Glencoe Mathematics

Geometry

Chapter 11 Resource Masters

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Consumable Workbooks Many of the worksheets contained in the Chapter Resource Masters are available as consumable workbooks in both English and Spanish.

	ISBN10	ISBN13
Study Guide and Intervention Workbook	0-07-877344-X	978-0-07-877344-0
Skills Practice Workbook	0-07-877346-6	978-0-07-877346-4
Practice Workbook	0-07-877347-4	978-0-07-877347-1
Word Problem Practice Workbook	0-07-877349-0	978-0-07-877349-5
Spanish Versions		
Study Guide and Intervention Workbook	0-07-877345-8	978-0-07-877345-7
Practice Workbook	0-07-877348-2	978-0-07-877348-8

Answers for Workbooks The answers for Chapter 11 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

StudentWorks Plus™ This CD-ROM includes the entire Student Edition test along with the English workbooks listed above.

TeacherWorks Plus™ All of the materials found in this booklet are included for viewing, printing, and editing in this CD-ROM.

Spanish Assessment Masters (ISBN10: 0-07-877350-4, ISBN13: 978-0-07-877350-1) These masters contain a Spanish version of Chapter 11 Test Form 2A and Form 2C.

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CONTENTS

Teacher's Guide to Using the *Chapter 11 Resource Masters*

The *Chapter 11 Resource Masters* includes the core materials needed for Chapter 11. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing on the *TeacherWorks Plus*™ CD-ROM.

Chapter Resources

Student-Built Glossary (pages 1–2) These masters are a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar. Give this to students before beginning Lesson 11–1. Encourage them to add these pages to their mathematics study notebooks. Remind them to complete the appropriate words as they study each lesson.

Anticipation Guide (pages 7–8) This master, presented in both English and Spanish, is a survey used before beginning the chapter to pinpoint what students may or may not know about the concepts in the chapter. Students will revisit this survey after they complete the chapter to see if their perceptions have changed.

Lesson Resources

Lesson Reading Guide Get Ready for the Lesson extends the discussion from the beginning of the Student Edition lesson. Read the Lesson asks students to interpret the context of and relationships among terms in the lesson. Finally, Remember What You Learned asks students to summarize what they have learned using various representation techniques. Use as a study tool for note taking or as an informal reading assignment. It is also a helpful tool for ELL (English Language Learners).

Study Guide and Intervention These masters provide vocabulary, key concepts, additional worked-out examples and Check Your Progress exercises to use as a reteaching activity. It can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice This master focuses more on the computational nature of the lesson. Use as an additional practice option or as homework for second-day teaching of the lesson.

Practice This master closely follows the types of problems found in the Exercises section of the Student Edition and includes word problems. Use as an additional practice option or as homework for second-day teaching of the lesson.

Word Problem Practice This master includes additional practice in solving word problems that apply the concepts of the lesson. Use as an additional practice or as homework for second-day teaching of the lesson.

Enrichment These activities may extend the concepts of the lesson, offer a historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. They are written for use with all levels of students.

Graphing Calculator, Scientific Calculator, or Spreadsheet Activities These activities present ways in which technology can be used with the concepts in some lessons of this chapter. Use as an alternative approach to some concepts or as an integral part of your lesson presentation.

Assessment Options

The assessment masters in the *Chapter 11 Resource Masters* offer a wide range of assessment tools for formative (monitoring) assessment and summative (final) assessment.

Student Recording Sheet This master corresponds with the standardized test practice at the end of the chapter.

Pre-AP Rubric This master provides information for teachers and students on how to assess performance on open-ended questions.

Quizzes Four free-response quizzes offer assessment at appropriate intervals in the chapter.

Mid-Chapter Test This 1-page test provides an option to assess the first half of the chapter. It parallels the timing of the Mid-Chapter Quiz in the Student Edition and includes both multiple-choice and free-response questions.

Vocabulary Test This test is suitable for all students. It includes a list of vocabulary words and 10 questions to assess students' knowledge of those words. This can also be used in conjunction with one of the leveled chapter tests.

Leveled Chapter Tests

- *Form 1* contains multiple-choice questions and is intended for use with below grade level students.
- *Forms 2A and 2B* contain multiple-choice questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- *Forms 2C and 2D* contain free-response questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- *Form 3* is a free-response test for use with above grade level students.

All of the above mentioned tests include a free-response Bonus question.

Extended-Response Test Performance assessment tasks are suitable for all students. Sample answers and a scoring rubric are included for evaluation.

Standardized Test Practice These three pages are cumulative in nature. It includes three parts: multiple-choice questions with bubble-in answer format, griddable questions with answer grids, and short-answer free-response questions.

Answers

- The answers for the Anticipation Guide and Lesson Resources are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment masters.

11 Student-Built Glossary

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 11. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Geometry Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
apothem		
composite figure		
geometric probability		
height of a parallelogram		

(continued on the next page)

11 Student-Built Glossary *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
sector		
segment		

11 Anticipation Guide

Areas of Polygons and Circles

Step 1 ***Before you begin Chapter 11***

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

STEP 1 A, D, or NS	Statement	STEP 2 A or D
	1. The area of a parallelogram whose sides measure 5 cm and 9 cm is 5 cm $\times$ 9 cm or 45 cm^2.	
	2. The area of a triangle is one-half its base times its height.	
	3. Given the coordinates of the vertices of any quadrilateral *ABCD*, its area can be found by multiplying the length of side *AB* by the length of side *BC*.	
	4. The area of a rhombus equals half the product of the lengths of its diagonals.	
	5. A segment drawn from the center to a vertex of a regular polygon is called an apothem.	
	6. The formula for the area of a circle is $A = \pi r^2$.	
	7. The area of an irregular figure can be found by separating the figure into shapes with known area formulas.	
	8. If an irregular figure is in the shape of a pentagon, then the formula for the area of a regular pentagon can be used to find its area.	
	9. A sector of a circle with a central angle of 35° will have an area of $\frac{35}{360}\pi r^2$.	
	10. A segment of a circle is the triangular region of a circle bound by a chord and two radii.	

Step 2 ***After you complete Chapter 11***

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

NOMBRE ______________________________ FECHA ____________ PERÍODO ___

11 Ejercicios preparatorios

Áreas de polígonos y círculos

PASO 1 ***Antes de comenzar el Capítulo 11***

- Lee cada enunciado.
- Decide si estás de acuerdo (A) o en desacuerdo (D) con el enunciado.
- Escribe A o D en la primera columna O si no estás seguro(a) de la respuesta, escribe NS (No estoy seguro(a).

PASO 1 A, D o NS	Enunciado	PASO 2 A o D
	1. El área de un paralelogramo con lados 5 cm y 9 cm es 5 cm $\times$ 9 cm ó 45 cm^2.	
	2. El área de un triángulo es la mitad de su base por la altura.	
	3. Dadas las coordenadas de los vértices de cualquier paralelogramo *ABCD*, su área se calcula multiplicando la longitud del lado *AB* por la longitud del lado *BC*.	
	4. El área de un rombo es igual a la mitad del producto de las longitudes de sus diagonales.	
	5. Un segmento que se dibuja del centro al vértice de un polígono regular se llama apotema.	
	6. La fórmula para el área de un círculo es $A = \pi r^2$.	
	7. El área de una figura irregular se puede calcular separando la figura en figuras con fórmulas de área conocidas.	
	8. Si una figura irregular tiene forma pentagonal, entonces se puede usar la fórmula del área de un pentágono regular para calcular su área.	
	9. El sector de un círculo con ángulo central de 35° tendrá un área de $\frac{35}{360}\pi r^2$.	
	10. Un segmento de círculo es la región triangular de un círculo.	

PASO 2 ***Después de completar el Capítulo 11***

- Vuelve a leer cada enunciado y completa la última columna con una A o una D.
- ¿Cambió cualquiera de tus opiniones sobre los enunciados de la primera columna?
- En una hoja de papel aparte, escribe un ejemplo de por qué estás en desacuerdo con los enunciados que marcaste con una D.

11-1 Lesson Reading Guide

Areas of Parallelograms

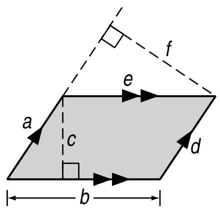
Get Ready for the Lesson

Read the introduction to Lesson 11-1 in your textbook.

How many 22-yard squares could fit in an acre?

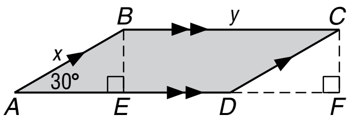
Read the Lesson

1. Which expression gives the area of the parallelogram? (Hint: There can be more than one correct response.)



A. *ab*	**B.** *cb*	**C.** *ed*
D. *af*	**E.** *ce*	**F.** *cd*
G. *df*	**H.** *bf*	**I.** *cf*

2. Refer to the figure. Determine whether each statement is *true* or *false*. If the statement is false, explain why.



a. $\overline{AB}$ is an altitude of the parallelogram.

b. $\overline{CD}$ is a base of parallelogram *ABCD*.

c. The perimeter of *ABCD* is $(2x + 2y)$ units^2.

d. $BE = CF$

e. $BE = \frac{\sqrt{3}}{2}x$

f. The area of *ABCD* is $2xy$ units^2.

Remember What You Learned

3. A good way to remember a new formula in geometry is to relate it to a formula you already know. How can you use the formula for the area of a rectangle to help you remember the formula for the area of a parallelogram?

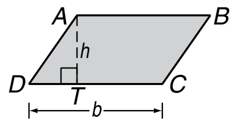
NAME ____ DATE ____________ PERIOD _____

11-1 Study Guide and Intervention

Areas of Parallelograms

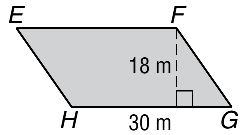
Areas of Parallelograms A parallelogram is a quadrilateral with both pairs of opposite sides parallel. Any side of a parallelogram can be called a **base**. Each base has a corresponding **altitude**, and the length of the altitude is the **height** of the parallelogram. The area of a parallelogram is the product of the base and the height.

Area of a Parallelogram	If a parallelogram has an area of A square units, a base of b units, and a height of h units, then $A = bh$.



The area of parallelogram *ABCD* is $CD \cdot AT$.

Example **Find the area of parallelogram *EFGH*.**



$A = bh$ Area of a parallelogram

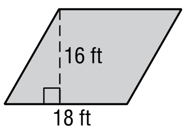
$= 30(18)$ $b = 30$, $h = 18$

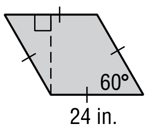
$= 540$ Multiply.

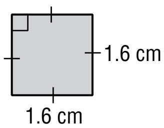
The area is 540 square meters.

Exercises

Find the area of each parallelogram.

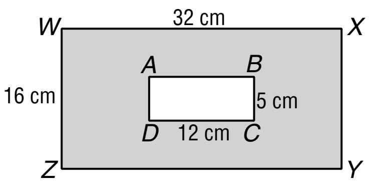
1. 

2. 

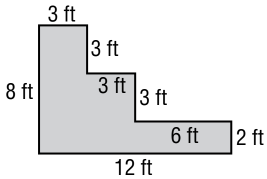
3. 

Find the area of each shaded region.

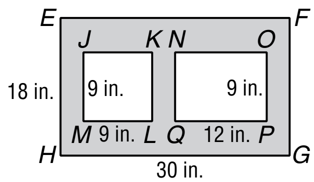
4. *WXYZ* and *ABCD* are rectangles.



5. All angles are right angles.



6. *EFGH* and *NOPQ* are rectangles; *JKLM* is a square.



7. The area of a parallelogram is 3.36 square feet. The base is 2.8 feet. If the measures of the base and height are each doubled, find the area of the resulting parallelogram.

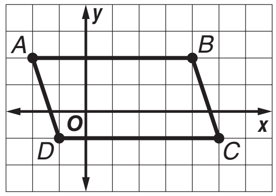
8. A rectangle is 4 meters longer than it is wide. The area of the rectangle is 252 square meters. Find the length.

11-1 Study Guide and Intervention *(continued)*

Areas of Parallelograms

Parallelograms on the Coordinate Plane To find the area of a quadrilateral on the coordinate plane, use the Slope Formula, the Distance Formula, and properties of parallelograms, rectangles, squares, and rhombi.

Example **The vertices of a quadrilateral are $A(-2, 2)$, $B(4, 2)$, $C(5, -1)$, and $D(-1, -1)$.**



a. Determine whether the quadrilateral is a *square*, a *rectangle*, or a *parallelogram*.

Graph the quadrilateral. Then determine the slope of each side.

slope of $\overline{AB} = \frac{2 - 2}{4 - (-2)}$ or 0

slope of $\overline{CD} = \frac{-1 - (-1)}{-1 - 5}$ or 0

slope $\overline{AD} = \frac{2 - (-1)}{-2 - (-1)}$ or -3

slope $\overline{BC} = \frac{-1 - 2}{5 - 4}$ or -3

Opposite sides have the same slope. The slopes of consecutive sides are not negative reciprocals of each other, so consecutive sides are not perpendicular. *ABCD* is a parallelogram; it is not a rectangle or a square.

b. Find the area of *ABCD*.

From the graph, the height of the parallelogram is 3 units and $AB = |4 - (-2)| = 6$.

$A = bh$ Area of a parallelogram

$= 6(3)$ $b = 6$, $h = 3$

$= 18 \text{ units}^2$ Multiply.

Exercises

Given the coordinates of the vertices of a quadrilateral, determine whether the quadrilateral is a *square*, a *rectangle*, or a *parallelogram*. Then find the area.

1. $A(-1, 2)$, $B(3, 2)$, $C(3, -2)$, and $D(-1, -2)$

2. $R(-1, 2)$, $S(5, 0)$, $T(4, -3)$, and $U(-2, -1)$

3. $C(-2, 3)$, $D(3, 3)$, $E(5, 0)$, and $F(0, 0)$

4. $A(-2, -2)$, $B(0, 2)$, $C(4, 0)$, and $D(2, -4)$

5. $M(2, 3)$, $N(4, -1)$, $P(-2, -1)$, and $R(-4, 3)$

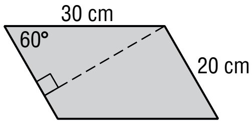
6. $D(2, 1)$, $E(2, -4)$, $F(-1, -4)$, and $G(-1, 1)$

11-1 Skills Practice

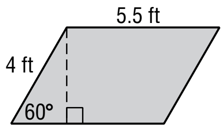
Areas of Parallelograms

Find the perimeter and area of each parallelogram. Round to the nearest tenth if necessary.

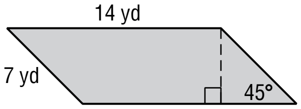
1.



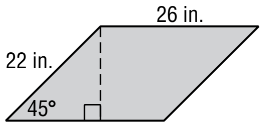
2.



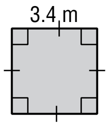
3.



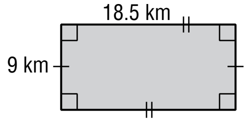
4.



5.

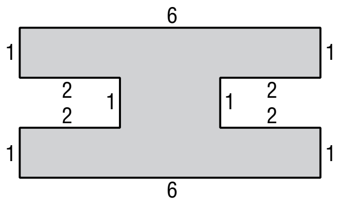


6.

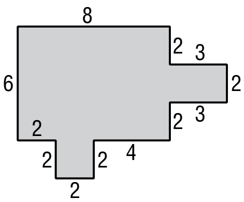


Find the area of each figure.

7.



8.



COORDINATE GEOMETRY Given the coordinates of the vertices of a quadrilateral, determine whether it is a *square*, a *rectangle*, or a *parallelogram*. Then find the area of the quadrilateral.

9. $A(-4, 2)$, $B(-1, 2)$, $C(-1, -1)$, $D(-4, -1)$

10. $P(-3, 3)$, $Q(1, 3)$, $R(1, -3)$, $S(-3, -3)$

11. $D(-5, 1)$, $E(7, 1)$, $F(4, -4)$, $G(-8, -4)$

12. $R(2, 3)$, $S(4, 10)$, $T(12, 10)$, $U(10, 3)$

NAME ____ DATE ____________ PERIOD _____

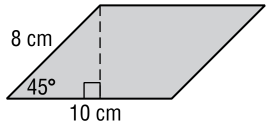
11-1 Practice

Areas of Parallelograms

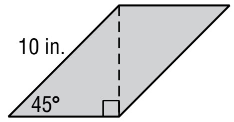
Find the perimeter and area of each parallelogram. Round to the nearest tenth if necessary.

