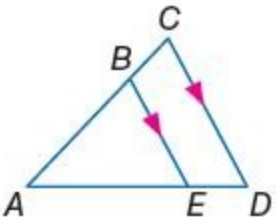
12. If $AC = 14$, $BC = 8$, and $AD = 21$, find ED.



SOLUTION:

Triangle Proportionality Theorem:
If a line is parallel to one side of a triangle and intersects the other two sides, then it divides the sides into segments of proportional lengths.

Here, $BC = 8$. So, $AB = 14 - 8 = 6$. Let x be the length of the segment AE. So, $ED = 21 - x$.

Use the Triangle Proportionality Theorem.

$$\frac{AB}{BC} = \frac{AE}{ED}$$

Substitute.

$$\frac{6}{8} = \frac{x}{21-x}$$

Solve for x.

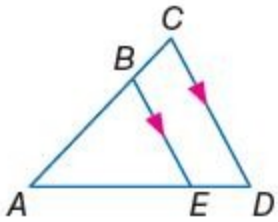
$$6(21-x) = 8x$$
$$126 - 6x = 8x$$
$$-14x = -126$$
$$x = 9$$

So, $AE = 9$ and $ED = 21 - 9 = 12$.

ANSWER:

12

13. If $AD = 27$, $AB = 8$, and $AE = 12$, find BC.



SOLUTION:

Triangle Proportionality Theorem:
If a line is parallel to one side of a triangle and intersects the other two sides, then it divides the sides into segments of proportional lengths.

Here, $AE = 12$. So, $ED = 27 - 12 = 15$.

Use the Triangle Proportionality Theorem.

$$\frac{AB}{BC} = \frac{AE}{ED}$$

Substitute in values and solve for BC.

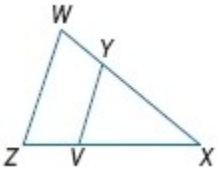
$$\frac{8}{BC} = \frac{12}{15}$$

$$BC = \frac{120}{12}$$
$$BC = 10$$

ANSWER:

10

Determine whether $\overline{VY} \parallel \overline{ZW}$. Justify your answer.



14. $ZX = 18$, $ZV = 6$, $WX = 24$, and $YX = 16$

SOLUTION:

$ZV = 6$ and $YX = 16$. Therefore, $VX = 18 - 6 = 12$ and $WY = 24 - 16 = 8$.

Use the Converse of the Triangle Proportionality Theorem.

$$\frac{ZV}{VX} = \frac{6}{12} = \frac{1}{2}$$
$$\frac{WY}{YX} = \frac{8}{16} = \frac{1}{2}$$

Since $\frac{ZV}{VX} = \frac{WY}{YX} = \frac{1}{2}$, then $\overline{VY} \parallel \overline{ZW}$.

ANSWER:

yes; $\frac{ZV}{VX} = \frac{WY}{YX} = \frac{1}{2}$

15. $VX = 7.5$, $ZX = 24$, $WY = 27.5$, and $WX = 40$

SOLUTION:

$VX = 7.5$ and $WY = 27.5$. So, $ZV = 24 - 7.5 = 16.5$ and $YX = 40 - 27.5 = 12.5$.

Use the Converse of the Triangle Proportionality Theorem.

$$\frac{ZV}{VX} = \frac{16.5}{7.5} = \frac{11}{5}$$
$$\frac{WY}{YX} = \frac{27.5}{12.5} = \frac{11}{5}$$

Since $\frac{ZV}{VX} = \frac{WY}{YX} = \frac{11}{5}$, so $\overline{VY} \parallel \overline{ZW}$.

ANSWER:

yes; $\frac{ZV}{VX} = \frac{WY}{YX} = \frac{11}{5}$

16. $ZV = 8$, $VX = 2$, and $YX = \frac{1}{2}WY$

SOLUTION:

Use the Converse of the Triangle Proportionality Theorem.

$$\frac{ZV}{VX} = \frac{8}{2} = \frac{4}{1}$$

$$\frac{WY}{YX} = \frac{WY}{\frac{WY}{2}} = WY \cdot \frac{2}{WY} = \frac{2}{1}$$

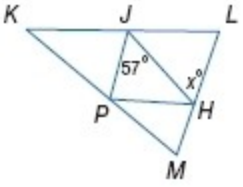
Because $\frac{ZV}{VX} \neq \frac{WY}{YX}$, $\overline{VY}$ and $\overline{ZW}$ are not parallel.

