1-7 Three-Dimensional Figures

ANALYZE RELATIONSHIPS Determine whether the solid is a polyhedron. Then identify the solid. If it is a polyhedron, name the bases, faces, edges, and vertices.

13.

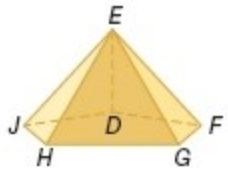
SOLUTION:

A solid with all flat surfaces that enclose a single region of space is called a polyhedron. This solid has a curved surface, so it is not a polyhedron. The given figure is a solid with a circular base connected by a curved surface to a single vertex. So it is a cone.

ANSWER:

not a polyhedron; cone

17.



SOLUTION:

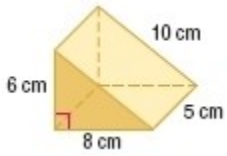
The solid is formed by polygonal faces, so it is a polyhedron. The given pyramid has a pentagonal base, so it is a pentagonal pyramid.
Faces: Each flat surface is called face.
Edges: The line segments where the faces intersect are called edges.
Vertex: The point where three or more edges intersect is called a vertex.
Base: *JHGFD*
Faces: $JHGFD, \Delta JEH, \Delta HEG, \Delta GEF, \Delta FED, \Delta EDJ$
Edges: $\overline{HG}, \overline{GF}, \overline{FD}, \overline{DJ}, \overline{JH}, \overline{EJ}, \overline{EH}, \overline{EG}, \overline{EF}, \overline{ED}$
Vertices: J, H, G, F, D, E

ANSWER:

a polyhedron; pentagonal pyramid; base: *JHGFD*; faces: *JHGFD*, $\Delta JEH, \Delta HEG, \Delta GEF, \Delta FED, \Delta EDJ$; edges: $\overline{HG}, \overline{GF}, \overline{FD}, \overline{DJ}, \overline{JH}, \overline{EJ}, \overline{EH}, \overline{EG}, \overline{EF}, \overline{ED}$ vertices: *J, H, G, F, D, E*

Find the surface area and volume of each solid to the nearest tenth.

21.



SOLUTION:

The formulas for finding the volume and surface area of a prism are $V = Bh$ and $S = Ph + 2B$, where *S* = total surface area, *V* = volume, *h* = height, *B* = area of the base, and *P* = perimeter of the base. Since the base of the prism is a triangle, the perimeter *P* of the base is $8 + 6 + 10$ or 24 centimeters. The area of the base *B* is $\frac{1}{2}(8 \times 6)$ or 24 square centimeters. The height of the prism is 5 centimeters.

$S = Ph + 2B$	Surface Area formula.
$= (24 \cdot 5) + 2(24)$	Substitution.
$= 120 + 48$	Multiply.
$= 168$	Addition.

The surface area of the triangular prism is 168 square centimeters.

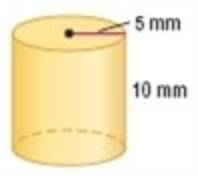
$V = Bh$	Volume Formula
$= 24 \cdot 5$	Substitution.
$= 120$	Multiply.

The volume of the prism is 120 cubic centimeters.

ANSWER:

168 cm^2 ; 120 cm^3

1-7 Three-Dimensional Figures

23. 

SOLUTION:

The formulas for finding the volume and surface area of a cylinder are $V = \pi r^2 h$ and $S = 2\pi rh + 2\pi r^2$, where S = total surface area, V = volume, r = radius, and h = height.

Here, $r = 5$ mm and $h = 10$ mm.

$S = 2\pi rh + 2\pi r^2$	Surface Area formula
$= 2\pi(5)(10) + 2\pi(5)^2$	Substitution.
$= 100\pi + 50\pi$	Simplify.
$= 150\pi$	Addition.
≈ 471.2	Use a calculator.

The surface area of the cylinder is 150π or about 471.2 mm^2.

$V = \pi r^2 h$	Volume Formula
$= \pi(5)^2(10)$	Substitution.
$= 250\pi$	Simplify.
≈ 785.4	Use a calculator.

The volume of the cylinder is 250π or about 785.4 mm^3.

ANSWER:

150π or about 471.2 mm^2 ; 250π or about 785.4 mm^3

28. **ALGEBRA** The volume of a cube is 729 cubic centimeters. Find the length of each edge.