1.

2.



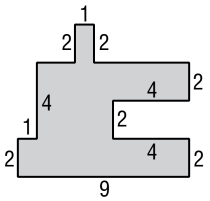
3.



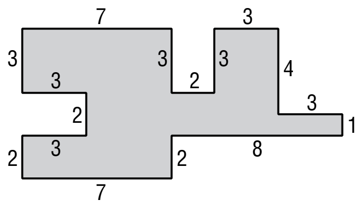
Lesson 11-1

Find the area of each figure.

4.



5.



COORDINATE GEOMETRY Given the coordinates of the vertices of a quadrilateral, determine whether it is a *square*, a *rectangle*, or a *parallelogram*. Then find the area of the quadrilateral.

6. $C(-4, -1), D(-4, 2), F(1, 2), G(1, -1)$

7. $W(2, 2), X(1, -2), Y(-2, -2), Z(-1, 2)$

8. $M(0, 4), N(4, 6), O(6, 2), P(2, 0)$

9. $P(-5, 2), Q(4, 2), R(5, 5), S(-4, 5)$

FRAMING For Exercises 10–12, use the following information.

A rectangular poster measures 42 inches by 26 inches. A frame shop fitted the poster with a half-inch mat border.

10. Find the area of the poster.

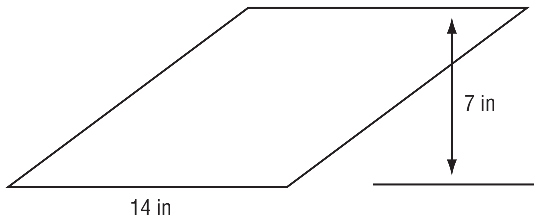
11. Find the area of the mat border.

12. Suppose the wall is marked where the poster will hang. The marked area includes an additional 12-inch space around the poster and frame. Find the total wall area that has been marked for the poster.

11-1 Word Problem Practice

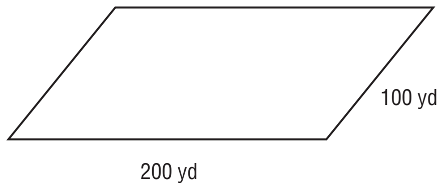
Areas of Parallelograms

1. **PACKAGING** A box with a square opening is squashed into the rhombus shown below.



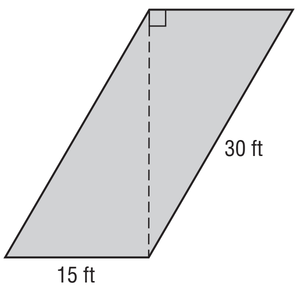
What is the area of the opening?

2. **RUNNING** Jason jogs once around a city block shaped like a parallelogram.

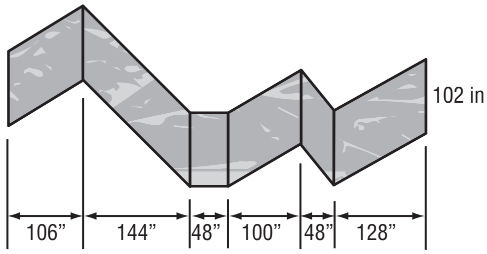


How far did Jason jog?

3. **SHADOWS** A rectangular billboard casts a shadow on the ground in the shape of a parallelogram. What is the area of the ground covered by the shadow? Round your answer to the nearest tenth.



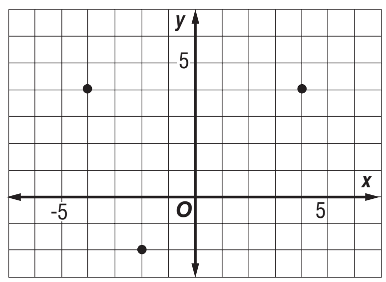
4. **PATHS** A concrete path shown below is made by joining several parallelograms.



What is the total area of the path?

HIGHWAY SUPPORTS For Exercises 5 and 6, use the following information.

Four columns are being placed at the vertices of a parallelogram to support a highway. Three of the columns are marked on the coordinate plane shown.



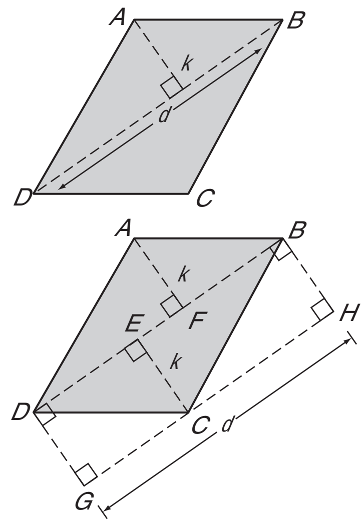
5. What are the coordinates of the three possible locations of the fourth column?

6. What is the area in square units of each of the three parallelograms that result from the possibilities you found in Exercise 5? Explain.

11-1 Enrichment

Area of a Parallelogram

You can prove some interesting results using the formula you have proved for the area of a parallelogram by drawing auxiliary lines to form congruent regions. Consider the top parallelogram shown at the right. In the figure, d is the length of the diagonal $\overline{BD}$, and k is the length of the perpendicular segment from A to $\overline{BD}$. Now consider the second figure, which shows the same parallelogram with a number of auxiliary perpendiculars added. Use what you know about perpendicular lines, parallel lines, and congruent triangles to answer the following.



1. What kind of figure is $DBHG$?

2. If you moved $\triangle AFB$ to the lower-left end of figure $DBHG$, would it fit perfectly on top of $\triangle DGC$? Explain your answer.

3. Which two triangular pieces of $\square ABCD$ are congruent to $\triangle CBH$?

4. The area of $\square ABCD$ is the same as that of figure $DBHG$, since the pieces of $\square ABCD$ can be rearranged to form $DBHG$. Express the area of $\square ABCD$ in terms of the measurements k and d.

Lesson 11-1

NAME ______________________________ DATE __________ PERIOD _____

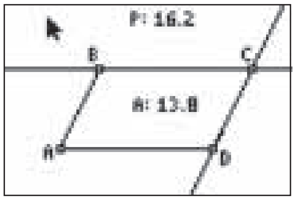
11-1 Graphing Calculator Activity

Cabri Junior: Areas of Parallelograms

Cabri Junior can be used to find the perimeters and areas of parallelograms.

Step 1 Draw a parallelogram.
- Select **F2 Segment** to draw a segment.
- Select **F5 Alph-num** to label the endpoints of the segment *A* and *B*.
- Draw segment *AD*.
- Select **F3 Parallel** to draw a line parallel to segment *AB* through *D*. Select point *D,* and then segment *AB*.
- Draw a line parallel to segment *AD* through *B*.
- Select **F2 Point, Intersection** to place a point at the intersection of the two lines drawn. Label the point *C*.
- Select **F2 Quad** and draw a quadrilateral by selecting points *A, B, C,* and *D*.

Step 2 Find the measure of the area of parallelogram *ABCD*.
- Select **F5 Measure, Area.**
- Place the cursor on any segment of parallelogram *ABCD*. Then press ENTER .
- The area appears with the hand attached. Move the number to an appropriate place.



Step 3 Find the measure of the perimeter of parallelogram *ABCD*.
- Select **F5 Measure, D. & Length.**
- Place the cursor on any segment of parallelogram *ABCD*. Then press ENTER .
- The area appears with the hand attached. Move the number to an appropriate place.

The perimeter of the parallelogram is 16.2 units and the area is 13.8 square units.

Exercises

Analyze your drawing.

1. Find the lengths of all four sides of the parallelogram.
2. Using the information from Exercise 1, what is the perimeter of the parallelogram? Does this measurement match that found by Cabri Jr.?
3. Construct the altitude of the parallelogram. What is the length of the altitude?
4. What is the measure of the base?
5. Using the information from Exercises 3 and 4, what is the area of the parallelogram? Does this measurement match the one found by Cabri Jr.?
6. Select one of the vertices and drag it to change the dimensions of the parallelogram. (Press CLEAR so the pointer becomes a black arrow. Move the pointer close to a vertex until the arrow becomes transparent and the vertex is blinking. Press ALPHA to change the arrow to a hand. Then move the vertex.) Do you see any patterns or relationships?

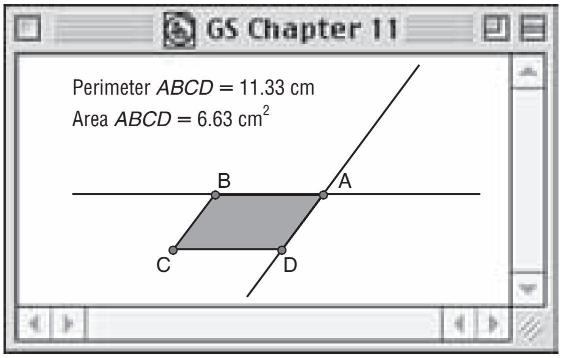
11-1 Geometer's Sketchpad Activity

Areas of Parallelograms

The Geometer's Sketchpad can be used to find the perimeters and areas of parallelograms.

Step 1 Use The Geometer's Sketchpad to draw a parallelogram.
- Construct a segment by selecting the Segment tool from the toolbar. First, click the first point. Then click on a second point to draw the segment.
- Next, use one of the endpoints of the original segment as the first point for the new segment and click on a second point to construct the new segment.
- Construct a parallel line to the original segment by first highlighting the original segment and the endpoint not on that segment. Then select **Parallel Line** from the **Construct** menu.
- Construct a parallel line to the second segment by highlighting the second segment and the point not on it. Then select **Parallel Line** from the **Construct** menu.
- Next, construct a point on the intersection of the two lines. Use the Point tool from the toolbar to select the point where the two lines intersect.
- Construct the interior of the parallelogram by highlighting all four points and selecting **Quadrilateral Interior** under the **Construct** menu.

Step 2 Use The Geometer's Sketchpad to find the perimeter of the parallelogram.
- Highlight the interior of the parallelogram using the Selection Arrow tool from the toolbar.
- Next, find the perimeter by selecting **Perimeter** under the **Measure** menu.



Step 3 Use The Geometer's Sketchpad to find the area of the parallelogram.
- Highlight the interior of the parallelogram using the Selection Arrow tool from the toolbar.
- Next, find the area by selecting **Area** under the **Measure** menu.

The perimeter of the parallelogram is 11.33 cm and the area is 6.63 cm^2.

Exercises

Analyze your drawing.

1. Find the lengths of all four sides of the parallelogram.

2. Using the information from Exercise 1, what is the perimeter of the parallelogram? Does this measurement match that found by The Geometer's Sketchpad?

3. Construct the altitude of the parallelogram. What is the length of the altitude?

4. What is the measure of the base?

5. Using the information from Exercises 3 and 4, what is the area of the parallelogram? Does this measurement match the one found by The Geometer's Sketchpad?

6. Select one of the vertices and drag it to change the dimensions of the parallelogram. Do you see any patterns or relationships?

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11-2 Lesson Reading Guide

Areas of Triangles, Trapezoids, and Rhombi

Get Ready for the Lesson

Read the introduction to Lesson 11-2 in your textbook.

Classify the polygons in the panels of the beach umbrella.

Read the Lesson

1. Match each area formula from the first column with the corresponding polygon in the second column.

a. $A = \ell w$	**i.** triangle
b. $A = \frac{1}{2}d_1d_2$	**ii.** parallelogram
c. $A = s^2$	**iii.** trapezoid
d. $A = \frac{1}{2}h(b_1 + b_2)$	**iv.** rhombus
e. $A = \frac{1}{2}bh$	**v.** square
f. $A = bh$	**vi.** rectangle

2. Determine whether each statement is *always*, *sometimes*, or *never* true. In each case, explain your reasoning.

a. The area of a square is half the product of its diagonals.

b. The area of a triangle is half the product of two of its sides.

c. You can find the area of a rectangle by multiplying base times height.

d. You can find the area of a rectangle by multiplying the lengths of any two of its sides.

e. The area of a trapezoid is the product of its height and the sum of the bases.

f. The square of the length of a side of a square is equal to half the product of its diagonals.

Remember What You Learned

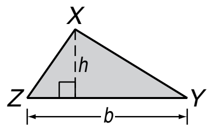
3. A good way to remember a new geometric formula is to state it in words. Write a short sentence that tells how to find the area of a trapezoid in a way that is easy to remember.

11-2 Study Guide and Intervention

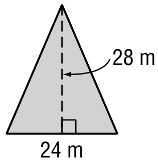
Areas of Triangles, Trapezoids, and Rhombi

Areas of Triangles The area of a triangle is half the area of a rectangle with the same base and height as the triangle.

If a triangle has an area of A square units, a base of b units, and a corresponding height of h units, then $A = \frac{1}{2}bh$.



Example **Find the area of the triangle.**



$A = \frac{1}{2}bh$ Area of a triangle

$= \frac{1}{2}(24)(28)$ $b = 24$, $h = 28$

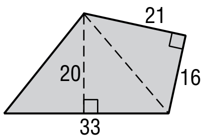
$= 336$ Multiply.

The area is 336 square meters.

Exercises

Find the area of each figure.

1.



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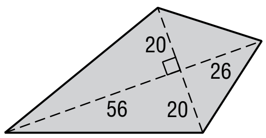

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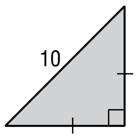

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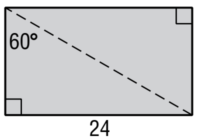

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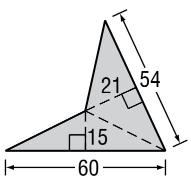

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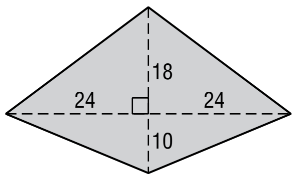

7. The area of a triangle is 72 square inches. If the height is 8 inches, find the length of the base.

8. A right triangle has a perimeter of 36 meters, a hypotenuse of 15 meters, and a leg of 9 meters. Find the area of the triangle.

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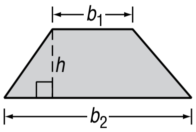
11-2 Study Guide and Intervention *(continued)*

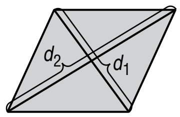
Areas of Triangles, Trapezoids, and Rhombi

Areas of Trapezoids and Rhombi The area of a trapezoid is the product of half the height and the sum of the lengths of the bases. The area of a rhombus is half the product of the diagonals.

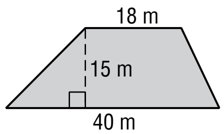
If a trapezoid has an area of A square units, bases of b_1 and b_2 units, and a height of h units, then
$A = \frac{1}{2}h(b_1 + b_2)$.



If a rhombus has an area of A square units and diagonals of d_1 and d_2 units, then
$A = \frac{1}{2}d_1d_2$.



Example **Find the area of the trapezoid.**



$A = \frac{1}{2}h(b_1 + b_2)$ Area of a trapezoid

$= \frac{1}{2}(15)(18 + 40)$ $h = 15$, $b_1 = 18$, $b_2 = 40$

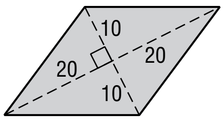
$= 435$ Simplify.

The area is 435 square meters.

Exercises

Find the area of each quadrilateral.

1.



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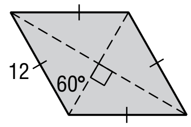
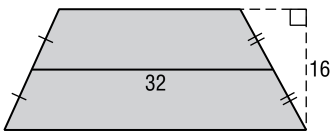


3.



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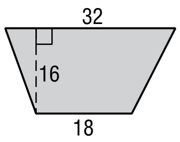

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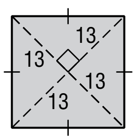
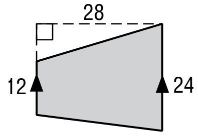


6.



7. The area of a trapezoid is 144 square inches. If the height is 12 inches, find the length of the median.

8. A rhombus has a perimeter of 80 meters and the length of one diagonal is 24 meters. Find the area of the rhombus.

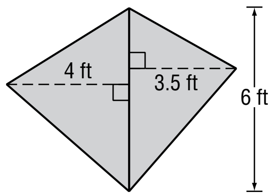
NAME ____ DATE ____________ PERIOD _____

11-2 Skills Practice

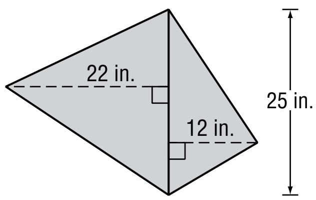
Areas of Triangles, Trapezoids, and Rhombi

Find the area of each figure. Round to the nearest tenth if necessary.

