,000	0
$100,00–$120,000	1
$121,00–$140,000	3
$141,00–$160,000	7
$161,00–$180,000	8
$181,00–$200,000	6
over $200,000	12

negatively skewed

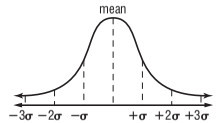


NAME _____ DATE _____ PERIOD ____

12-7 Study Guide and Intervention *(continued)*

The Normal Distribution

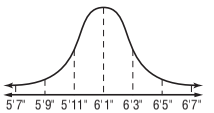
Use Normal Distributions

Normal Distribution	Normal distributions have these properties.
	The graph is maximized at the mean. The mean, median, and mode are about equal. About 68% of the values are within one standard deviation of the mean. About 95% of the values are within two standard deviations of the mean. About 99% of the values are within three standard deviations of the mean.

Example **The heights of players in a basketball league are normally distributed with a mean of 6 feet 1 inch and a standard deviation of 2 inches.**

a. What is the probability that a player selected at random will be shorter than 5 feet 9 inches?

Draw a normal curve. Label the mean and the mean plus or minus multiples of the standard deviation.



The value of 5 feet 9 inches is 2 standard deviations below the mean, so approximately 2.5% of the players will be shorter than 5 feet 9 inches.

b. If there are 240 players in the league, about how many players are taller than 6 feet 3 inches?

The value of 6 feet 3 inches is one standard deviation above the mean. Approximately 16% of the players will be taller than this height.

$240 \times 0.16 \approx 38$

About 38 of the players are taller than 6 feet 3 inches.

Exercises

EGG PRODUCTION The number of eggs laid per year by a particular breed of chicken is normally distributed with a mean of 225 and a standard deviation of 10 eggs.

1. About what percent of the chickens will lay between 215 and 235 eggs per year? 68%

2. In a flock of 400 chickens, about how many would you expect to lay more than 245 eggs per year? 10 chickens

MANUFACTURING The diameter of bolts produced by a manufacturing plant is normally distributed with a mean of 18 mm and a standard deviation of 0.2 mm.

3. What percent of bolts coming off of the assembly line have a diameter greater than 18.4 mm? 2.5%

4. What percent have a diameter between 17.8 and 18.2 mm? 68%

NAME _____ DATE _____ PERIOD ____

12-7 Skills Practice

The Normal Distribution

Determine whether the data in each table appear to be *positively skewed, negatively skewed, or normally distributed.*

1.

Miles Run	Track Team Members
0–4	3
5–9	4
10–14	7
15–19	5
20–23	2

normally distributed

2.

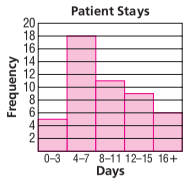
Speeches Given	Political Candidates
0–5	1
6–11	2
12–17	3
18–23	8
24–29	8

negatively skewed

For Exercises 3 and 4, use the frequency table that shows the average number of days patients spent on the surgical ward of a hospital last year.

Days	Number of Patients
0–3	5
4–7	18
8–11	11
12–15	9
16+	6

3. Make a histogram of the data.



4. Do the data appear to be *positively skewed*, *negatively skewed*, or *normally distributed*? Explain.
Positively skewed; the histogram is high at the left and has a tail to the right.

DELIVERY For Exercises 5–7, use the following information.
The time it takes a bicycle courier to deliver a parcel to his farthest customer is normally distributed with a mean of 40 minutes and a standard deviation of 4 minutes.

5. About what percent of the courier's trips to this customer take between 36 and 44 minutes? 68%

6. About what percent of the courier's trips to this customer take between 40 and 48 minutes? 47.5%

7. About what percent of the courier's trips to this customer take less than 32 minutes? 2.5%

TESTING For Exercises 8–10, use the following information.
The average time it takes sophomores to complete a math test is normally distributed with a mean of 63.3 minutes and a standard deviation of 12.3 minutes.

8. About what percent of the sophomores take more than 75.6 minutes to complete the test? 16%

9. About what percent of the sophomores take between 51 and 63.3 minutes? 34%

10. About what percent of the sophomores take less than 63.3 minutes to complete the test? 50%

NAME _____ DATE _____ PERIOD ____

12-7 Practice

The Normal Distribution

Determine whether the data in each table appear to be *positively skewed, negatively skewed, or normally distributed.*

1.

Time Spent at a Museum Exhibit	
Minutes	Frequency
0–25	27
26–50	46
51–75	89
75–100	57
100+	24

normally distributed

2.

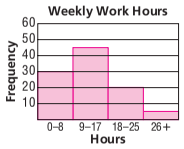
Average Age of High School Principals	
Age in Years	Number
31–35	3
36–40	8
41–45	15
46–50	32
51–55	40
56–60	38
60+	4

negatively skewed

For Exercises 3 and 4, use the frequency table that shows the number of hours worked per week by 100 high school seniors.

Hours	Number of Students
0–8	30
9–17	45
18–25	20
26+	5

3. Make a histogram of the data.



4. Do the data appear to be *positively skewed*, *negatively skewed*, or *normally distributed*? Explain.
Positively skewed; the histogram is high at the left and has a tail to the right.

TESTING For Exercises 5–10, use the following information.
The scores on a test administered to prospective employees are normally distributed with a mean of 100 and a standard deviation of 15.

5. About what percent of the scores are between 70 and 130? 95%

6. About what percent of the scores are between 85 and 130? 81.5%

7. About what percent of the scores are over 115? 16%

8. About what percent of the scores are lower than 85 or higher than 115? 32%

9. If 80 people take the test, how many would you expect to score higher than 130? 2

10. If 75 people take the test, how many would you expect to score lower than 85? 12

11. TEMPERATURE The daily July surface temperature of a lake at a resort has a mean of 82° and a standard deviation of 4.2°. If you prefer to swim when the temperature is at least 77.8°, about what percent of the days does the temperature meet your preference? 84%

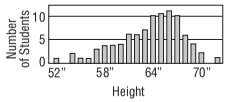
NAME _____ DATE _____ PERIOD ____

12-7 Word Problem Practice

The Normal Distribution

1. **PARKING** Over several years, Bertram conducted a study of how far into parking spaces people tend to park by measuring the distance from the end of a parking space to the front fender of a car parked in the space. He discovered that the distribution of the data closely approximated a normal distribution with mean 8.5 inches. He found that about 5% of cars parked more than 11.5 inches away from the end of the parking space. What percentage of cars would you expect parked less than 5.5 inches away from the end of the parking space?
5%

2. **HEIGHT** Chandra's graph of the number of tenth grade students of different heights is shown below.



