11-5 Areas of Similar Figures

For each pair of similar figures, find the area of the green figure.

6. 

SOLUTION:

The scale factor between the blue triangle and the green triangle is $\frac{10}{18}$ or $\frac{5}{9}$, so the ratio of their areas is $\left(\frac{5}{9}\right)^2$.

$$\frac{\text{Area of the blue triangle}}{\text{Area of the green triangle}} = \left(\frac{5}{9}\right)^2$$

$$\frac{25}{\text{Area of the green triangle}} = \frac{25}{81}$$

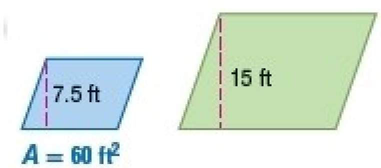
$$\text{Area of the green triangle} = \frac{25 \times 81}{25}$$

$$\text{Area of the green triangle} = 81$$

The area of the green triangle is 81 mm^2.

ANSWER:

81 mm^2

7. 

SOLUTION:

The scale factor between the blue parallelogram and the green parallelogram is $\frac{7.5}{15}$ or $\frac{1}{2}$, so the ratio of their areas is $\left(\frac{1}{2}\right)^2$.

$$\frac{\text{Area of the blue parallelogram}}{\text{Area of the green parallelogram}} = \left(\frac{1}{2}\right)^2$$

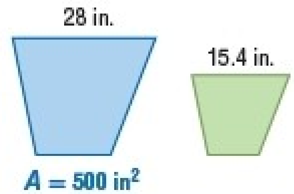
$$\frac{60}{\text{Area of the green parallelogram}} = \frac{1}{4}$$

$$\text{Area of the green parallelogram} = 240$$

The area of the green parallelogram is 240 ft^2.

ANSWER:

240 ft^2

8. 

SOLUTION:

The scale factor between the blue trapezoid and the green trapezoid is $\frac{28}{15.4}$, so the ratio of their areas is $\left(\frac{28}{15.4}\right)^2$.

$$\frac{\text{Area of the blue trapezoid}}{\text{Area of the green trapezoid}} = \left(\frac{28}{15.4}\right)^2$$

$$\frac{500}{\text{Area of the green trapezoid}} = \frac{784}{237.16}$$

$$\text{Area of the green trapezoid} = 151.25$$

The area of green triangle is 151.25 in^2.

ANSWER:

151.25 in^2

9. 

SOLUTION:

The scale factor between the blue pentagon and the green pentagon is $\frac{35}{28}$ or $\frac{5}{4}$, so the ratio of their areas is $\left(\frac{5}{4}\right)^2$.

$$\frac{\text{Area of the blue pentagon}}{\text{Area of the green pentagon}} = \left(\frac{5}{4}\right)^2$$

$$\frac{1050}{\text{Area of the green pentagon}} = \frac{25}{16}$$

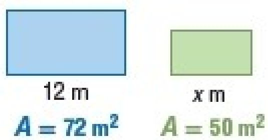
$$\text{Area of the green pentagon} = 672$$

The area of green pentagon is 672 cm^2.

ANSWER:

672 cm^2

ANALYZE RELATIONSHIPS **For each pair of similar figures, use the given areas to find the scale factor of the blue to the green figure. Then find *x*.**

10.


SOLUTION:

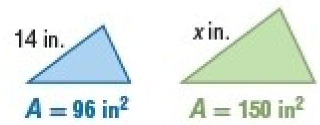
The scale factor between the blue figure and the green figure is $\frac{12}{x}$, so the ratio of their areas is $\left(\frac{12}{x}\right)^2$.

$$\frac{\text{Area (blue)}}{\text{Area (green)}} = \left(\frac{12}{x}\right)^2$$
$$\frac{72}{50} = \frac{144}{x^2}$$
$$72x^2 = 144 \cdot 50$$
$$x^2 = \frac{144 \cdot 50}{72}$$
$$x^2 = 100$$
$$x = 10 \text{ m}$$

The scale factor is $\frac{12}{10}$ or $\frac{6}{5}$.

ANSWER:

$\frac{6}{5}$;10

11.


SOLUTION:

The scale factor between the blue triangle and the green triangle is $\frac{14}{x}$, so the ratio of their areas is $\left(\frac{14}{x}\right)^2$.

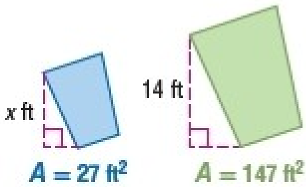
$$\frac{\text{Area (blue)}}{\text{Area (green)}} = \left(\frac{14}{x}\right)^2$$
$$\frac{96}{150} = \frac{196}{x^2}$$
$$96x^2 = 196 \cdot 150$$
$$x^2 = \frac{196 \cdot 150}{96}$$
$$x^2 = 306.25$$
$$x = 17.5 \text{ in}$$

The scale factor is $\frac{14}{17.5}$ or $\frac{4}{5}$.

ANSWER:

$\frac{4}{5}$;17.5

11-5 Areas of Similar Figures

12. 