ANSWER:

no; $\frac{ZV}{VX} \neq \frac{WY}{YX}$

$\overline{JH}$, $\overline{JP}$, and $\overline{PH}$ are midsegments of $\triangle KLM$. Find the value of x.

18.



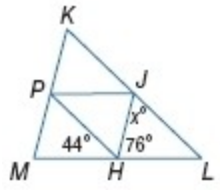
SOLUTION:

By the Triangle Midsegment Theorem, $\overline{JP} \parallel \overline{LM}$. By the Alternate Interior Angles Theorem, $x = 57$.

ANSWER:

57

7-4 Parallel Lines and Proportional Parts

19. 

SOLUTION:

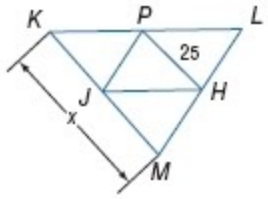
By the Triangle Midsegment Theorem, $\overline{PH} \parallel \overline{KL}$.

$$\begin{aligned} m\angle PHJ &= 180 - (44 + 76) \\ &= 180 - 120 \\ &= 60 \end{aligned}$$

By the Alternate Interior Angles Theorem, $x = m\angle PHJ = 60$.

ANSWER:

60

20. 

SOLUTION:

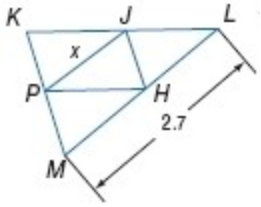
By the Triangle Midsegment Theorem, $PH = \frac{1}{2}KM$.

Substitute.

$$\begin{aligned} 25 &= \frac{1}{2}(KM) \\ KM &= 50 \end{aligned}$$

ANSWER:

50

21. 

SOLUTION:

By the Triangle Midsegment Theorem, $PJ = \frac{1}{2}ML$.

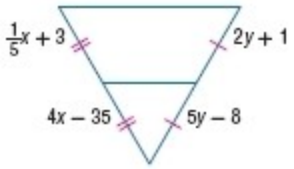
Substitute.

$$\begin{aligned} x &= \frac{1}{2}(2.7) \\ &= 1.35 \end{aligned}$$

ANSWER:

1.35

ALGEBRA Find x and y.

26. 

SOLUTION:

It is given that $5y - 8 = 2y + 1$ and $\frac{1}{5}x + 3 = 4x - 35$.

Solve for x.

$$\begin{aligned} \frac{1}{5}x + 3 &= 4x - 35 \\ \frac{1}{5}x + 38 &= 4x \\ 5\left(\frac{1}{5}x + 38\right) &= 5(4x) \\ x + 190 &= 20x \\ -19x &= -190 \\ x &= 10 \end{aligned}$$

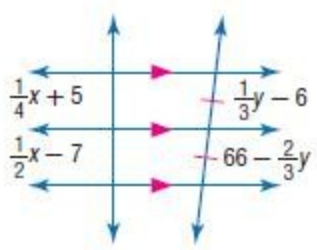
Solve for y.

$$\begin{aligned} 5y - 8 &= 2y + 1 \\ 3y &= 9 \\ y &= 3 \end{aligned}$$

ANSWER:

$x = 10; y = 3$

7-4 Parallel Lines and Proportional Parts

27. 

SOLUTION:

We are given that $\frac{1}{3}y-6=66-\frac{2}{3}y$.

Solve for y.

$$\frac{1}{3}y-6=66-\frac{2}{3}y$$
$$\frac{1}{3}y+\frac{2}{3}y=66+6$$
$$\frac{3}{3}y=72$$
$$y=72$$

By Corollary 7.2, $\frac{1}{2}x-7=\frac{1}{4}x+5$.

Solve for x.

$$\frac{1}{2}x-7=\frac{1}{4}x+5$$
$$4\left(\frac{1}{2}x-7\right)=4\left(\frac{1}{4}x+5\right)$$
$$2x-28=x+20$$
$$2x-x=20+28$$
$$x=48$$

ANSWER:

$x = 48; y = 72$