Is the data positively skewed, negatively skewed, or normally distributed?
Negatively skewed

3. **OVENS** An oven manufacturer tries to make the temperature setting on its ovens as accurate as possible. However, if one measures the actual temperatures in the ovens when the temperature setting is 350°F, they will differ slightly from 350°F. The set of actual temperatures for all the ovens is normally distributed around 350°F with a standard deviation of 0.5°F. About what percentage of ovens will be between 350°F and 351°F when their temperature setting is 350°F?
47.5%

4. **LIGHT BULBS** The time that a certain brand of light bulb will last before burning out is normally distributed. About 2.5% of the bulbs last longer than 6800 hours and about 16% of the bulbs last longer than 6500 hours. How long does the average bulb last?
6200 hours

DOGS For Exercises 5-8, use the following information.

The weights of adult greyhound dogs are normally distributed. The mean weight is about 69 pounds and the standard deviation is about 10 pounds.

5. Approximately what percentage of adult greyhound dogs would you expect weigh between 59 and 79 pounds?
68%

6. Approximately what percentage of adult greyhound dogs would you expect weigh more than 99 pounds?
0.5%

7. Approximately what percentage of adult greyhound dogs would you expect weigh less than 49 pounds?
2.5%

8. What would you expect an adult greyhound dog to weigh if it weighed less than 0.5% of an average adult greyhound?
39 lbs or less.

NAME _____ DATE _____ PERIOD ____

12-7 Enrichment

Calculating Z-Scores

The normal distribution is the most important probability distribution. Many physical measurements have distributions approximately normal. Examples include height, weight, and measures of intelligence. More importantly, even if the individual variables are not normally distributed, sums and averages tend to still be normally distributed. Unfortunately, normal probability distribution functions are difficult to calculate. Fortunately, statisticians have compiled a table for a normal distribution with mean of zero and standard deviation of one. This is called the Standard Normal Distribution and is typically denoted by N(0, 1), where the N indicates a normal distribution which has mean, μ (mu) = 0, and standard deviation, σ (sigma) = 1.

Suppose the variable x is normally distributed with mean μ and standard deviation σ. In order to calculate probabilities of this normal distribution, we must standardize the variable x by an appropriate transformation. The letter Z denotes the transformed variable and is called the Z-score, which is a measure of relative standing. The following steps are needed to complete the transformation.

- If the mean and standard deviation are not given, then calculate the mean and standard deviation of the given (population) data.
- Define $Z = \frac{x - \mu}{\sigma}$.

Example Find the standard normal variable Z given and $\mu = 15$ and $\sigma = 3$.

Apply the transform to the variable X using the definition above, that is: $Z = \frac{X - 15}{3}$.

1. Suppose that the time, X, to complete an exam is normally distributed. The time, in minutes, of a class of 12 to complete the exam is given in the table. Transform X to a Z-score.

Student	1	2	3	4	5	6	7	8	9	10	11	12
Time	35	42	48	33	32	39	40	52	48	34	36	44

Mean = 40.25, Standard Deviation = 6.34
$Z = (X - 40.25) / 6.34$

2. Suppose that a random variable X is normally distributed with $\mu = 20$ and $\sigma = 5$. Convert the following probability statements to the equivalent statements by standardizing X.

Example $P(X < 25) = P\left(\frac{X - 20}{5} < 25\right) = P(Z < 25)$

a. $P(X > 18)$ $P\left(Z > \frac{18 - 20}{5}\right) = P\left(Z > -\frac{2}{5}\right)$

b. $P(17 < X < 23)$ $P\left(\frac{17 - 20}{5} < Z < \frac{23 - 20}{5}\right) = P\left(-\frac{3}{5} < Z < \frac{3}{5}\right)$

c. $P(X < 19)$ $P\left(Z < \frac{19 - 20}{5}\right) = P\left(Z < -\frac{1}{5}\right)$

NAME _____ DATE_____ PERIOD ____

12-8 Lesson Reading Guide

Binomial Experiments

Get Ready for the Lesson

Read the introduction to Lesson 12-8 in your textbook.

Suppose you are taking a 50-question multiple-choice test in which there are 5 answer choices for each question. You are told that no points will be deducted for wrong answers. Should you guess the answers to the questions you do not know? Explain your reasoning.

Sample answer: Yes; the probability of guessing the right answer to a question is $\frac{1}{5}$, so you have a chance to get some points by guessing, and you have nothing to lose.

Read the Lesson

1. Indicate whether each of the following is a *binomial experiment* or *not a binomial experiment*. If the experiment is not a binomial experiment, explain why.

   a. A fair coin is tossed 10 times and "heads" or "tails" is recorded each time. binomial experiment

   b. A pair of dice is thrown 5 times and the sum of the numbers that come up is recorded each time. Not a binomial experiment; there are more than two possible outcomes for each trial.

   c. There are 5 red marbles and 6 blue marbles in a bag. One marble is drawn from the bag and its color recorded. The marble is not put back in the bag. A second marble is drawn and its color recorded. Not a binomial experiment; the trials are not independent (or, the probabilities for the two trials are not the same).

   d. There are 5 red marbles and 6 blue marbles in a bag. One marble is drawn from the bag and its color recorded. The marble is put back in the bag. A second marble is drawn and its color recorded. binomial experiment

2. Len randomly guesses the answers to all 6 multiple-choice questions on his chemistry test. Each question has 5 choices. Which of the following expressions gives the probability that he will get at least 4 of the answers correct? B

   A. $P(6, 4)\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^2 + P(6, 5)\left(\frac{1}{5}\right)^5\left(\frac{4}{5}\right)^1 + P(6, 6)\left(\frac{1}{5}\right)^6\left(\frac{4}{5}\right)^0$

   B. $C(6, 4)\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^2 + C(6, 5)\left(\frac{1}{5}\right)^5\left(\frac{4}{5}\right)^1 + C(6, 6)\left(\frac{1}{5}\right)^6\left(\frac{4}{5}\right)^0$

   C. $C(6, 4)\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^4 + C(6, 5)\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^5 + C(6, 6)\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^6$

Remember What You Learned

3. Some students have trouble remembering how to calculate binomial probabilities. What is an easy way to remember which numbers to put into an expression like $C(6, 4)\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^4$?

   Sample answer: The binomial coefficient is $C(n, r)$, where n is the number of trials and r is the number of successes. The probability of success is raised to the rth power and the probability of failure is raised to the $(n - r)$th power.

NAME _____ DATE_____ PERIOD ____

12-8 Study Guide and Intervention

Binomial Experiments

Binomial Expansions For situations with only 2 possible outcomes, you can use the Binomial Theorem to find probabilities. The coefficients of terms in a binomial expansion can be found by using combinations.

