12-2 Study Guide and Intervention

Permutations and Combinations

Permutations When a group of objects or people are arranged in a certain order, the arrangement is called a **permutation**.

Permutations	The number of permutations of n distinct objects taken r at a time is given by $P(n, r) = \frac{n!}{(n-r)!}$.
Permutations with Repetitions	The number of permutations of n objects of which p are alike and q are alike is $\frac{n!}{p!q!}$.

The rule for permutations with repetitions can be extended to any number of objects that are repeated.

Example **From a list of 20 books, each student must choose 4 books for book reports. The first report is a traditional book report, the second a poster, the third a newspaper interview with one of the characters, and the fourth a timeline of the plot. How many different orderings of books can be chosen?**

Since each book report has a different format, order is important. You must find the number of permutations of 20 objects taken 4 at a time.

$P(n, r) = \frac{n!}{(n-r)!}$ Permutation formula

$P(20, 4) = \frac{20!}{(20-4)!}$ $n = 20$, $r = 4$

$= \frac{20!}{16!}$ Simplify.

$= \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot \cancel{16}^{1} \cdot \cancel{15}^{1} \cdot \ldots \cdot \cancel{1}^{1}}{\cancel{16}_{1} \cdot \cancel{15}_{1} \cdot \ldots \cdot \cancel{1}_{1}}$ Divide by common factors.

$= 116{,}280$

Books for the book reports can be chosen 116,280 ways.

Exercises

Evaluate each expression.

1. $P(6, 3)$ **2.** $P(8, 5)$ **3.** $P(9, 4)$ **4.** $P(11, 6)$

How many different ways can the letters of each word be arranged?

5. MOM **6.** MONDAY **7.** STEREO

8. SCHOOL The high school chorus has been practicing 12 songs, but there is time for only 5 of them at the spring concert. How may different orderings of 5 songs are possible?

Lesson 12-2

12-2 Study Guide and Intervention *(continued)*

Permutations and Combinations

Combinations An arrangement or selection of objects in which order is *not* important is called a combination.

Combinations	The number of combinations of n distinct objects taken r at a time is given by $C(n, r) = \frac{n!}{(n - r)!r!}$.

Example 1 **SCHOOL How many groups of 4 students can be selected from a class of 20?**

Since the order of choosing the students is not important, you must find the number of combinations of 20 students taken 4 at a time.

$C(n, r) = \frac{n!}{(n - r)!r!}$ Combination formula

$C(20, 4) = \frac{20!}{(20 - 4)!4!}$ $n = 20$, $r = 4$

$= \frac{20!}{16!4!}$ or 4845

There are 4845 possible ways to choose 4 students.

Example 2 **In how many ways can you choose 1 vowel and 2 consonants from a set of 26 letter tiles? (Assume there are 5 vowels and 21 consonants.)**

By the Fundamental Counting Principle, you can multiply the number of ways to select one vowel and the number of ways to select 2 consonants. Only the letters chosen matter, not the order in which they were chosen, so use combinations.

$C(5, 1)$ One of 5 vowels are drawn.

$C(21, 2)$ Two of 21 consonants are drawn.

$C(5, 1) \cdot C(21, 2) = \frac{5!}{(5 - 1)!1!} \cdot \frac{21!}{(21 - 2)!2!}$ Combination formula

$= \frac{5!}{4!} \cdot \frac{21!}{19!2!}$ Subtract.

$= 5 \cdot 210$ or 1050 Simplify.

There are 1050 combinations of 1 vowel and 2 consonants.

Exercises

Evaluate each expression.

1. $C(5, 3)$ **2.** $C(7, 4)$ **3.** $C(15, 7)$ **4.** $C(10, 5)$

5. PLAYING CARDS From a standard deck of 52 cards, in how many ways can 5 cards be drawn?

6. HOCKEY How many hockey teams of 6 players can be formed from 14 players without regard to position played?

7. COMMITTEES From a group of 10 men and 12 women, how many committees of 5 men and 6 women can be formed?