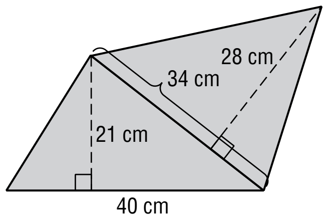
1.



2.



3.



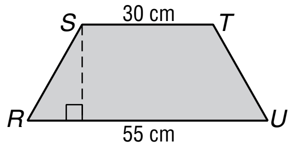
Find the area of each quadrilateral given the coordinates of the vertices.

4. trapezoid $WXYZ$
$W(-5, 3), X(3, 3), Y(6, -3), Z(-8, -3)$

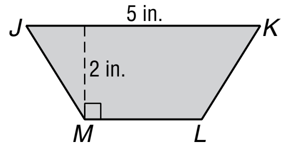
5. rhombus $HIJK$
$H(4, -3), I(2, -7), J(0, -3), K(2, 1)$

Find the missing measure for each figure.

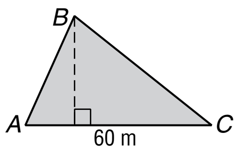
6. Trapezoid $RSTU$ has an area of 935 square centimeters. Find the height of $RSTU$.



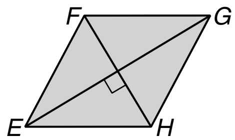
7. Trapezoid $JKLM$ has an area of 7.5 square inches. Find ML.



8. Triangle ABC has an area of 1050 square meters. Find the height of $\triangle ABC$.



9. Rhombus $EFGH$ has an area of 750 square feet. If EG is 50 feet, find FH.



Lesson 11-2

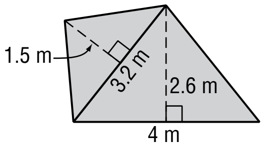
NAME ____ DATE ____________ PERIOD _____

11-2 Practice

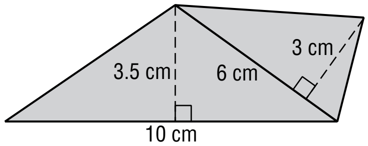
Areas of Triangles, Trapezoids, and Rhombi

Find the area of each figure. Round to the nearest tenth if necessary.

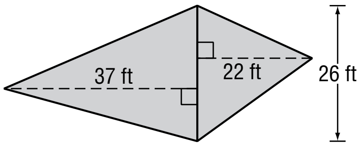
1.



2.



3.



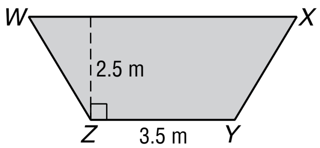
Find the area of each quadrilateral given the coordinates of the vertices.

4. trapezoid *ABCD*
$A(-7, 1)$, $B(-4, 4)$, $C(-4, -6)$, $D(-7, -3)$

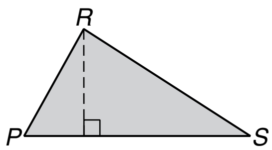
5. rhombus *LMNO*
$L(6, 8)$, $M(14, 4)$, $N(6, 0)$, $O(-2, 4)$

Find the missing measure for each figure.

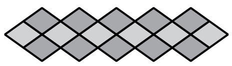
6. Trapezoid *WXYZ* has an area of 13.75 square meters. Find *WX*.



7. Triangle *PRS* has an area of 68 square yards. If the height of $\triangle PRS$ is 8 yards, find the base.



DESIGN For Exercises 8 and 9, use the following information.
Mr. Hagarty used 16 congruent rhombi-shaped tiles to design the midsection of the backsplash area above a kitchen sink. The length of the design is 27 inches and the total area is 108 square inches.



8. Find the area of one rhombus.

9. Find the length of each diagonal.

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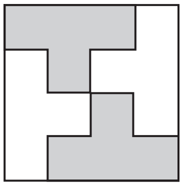
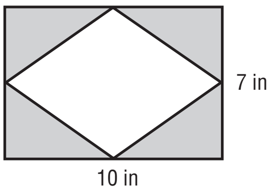
11-2 Word Problem Practice

Areas of Triangles, Trapezoids, and Rhombi

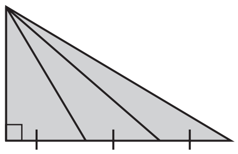
1. INTERIOR DESIGN The 10 by 10 square shows an office floor plan composed of four congruent 8-sided cubicles. What is the area of one of these irregular 8-sided cubicles?



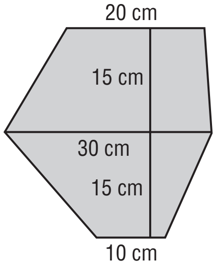
2. CUTOUTS Jeremy cut a rhombus out of a 10-inch by 7-inch rectangle. The diagonals of the rhombus are parallel and perpendicular to the sides of the rectangle and are congruent to the length and width of the rectangle, respectively. What is the total area of the four shaded triangles?



3. SHARING Bernard has a piece of cake that is shaped like a right triangle. He needs to cut it into three pieces to share it with two friends. He divides one of the legs into thirds and connects the division points to the opposite vertex of the triangle as shown in the figure. Which piece is the largest?

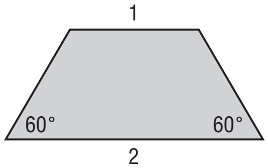


4. HEXAGONS Heather makes a hexagon by attaching two trapezoids together as shown. What is the area of the hexagon?

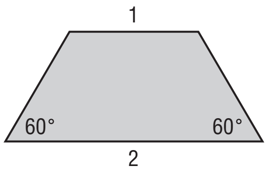


TILINGS For Exercises 5 and 6, use the following information.

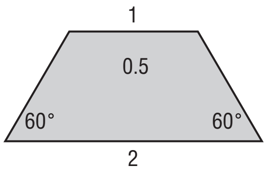
Tile making often requires an artist to find clever ways of dividing a shape into several smaller, congruent shapes. Consider the isosceles trapezoid shown below.



5. Show how to divide the trapezoid into 3 congruent triangles. What is the area of each triangle?



6. Show how to divide the trapezoid into 4 congruent trapezoids. What is the area of each of the smaller trapezoids?



Lesson 11-2

NAME ______________________________ DATE ____________ PERIOD _____

11-2 Enrichment

Areas of Similar Triangles

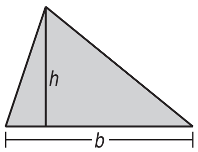
You have learned that if two triangles are similar, the ratio of the lengths of corresponding altitudes is equal to the ratio of the lengths of a pair of corresponding sides. However, there is a different relationship between the areas of the two triangles.

Theorem If two triangles are similar, the ratio of their areas is the square of the ratio of the lengths of a pair of corresponding sides.

Triangle II is k times larger than Triangle I. Thus, its base is k times as large as that of Triangle I and its height is k times as large as that of Triangle I.

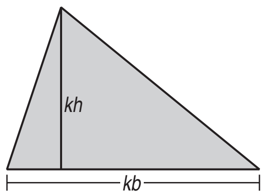
$$\frac{\text{side of } \triangle \text{II}}{\text{side of } \triangle \text{I}} = \frac{kb}{b} \text{ or } \frac{k}{1}$$

$$\frac{\text{area of } \triangle \text{II}}{\text{area of } \triangle \text{I}} = \frac{\frac{1}{2}k^2bh}{\frac{1}{2}bh} \text{ or } \frac{k^2}{1}$$



Triangle I

area $\triangle \text{I} = \frac{1}{2}bh$



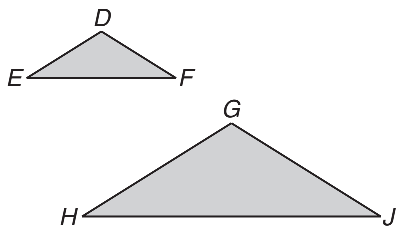
Triangle II

area $\triangle \text{II} = \frac{1}{2}(kb)(kh)$

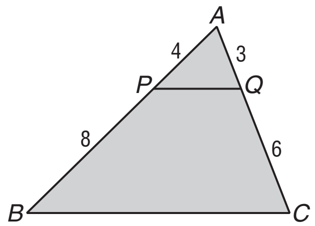
$= \frac{1}{2}k^2bh$

Solve.

1. $\triangle DEF \sim \triangle GHJ$, $HJ = 16$, and $EF = 8$. The area of $\triangle GHJ$ is 40. Find the area of $\triangle DEF$.



2. In the figure below, $\overline{PQ} \parallel \overline{BC}$. The area of $\triangle ABC$ is 72. Find the area of $\triangle APQ$.



3. Two similar triangles have areas of 16 and 36. The length of a side of the smaller triangle is 10 feet. Find the length of the corresponding side of the larger triangle.

4. Find the ratio of the areas of two similar triangles if the lengths of two corresponding sides of the triangles are 3 centimeters and 5 centimeters.

11-3 Lesson Reading Guide

Areas of Regular Polygons and Circles

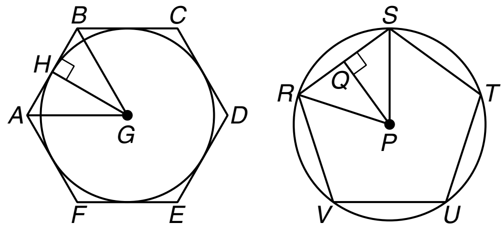
Get Ready for the Lesson

Read the introduction to Lesson 11-3 in your textbook.

Describe what type of triangles you could form by drawing the radii from the center of the octagon.

Read the Lesson

1. *ABCDEF* and *RSTUV* are regular polygons. Name each of the following in one of the figures.



a. a circumscribed polygon

b. an inscribed polygon

c. an apothem of a regular hexagon

d. an isosceles triangle

e. a 30°-60°-90° triangle

f. a central angle with a measure of 72

2. Refer to the figures in Exercise 1. Match each item in the first column with an expression in the second column.

a. perimeter of *ABCDEF*	**i.** $\pi(PS)^2$
b. circumference of circle *G*	**ii.** $2\pi(PR)$
c. perimeter of *RSTUV*	**iii.** $\frac{5}{2}(RS)(PQ)$
d. area of circle *G*	**iv.** $3(AB)(HG)$
e. area of *RSTUV*	**v.** $6(CD)$
f. area of *ABCDEF*	**vi.** $\pi(GH)^2$
g. area of circle *P*	**vii.** $5(UV)$
h. circumference of circle *P*	**viii.** $2\pi(GH)$

3. Explain in your own words how to find the area of a circle if you know the circumference.

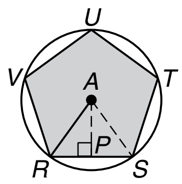
Remember What You Learned

4. A good way to remember something is to explain it to someone else. Suppose your classmate Joelle is having trouble remembering which formula is for circumference and which is for area. How can you help her?

11-3 Study Guide and Intervention

Areas of Regular Polygons and Circles

Areas of Regular Polygons In a regular polygon, the segment drawn from the center of the polygon perpendicular to the opposite side is called the **apothem**. In the figure at the right, $\overline{AP}$ is the apothem and $\overline{AR}$ is the radius of the circumscribed circle.



Area of a Regular Polygon	If a regular polygon has an area of A square units, a perimeter of P units, and an apothem of a units, then $A = \frac{1}{2}Pa$.

Example 1 **Verify the formula $A = \frac{1}{2}Pa$ for the regular pentagon above.**

For $\triangle RAS$, the area is $A = \frac{1}{2}bh = \frac{1}{2}(RS)(AP)$. So the area of the pentagon is $A = 5\left(\frac{1}{2}\right)(RS)(AP)$. Substituting P for $5RS$ and substituting a for AP, then $A = \frac{1}{2}Pa$.

Example 2 **Find the area of regular pentagon *RSTUV* above if its perimeter is 60 centimeters.**

First find the apothem.

The measure of central angle RAS is $\frac{360}{5}$ or 72. Therefore, $m\angle RAP = 36$. The perimeter is 60, so $RS = 12$ and $RP = 6$.

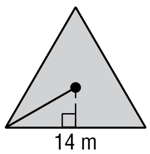
$$\tan \angle RAP = \frac{RP}{AP}$$
$$\tan 36° = \frac{6}{AP}$$
$$AP = \frac{6}{\tan 36°}$$
$$\approx 8.26$$

So, $A = \frac{1}{2}Pa = \frac{1}{2}60(8.26)$ or 247.8.

The area is about 248 square centimeters.

Exercises

Find the area of each regular polygon. Round to the nearest tenth.

1.


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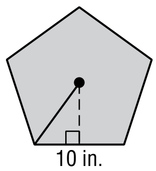

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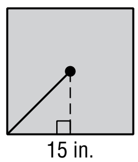

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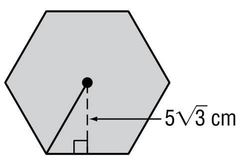

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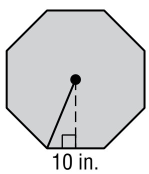

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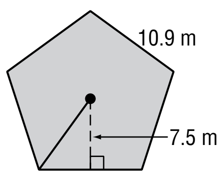
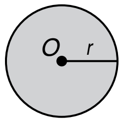


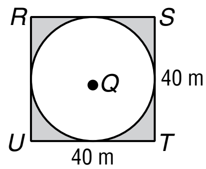
11-3 Study Guide and Intervention *(continued)*

Areas of Regular Polygons and Circles

Areas of Circles As the number of sides of a regular polygon increases, the polygon gets closer and closer to a circle and the area of the polygon gets closer to the area of a circle.

Area of a Circle	If a circle has an area of A square units and a radius of r units, then $A = \pi r^2$. 

Example **Circle Q is inscribed in square $RSTU$. Find the area of the shaded region.**



A side of the square is 40 meters, so the radius of the circle is 20 meters.

The shaded area is
Area of $RSTU$ − Area of circle Q

$= 40^2 - \pi r^2$

$= 1600 - 400\pi$

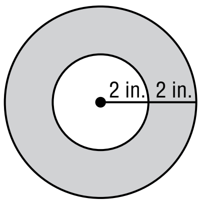
$\approx 1600 - 1256.6$

$= 343.4 \text{ m}^2$

Exercises

Find the area of each shaded region. Assume that all polygons are regular. Round to the nearest tenth.

1.



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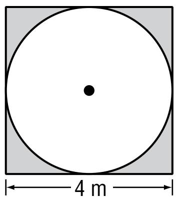

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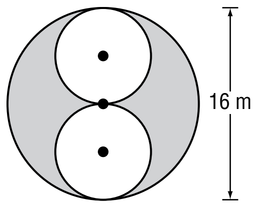

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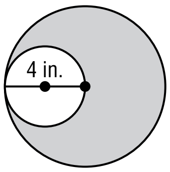

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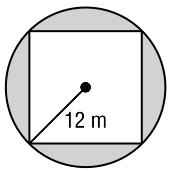

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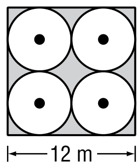


11-3 Skills Practice

Areas of Regular Polygons and Circles

Find the area of each regular polygon. Round to the nearest tenth.

1. a pentagon with a perimeter of 45 feet

2. a hexagon with a side length of 4 inches

3. a nonagon with a side length of 8 meters

4. a triangle with a perimeter of 54 centimeters

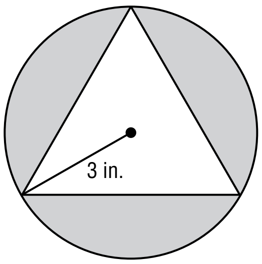
Find the area of each circle. Round to the nearest tenth.

5. a circle with a radius of 6 yards

6. a circle with a diameter of 18 millimeters

Find the area of each shaded region. Assume that all polygons are regular. Round to the nearest tenth.

7.



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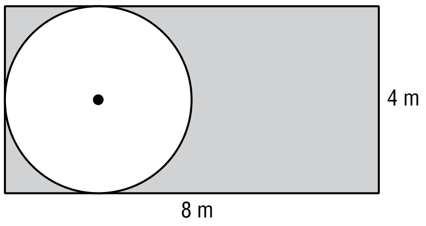
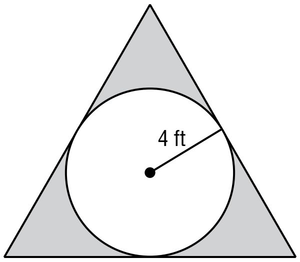
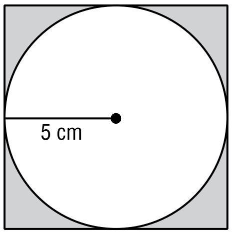


9.