Example **What is the probability that 3 coins show heads and 3 show tails when 6 coins are tossed?**

There are 2 possible outcomes that are equally likely: heads (H) and tails (T). The tosses of 6 coins are independent events. When $(H + T)^6$ is expanded, the term containing H^3T^3, which represents 3 heads and 3 tails, is used to get the desired probability. By the Binomial Theorem the coefficient of H^3T^3 is $C(6, 3)$.

$$P(3 \text{ heads, } 3 \text{ tails}) = \frac{6!}{3!3!}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^3 \quad P(\text{H}) = \frac{1}{2} \text{ and } P(\text{T}) = \frac{1}{2}$$
$$= \frac{20}{64}$$
$$= \frac{5}{16}$$

The probability of getting 3 heads and 3 tails is $\frac{5}{16}$ or 0.3125.

Exercises

Find each probability if a coin is tossed 8 times.

1. P(exactly 5 heads)
   about 22%

2. P(exactly 2 heads)
   about 11%

3. P(even number of heads)
   50%

4. P(at least 6 heads)
   about 14%

Mike guesses on all 10 questions of a true-false test. If the answers true and false are evenly distributed, find each probability.

5. Mike gets exactly 8 correct answers.
   $\frac{45}{1024}$ or 0.044

6. Mike gets at most 3 correct answers.
   $\frac{11}{64}$ or 0.172

7. A die is tossed 4 times. What is the probability of tossing exactly two sixes?
   $\frac{25}{216}$ or 0.116

Answers

NAME _____ DATE _____ PERIOD ____

12-8 Study Guide and Intervention *(continued)*

Binomial Experiments

Binomial Experiments

Binomial Experiments	A binomial experiment is possible if and only if all of these conditions occur. • There are exactly two outcomes for each trial. • There is a fixed number of trials. • The trials are independent. • The probabilities for each trial are the same.

Example **Suppose a coin is weighted so that the probability of getting heads in any one toss is 90%. What is the probability of getting exactly 7 heads in 8 tosses?**

The probability of getting heads is $\frac{9}{10}$, and the probability of getting tails is $\frac{1}{10}$. There are $C(8, 7)$ ways to choose the 7 heads.

$$P(7 \text{ heads}) = C(8, 7)\left(\frac{9}{10}\right)^7\left(\frac{1}{10}\right)^1$$

$$= 8 \cdot \frac{9^7}{10^8}$$

$$\approx 0.38$$

The probability of getting 7 heads in 8 tosses is about 38%.

Exercises

1. **BASKETBALL** For any one foul shot, Derek has a probability of 0.72 of getting the shot in the basket. As part of a practice drill, he shoots 8 shots from the foul line.
   a. What is the probability that he gets in exactly 6 foul shots? **about 31%**
   b. What is the probability that he gets in at least 6 foul shots? **about 60%**
2. **SCHOOL** A teacher is trying to decide whether to have 4 or 5 choices per question on her multiple choice test. She wants to prevent students who just guess from scoring well on the test.
   a. On a 5-question multiple-choice test with 4 choices per question, what is the probability that a student can score at least 60% by guessing? **10.4%**
   b. What is the probability that a student can score at least 60% by guessing on a test of the same length with 5 choices per question? **5.8%**
3. Julie rolls two dice and adds the two numbers.
   a. What is the probability that the sum will be divisible by 3? $\frac{1}{3}$
   b. If she rolls the dice 5 times what is the chance that she will get exactly 3 sums that are divisible by 3? **about 16%**
4. **SKATING** During practice a skater falls 15% of the time when practicing a triple axel. During one practice session he attempts 20 triple axels.
   a. What is the probability that he will fall only once? **about 14%**
   b. What is the probability that he will fall 4 times? **about 18%**

NAME _____ DATE _____ PERIOD ____

12-8 Skills Practice

Binomial Experiments

Find each probability if a coin is tossed 4 times.

1. $P(4 \text{ heads})$ $\frac{1}{16}$
2. $P(0 \text{ heads})$ $\frac{1}{16}$
3. $P(\text{exactly 3 heads})$ $\frac{1}{4}$
4. $P(\text{exactly 2 heads})$ $\frac{3}{8}$
5. $P(\text{exactly 1 head})$ $\frac{1}{4}$
6. $P(\text{at least 3 heads})$ $\frac{5}{16}$

Find each probability if a die is rolled 3 times.

7. $P(\text{exactly one 2})$ $\frac{25}{72}$
8. $P(\text{exactly two 2s})$ $\frac{5}{72}$
9. $P(\text{exactly three 2s})$ $\frac{1}{216}$
10. $P(\text{at most one 2})$ $\frac{25}{27}$

A town that presents a fireworks display during its July 4 celebration found the probability that a family with two or more children will watch the fireworks is $\frac{3}{5}$. If 5 of these families are selected at random, find each probability.

11. $P(\text{exactly 3 families watch the fireworks})$ $\frac{216}{625}$
12. $P(\text{exactly 2 families watch the fireworks})$ $\frac{144}{625}$
13. $P(\text{exactly 5 families watch the fireworks})$ $\frac{243}{3125}$
14. $P(\text{no families watch the fireworks})$ $\frac{32}{3125}$
15. $P(\text{at least 4 families watch the fireworks})$ $\frac{1053}{3125}$
16. $P(\text{at most 1 family watches the fireworks})$ $\frac{272}{3125}$

One section of a standardized English language test has 10 true/false questions. Find each probability when a student guesses at all ten questions.

17. $P(\text{exactly 8 correct})$ $\frac{45}{1024}$
18. $P(\text{exactly 2 correct})$ $\frac{45}{1024}$
19. $P(\text{exactly half correct})$ $\frac{63}{256}$
20. $P(\text{all 10 correct})$ $\frac{1}{1024}$
21. $P(0 \text{ correct})$ $\frac{1}{1024}$
22. $P(\text{at least 8 correct})$ $\frac{7}{128}$

NAME _____ DATE_____ PERIOD ____

12-8 Practice

Binomial Experiments

Find each probability if a coin is tossed 6 times.

1. P(exactly 3 tails) $\frac{5}{16}$
2. P(exactly 5 tails) $\frac{3}{32}$
3. P(0 tails) $\frac{1}{64}$
4. P(at least 4 heads) $\frac{11}{32}$
5. P(at least 4 tails) $\frac{11}{32}$
6. P(at most 2 tails) $\frac{11}{32}$

The probability of Chris making a free throw is $\frac{2}{3}$. If she shoots 5 times, find each probability.