SOLUTION:

The formula for finding the volume of the prism is $V = Bh$.

The base of the cube is a square, so the area of the base is a^2. The length of height is equal to the length of the side, since all the sides are congruent in a cube.

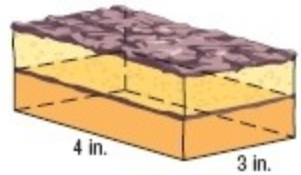
$V = a^3$	Surface Area formula
$729 = a^3$	Substitution.
$\sqrt[3]{729} = \sqrt[3]{a^3}$	Square root
$9 = a$	Simplify.

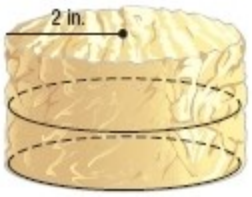
The length of each edge is 9 cm.

ANSWER:

9 cm

31. **CAKES** Cakes come in many shapes and sizes. Often they are stacked in two or more layers, like those in the diagrams shown below.





a. If each layer of the rectangular prism cake is 3 inches high, calculate the area of the cake that will be frosted assuming there is no frosting between layers.

b. Calculate the area of the cylindrical cake that will be frosted, if each layer is 4 inches in height.

c. If one can of frosting will cover 50 square inches of cake, how many cans of frosting will be needed for each cake?

d. If the height of each layer of cake is 5 inches, what does the radius of the cylindrical cake need to be, so the same amount of frosting is used for both cakes? Explain your reasoning.

SOLUTION:

a. The formula for finding the surface area of a prism is $S = Ph + 2B$, where S = total surface area, h = height, B = area of the base, and P = perimeter of the base

Since the base of the prism is a rectangle, the perimeter P of the base is $2(3) + 2(4)$ or 14 inches. The area of the base B is 4×3 or 12 square inches. Each cake is 3 inches high. So, the height is 6 inches.

The top is not going to be frosted. So, the area to be frosted is given by $S = Ph + B$.

Substitute.

$S = Ph + B$ Surface Area formula
$= (14)(6) + 12$ Substitution.
$= 84 + 12$ Multiply.
$= 96$ Addition.

The area of the cake to be frosted is 96 in^2.

b. The formula for finding the surface area of a cylinder is $S = 2\pi rh + 2\pi r^2$, where S = total surface area, r = radius, and h = height.

Here, $r = 2$. The height of each cylindrical cake is 4 in. So, the total height is 8 in.
Since the top is not going to be frosted, the area to be frosted is given by $S = 2\pi rh + \pi r^2$.

$S = 2\pi rh + \pi r^2$ Surface Area formula
$= 2\pi(2)(8) + \pi(2)^2$ Substitution.
$= 32\pi + 4\pi$ Simplify.
$= 36\pi$ Addition.
≈ 113.1 Use a calcualtor.

The area of the cylindrical cake to be frosted is about 113.1 in^2.

c. Divide the area to be frosted by 50.

$$\frac{96}{50} = 1.92$$

So, 2 cans of frosting are needed for the rectangular prism cake.

$$\frac{113.1}{50} = 2.262$$

So, 3 cans of frosting are needed for the cylindrical cake.

d. Find the surface area of the rectangular cake if the height of the each layer 5 in.

$S = Ph + B$ Surface Area Formula
$= (14)(10) + 12$ Substitution.
$= 140 + 12$ Multiply.
$= 152$ Addition.

The surface area of the rectangular cake is 152 in^2.

To find the radius of a cylindrical cake with the same height, solve the equation $152 = \pi r^2 + 20\pi r$. Solving the equation using the quadratic formula gives $r = -22.18$ and $r = 2.18$.

Since the radius can never be negative, $r = 2.18$.

The same amount of frosting will be needed if the radius of the cake is 2.18 in.

ANSWER:

a. 96 in^2
b. 113.1 in^2
c. prism: 2 cans; cylinder: 3 cans
d. 2.18 in.; if the height is 10 in., then the surface area of the rectangular cake is 152 in^2. To find the radius of a cylindrical cake with the same height, solve the equation $152 = \pi r^2 + 20\pi r$. The solutions are $r = -22.18$ or $r = 2.18$. Using a radius of 2.18 in. gives surface area of about 152 in^2.