10.



NAME ____ DATE ____________ PERIOD _____

11-3 Practice

Areas of Regular Polygons and Circles

Find the area of each regular polygon. Round to the nearest tenth.

1. a nonagon with a perimeter of 117 millimeters

2. an octagon with a perimeter of 96 yards

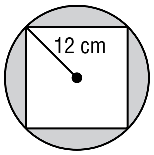
Find the area of each circle. Round to the nearest tenth.

3. a circle with a diameter of 26 feet

4. a circle with a circumference of 88 kilometers

Find the area of each shaded region. Assume that all polygons are regular. Round to the nearest tenth.

5.



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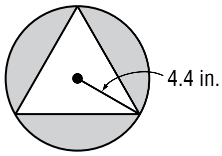

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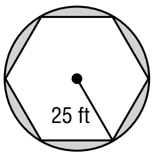

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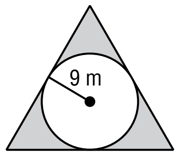


DISPLAYS For Exercises 9 and 10, use the following information.

A display case in a jewelry store has a base in the shape of a regular octagon. The length of each side of the base is 10 inches. The owners of the store plan to cover the base in black velvet.

9. Find the area of the base of the display case.

10. Find the number of square yards of fabric needed to cover the base.

Lesson 11-3

NAME ____ DATE ____________ PERIOD _____

11-3 Word Problem Practice

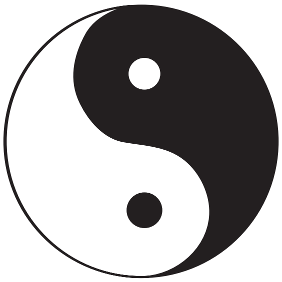
Areas of Regular Polygons and Circles

1. LOBBY The lobby of a bank features a large marble regular octagon. Each side of the octagon is 15 feet long.

What is the area of the octagon? Round your answer to the nearest tenth.

2. PORTHOLES A circular window on a ship has a radius of 8 inches. What is the area of the window? Round your answer to the nearest hundredth.

3. YIN-YANG SYMBOL A well-known symbol from Chinese culture is the yin-yang symbol, shown below.

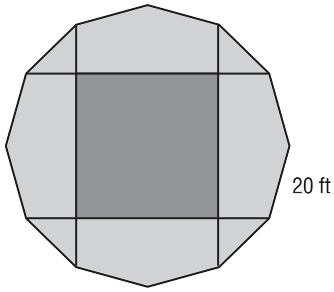


Suppose the large circle has radius r, the small circles have radius $\frac{r}{8}$, and the S-curve is two semicircles each with radius $\frac{r}{2}$. In terms of r, what is the area of the black region?

4. PYRAMIDS Martha's clubhouse is shaped like a square pyramid with four congruent equilateral triangles for its sides. All of the edges are 6 feet long. What is the total surface area of the clubhouse including the floor? Round your answer to the nearest hundredth.

POOL DECKS For Exercises 5-7, use the following information.

Ricardo designs a square pool with surrounding pool deck according to the plan shown. The outer edge of the deck is a regular dodecagon with side length 20 feet.



5. What is the length of the apothem of the dodecagonal deck?

6. What is the length of the diagonal of the square pool?

7. What is the area of the deck?

11-3 Enrichment

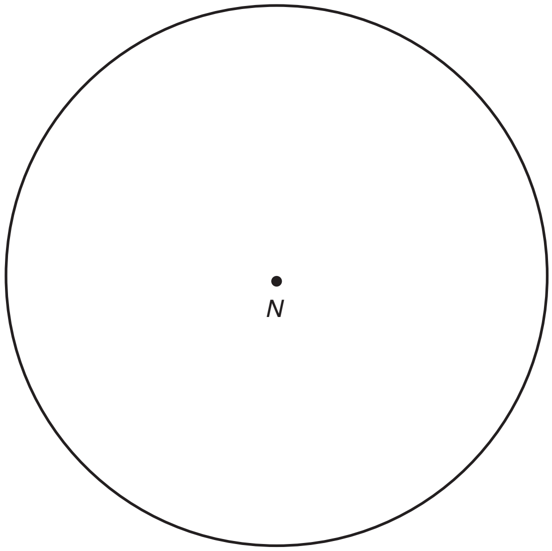
Areas of Inscribed Polygons

A protractor can be used to inscribe a regular polygon in a circle. Follow the steps below to inscribe a regular nonagon in ⊙*N*.

Step 1 Find the degree measure of each of the nine congruent arcs.

Step 2 Draw 9 radii to form 9 angles with the measure you found in Step 1. The radii will intersect the circle in 9 points.

Step 3 Connect the nine points to form the nonagon.



1. Find the length of one side of the nonagon to the nearest tenth of a centimeter. What is the perimeter of the nonagon?

2. Measure the distance from the center perpendicular to one of the sides of the nonagon.

3. What is the area of one of the nine triangles formed?

4. What is the area of the nonagon?

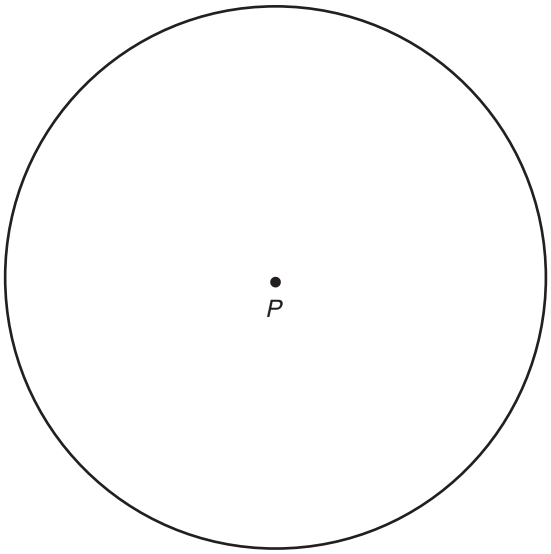
Make the appropriate changes in Steps 1–3 above to inscribe a regular pentagon in ⊙*P*. Answer each of the following.

5. Use a protractor to inscribe a regular pentagon in ⊙*P*.

6. What is the measure of each of the five congruent arcs?

7. What is the perimeter of the pentagon to the nearest tenth of a centimeter?

8. What is the area of the pentagon to the nearest tenth of a centimeter?



Lesson 11-3

NAME ______________________ DATE __________ PERIOD _____

11-4 Lesson Reading Guide

Areas of Composite Figures

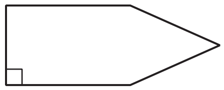
Get Ready for the Lesson

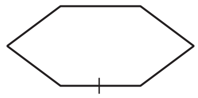
Read the introduction to Lesson 11-4 in your textbook.

How do you think the areas of the figures outlined in the picture of the sail are related?

Read the Lesson

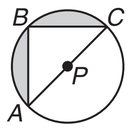
1. Use dashed segments to show how each figure can be subdivided into figures for which you have learned area formulas. Name the smaller figures that you have formed as specifically as possible and indicate whether any of them are congruent to each other.

a. 

b. 

c. 

2. In the figure, B is the midpoint of $\widehat{ABC}$. Complete the following steps to derive a formula for the area of the shaded region in terms of the radius r of the circle.



The area of circle P is ________.

$m\angle ABC =$ ________ because

____.

$m\widehat{AB} = m\widehat{BC}$ because

____.

$\overline{AB} \cong \overline{BC}$ because ______________________________

____.

Therefore, $\triangle ABC$ is a(n) ____________________ triangle.

$AC =$ __________, so $AB =$ __________ and $BC =$ __________.

The area of $\triangle ABC$ is $\frac{1}{2} \cdot$ __________ $\cdot$ __________ $=$ __________.

Therefore, the area of the shaded region is given by

$A =$ __________ $-$ __________ $=$ __________.

Remember What You Learned

3. Rolando is having trouble remembering when to subtract an area when finding the area of a composite figure. How can you help him remember?

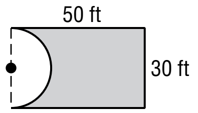
NAME ____ DATE ____________ PERIOD _____

11-4 Study Guide and Intervention

Areas of Composite Figures

Composite Figures A composite figure is a figure that can be seprated into regions that are basic figures. To find the area of a composite figure separate the figure into basic figures of which we can find the area. The sum of the areas of the basic figures is the area of the figure.

Example 1 **Find the area of the composite figure.**



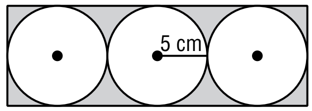
The figure is a rectangle minus one half of a circle. The radius of the circle is one half of 30 or 15.

$A = lw - \frac{1}{2}\pi r^2$

$= 50(30) - 0.5\pi(15)^2$

≈ 1146.6 or about 1147 ft^2

Example 2 **Find the area of the shaded region.**



The dimensions of the rectangle are 10 centimeters and 30 centimeters. The area of the shaded region is

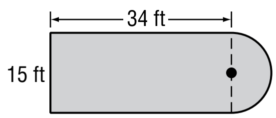
$(10)(30) - 3\pi(5^2) = 300 - 75\pi$

$\approx 64.4\ cm^2$

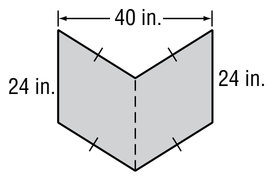
Exercises

Find the area of each figure. Round to the nearest tenth if necessary.

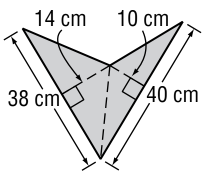
1.



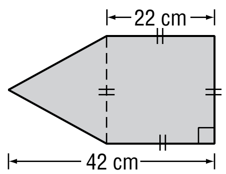
2.



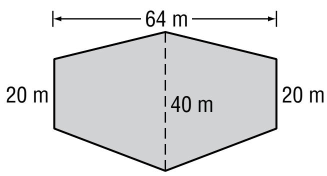
3.



4.



5.



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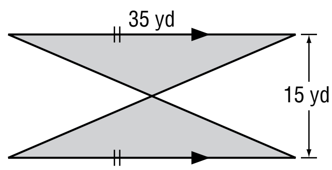


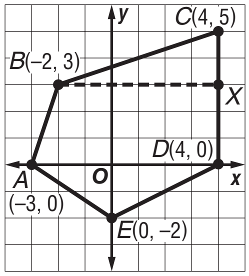
7. Refer to Example 2 above. Draw the largest possible square inside each of the three circles. What is the total area of the three squares?

11-4 Study Guide and Intervention *(continued)*

Areas of Composite Figures

Composite Figures on the Coordinate Plane To find the area of a composite figure on the coordinate plane, separate the figure into basic figures.

Example **Find the area of pentagon *ABCDE*.**



Draw $\overline{BX}$ between $B(-2, 3)$ and $X(4, 3)$ and draw $\overline{AD}$. The area of $ABCDE$ is the sum of the areas of $\triangle BCX$, trapezoid $BXDA$, and $\triangle ADE$.

A = area of $\triangle BCX$ + area of $BXDA$ + area of $\triangle ADE$

$= \frac{1}{2}(2)(6) + \frac{1}{2}(3)(6 + 7) + \frac{1}{2}(2)(7)$

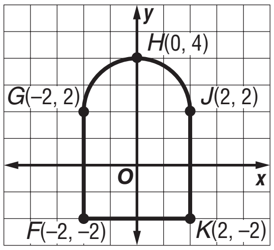
$= 6 + \frac{39}{2} + 7$

$= 32.5$ square units

Exercises

Find the area of each figure. Round to the nearest tenth.

1.



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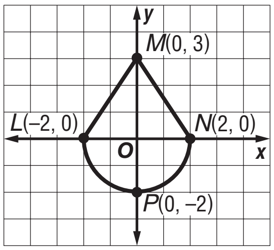
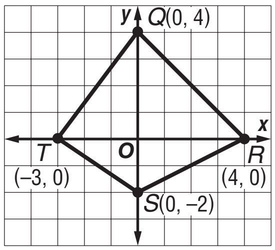


3.



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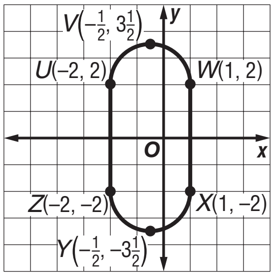

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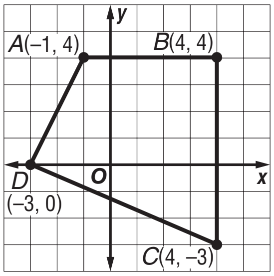

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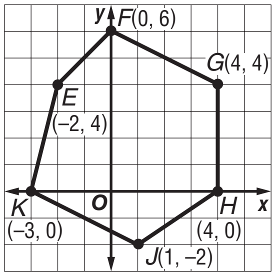
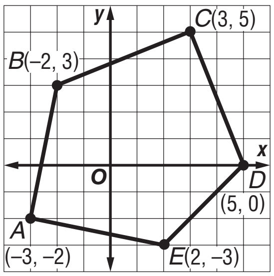
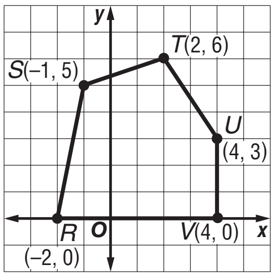


7. pentagon *ABCDE*



8. pentagon *RSTUV*



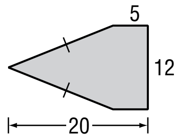
NAME ____ DATE ____________ PERIOD _____

11-4 Skills Practice

Areas of Composite Figures

Find the area of each figure. Round to the nearest tenth if necessary.

1.



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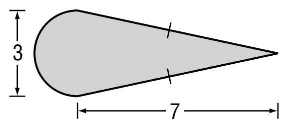

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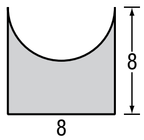

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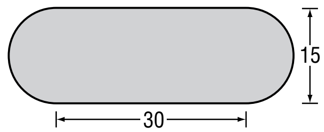

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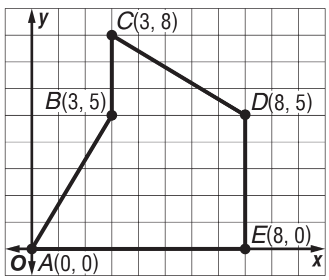
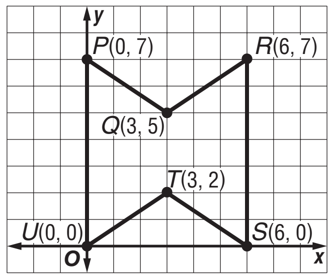


6.



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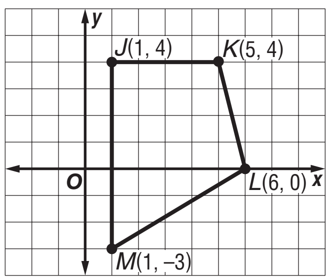

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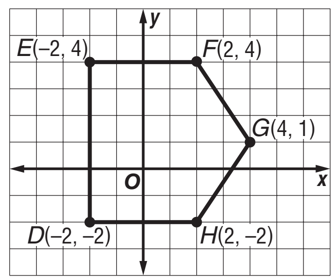


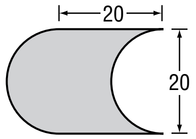
NAME ______________________________ DATE ____________ PERIOD _____

11-4 Practice

Areas of Composite Figures

Find the area of each figure. Round to the nearest tenth if necessary.

1.



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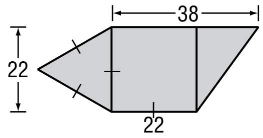

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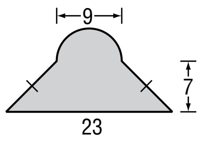

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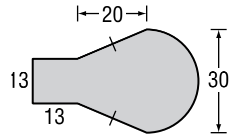

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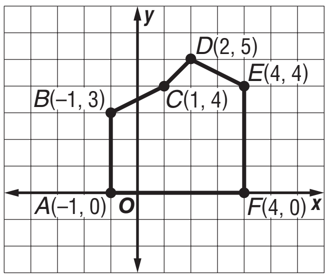

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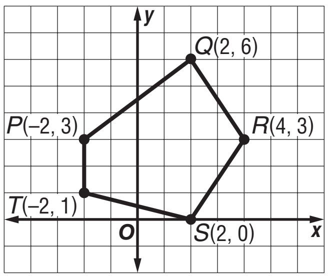
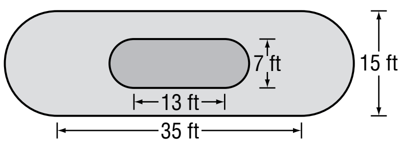


LANDSCAPING For Exercises 7 and 8, use the following information.
One of the displays at a botanical garden is a koi pond with a walkway around it. The figure shows the dimensions of the pond and the walkway.