7. P(all missed) $\frac{1}{243}$
8. P(all made) $\frac{32}{243}$
9. P(exactly 2 made) $\frac{40}{243}$
10. P(exactly 1 missed) $\frac{80}{243}$
11. P(at least 3 made) $\frac{64}{81}$
12. P(at most 2 made) $\frac{17}{81}$

When Tarin and Sam play a certain board game, the probability that Tarin will win a game is $\frac{3}{4}$. If they play 5 games, find each probability.

13. P(Sam wins only once) $\frac{405}{1024}$
14. P(Tarin wins exactly twice) $\frac{45}{512}$
15. P(Sam wins exactly 3 games) $\frac{45}{512}$
16. P(Sam wins at least 1 game) $\frac{781}{1024}$
17. P(Tarin wins at least 3 games) $\frac{459}{512}$
18. P(Tarin wins at most 2 games) $\frac{53}{512}$

19. **SAFETY** In August 2001, the American Automobile Association reported that 73% of Americans use seat belts. In a random selection of 10 Americans in 2001, what is the probability that exactly half of them use seat belts? about 7.5%

HEALTH For Exercises 20 and 21, use the following information.

In 2001, the American Heart Association reported that 50 percent of the Americans who receive heart transplants are ages 50–64 and 20 percent are ages 35–49.

20. In a randomly selected group of 10 heart transplant recipients, what is the probability that at least 8 of them are ages 50–64? $\frac{7}{128}$
21. In a randomly selected group of 5 heart transplant recipients, what is the probability that 2 of them are ages 35–49? $\frac{128}{625}$

NAME _____ DATE_____ PERIOD ____

12-8 Word Problem Practice

Binomial Experiments

1. **GENETICS** Dagmar is conducting a genetic experiment. Before she performs the experiment, she would like to compute theoretically probabilities for some of the outcomes. One of these computations involves expanding $(p + q)^4$. What is this expansion?
$p^4 + 4p^3q + 6p^2q^2 + 4pq^3 + q^4$

2. **GAMES** The probability that Kendra will win a card game is $\frac{2}{3}$. If Kendra plays 7 games what is the probability she wins exactly 4 games? Round your answer to the nearest thousandth.
about 0.256

3. **DEFECTS** An electronics parts manufacturer produces capacitors for electronic circuits. The probability that a capacitor comes out defective is 1 in 1,000. In a batch of 10,000 capacitors, write an expression for the probability that 10 of the capacitors are defective.
$C(10000, 10)\left(\frac{1}{1000}\right)^{10}\left(\frac{999}{1000}\right)^{9990}$

4. **SUBWAYS** Fiona uses the subway to commute to work. During the morning commute, the trains run frequently and there is a 1 in 8 chance that she will find a train waiting for her as soon as she gets to the platform. Over the course of a five-day work week, what is the probability that she found a train waiting for her at least twice? Round your answer to the nearest thousandth.
0.121

5. **SOCCER** The boys varsity soccer team at Lincoln High School has a 75% probability of winning each of their 17 games this season. What is the probability that the team will win at least 13 games this season? Round your answer to the nearest thousandth.
0.574

CHESS For Exercises 6-8, use the following information.

Gary and Howard play chess. Gary's chess rating is 2050 and Howard's chess rating is 1948. This means that whenever they play, Gary has a 64% chance of defeating Howard. One day, Gary and Howard play three games against each other. Round your answers to the nearest thousandth.

6. What is the probability that Gary will win all three of the matches?
0.262
7. What is the probability that Gary will win at least two of the three matches?
0.705
8. What is the probability that Gary will win only one of the matches?
0.249

Answers

NAME _____ DATE_____ PERIOD ____

12-8 Enrichment

Multinomial Experiments

A multinomial is a generalization of a binomial. For example, $(a + b + c)^2$ is a multinomial. One way to determine the coefficients is by direct multiplication using the distributive property. Take each term in the first factor and multiply by each term (a, b, and c) in the second factor, then combine like terms. (Notice that the sum of the exponents is always equal to two.)

$$\begin{aligned}(a + b + c)^2 = (a + b + c)(a + b + c) &= a(a + b + c) + b(a + b + c) + c(a + b + c)\\ &= a^2 + ab + ac + ab + b^2 + bc + ac + bc + c^2\\ &= a^2 + b^2 + c^2 + 2ab + 2ac + 2bc\end{aligned}$$

Underlying this expansion is the notion of a *partition* into categories. The example partitions two 'items' among three categories. In this case the categories are the variables a, b, and c and the items are exponents. For example the partition, {1, 0, 1} represents the term ac in the expansion, which could also be written as $a^1b^0c^1$, whereas the partition {0, 2, 0} represents the b^2 term. The coefficients of each term can be computed by the formula:

$\frac{n!}{n_1! \cdot n_2! \cdot n_3! \cdots n_k!}$, where n is the exponent and $n_1 + n_2 + n_3 + \cdots + n_k = n$. Recall, $0! = 1$.

Term	Partition	Coefficient
a^2	{2,0,0}	$\frac{2!}{2! \cdot 0! \cdot 0!} = 1$
b^2	{0,2,0}	$\frac{2!}{0! \cdot 2! \cdot 0!} = 1$
c^2	{0,0,2}	$\frac{2!}{0! \cdot 0! \cdot 2!} = 1$
ab	{1,1,0}	$\frac{2!}{1! \cdot 1! \cdot 0!} = 2$
ac	{1,0,1}	$\frac{2!}{1! \cdot 0! \cdot 1!} = 2$
bc	{0,1,1}	$\frac{2!}{0! \cdot 1! \cdot 1!} = 2$

1. Determine the all the *partitions* of $(x + y + z)^3$.

   {3,0,0}, {0,3,0}, {0,0,3}, {2,0,1}, {2,1,0}, {1, 2, 0}, {1, 0, 2}, {0, 1, 2}, {0, 2, 1}, {1,1,1}

2. Determine the coefficients in the expansion of $(x + y + z)^3$ associated with each partition.

   {3,0,0} → 1, {0,3,0} → 1, {0,0,3} → 1, {2,0,1} → 3, {2,1,0} → 3, {1, 2, 0} → 3, {1, 0, 2} → 3, {0, 1, 2} → 3, {0, 2, 1} → 3, {1,1,1} → 6

3. How can you build and interpret a trinomial distribution?

   The trinomial distribution is interpreted as independent events with 3 outcomes whose individual probabilities are for example $x = 0.25$, $y = 0.45$, and $y = 0.30$.

NAME _____ DATE_____ PERIOD ____

12-9 Lesson Reading Guide

Sampling and Error

Get Ready for the Lesson

Read the introduction to Lesson 12-9 in your textbook.