SOLUTION:

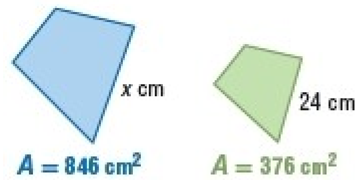
The scale factor between the blue figure and the green figure is $\frac{x}{14}$, so the ratio of their areas is $\left(\frac{x}{14}\right)^2$.

$$\frac{\text{Area (blue)}}{\text{Area (green)}} = \left(\frac{x}{14}\right)^2$$
$$\frac{27}{147} = \frac{x^2}{196}$$
$$147x^2 = 196 \cdot 27$$
$$x^2 = \frac{196 \cdot 27}{147}$$
$$x^2 = 36$$
$$x = 6 \text{ ft}$$

The scale factor is $\frac{6}{14}$ or $\frac{3}{7}$.

ANSWER:

$\frac{3}{7}$;6

13. 

SOLUTION:

The scale factor between the blue figure and the green figure is $\frac{x}{24}$, so the ratio of their areas is $\left(\frac{x}{24}\right)^2$.

$$\frac{\text{Area (blue)}}{\text{Area (green)}} = \left(\frac{x}{24}\right)^2$$
$$\frac{846}{376} = \frac{x^2}{576}$$
$$376x^2 = 576 \cdot 846$$
$$x^2 = \frac{576 \cdot 846}{376}$$
$$x = 36 \text{ m}$$

The scale factor is $\frac{36}{24}$ or $\frac{3}{2}$.

ANSWER:

$\frac{3}{2}$;36

15. **CHANGING DIMENSIONS** A circle has a radius of 24 inches.

a. If the area is doubled, how does the radius change?

b. How does the radius change if the area is tripled?

c. What is the change in the radius if the area is increased by a factor of *x*?

SOLUTION:

a. Let *A* be the area. The new area is 2*A*. From theorem 11.1:

$$\frac{A_{new}}{A} = \left(\frac{r_{new}}{r}\right)^2$$
$$\frac{2A}{A} = \left(\frac{r_{new}}{r}\right)^2$$
$$\frac{r_{new}}{r} = \sqrt{2}$$
$$r_{new} = \sqrt{2}r$$
$$r_{new} = \sqrt{2}(24)$$
$$r_{new} = 33.9$$

If the area is doubled, the radius changes from 24 in. to 33.9 in.

b. Let A be the area. The new area is 3*A*. From theorem 11.1:

$$\frac{A_{new}}{A} = \left(\frac{r_{new}}{r}\right)^2$$
$$\frac{3A}{A} = \left(\frac{r_{new}}{r}\right)^2$$
$$\frac{r_{new}}{r} = \sqrt{3}$$
$$r_{new} = \sqrt{3}r$$
$$r_{new} = \sqrt{3}(24)$$
$$r_{new} = 41.6$$

If the area is tripled, the radius changes from 24 in. to 41.6 in.

c. Let A be the area. The new area is *xA*. From theorem 11.1:

$$\frac{A_{new}}{A} = \left(\frac{r_{new}}{r}\right)^2$$
$$\frac{xA}{A} = \left(\frac{r_{new}}{r}\right)^2$$
$$\frac{r_{new}}{r} = \sqrt{x}$$
$$r_{new} = \sqrt{x}r$$
$$r_{new} = 24\sqrt{x}$$

If the area changes by a factor of *x*, then the radius changes from 24 in. to $24\sqrt{x}$ in.

ANSWER:

a. If the area is doubled, the radius changes from 24 in. to 33.9 in.
b. If the area is tripled, the radius changes from 24 in. to 41.6 in.
c. If the area changes by a factor of *x*, then the radius changes from 24 in. to $24\sqrt{x}$ in.

16. **CHANGING DIMENSIONS** A polygon has an area of 144 square meters.
a. If the area is doubled, how does each side length change?
b. How does each side length change if the area is tripled?
c. What is the change in each side length if the area is increased by a factor of *x*?

SOLUTION:

a. Let A be the original area. The the new area is 2*A*. From theorem 11.1:

$$\frac{A_{new}}{A} = \left(\frac{s_{new}}{s}\right)^2$$
$$\frac{2A}{A} = \left(\frac{s_{new}}{s}\right)^2$$
$$\frac{s_{new}}{s} = \sqrt{2}$$

If the area is doubled, each side length will increase by a factor of $\sqrt{2}$.

b. Let A be the original area. The the new area is 3*A*. From theorem 11.1:

$$\frac{A_{new}}{A} = \left(\frac{s_{new}}{s}\right)^2$$
$$\frac{3A}{A} = \left(\frac{s_{new}}{s}\right)^2$$
$$\frac{s_{new}}{s} = \sqrt{3}$$

If the area is tripled, each side length will increase by a factor of $\sqrt{3}$.

c. Let A be the original area. The the new area is *xA*. From theorem 11.1:

$$\frac{A_{\text{new}}}{A} = \left(\frac{s_{\text{new}}}{s}\right)^2$$
$$\frac{xA}{A} = \left(\frac{s_{\text{new}}}{s}\right)^2$$
$$\frac{s_{\text{new}}}{s} = \sqrt{x}$$

If the area changes by a factor of x, then each side length will change by a factor of $\sqrt{x}$.

ANSWER:

a. If the area is doubled, each side length will increase by a factor of $\sqrt{2}$.
b. If the area is tripled, each side length will increase by a factor of $\sqrt{3}$.
c. If the area changes by a factor of x, then each side length will change by a factor of $\sqrt{x}$.