7. Find the area of the pond to the nearest tenth.

8. Find the area of the walkway to the nearest tenth.

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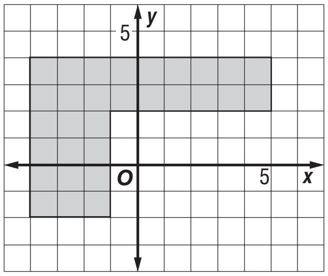
11-4 Word Problem Practice

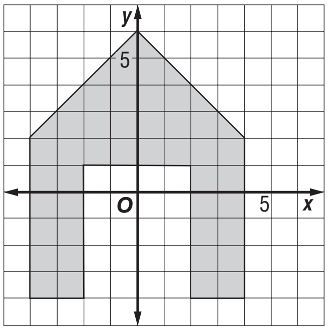
Areas of Composite Figures

1. FLOOR PLANS The floor plan of an L-shaped building is shown in the coordinate plane. Each unit represents 5 meters.



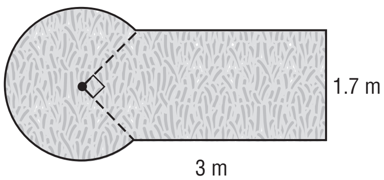
What is the area of the building?

2. DOG HOUSES Miranda is building a dog house out of wood. The front view of the dog house is shown on the coordinate plane below.



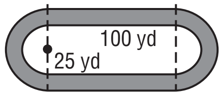
If each unit corresponds to 5 inches, what is the area of the front?

3. MINIATURE GOLF The plan for a miniature golf hole is shown below. The right angle in the drawing is a central angle.



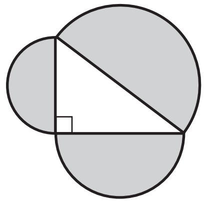
What is the area of the playing surface? Round your answer to the nearest hundredth of a square meter.

4. TRACK A running track has an inner and outer edge. Both the inner and outer edge consists of two semicircles joined by two straight line segments. The straight line segments are 100 yards long. The radii of the inner edge semicircles are 25 yards and the radii of the outer edge semicircles are 32 yards. What is the area of the track? Round your answer to the nearest hundredth of a yard.



SEMICIRCLES For Exercises 5 and 6, use the following information.

Bridget arranged three semicircles in the pattern shown.



The right triangle has side lengths 6, 8, and 10 inches.

5. What is the total area of the three semicircles? Round your answer to the nearest hundredth of a square inch.

6. If the right triangle had side lengths $\sqrt{21}$, $\sqrt{79}$, and 10 inches, what would the total area of the three semicircles be? Round your answer to the nearest hundredth of a square inch.

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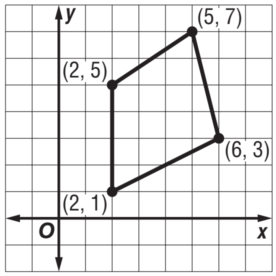
11-4 Enrichment

Aerial Surveyors and Area

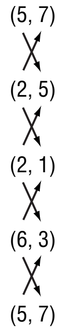
Many land regions have irregular shapes. Aerial surveyors often use coordinates when finding areas of such regions. The coordinate method described in the steps below can be used to find the area of *any* polygonal region. Study how this method is used to find the area of the region at the right.



Step 1 List the ordered pairs for the vertices in counter-clockwise order, repeating the first ordered pair at the bottom of the list.

Step 2 Find D, the sum of the downward diagonal products (from left to right).

$D = (5 \cdot 5) + (2 \cdot 1) + (2 \cdot 3) + (6 \cdot 7)$
$= 25 + 2 + 6 + 42$ or 75



Step 3 Find U, the sum of the upward diagonal products (from left to right).

$U = (2 \cdot 7) + (2 \cdot 5) + (6 \cdot 1) + (5 \cdot 3)$
$= 14 + 10 + 6 + 15$ or 45

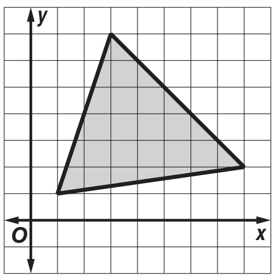
Step 4 Use the formula $A = \frac{1}{2}(D - U)$ to find the area.

$A = \frac{1}{2}(D - U)$
$= \frac{1}{2}(75 - 45)$
$= \frac{1}{2}(30)$ or 15

The area is 15 square units. Count the number of square units enclosed by the polygon. Does this result seem reasonable?

Use the coordinate method to find the area of each region in square units.

1.



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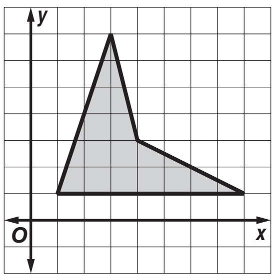

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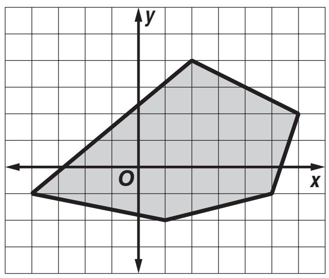

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11-5 Lesson Reading Guide

Geometric Probability and Areas of Sectors

Get Ready for the Lesson

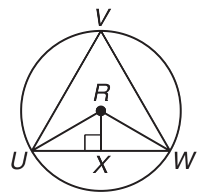
Read the introduction to Lesson 11-5 in your textbook.

To find the probability of winning at darts, would you use geometric probability to compare areas or lengths?

Read the Lesson

1. Explain the difference between a sector of a circle and a segment of a circle

2. Suppose you are playing a game of darts with a target like the one shown at the right. If your dart lands inside equilateral $\triangle UVW$, you get a point. Assume that every dart will land on the target. The radius of the circle is 1. Complete the following steps to figure out the probability of getting a point.



The area of circle R is ________.

$\triangle URW$ is a(n) ________________ triangle because $\overline{RU}$ and $\overline{RW}$ are ________________ of the same ________________.

$\angle URW$ is a(n) ________________ angle of the circle, and $m\angle URW$ = ________.

$m\angle RUX$ = ________ and $m\angle RWX$ = ________.

The angle measures in $\triangle RUX$ are ________, ________, and ________.

$\overline{RU}$ is a ________________ of the circle, so RU = ________.

$\overline{RX}$ is the leg of $\triangle RUX$ opposite the ________ angle, so RX = ________.

Also, $\overline{UX}$ is the leg of $\triangle RUX$ opposite the ________ angle, so UX = ________.

UW = ________, so the area of $\triangle URW$ is $\frac{1}{2}$ · ________ · ________ = ________.

Then, the area of $\triangle UVW$ = 3 · ________ = ________.

Therefore, the probability that the dart will fall inside the triangle is the ratio of ________ to ________, which is approximately ________ (to the nearest thousandth).

Remember What You Learned

3. Many students find it difficult to remember a large number of geometric formulas. How can you use the formula for the area of a circle to find the area of a sector of a circle without having to learn a new formula?

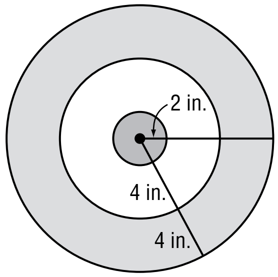
11-5 Study Guide and Intervention

Geometric Probability and Areas of Sectors

Geometric Probability The probability that a point in a figure will lie in a particular part of the figure can be calculated by dividing the area of the part of the figure by the area of the entire figure. The quotient is called the **geometric probability** for the part of the figure.

> If a point in region A is chosen at random, then the probability $P(B)$ that the point is in region B, which is in the interior of region A, is
> $P(B) = \frac{\text{area of region } B}{\text{area of region } A}$.

Example **Darts are thrown at a circular dartboard. If a dart hits the board, what is the probability that the dart lands in the bull's-eye?**



Area of bull's-eye: $A = \pi(2)^2$ or 4π

Area of entire dartboard: $A = \pi(10)^2$ or 100π

The probability of landing in the bull's-eye is

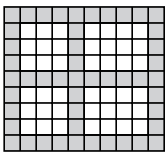
$$\frac{\text{area of bull's-eye}}{\text{area of dartboard}} = \frac{4\pi}{100\pi}$$

$$= \frac{1}{25} \text{ or } 0.04.$$

Exercises

Find the probability that a point chosen at random lies in the shaded region. Round to the nearest hundredth if necessary.

1.



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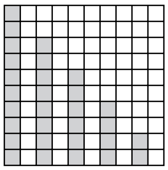

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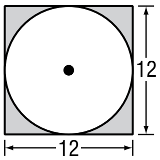

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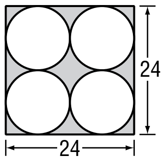

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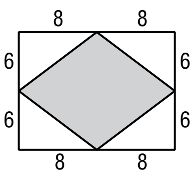

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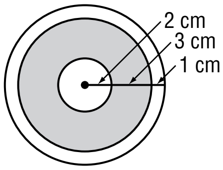
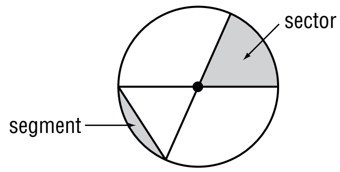


11-5 Study Guide and Intervention *(continued)*

Geometric Probability and Areas of Sectors

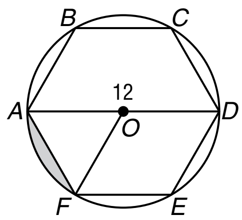
Sectors and Segments of Circles

A **sector of a circle** is a region of a circle bounded by a central angle and its intercepted arc. A **segment of a circle** is bounded by a chord and its arc. Geometric probability problems sometimes involve sectors or segments of circles.



> If a sector of a circle has an area of A square units, a central angle measuring $N°$, and a radius of r units, then $A = \frac{N}{360}\pi r^2$.

Example **A regular hexagon is inscribed in a circle with diameter 12. Find the probability that a point chosen at random in the circle lies in the shaded region.**



The area of the shaded segment is the area of sector AOF − the area of $\triangle AOF$.

$$\text{Area of sector } AOF = \frac{N}{360}\pi r^2 = \frac{60}{360}\pi(6^2) = 6\pi$$

$$\text{Area of } \triangle AOF = \frac{1}{2}bh = \frac{1}{2}(6)\left(3\sqrt{3}\right) = 9\sqrt{3}$$

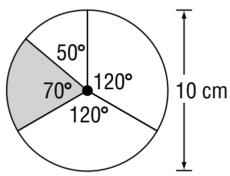
The shaded area is $6\pi - 9\sqrt{3}$ or about 3.26.

The probability is $\frac{\text{area of segment}}{\text{area of circle}} = \frac{3.26}{36\pi}$ or about 0.03.

Exercises

Find the probability that a point in the circle chosen at random lies in the shaded region. Round to the nearest hundredth.

1.



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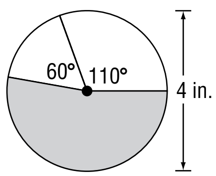

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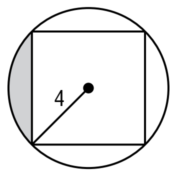

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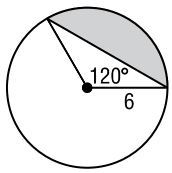

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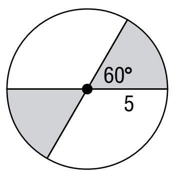

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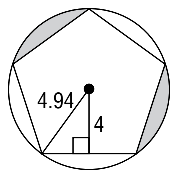
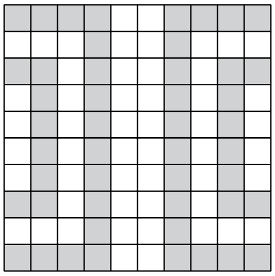
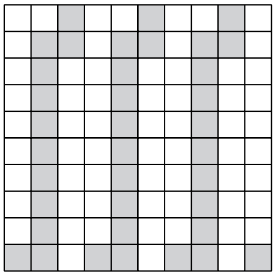


11-5 Skills Practice

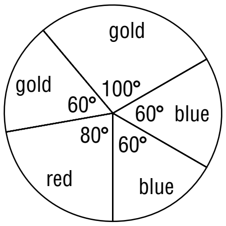
Geometric Probability and Areas of Sectors

Find the probability that a point chosen at random lies in the shaded region.

1. 

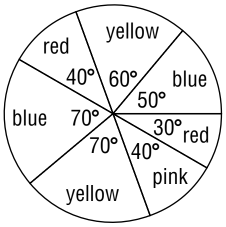
2. 

Find the area of the indicated sector. Then find the probability of spinning the color indicated if the diameter of each spinner is 6 inches.



3. red

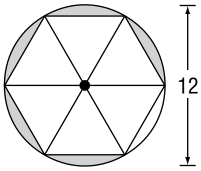
4. gold

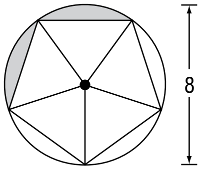


5. blue

6. yellow

Find the area of the shaded region. Then find the probability that a point chosen at random is in the shaded region. Assume that all inscribed polygons are regular.

7. 

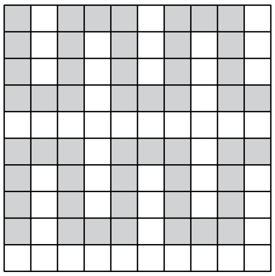
8. 

11-5 Practice

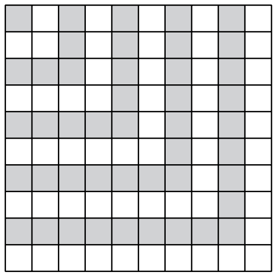
Geometric Probability and Areas of Sectors

Find the probability that a point chosen at random lies in the shaded region.

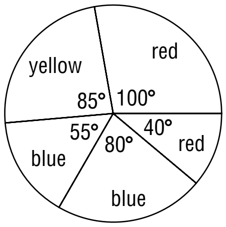
1.



2.



Find the area of the indicated sector. Then find the probability of spinning the color indicated if the diameter of the spinner is 9 meters.



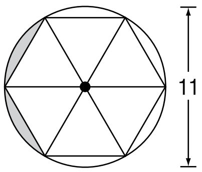
3. red

4. blue

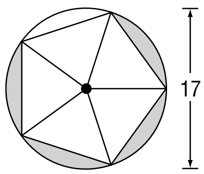
5. yellow

Find the area of the shaded region. Then find the probability that a point chosen at random is in the shaded region. Assume that all inscribed polygons are regular.

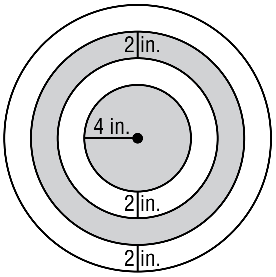
6.



7.



8. ARCHERY A target consists of four concentric rings. The radius of the center circle is 4 inches, and the circles are spaced 2 inches apart. Find the probability that an arrow shot at random by an inexperienced archer will land in a shaded region.

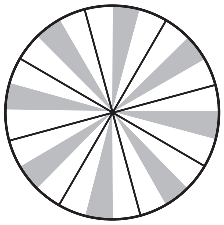


NAME ______________________________ DATE __________ PERIOD _____

11-5 Word Problem Practice

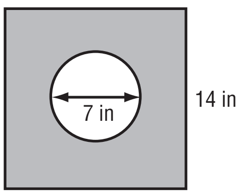
Geometric Probability and Areas of Sectors

1. DARTS A dart is thrown at the dartboard shown. Each sector has the same central angle. The dart has equal probability of hitting any point on the dartboard. What is the probability that the dart will land in a shaded sector?



2. SPINNERS Jamie, Joe, and Pat celebrate the end of each work week by ordering spring rolls from a Chinese restaurant. The order comes with 4 spring rolls so somebody gets an extra roll. Because Jamie works full time and Joe and Pat work half time, they decide who gets the extra roll by using a spinner that has a 50% chance of coming up Jamie, and 25% chances of coming up either Joe or Pat. Design such a spinner.

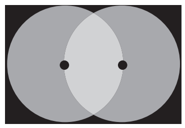
3. RAIN A container has a square top with a hole as shown. What is the probability that a raindrop that hits the container falls into the hole? Round your answer to the nearest thousandth.



