12-8 Study Guide and Intervention

Binomial Experiments

Binomial Expansions For situations with only 2 possible outcomes, you can use the Binomial Theorem to find probabilities. The coefficients of terms in a binomial expansion can be found by using combinations.

Example **What is the probability that 3 coins show heads and 3 show tails when 6 coins are tossed?**

There are 2 possible outcomes that are equally likely: heads (H) and tails (T). The tosses of 6 coins are independent events. When $(H + T)^6$ is expanded, the term containing H^3T^3, which represents 3 heads and 3 tails, is used to get the desired probability. By the Binomial Theorem the coefficient of H^3T^3 is $C(6, 3)$.

$$P(3 \text{ heads, 3 tails}) = \frac{6!}{3!3!}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^3 \quad P(\text{H}) = \frac{1}{2} \text{ and } P(\text{T}) = \frac{1}{2}$$

$$= \frac{20}{64}$$

$$= \frac{5}{16}$$

The probability of getting 3 heads and 3 tails is $\frac{5}{16}$ or 0.3125.

Exercises

Find each probability if a coin is tossed 8 times.

1. P(exactly 5 heads)

2. P(exactly 2 heads)

3. P(even number of heads)

4. P(at least 6 heads)

Mike guesses on all 10 questions of a true-false test. If the answers true and false are evenly distributed, find each probability.

5. Mike gets exactly 8 correct answers.

6. Mike gets at most 3 correct answers.

7. A die is tossed 4 times. What is the probability of tossing exactly two sixes?

Lesson 12-8

12-8 Study Guide and Intervention *(continued)*
Binomial Experiments

Binomial Experiments

Binomial Experiments	A binomial experiment is possible if and only if all of these conditions occur. • There are exactly two outcomes for each trial. • There is a fixed number of trials. • The trials are independent. • The probabilities for each trial are the same.

Example **Suppose a coin is weighted so that the probability of getting heads in any one toss is 90%. What is the probability of getting exactly 7 heads in 8 tosses?**

The probability of getting heads is $\frac{9}{10}$, and the probability of getting tails is $\frac{1}{10}$. There are $C(8, 7)$ ways to choose the 7 heads.

$$P(7 \text{ heads}) = C(8, 7)\left(\frac{9}{10}\right)^7\left(\frac{1}{10}\right)^1$$

$$= 8 \cdot \frac{9^7}{10^8}$$

$$\approx 0.38$$

The probability of getting 7 heads in 8 tosses is about 38%.

Exercises

1. **BASKETBALL** For any one foul shot, Derek has a probability of 0.72 of getting the shot in the basket. As part of a practice drill, he shoots 8 shots from the foul line.

   a. What is the probability that he gets in exactly 6 foul shots?

   b. What is the probability that he gets in at least 6 foul shots?

2. **SCHOOL** A teacher is trying to decide whether to have 4 or 5 choices per question on her multiple choice test. She wants to prevent students who just guess from scoring well on the test.

   a. On a 5-question multiple-choice test with 4 choices per question, what is the probability that a student can score at least 60% by guessing?

   b. What is the probability that a student can score at least 60% by guessing on a test of the same length with 5 choices per question?

3. Julie rolls two dice and adds the two numbers.

   a. What is the probability that the sum will be divisible by 3?

   b. If she rolls the dice 5 times what is the chance that she will get exactly 3 sums that are divisible by 3?

4. **SKATING** During practice a skater falls 15% of the time when practicing a triple axel. During one practice session he attempts 20 triple axels.

   a. What is the probability that he will fall only once?

   b. What is the probability that he will fall 4 times?