Do you think the results of the survey show that more mothers spend \$249 or less than \$250–\$349? If there is not enough information given to determine this, list at least two questions you would ask about the survey that would help you determine the significance of the survey. Sample answer: There is not enough information to tell. 1. How many people were surveyed? 2. How was the sample for the survey selected? 3. What is the margin of error for this survey?

Read the Lesson

1. Determine whether each situation would produce a random sample. Write *yes* or *no* and explain your answer.

   a. asking all the customers at five restaurants on the same evening how many times a month they eat dinner in restaurants to determine how often the average American eats dinner in a restaurants No; people surveyed at a restaurant might be likely to eat dinner in restaurants more often than other people.

   b. putting the names of all seniors at your high school in a hat and then drawing 20 names for a survey to find out where seniors would like to hold their prom Yes; every senior would have an equal chance of being chosen for the survey.

2. A survey determined that 58% of registered voters in the United States support increased federal spending for education. The margin of error for this survey is 4%. Explain in your own words what this tells you about the actual percentage of registered voters who support increased spending for education. Sample answer: There is a 95% chance that the actual percentage of voters supporting increased federal spending for education is between 54% and 62%.

Remember What You Learned

3. The formula for margin of sampling error may be tricky to remember. A good way to start is to think about the variables that must be included in the formula. What are these variables, and what do they represent? What is an easy way to remember which variable goes in the denominator in the formula? Sample answer: p is the probability of a certain response and n is the sample size. The larger the sample size, the smaller the margin of error, so n must go in the denominator since dividing by a larger number gives a smaller number. The square root of a smaller number is a smaller number, and twice the square root of a smaller number is a smaller number.

NAME _____ DATE_____ PERIOD ____

12-9 Study Guide and Intervention

Sampling and Error

Bias A sample of size n is random (or **unbiased**) when every possible sample of size n has an equal chance of being selected. If a sample is biased, then information obtained from it may not be reliable.

Example **To find out how people in the U.S. feel about mass transit, people at a commuter train station are asked their opinion. Does this situation represent a random sample?**

No; the sample includes only people who actually use a mass-transit facility. The sample does not include people who ride bikes, drive cars, or walk.

Exercises

Determine whether each situation would produce a random sample. Write *yes* or *no* and explain your answer.

1. asking people in Phoenix, Arizona, about rainfall to determine the average rainfall for the United States **No; it rains less in Phoenix than most places in the U.S.**

2. obtaining the names of tree types in North America by surveying all of the U.S. National Forests **Yes; there are National Forests in about every state in the U.S.**

3. surveying every tenth person who enters the mall to find out about music preferences in that part of the country **Yes; mall customers should be fairly representative in terms of music tastes.**

4. interviewing country club members to determine the average number of televisions per household in the community **No; country club members would tend to be more affluent and thus not a representative sample of the community.**

5. surveying all students whose ID numbers end in 4 about their grades and career counseling needs **Yes; ID numbers are probably assigned alphabetically or by some other method not connected to students' grades or counseling needs.**

6. surveying parents at a day care facility about their preferences for brands of baby food for a marketing campaign **Yes; choice of a daycare facility would probably not influence baby food preferences.**

7. asking people in a library about the number of magazines to which they subscribe in order to describe the reading habits of a town **No; library visitors tend to read more than most citizens.**

NAME _____ DATE_____ PERIOD ____

12-9 Study Guide and Intervention *(continued)*

Sampling and Error

Margin of Error The **margin of sampling error** gives a limit on the difference between how a sample responds and how the total population would respond.

Margin of Error	If the percent of people in a sample responding in a certain way is p and the size of the sample is n, then 95% of the time, the percent of the population responding in that same way will be between $p - ME$ and $p + ME$, where $ME = 2\sqrt{\frac{p(1-p)}{n}}$.

Example 1 **In a survey of 4500 randomly selected voters, 62% favored candidate A. What is the margin of error?**

$ME = 2\sqrt{\frac{p(1-p)}{n}}$ Formula for margin of sampling error

$= 2\sqrt{\frac{0.62 \cdot (1 - 0.62)}{4500}}$ p = 62% or 0.62, n = 4500

≈ 0.01447 Use a calculator.

The margin of error is about 1%. This means that there is a 95% chance that the percent of voters favoring candidate A is between 62 − 1 or 61% and 62 + 1 or 63%.

Example 2 **The CD that 32% of teenagers surveyed plan to buy next is the latest from the popular new group BFA. If the margin of error of the survey is 2%, how many teenagers were surveyed?**

$ME = 2\sqrt{\frac{p(1-p)}{n}}$ Formula for margin of sampling error

$0.02 = 2\sqrt{\frac{0.32 \cdot (1 - 0.32)}{n}}$ ME = 0.02, p = 0.32

$0.01 = \sqrt{\frac{0.32(0.68)}{n}}$ Divide each side by 2.

$0.0001 = \frac{0.32(0.68)}{n}$ Square each side.

$n = \frac{0.32(0.68)}{0.0001}$ Multiply by n and divide by 0.0001

$n = 2176$

2176 teenagers were surveyed.

Exercises

Find the margin of sampling error to the nearest percent.

1. $p = 45\%, n = 350$ **about 5%**
2. $p = 12\%, n = 1500$ **about 2%**
3. $p = 86\%, n = 600$ **about 3%**

4. A study of 50,000 drivers in Indiana, Illinois, and Ohio showed that 68% preferred a speed limit of 75 mph over 65 mph on highways and country roads. What was the margin of sampling error to the nearest tenth of a percent? **about 0.4%**

Answers

NAME _____ DATE_____ PERIOD ____

12-9 Skills Practice

Sampling and Error

Determine whether each situation would produce a random sample. Write *yes* or *no* and explain your answer.

1. calling households at 3:30 P.M. on Tuesday to determine a political candidate's support
   No; since most registered voters are likely to be at work at this time, this sample would not be representative of all registered voters.

2. polling customers as they exit a sporting goods store about their attitudes about exercise
   No; these customers are likely to value exercise more than those who do not shop at sporting goods stores, who are not represented in this survey.

3. recording the number of sit-ups performed by 15-year old girls in the high schools of a large school district to determine the fitness of all high-school girls in the district
   No; 15-year old girls may not have the same abilities as 18-year old seniors, for example.

4. selecting two of a city's 20 apartment buildings for a survey to determine the desire of apartment dwellers in the city to own a home **No; the residents of the two buildings selected might, for example, have nicer apartments or be in a nicer area of town, and thus would not well represent the desires of people in other buildings.**

5. In a large school district, the superintendent of schools interviews two teachers at random from each school to determine whether teachers in the district think students are assigned too much or too little homework. **Yes; since a cross section of teachers from all levels was selected at random, the sample should well represent the population of teachers in the district.**

6. For seven consecutive days, one hour each in the morning, afternoon, and evening, every tenth customer who enters a mall is asked to choose her or his favorite store. **Yes; because the sample is chosen over the course of a whole week, during hours when different consumer groups shop, and because the selection is systematic, the sample should well represent the general population that shops at the mall stores.**

Find the margin of sampling error to the nearest percent.