4. ELECTRON MICROSCOPES Crystal places a 7 millimeter by 10 millimeter rectangular plate into the sample chamber of an electron microscope. A black and white checkerboard pattern of 1-millimeter squares was painted over the plate to identify different treatments of the material. When she turns on the monitor, she has no idea at what point on the plate she is looking because the white and black contrast does not show up on the screen. If there are 2 more black squares than white squares, what is the probability that she is looking at a white square?

ENTERTAINMENT For Exercises 5 and 6, use the following information.

A rectangular dance stage is lit by two lights that light up circular regions of the stage. The circles have the same radius and each circle passes through the center of the other. The stage perfectly circumscribes the two circles. A spectator throws a bouquet of flowers onto the stage. Assume the bouquet has an equal chance of landing anywhere on the stage. (*Hint:* Use inscribed equilateral triangles.)



5. What is the probability that the flowers land on a lit part of the stage?

6. What is the probability that the flowers land on the part of the stage where the spotlights overlap?

NAME ____ DATE ____________ PERIOD _____

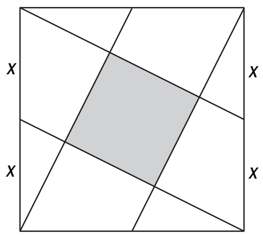
11-5 Enrichment

Polygon Probability

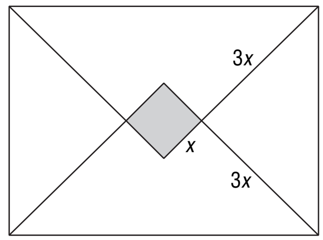
Each problem on this page involves one or more regular polygons. To find the probability of a point chosen at random being in the shaded region, you need to find the ratio of the shaded area to the total area. If you wish, you may substitute numbers for the variables.

Find the probability that a point chosen at random in each figure is in the shaded region. Assume polygons that appear to be regular are regular. Round your answer to the nearest hundredth.

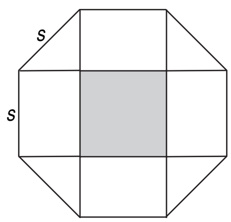
1.



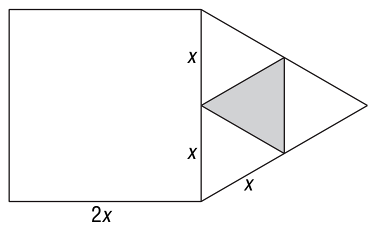
2.



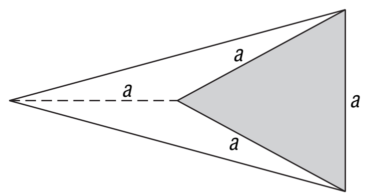
3.



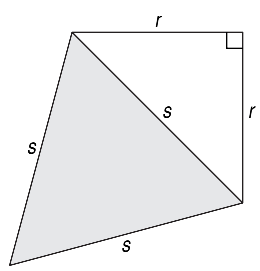
4.



5.



6.



NAME ____ DATE ____________ PERIOD _____

11-1 Student Recording Sheet

Use this recording sheet with pages 676–677 of the Student Edition.

Assessment

Read each question. Then fill in the correct answer.

1. Ⓐ Ⓑ Ⓒ Ⓓ
2. Ⓕ Ⓖ Ⓗ Ⓙ
3. Record your answer and fill in the bubbles in the grid below. Be sure to use the correct place value.

				.			
⓪	⓪	⓪	⓪		⓪	⓪	⓪
①	①	①	①		①	①	①
②	②	②	②		②	②	②
③	③	③	③		③	③	③
④	④	④	④		④	④	④
⑤	⑤	⑤	⑤		⑤	⑤	⑤
⑥	⑥	⑥	⑥		⑥	⑥	⑥
⑦	⑦	⑦	⑦		⑦	⑦	⑦
⑧	⑧	⑧	⑧		⑧	⑧	⑧
⑨	⑨	⑨	⑨		⑨	⑨	⑨

4. Ⓐ Ⓑ Ⓒ Ⓓ
5. Ⓕ Ⓖ Ⓗ Ⓙ
6. Ⓐ Ⓑ Ⓒ Ⓓ
7. Ⓕ Ⓖ Ⓗ Ⓙ
8. Ⓐ Ⓑ Ⓒ Ⓓ
9. Ⓕ Ⓖ Ⓗ Ⓙ
10. Ⓐ Ⓑ Ⓒ Ⓓ
11. Ⓕ Ⓖ Ⓗ Ⓙ

Pre-AP

Record your answers for Question 12 on the back of this paper.

11 Rubric for Scoring Pre-AP

(Use to score the Pre-AP question on page 677 of the Student Edition.)

General Scoring Guidelines

- If a student gives only a correct numerical answer to a problem but does not show how he or she arrived at the answer, the student will be awarded only 1 credit. All extended-response questions require the student to show work.
- A fully correct answer for a multiple-part question requires correct responses for all parts of the question. For example, if a question has three parts, the correct response to one or two parts of the question that required work to be shown is *not* considered a fully correct response.
- Students who use trial and error to solve a problem must show their method. Merely showing that the answer checks or is correct is not considered a complete response for full credit.

Exercise 12 Rubric

Score	Specific Criteria
4	A correct solution that is supported by well-developed, accurate explanations. D's coordinates are (5, 0). The height of $ABCD$ is 4 units and the base of $ABCD$ is 5 units, so the area is 20 units^2.
3	A generally correct solution, but may contain minor flaws in reasoning or computation.
2	A partially correct interpretation and/or solution to the problem.
1	A correct solution with no supporting evidence or explanation.
0	An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given.

NAME ______________________ DATE __________ PERIOD _____

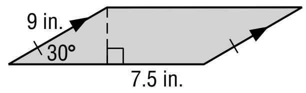
11 Chapter 11 Quiz 1

SCORE _____

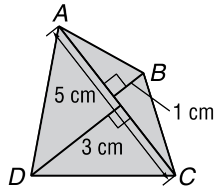
(Lessons 11–1 and 11–2)

Find the area of each figure. Round to the nearest tenth.

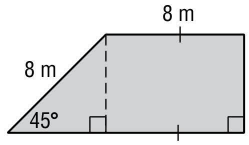
1.



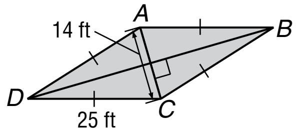
2.



3.



4.



5. The vertices of a quadrilateral are $A(-1, 3)$, $B(4, 3)$, $C(4, -2)$, and $D(-1, -2)$. Determine whether the quadrilateral is a *square*, a *rectangle*, or a *parallelogram*.

1. ____________
2. ____________
3. ____________
4. ____________
5. ____________

Assessment

NAME ______________________ DATE __________ PERIOD _____

11 Chapter 11 Quiz 2

SCORE _____

(Lesson 11–3)

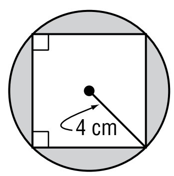
Find the area of each polygon. Round to the nearest tenth.

1. an equilateral triangle with a side length of $8\sqrt{3}$ inches

2. a regular hexagon with apothem length of 5 centimeters

3. a regular octagon with a perimeter of 80 meters

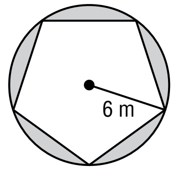
4. Find the area of the shaded region to the nearest tenth. Assume that the polygon is regular.



5. MULTIPLE CHOICE Find the area of the shaded region to the nearest tenth. Assume that the polygon is regular.

A. 113 m^2 **C.** 70.1 m^2

B. 105.5 m^2 **D.** 27.5 m^2



1. ____________
2. ____________
3. ____________
4. ____________
5. ____________

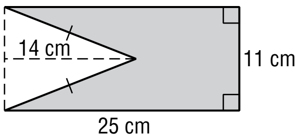
NAME ____________________ DATE __________ PERIOD _____

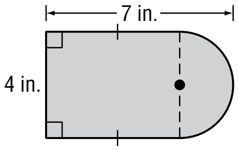
11 Chapter 11 Quiz 3

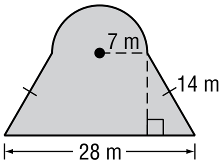
SCORE _____

(Lesson 11–4)

Find the area of each figure. Round to the nearest tenth if necessary.

1. 

2. 

3. 

4. quadrilateral $PQRS$ with vertices $P(0, 6)$, $Q(4, 0)$, $R(0, -4)$, and $S(-4, 0)$

5. figure $ABCDEF$ with vertices $A(1, 3)$, $B(4, -1)$, $C(2, -4)$, $D(-4, -4)$, $E(-2, -1)$, and $F(1, -1)$

1. __________
2. __________
3. __________
4. __________
5. __________

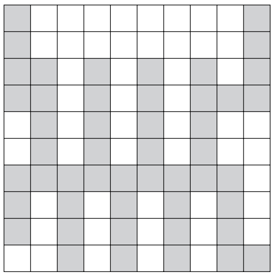
NAME ____________________ DATE __________ PERIOD _____

11 Chapter 11 Quiz 4

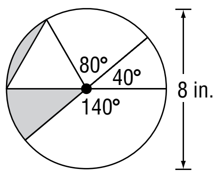
SCORE _____

(Lesson 11–5)

1. Find the probability that a point chosen at random lies in the shaded region.

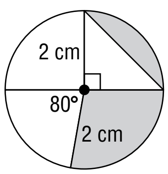


For Questions 2 and 3, refer to the figure.



2. Find the probability that a point chosen at random lies in the shaded sector of the circle.
3. Find the probability that a point chosen at random lies in the shaded segment of the circle.

For Questions 4 and 5, refer to the figure.



4. Find the probability that a point chosen at random lies in the shaded sector of the circle.
5. Find the probability that a point chosen at random lies in the shaded segment of the circle.

1. __________
2. __________
3. __________
4. __________
5. __________

NAME ______________________________ DATE __________ PERIOD _____

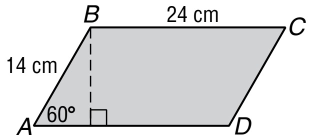
11 Chapter 11 Mid-Chapter Test

SCORE _____

(Lessons 11–1 through 11–3)

Part I ***Write the letter for the correct answer in the blank at the right of each question.***

1. Find the area of parallelogram *ABCD*. Round to the nearest tenth. **1.** ______

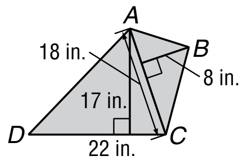


A. 145.5 cm^2 **C.** 190.5 cm^2
B. 168.0 cm^2 **D.** 291.0 cm^2

2. What is the best classification of quadrilateral *WXYZ* with vertices *W*(1, 1), *X*(4, 1), *Y*(4, −2), and *Z*(1, −2)? **2.** ______

F. square **H.** parallelogram
G. rectangle **J.** none of these

3. Find the area of quadrilateral *ABCD*. Round to the nearest tenth. **3.** ______



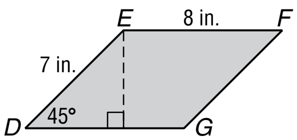
A. 187 in^2 **C.** 374 in^2
B. 259 in^2 **D.** 518 in^2

4. Find the area of trapezoid *ABCD* with vertices *A*(−1, 3), *B*(3, 3), *C*(4, −2), and *D*(−4, −2). **4.** ______

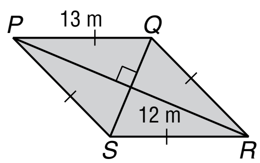
F. 15 $units^2$ **H.** 30 $units^2$
G. 20 $units^2$ **J.** 60 $units^2$

Part II

5. Find the area of parallelogram *DEFG*. Round to the nearest tenth if necessary. **5.** ________________



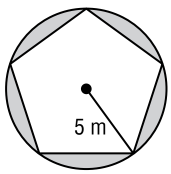
6. Find the area of rhombus *PQRS*. Round to the nearest tenth if necessary. **6.** ________________



7. Find the area of an equilateral triangle with an apothem having a length of 4 feet. Round to the nearest tenth. **7.** ________________

8. A circular tablecloth is used to cover an octagonal dining table. The radius of the tablecloth and the apothem of the table are both 24 inches. Each side of the table is 22.5 inches. Find the area of the table left uncovered by the tablecloth. **8.** ________________

9. Find the area of the shaded region to the nearest tenth. Assume that the pentagon is regular. **9.** ________________



Assessment

NAME ______________________ DATE __________ PERIOD _____

11 Chapter 11 Vocabulary Test

SCORE _____

apothem	geometric probability	segment
composite figure	sector	

Select the correct formula to complete each sentence.

1. The area of a circle can be found by ($A = \frac{1}{2}Pa$, $A = \pi r^2$, $A = bh$). **1.** __________

2. ($A = \frac{1}{2}d_1d_2$, $A = \frac{1}{2}Pa$, $A = \frac{1}{2}bh$) can be used to find the area of a triangle. **2.** __________

3. The formula ($A = bh$, $A = \frac{1}{2}d_1d_2$, $A = \frac{1}{2}h(b_1 + b_2)$) is used to find the area of a trapezoid. **3.** __________

4. The area of all regular polygons can be found using ($A = \frac{1}{2}Pa$, $A = \frac{1}{2}d_1d_2$, $A = \frac{1}{2}bh$). **4.** __________

5. The formula to find the area of a sector is ($A = \frac{N}{360}\pi r^2 - \frac{1}{2}bh$, $A = \frac{N}{360}\pi r^2$, $A = \pi r^2$). **5.** __________

?

Choose from the terms above to complete each sentence.

6. In terms of geometric shapes, a slice of pizza closely resembles a ___?___. **6.** __________

7. The region of a circle bounded by an arc and a chord is called a ___?___ of a circle. **7.** __________

8. A line drawn from the center of a regular polygon perpendicular to a side of the polygon is called a(n) ___?___. **8.** __________

Define each term in your own words.

9. irregular figure **9.** __________

10. geometric probability **10.** __________

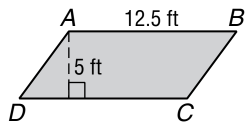
NAME ______________________ DATE ____________ PERIOD _____

11 Chapter 11 Test, Form 1

SCORE _____

Write the letter for the correct answer in the blank at the right of each question.

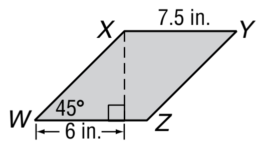
1. Find the area of parallelogram *ABCD*. Round to the nearest tenth. **1.** _____



A. 17.5 ft^2 **C.** 35 ft^2

B. 31.25 ft^2 **D.** 62.5 ft^2

2. Find the area of parallelogram *WXYZ*. Round to the nearest tenth. **2.** _____



F. 27 in^2 **H.** 63.6 in^2

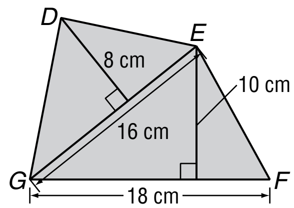
G. 45 in^2 **J.** 81 in^2

3. What is the best classification of quadrilateral *PQRS* with vertices $P(2, 2)$, $Q(-1, 2)$, $R(-1, -3)$, and $S(2, -3)$? **3.** _____

A. square **C.** parallelogram

B. rectangle **D.** none of these

4. Find the area of quadrilateral *DEFG*. **4.** _____



F. 154 cm^2 **H.** 244 cm^2

G. 218 cm^2 **J.** 308 cm^2

5. Find the area of trapezoid *ABCD* with vertices $A(1, -2)$, $B(5, -2)$, $C(4, 4)$, and $D(1, 4)$. **5.** _____

A. 6.5 $units^2$ **C.** 21 $units^2$

B. 14 $units^2$ **D.** 36 $units^2$

6. Find the area of rhombus *ABCD* with vertices $A(-1, 3)$, $B(3, 0)$, $C(-1, -3)$, and $D(-5, 0)$. **6.** _____

F. 8 $units^2$ **H.** 26 $units^2$

G. 24 $units^2$ **J.** 32 $units^2$

7. Find the area of a regular octagon with a perimeter of 96 centimeters. **7.** _____

A. about 695.3 cm^2 **C.** about 532 cm^2

B. about 576 cm^2 **D.** about 119.3 cm^2

8. Find the area of an equilateral triangle with a side length of 14 inches. **8.** _____

F. about 12.1 in^2 **H.** about 84.9 in^2

G. about 42 in^2 **J.** about 254.6 in^2

9. Find the area of a circle with a circumference of 20π. **9.** _____

A. 400π **C.** 200π

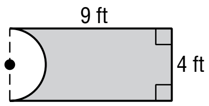
B. 314π **D.** 100π

Assessment

10. Serena is wearing a pendant that was made by inscribing a square ruby in a sterling silver circle. The distance from the center of the pendant to its edge is 5.3 cm. Find the area of the pendant that is not covered by the ruby. Round to the nearest tenth. **10.** ______

F. 32.06 **G.** 56.18 **H.** 60.19 **J.** 88.24

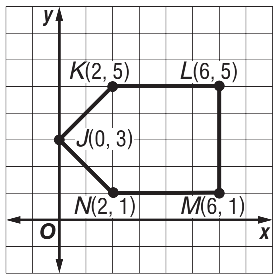
11. Find the area of the figure. Round to the nearest tenth. **11.** ______



A. 23.4 ft^2 **C.** 29.7 ft^2

B. 28.3 ft^2 **D.** 36 ft^2

12. Find the area of the figure. **12.** ______



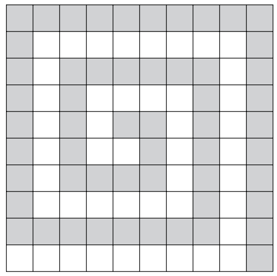
F. 22 $units^2$

G. 20 $units^2$

H. 18 $units^2$

J. 16 $units^2$

13. Find the probability that a point on the grid chosen at random lies in the shaded region. **13.** ______



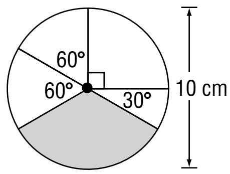
A. 0.46

B. 0.50

C. 0.55

D. 0.85

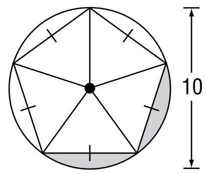
14. Find the probability that a point chosen at random lies in the shaded sector. **14.** ______



F. 0.50 **H.** 0.17

G. 0.33 **J.** 0.08

15. Find the area of the shaded segments. **15.** ______



A. about 15.3 $units^2$

B. about 7.6 $units^2$

C. about 3.8 $units^2$

D. about 3.1 $units^2$

B: ______________

Bonus A rhombus has an area of 165 square units. If the length of one of its diagonals is 15 units, find the length of its other diagonal.

NAME ________________________________ DATE ____________ PERIOD _____

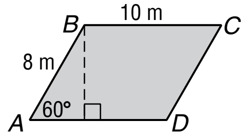
11

Chapter 11 Test, Form 2A

SCORE _____

Write the letter for the correct answer in the blank at the right of each question.

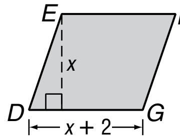
1. Find the area of parallelogram *ABCD*. Round to the nearest tenth. **1.** _____



A. $55.4\ m^2$ **C.** $69.3\ m^2$

B. $60\ m^2$ **D.** $80\ m^2$

2. The area of parallelogram *DEFG* is 143 square units. Find the lengths of the height and the base. Round to the nearest tenth. **2.** _____



F. 8, 10 **H.** 47, 49

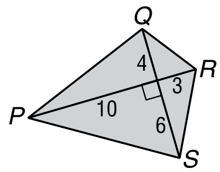
G. 11, 13 **J.** 70.5, 72.5

3. What is the best classification of quadrilateral *WXYZ* with vertices $W(-1, 0)$, $X(4, 0)$, $Y(2, -3)$, and $Z(-3, -3)$? **3.** _____

A. parallelogram **C.** square

B. rectangle **D.** none of these

4. Find the area of quadrilateral *PQRS*. **4.** _____



F. $34.1\ units^2$ **H.** $130\ units^2$

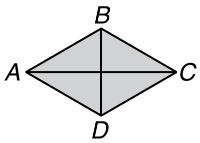
G. $65\ units^2$ **J.** $360\ units^2$

5. Find the area of trapezoid *ABCD* with vertices $A(2, 2)$, $B(4, 6)$, $C(4, -3)$, and $D(2, -1)$. **5.** _____

A. $27\ units^2$ **C.** $18\ units^2$

B. $22.5\ units^2$ **D.** $12\ units^2$

6. Rhombus *ABCD* has an area of 264 square units. If $DB = 12$ units, find *AC*. **6.** _____



F. 44 units **H.** 18 units

G. 22 units **J.** 12 units

7. Find the area of a regular hexagon with side length of 10 centimeters. Round to the nearest tenth. **7.** _____

A. $129.9\ cm^2$ **C.** $259.8\ cm^2$

B. $150\ cm^2$ **D.** $519.6\ cm^2$

8. Find the area of a nonagon with a perimeter of 126 inches. Round to the nearest tenth. **8.** _____

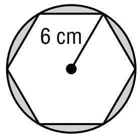
F. $1289.4\ in^2$ **H.** $466.2\ in^2$

G. $1211.6\ in^2$ **J.** $157.5\ in^2$

Assessment

11 Chapter 11 Test, Form 2A *(continued)*

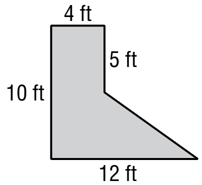
9. Find the area of the shaded region. Round to the nearest tenth. **9.** _____



A. 59.1 cm^2 **C.** 25.7 cm^2

B. 57.5 cm^2 **D.** 19.6 cm^2

10. Find the area of the figure. **10.** _____



F. 31 ft^2 **H.** 60 ft^2

G. 40 ft^2 **J.** 80 ft^2

11. A running track consists of two parallel lines that are connected at each end by the curved boundary of a semicircle. The parallel lines are 30 meters long and 7 meters apart. Find the area of the running track. **11.** _____

A. 229.24 **C.** 312.46

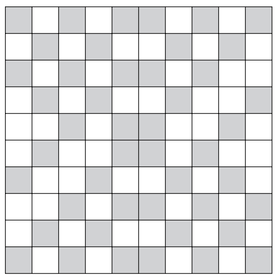
B. 248.48 **D.** 363.93

12. Find the area of pentagon *ABCDE* with vertices *A*(1, 1), *B*(4, 1), *C*(4, −3), *D*(2.5, −5) and *E*(1, −3). **12.** _____

F. 15 $units^2$ **H.** 21 $units^2$

G. 19 $units^2$ **J.** 23 $units^2$

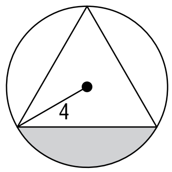
13. Find the probability that a point chosen at random lies in the shaded region. **13.** _____



A. about 0.92 **C.** about 0.55

B. about 0.75 **D.** about 0.46

14. Find the probability that a point chosen at random lies in the shaded region. **14.** _____



F. about 0.17 **H.** about 0.25

G. about 0.20 **J.** about 0.33

Bonus Find the area of a circle circumscribed about a regular pentagon with a perimeter of 50 inches. Round to the nearest tenth. **B:** _______________

NAME ____________________ DATE __________ PERIOD _____

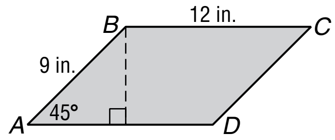
11 Chapter 11 Test, Form 2B

SCORE _____

Assessment

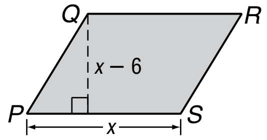
Write the letter for the correct answer in the blank at the right of each question.

1. Find the area of parallelogram *ABCD*. Round to the nearest tenth. **1.** _____



A. 54 in^2 **C.** 95.2 in^2
B. 76.4 in^2 **D.** 152.7 in^2

2. The area of parallelogram *PQRS* is 187 square units. Find the lengths of the height and the base. Round to the nearest tenth. **2.** _____

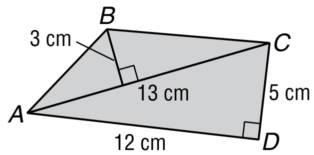


F. 15, 12.5 **H.** 12, 15.6
G. 13.5, 7.5 **J.** 11, 17

3. What is the best classification of quadrilateral *WXYZ* with vertices $W(1, 3)$, $X(2, 1)$, $Y(0, 0)$, and $Z(-1, 2)$? **3.** _____

A. parallelogram **C.** square
B. rectangle **D.** none of these

4. Find the area of quadrilateral *ABCD*. **4.** _____

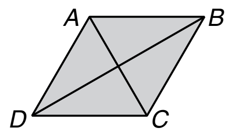


F. 49.5 cm^2 **H.** 60 cm^2
G. 52 cm^2 **J.** 97.5 cm^2

5. Find the area of trapezoid *ABCD* with vertices $A(2, 2)$, $B(4, -2)$, $C(-3, -2)$, and $D(-1, 2)$. **5.** _____

A. 30 $units^2$ **C.** 24 $units^2$
B. 25 $units^2$ **D.** 20 $units^2$

6. Rhombus *ABCD* has an area of 126 square units. If $DB = 18$ units, find *AC*. **6.** _____



F. 18 units **H.** 7 units
G. 14 units **J.** 3.5 units

7. Find the area of an equilateral triangle with a side length of 12 centimeters. Round to the nearest tenth. **7.** _____

A. 187.1 cm^2 **C.** 62.4 cm^2
B. 93.5 cm^2 **D.** 54 cm^2

8. Find the area of an octagon with a perimeter of 80 inches. Round to the nearest tenth. **8.** _____

F. 965.7 in^2 **H.** 165.7 in^2
G. 482.8 in^2 **J.** 82.8 in^2

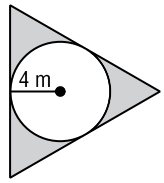
NAME ______________________ DATE ____________ PERIOD _____

11 Chapter 11 Test, Form 2B *(continued)*

9. Find the area of the shaded region. Round to the nearest tenth.

A. 12.6 m^2 **C.** 32.9 m^2

B. 24.6 m^2 **D.** 44.9 m^2

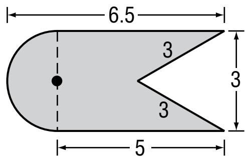


9. ______

10. Find the area of the figure. Round to the nearest tenth.

F. 14.6 units^2 **H.** 18.2 units^2

G. 15 units^2 **J.** 22.4 units^2



10. ______

11. Gerry wants to have a cover made for his swimming pool which consists of two parallel lines that are connected at each end by the curved boundary of a semicircle. The parallel lines are 12 feet long and 10 feet apart. Find the area of the swimming pool cover.

A. 572.39 **C.** 434.16

B 233.02 **D.** 198.54

11. ______

12. Find the area of pentagon *ABCDE* with vertices $A(0, 4)$, $B(3, 2)$, $C(3, -1)$, $D(-3, -1)$ and $E(-3, 2)$.

F. 24 units^2 **H.** 30 units^2

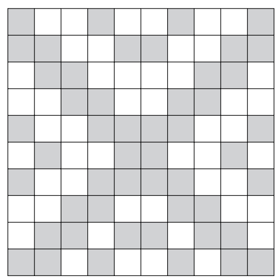
G. 27 units^2 **J.** 33 units^2

12. ______

13. Find the probability that a point chosen at random lies in the shaded region.

A. 0.47 **C.** 0.53

B. 0.50 **D.** 0.75

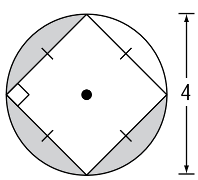


13. ______

14. Find the probability that a point chosen at random lies in the shaded region.

F. about 0.09 **H.** about 0.50

G. about 0.27 **J.** about 0.75



14. ______

Bonus Find the area of a circle circumscribed about a regular hexagon with an apothem of 5 inches. Round to the nearest tenth.

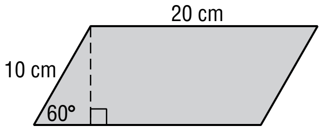
B: ______________

NAME ______________________ DATE __________ PERIOD _____

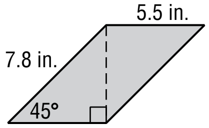
11 Chapter 11 Test, Form 2C

SCORE _____

For Questions 1 and 2, find the area of each parallelogram. Round to the nearest tenth if necessary.

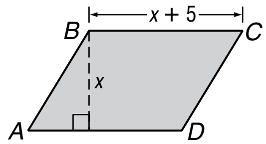
1. 

1. ____________

2. 

2. ____________

3. The area of parallelogram *ABCD* is 2250 square meters. Find the lengths of the height and base.

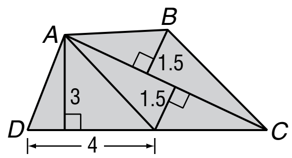


3. ____________

4. Classify quadrilateral *ABCD*, with vertices $A(-1, 4)$, $B(3, 4)$, $C(3, 1)$, and $D(-1, 1)$. List all that apply.

4. ____________

5. Find the area of quadrilateral *ABCD* if $AC = 7$. Round to the nearest tenth.



5. ____________

For Questions 6 and 7, find the area of each quadrilateral given the coordinates of the vertices.

6. trapezoid *GHIJ*; $G(-2, 3)$, $H(1, 3)$, $I(2, -1)$, and $J(-3, -1)$

6. ____________

7. rhombus *KLMN*; $K(4, 4)$, $L(6, 0)$, $M(4, -4)$, and $N(2, 0)$

7. ____________

For Questions 8–10, find the area of each polygon. Round to the nearest tenth.

8. a square with a perimeter of $16\sqrt{2}$ inches

8. ____________

9. a regular hexagon with apothem length of 4.3 centimeters

9. ____________

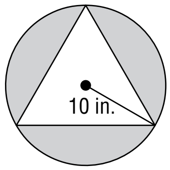
10. an equilateral triangle with side length of 10.4 meters

10. ____________

Assessment

11 Chapter 11 Test, Form 2C *(continued)*

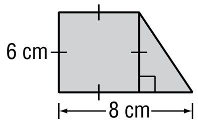
11. Find the area of the shaded region to the nearest tenth. Assume that the triangle is equilateral.



11. ____________

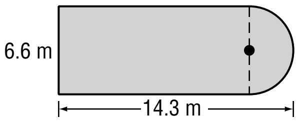
For Questions 12–14, find the area of each figure. Round to the nearest tenth if necessary.

12.



12. ____________

13.

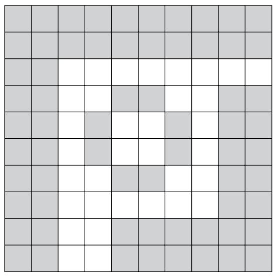


13. ____________

14. pentagon $RSTUV$ with vertices $R(5, 5)$, $S(5, 0)$, $T(2, -3)$, $U(-1, 0)$, and $V(-1, 2)$

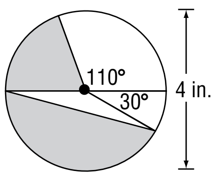
14. ____________

15. A children's game is won by tossing a coin so that it lands on the white part of this board. If one coin is tossed, what is the probability of winning?



15. ____________

For Questions 16 and 17, refer to the figure at the right. Round to the nearest tenth if necessary.



16. Find the probability that a point chosen at random lies in the shaded sector.

16. ____________

17. Find the probability that a point chosen at random lies in the shaded segment.

17. ____________

Bonus If the length of the height of a trapezoid is 4 meters, the length of one of its bases is 11 meters, and its area is 62 square meters, then what is the measure of the other base?

B: ____________

NAME ______________________ DATE __________ PERIOD _____

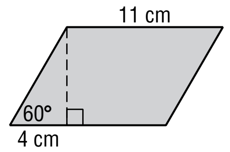
SCORE _____

11 Chapter 11 Test, Form 2D

Assessment

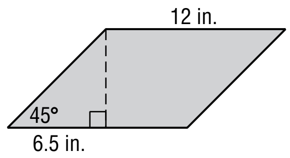
For Questions 1 and 2, find the area of each parallelogram. Round to the nearest tenth if necessary.

1.



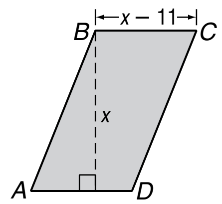
1. __________

2.



2. __________

3. If the area of parallelogram *ABCD* is 570 square meters, find the lengths of the height and base.

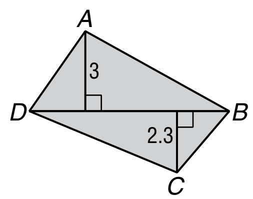


3. __________

4. Classify quadrilateral *ABCD*, with vertices $A(1, 1)$, $B(1, -3)$, $C(-3, -3)$, and $D(-3, 1)$. List all that apply.

4. __________

5. Find the area of quadrilateral *ABCD*, if $DB = 7.5$. Round to the nearest tenth.



5. __________

For Questions 6 and 7, find the area of each quadrilateral given the coordinates of the vertices.