7. $p = 85\%, n = 100$ **about 7%**
8. $p = 78\%, n = 100$ **about 8%**
9. $p = 15\%, n = 100$ **about 7%**
10. $p = 37\%, n = 500$ **about 4%**
11. $p = 12\%, n = 500$ **about 3%**
12. $p = 93\%, n = 500$ **about 2%**
13. $p = 23\%, n = 1000$ **about 3%**
14. $p = 56\%, n = 1000$ **about 3%**

15. **HEALTH** In a recent poll of cigarette smokers, 67% of those surveyed said they had tried to quit smoking within the last year. The margin of error was 3%. About how many people were surveyed? **about 983**

NAME _____ DATE_____ PERIOD ____

12-9 Practice

Sampling and Error

Determine whether each situation would produce a random sample. Write *yes* or *no* and explain your answer.

1. calling every twentieth registered voter to determine whether people own or rent their homes in your community **No; registered voters may be more likely to be homeowners, causing the survey to underrepresent renters.**

2. predicting local election results by polling people in every twentieth residence in all the different neighborhoods of your community **Yes; since all neighborhoods are represented proportionally, the views of the community should as a whole should be well represented.**

3. to find out why not many students are using the library, a school's librarian gives a questionnaire to every tenth student entering the library **No; she is polling only the students who are coming to the library, and will obtain no input from those who aren't using the library.**

4. testing overall performance of tires on interstate highways only **No; for overall performance, tires should be tested on many kinds of surfaces, and under many types of conditions.**

5. selecting every 50th hamburger from a fast-food restaurant chain and determining its fat content to assess the fat content of hamburgers served in fast-food restaurant chains throughout the country **No; the selected hamburgers are a random sample of the hamburgers served in one chain, and may represent the fat content for that chain, but will not necessarily represent the fat content of hamburgers served in other fast-food restaurant chains.**

6. assigning all shift workers in a manufacturing plant a unique identification number, and then placing the numbers in a hat and drawing 30 at random to determine the annual average salary of the workers **Yes; because the numbers are randomly chosen from among all shift workers, all workers have the same chance of being selected.**

Find the margin of sampling error to the nearest percent.

7. $p = 26\%, n = 100$ **about 9%**
8. $p = 55\%, n = 100$ **about 10%**
9. $p = 75\%, n = 500$ **about 4%**
10. $p = 14\%, n = 500$ **about 3%**
11. $p = 96\%, n = 1000$ **about 1%**
12. $p = 21\%, n = 1000$ **about 3%**
13. $p = 34\%, n = 1000$ **about 3%**
14. $p = 49\%, n = 1500$ **about 3%**
15. $p = 65\%, n = 1500$ **about 2%**

16. **COMPUTING** According to a poll of 500 teenagers, 43% said that they use a personal computer at home. What is the margin of sampling error? **about 4%**

17. **TRUST** A survey of 605 people, ages 13–33, shows that 68% trust their parents more than their best friends to tell them the truth. What is the margin of sampling error? **about 4%**

18. **PRODUCTIVITY** A study by the University of Illinois in 1995 showed an increase in productivity by 10% of the employees who wore headsets and listened to music of their choice while they were working. The margin of sampling error for the study was about 7%. How many employees participated in the study? **about 73**

NAME _____ DATE_____ PERIOD ____

12-9 Word Problem Practice

Sampling and Error

1. **COMICS** Isaac would like to know if people prefer reading comic books or novels. He decides to wait outside of a bookstore and ask people exiting whether they purchased comics or novels. Discuss whether this method of acquiring data would produce a biased or unbiased sample.
Sample answer: The results are likely to be biased because the customers of a bookstore may be more likely to purchase a novel over a comic book since bookstores do not generally offer a wide selection of comic books compared to novels.

2. **PARKING** A town wants to find out if people are happy with a proposal to tear down a section of a park and replace it with a parking lot. The town council decides to conduct a random survey of the town's citizens. They send a person to the location in the park where the proposed parking lot will be and have that person ask all passersby whether they would like to see a parking lot built at the location. Discuss whether or not this would produce a random sample.
It would not. Only people who use the park would be surveyed.

3. **PROMS** A poll asked 50 random seniors at a high school whether they would like to have the senior prom at a nearby hotel or at a local convention hall. Sixteen students responded that they would prefer the hotel. What is the margin of sampling error? Round your answer to the nearest percent.
13%

4. **AIRPORTS** In a large city, a random survey found that 18% of the city's population want a new runway built at the city airport. The survey had a margin of error of 5%. About how many people were surveyed?
about 236

INTERNET USE For Exercises 5-7, use the following information.

Two surveys were conducted to find out if people think that Americans are becoming more knowledgeable about the Internet. One survey polled 500 people and found that 395 of them felt that Americans are becoming more Internet savvy. A second survey concluded that 79% of those polled think that Americans are becoming more Internet savvy with a margin of error of 2%.

5. What was the margin of error for the first survey? Round your answer to the nearest percent.
4%

6. About how many people were polled in the second survey?
1659

7. Based on the results of the second survey, between what two percentages would you estimate is the true percentage of people who think that Americans are more Internet savvy, with 95% confidence?
77% and 81%

NAME _____ DATE_____ PERIOD ____

12-9 Enrichment

Sample Mean and Standard Error

The mean of a sample of size n is an estimate of the mean of the entire population under study. A different sample of size n probably will have a different mean, and a third sample yet another. For example, suppose the means for three different samples were 9, 5, and 11. What prediction would you make for the mean of a fourth sample? Clearly there is some uncertainty involved in the prediction. Therefore the sample mean must have a probability distribution associated with it. If the standard deviation of the population is known, σ (sigma), then the standard deviation of the sample mean, called the *standard error*, is given by: $SE = \frac{\sigma}{\sqrt{n}}$, where n is the sample size.

A 95% confidence interval for the population mean is given by: $(\bar{x} - 2SE, \bar{x} + 2SE) = \left(\bar{x} - 2\frac{\sigma}{\sqrt{n}}, \bar{x} + 2\frac{\sigma}{\sqrt{n}}\right)$. That is, there is a 95% chance that the *true* population mean will be contained in this interval.

Suppose a high school football team has 50 players. The coach wants to know what the average amount a player can bench press. The only exact way to calculate this average would be to measure, in the weight room, each player on the team. Instead the coach randomly samples 10 players and computes the average of these 10 to estimate the mean bench press weight for the entire team. Suppose the coach knew that $\sigma = 41.6$. (This data is unknown to the coach but presented here for the purpose of the exercises.)

185	185	200	205	215
235	240	200	205	180
175	190	195	215	205
250	275	250	230	250
205	260	190	210	230
180	175	165	165	195
290	300	305	315	210
315	210	230	205	200
150	155	145	200	215
160	190	195	185	180

Exercises

1. Calculate the standard error of the sample mean.
$SE = \frac{41.6}{\sqrt{10}} \approx 13.155$

2. Calculate a 95% confidence interval for the population mean.
Take any sample of 10, find the mean and add (and subtract) the standard error.

3. How can the coach make more accurate estimations?
Increase the sample size, which decreases the standard error.

4. Calculate the sample mean and standard error for a sample of size 20.
Answers will vary based upon the data chosen from the table.

Chapter 12 Assessment Answer Key

Quiz 1
(Lessons 12–1 through 12–3)

Page 73

1. D
2. 5040
3. combination; 175
4. $\frac{9}{20}$
5. $\frac{2}{7}$

Quiz 2 (Lessons 12–4 and 12–5)

Page 73

1. $\frac{4}{9}$
2. $\frac{16}{675}$
3. $\frac{8}{273}$
4. $\frac{1}{4}$
5. $\frac{11}{26}$

Quiz 3
(Lessons 12–6 and 12–7)

Page 74

1. 4.69
2. 2.17
3. positively skewed
4. 68%
5. 396

Quiz 4 (Lessons 12–8 and 12–9)

Page 74

1. $\frac{5}{72}$
2. $\frac{215}{216}$
3. $\frac{11}{243}$
4. No, the people surveyed are more likely to prefer basketball over other sports.
5. about 11%

Mid-Chapter Test

Page 75

1. C
2. G
3. D
4. J
5. A
6. G
7. $\frac{56}{969}$
8. $\frac{49}{99}$
9. $\frac{4}{5}$
10. 24
11. $\frac{16}{19}$
12. 1140

Chapter 12 Assessment Answer Key

Vocabulary Test
Page 76

1. false; combination
2. true
3. false; standard deviation
4. true
5. false; binomial experiment
6. true
7. true
8. false; compound events
9. false; standard deviation
10. true
11. Sample answer: Mutually exclusive events are events that cannot both happen at the same time.
12. The sample space is the set of all possible outcomes of an event.

Form 1
Page 77

1. A
2. J
3. C
4. G
5. D
6. F
7. B
8. H
9. A
10. J
11. D

Page 78

12. H
13. B
14. F
15. C
16. F
17. C
18. G
19. C
20. G

B: $4f^3n; \frac{5}{324}$

Answers

Chapter 12 Assessment Answer Key

Form 2A

Page 79

1. B
2. J
3. A
4. F
5. B
6. F
7. C
8. G
9. B
10. H
11. B

Page 80

12. H
13. B
14. F
15. B
16. F
17. D
18. H
19. C
20. F

B: Sample answer: {7, 10, 17, 24, 26, 28, 28}

Form 2B

Page 81

1. C
2. F
3. D
4. G
5. D
6. H
7. C
8. G
9. A
10. J
11. D

Page 82

12. F
13. C
14. J
15. B
16. F
17. A
18. H
19. C
20. F

B: Sample answer: {7, 9, 18, 20, 24, 40, 50}

Chapter 12 Assessment Answer Key

Form 2C

Page 83

1. 72
2. 120
3. 1320
4. 3003
5. $\frac{5}{28}$
6. $\frac{5}{12}$
7. $\frac{125}{216}$
8. $\frac{4}{663}$
9. $\frac{19}{91}$
10. $\frac{2}{5}$
11. $\frac{1}{16}$
12. $\frac{147}{512}$
13. 3360

Page 84

14. mode; it is the lowest
15. 106.0
16. 10.3°F
17. normally distributed
18. 47.5%
19. No, the opinions of one class may not be typical of all members of their age group.
20. about 9%

B: 74; 4

Chapter 12 Assessment Answer Key

Form 2D

Page 85

1. 105
2. 24
3. 60
4. 792
5. $\frac{1}{56}$
6. $\frac{4}{7}$
7. $\frac{1}{216}$
8. $\frac{1}{221}$
9. $\frac{8}{99}$
10. $\frac{9}{20}$
11. $\frac{9}{256}$
12. $\frac{54}{125}$

Page 86

13. 630
14. Sample answer: median; it is closer to most of the values.
15. 0.02
16. 0.15 in.
17. positively skewed
18. 47.5%
19. No, library card holders may not have opinions that are typical of the community.
20. about 13%

B: 77; 3

Chapter 12 Assessment Answer Key

Form 3

Page 87

1. 7776

2. 80,640

3. 70; combination; order does not matter.

4. 162,162

5. $\frac{4}{17}$

6. $\frac{1}{2}$

7. $\frac{1}{216}$

8. $\frac{169}{41,650}$

9. $\frac{13}{34}$

10. $\frac{55}{221}$

11. 34,560

Page 88

12. Sample answer: mean; the Maryland taxes are above the median, but they are below the mean.

13. 126,875.81

14. $356.20

15. positively skewed

16. 4890

17. $\frac{203}{23,328}$

18. 0.05

19. No, drivers may not have the same opinion as nondrivers in the town.

20. about 348 people

B: about 1275 students

Chapter 12 Assessment Answer Key

Page 89, Extended-Response Test
Scoring Rubric

Score	General Description	Specific Criteria
4	**Superior** A correct solution that is supported by well-developed, accurate explanations	• Shows thorough understanding of the concepts of *solving problems involving finding probability, independent* and *dependent events, permutations, combinations, mutually exclusive and inclusive events, statistical measures,* and *the normal distribution* • Uses appropriate strategies to solve problems. • Computations are correct. • Written explanations are exemplary. • Goes beyond requirements of some or all problems.
3	**Satisfactory** A generally correct solution, but may contain minor flaws in reasoning or computation	• Shows an understanding of the concepts of *solving problems involving finding probability, independent and dependent events, permutations, combinations, mutually exclusive and inclusive events, statistical measures,* and *the normal distribution* • Uses appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are effective. • Satisfies all requirements of problems.
2	**Nearly Satisfactory** A partially correct interpretation and/or solution to the problem	• Shows an understanding of most of the concepts of *solving problems involving finding probability, independent* and *dependent events, permutations, combinations, mutually exclusive and inclusive events, statistical measures,* and *the normal distribution* • May not use appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are satisfactory. • Satisfies the requirements of most of the problems.
1	**Nearly Unsatisfactory** A correct solution with no supporting evidence or explanation	• Final computation is correct. • No written explanations or work is shown to substantiate the final computation. • Satisfies minimal requirements of some of the problems.
0	**Unsatisfactory** An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given	• Shows little or no understanding of most of the concepts of *solving problems involving finding probability, independent* and *dependent events, permutations, combinations, mutually exclusive and inclusive events, statistical measures,* and *the normal distribution* • Does not use appropriate strategies to solve problems. • Computations are incorrect. • Written explanations are unsatisfactory. • Does not satisfy the requirements of problems. • No answer may be given.

Chapter 12 Assessment Answer Key

Page 89, Extended-Response Test
Sample Answers

In addition to the scoring rubric found on page A40, the following sample answers may be used as guidance in evaluating open-ended assessment items.

1a. Student responses must indicate that Alma's solution is correct. Explanations should indicate that, since A and B represent two independent events and they are looking for the probability that both events occurred, the two probabilities should be multiplied. Addition would be required if they were looking for the probability of either one of the events to occur.

1b. Sample answer for Steven's solution $P(A) + P(B) - P(A \text{ and } B) = \frac{2}{6} + \frac{3}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$: A die is rolled. Find the probability that a number greater than 4 or an even number is rolled.

2a. The student response should indicate that for grades listed, left to right, from lowest to highest, a negatively skewed distribution would include a greater number of high scores than low scores. Thus, the student should be happy!

2b. Students should explain that the mean, median, and mode of a normal distribution are the same, so the mean can be presumed to be $\frac{56 + 98}{2} = 77$, or very close to 77. The fact that there are three standard deviations between 77 and 98 (or between 56 and 77) means that the standard deviation is $\frac{98 - 77}{3} = 7$ $\left(\text{or } \frac{77 - 56}{3} = 7\right)$. Thus, scores in the range 77 ± 7, or between 70 and 84, would earn a grade of C.

3a. Sample answer: For 6 dinner guests, there would be 8 players including Greg and Jacqui, meaning that there would be 70 different ways to arrange the guests in two teams; students should indicate that this is a problem involving combinations, rather than permutations since changing the order in which players are selected for each team would not result in the formation of different teams.

3b. Students should state that this new condition would, in fact, change the number of arrangements. Taking Greg and Jacqui out of the situation for the moment, the question, for the sample answer in part **a**, would become: In how many ways can you divide a group of 6 people into two groups of 3 people each? The number of ways to do so would be $C(6, 3) = 20$. Then, since there are two ways to place Greg with one group and Jacqui with the other, there are only $20 \cdot 2 = 40$ possible arrangements if Greg and Jacqui cannot be on the same team.

Chapter 12 Assessment Answer Key

Standardized Test Practice

Page 90

1. Ⓐ Ⓑ Ⓒ Ⓓ

2. Ⓕ Ⓖ Ⓗ Ⓙ

3. Ⓐ Ⓑ Ⓒ Ⓓ

4. Ⓕ Ⓖ Ⓗ Ⓙ

5. Ⓐ Ⓑ Ⓒ Ⓓ

6. Ⓕ Ⓖ Ⓗ Ⓙ

7. Ⓐ Ⓑ Ⓒ Ⓓ

8. Ⓕ Ⓖ Ⓗ Ⓙ

9. Ⓐ Ⓑ Ⓒ Ⓓ

10. Ⓕ Ⓖ Ⓗ Ⓙ

Page 91

11. Ⓐ Ⓑ Ⓒ Ⓓ

12. Ⓕ Ⓖ Ⓗ Ⓙ

13. Ⓐ Ⓑ Ⓒ Ⓓ

14. Ⓕ Ⓖ Ⓗ Ⓙ

15. Ⓐ Ⓑ Ⓒ Ⓓ

16. Ⓕ Ⓖ Ⓗ Ⓙ

17. Ⓐ Ⓑ Ⓒ Ⓓ

18.

		3	1	.	5		
0	0	0	0		0	0	0
1	1	1	1		1	1	1
2	2	2	2		2	2	2
3	3	3	3		3	3	3
4	4	4	4		4	4	4
5	5	5	5		5	5	5
6	6	6	6		6	6	6
7	7	7	7		7	7	7
8	8	8	8		8	8	8
9	9	9	9		9	9	9

19.

		7	0	.			
0	0	0	0		0	0	0
1	1	1	1		1	1	1
2	2	2	2		2	2	2
3	3	3	3		3	3	3
4	4	4	4		4	4	4
5	5	5	5		5	5	5
6	6	6	6		6	6	6
7	7	7	7		7	7	7
8	8	8	8		8	8	8
9	9	9	9		9	9	9

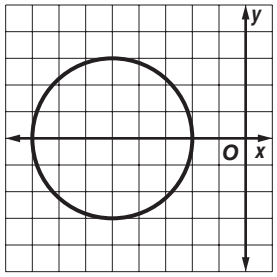
Chapter 12 Assessment Answer Key

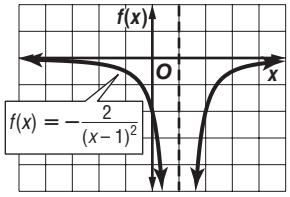
Standardized Practice Test
Page 92

20. $R'(-3, -1)$, $S'(4, 2)$, $T'(3, -4)$

21. $-3x^2 + 5x + 12$

22. no

23. 

24. 

25. $w = -1$

26. $1000^{-1/3} = \frac{1}{10}$

27. 7.3 yr

28. 2, −1, −4

29. $9, 3, 1, \frac{1}{3}$

30a. $\frac{1}{25}$

b. $\frac{3}{10}$

c. $\frac{9}{25}$

Answers

Chapter 12 Assessment Answer Key

Unit 4 Test

Page 93

1. 22, 28, 34, 40
2. 19, 17, 15
3. 7
4. 3200, 2560
5. 3072, 2304, 1728, 1296
6. 63
7. 96
8. 108
9. $\frac{245}{333}$
10. 5, 16, 49, 148, 445
11. 1, −3, −11
12. $810xy^4$
13. See students' answers.
14. Sample answer: $n = 2$
15. 240
16. 5040

Page 94

17. 66
18. 11,880
19. $\frac{1}{6}$
20. $\frac{1}{3}$
21. $\frac{7}{13}$
22. 374.88; 356; no mode; 243.67
23. positively skewed
24. 0.5%
25. $\frac{45}{512}$
26. Sample answer: No; those surveyed are more likely to listen to a station that airs the type of music being performed at the concert.