6. trapezoid *GHIJ*; $G(-2, 1)$, $H(2, 3)$, $I(2, -3)$, and $J(-2, -1)$

6. __________

7. rhombus *KLMN*; $K(-3, 7)$, $L(0, 3)$, $M(-3, -1)$, and $N(-6, 3)$

7. __________

For Questions 8–10, find the area of each polygon. Round to the nearest tenth if necessary.

8. a square with apothem length of 3 inches

8. __________

9. a regular hexagon with side length of 15 centimeters

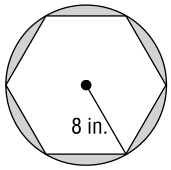
9. __________

10. an equilateral triangle with a perimeter of 42 meters

10. __________

11 Chapter 11 Test, Form 2D *(continued)*

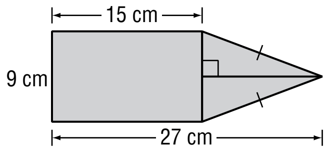
11. Find the area of the shaded region to the nearest tenth. Assume that the hexagon is regular.



11. ____________________

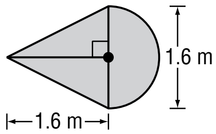
For Questions 12–14, find the area of each figure. Round to the nearest tenth if necessary.

12.



12. ____________________

13.

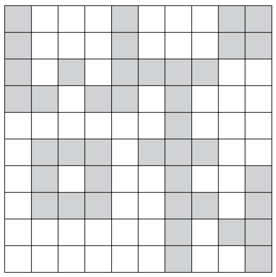


13. ____________________

14. pentagon *RSTUV* with vertices $R(4, -2)$, $S(0, -1)$, $T(-3, -1)$, $U(0, -4)$, and $V(4, -4)$

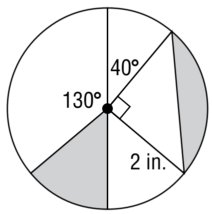
14. ____________________

15. A group of children are tossing a coin on this game board. To win the game the coin must land on a shaded part of the board. What is the probability of winning?



15. ____________________

For Questions 16 and 17, refer to the figure at the right. Round to the nearest hundredth.



16. Find the probability that a point selected at random lies in the shaded sector.

16. ____________________

17. Find the probability that a point selected at random lies in the shaded segment.

17. ____________________

Bonus If one diagonal of a rhombus is 15 meters long and its area is 157.5 square meters, find the measure of the other diagonal.

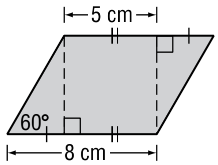
B: ____________________

11 Chapter 11 Test, Form 3

SCORE _____

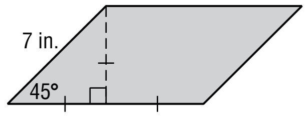
For Questions 1 and 2, find the area of each parallelogram. Round to the nearest tenth if necessary.

1.



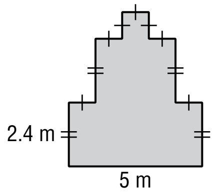
1. ____________

2.



2. ____________

3. Find the area of the figure.

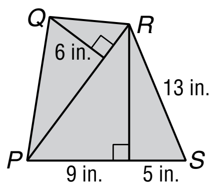


3. ____________

4. Classify quadrilateral *ABCD* with vertices *A*(4, 1), *B*(8, −2), *C*(4, −5), and *D*(0, −2). List all that apply.

4. ____________

5. Find the area of quadrilateral *PQRS*. Round to the nearest tenth.



5. ____________

6. Find the area of trapezoid *GHIJ* with vertices *G*(−2, 1), *H*(8, 7), *I*(6, −1), and *J*(1, −4).

6. ____________

7. Find the area of a rhombus with a perimeter of 100 meters and one diagonal with a length of 48 meters.

7. ____________

For Questions 8–10, find the area of each polygon. Round to the nearest tenth.

8. a regular octagon with perimeter of 96 meters

8. ____________

9. a regular pentagon with apothem length of 5 inches

9. ____________

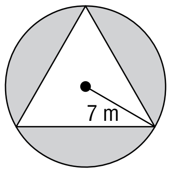
10. a regular nonagon with side length of 12 centimeters

10. ____________

Assessment

11 Chapter 11 Test, Form 3 *(continued)*

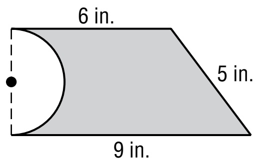
11. Find the area of the shaded region to the nearest tenth. Assume that the triangle is equiangular.



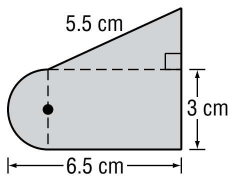
11. ____________________

For Questions 12–14, find the area of each figure. Round to the nearest tenth if necessary.

12.



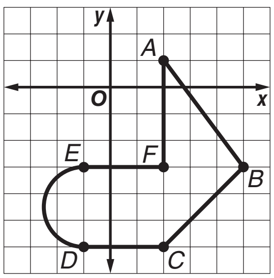
13.



12. ____________________

13. ____________________

14.

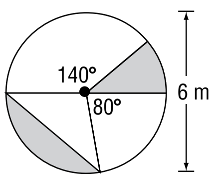


14. ____________________

15. To win a certain board game, a tossed token must land on a shaded square on the board. The probability of winning is approximately 23%. If the board has a total area of 130 congruent squares, how many of these squares are shaded?

15. ____________________

For Questions 16 and 17, use the figure at the right. Round to the nearest hundredth.



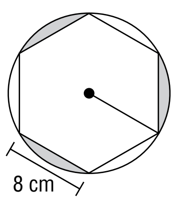
16. What is the probability that a point chosen at random lies in the shaded sector?

16. ____________________

17. What is the probability that a point chosen at random lies in the shaded segment?

17. ____________________

Bonus Find the area of the shaded region to the nearest tenth. Assume that the hexagon is regular.



B: ____________________

NAME ______________________________ DATE ____________ PERIOD _____

11 Chapter 11 Extended-Response Test

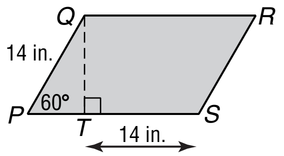
SCORE _____

Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. a. Explain how to determine whether quadrilateral *ABCD* with vertices *A*(−3, 3), *B*(0, 0), *C*(−3, −3), and *D*(−6, 0) is a *square*, a *rectangle* or a *parallelogram*.

b. Find the area of quadrilateral *ABCD*.

2. a. Explain how a 30°-60°-90° triangle is used in finding the area of parallelogram *PQRS*.

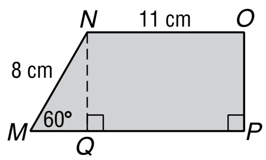


b. Find the area of parallelogram *PQRS* to the nearest tenth.

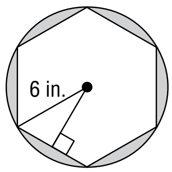
3. Explain how to find the area of each figure described below, then find each area. Round to the nearest tenth if necessary.

a. a rhombus with vertices *A*(4, 6), *B*(6, 2), *C*(4, −2), and *D*(2, 2)

b.

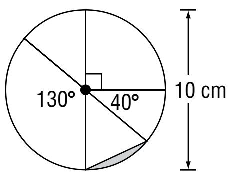


c. the shaded region



d. a pentagon with vertices *A*(1, 4), *B*(5, 4), *C*(5, −1), *D*(1, −2) and *E*(−3, −1)

4.



a. Explain how to find the probability that a point chosen at random lies in the shaded region.

b. Find the probability that a point chosen at random lies in the shaded region.

Assessment

NAME ______________________ DATE __________ PERIOD _____

11 Standardized Test Practice

SCORE _____

(Chapters 1–11)

Part 1: Multiple Choice

Instructions: Fill in the appropriate circle for the best answer.

1. Mai knows that if two arcs are congruent, then their corresponding central angles are congruent. She is given $\odot D$ with $\widehat{BC} \cong \widehat{GH}$, and she concludes that $\angle BDC \cong \angle GDH$. Which form of reasoning does she use? (Lesson 2-4)

A Inductive Reasoning **C** Law of Syllogism
B Law of Detachment **D** Formal Proof

1. Ⓐ Ⓑ Ⓒ Ⓓ

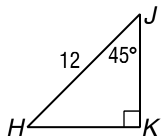
2. The distance between A and B is 17.8 centimeters, and the distance between B and C is 9.5 centimeters. If A, B, and C are noncollinear, which inequality represents the possible distance between A and C? (Lesson 5-4)

F 9.5 cm $< AC <$ 17.8 cm **H** 10 cm $< AC <$ 18 cm
G 8.3 cm $< AC <$ 27.3 cm **J** 8 cm $< AC <$ 27 cm

2. Ⓕ Ⓖ Ⓗ Ⓙ

3. Find HK. (Lesson 7-3)

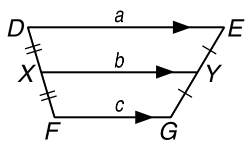
A $3\sqrt{2}$ **C** $6\sqrt{2}$
B 6 **D** $2\sqrt{3}$



3. Ⓐ Ⓑ Ⓒ Ⓓ

4. Which expression can you use to find a? (Lesson 8-6)

F $c^2 - b^2$ **H** $\frac{b + c}{2}$
G $2b - c$ **J** $2c - b$



4. Ⓕ Ⓖ Ⓗ Ⓙ

5. If $\overline{ST}$ with endpoints $S(3, -7)$ and $T(-5, -2)$ is reflected in the line $y = x$, find the coordinates of $\overline{S'T'}$. (Lesson 9-1)

A $S'(-3, -7)$ and $T'(5, -2)$ **C** $S'(-3, 7)$ and $T'(5, 2)$
B $S'(3, 7)$ and $T'(-5, 2)$ **D** $S'(-7, 3)$ and $T'(-2, -5)$

5. Ⓐ Ⓑ Ⓒ Ⓓ

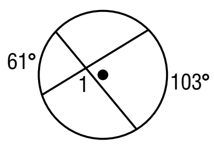
6. Find the circumference of a circle with a radius of 26.5 centimeters. (Lesson 10-1)

F 26.5π cm **G** 53π cm **H** 702.25π cm **J** 2809π cm

6. Ⓕ Ⓖ Ⓗ Ⓙ

7. Find $m\angle 1$. (Lesson 10-6)

A 61 **C** 82
B 98 **D** 103



7. Ⓐ Ⓑ Ⓒ Ⓓ

8. Find the area of a regular hexagon with a perimeter of 72 inches. Round to the nearest square inch. (Lesson 11-3)

F 72 in^2 **G** 432 in^2 **H** 374 in^2 **J** 864 in^2

8. Ⓕ Ⓖ Ⓗ Ⓙ

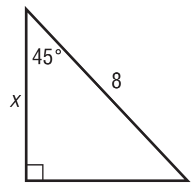
11 Standardized Test Practice *(continued)*

9. If $LM = 3x + 2$, $MN = 7x - 18$, and $NL = 22 - x$, find the length of the sides of equilateral triangle LMN. (Lesson 4-1)

A 5 **B** 10 **C** 12 **D** 17

9. Ⓐ Ⓑ Ⓒ Ⓓ

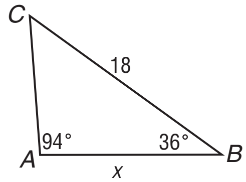
10. Find x. (Lesson 8-3)



F $\frac{4}{\sqrt{4}}$ **G** 4 **H** $4\sqrt{2}$ **J** $8\sqrt{2}$

10. Ⓕ Ⓖ Ⓗ Ⓙ

11. Find x to the nearest tenth. (Lesson 8-6)



A 4.70 **B** 12.77 **C** 13.82 **D** 21.17

11. Ⓐ Ⓑ Ⓒ Ⓓ

12. The diameter of a circle is 34 inches, and a chord of the circle 18 inches. Find the distance between the chord and the center of the circle to the nearest tenth. (Lesson 10-3)

F 14.4 **G** 15.6 **H** 17.3 **J** 19.23

12. Ⓕ Ⓖ Ⓗ Ⓙ

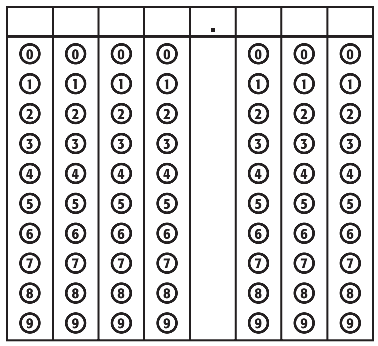
13. Ⓐ Ⓑ Ⓒ Ⓓ

Part 2: Griddable

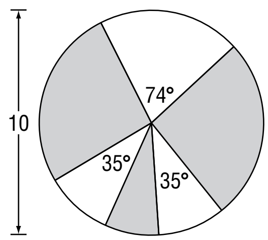
Instructions: Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate circle that corresponds to that entry.

13. Angles B and G are opposite angles of a parallelogram. Find $m\angle G$ if $m\angle B = 3x + 80$ and $m\angle G = 140 - x$. (Lesson 8-2)

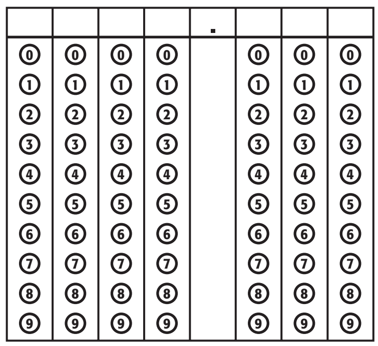
13.



14. Find the probability as a percent that a point chosen at random lies in the unshaded region. (Lesson 11-5)



14.



Assessment

NAME ______________________________ DATE ____________ PERIOD _____

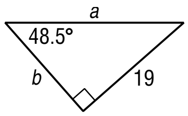
11 Standardized Test Practice *(continued)*

SCORE _____

Part 3: Short Response

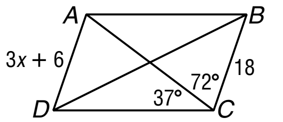
Instructions: Write your answer in the space provided.

15. Find the values of a and b to the nearest tenth. (Lesson 8-3)



15. ____________________

16. If $ABCD$ is a parallelogram, find $m\angle DAB$, $m\angle CDA$, and x. (Lesson 6-3)

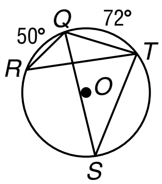


16. ____________________

17. A plane is flying due south at 310 miles per hour and the wind is blowing from the east at 40 miles per hour. Find the resultant speed and direction of the plane to the nearest tenth. (Lesson 9-6)

17. ____________________

18. Find $m\angle R$, $m\angle S$, $m\angle QTR$, and $m\angle QTS$ if $\overset{\frown}{SR} \cong \overset{\frown}{TS}$. (Lesson 10-4)

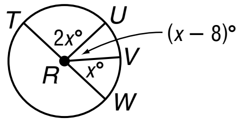


18. ____________________

19. If $\triangle XYZ$ with vertices $X(0, 3)$, $Y(4, 7)$, and $Z(-5, 4)$ is reflected in the x-axis and then the y-axis, find the coordinates of the rotated image. (Lesson 9-3)

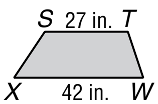
19. ____________________

20. Find $m\angle TRU$, $m\angle URV$, and $m\angle VRW$. (Lesson 10-2)



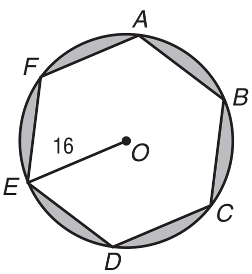
20. ____________________

21. Trapezoid $STWX$ has an area of 517.5 square inches. Find the height of $STWX$. (Lesson 11-2)



21. ____________________

22. Given the regular polygon $ABCDEF$:



a. Find the sum of the interior angles of $ABCDEF$. (Lesson 6-1)

22a. ____________________

b. Find the shortest distance from $\overline{AB}$ to $\overline{ED}$. Round to the nearest tenth. (Lesson 10-3)

22b. ____________________

c. Find the area of the shaded region. Round to the nearest tenth. (Lesson 11-3)

22c. ____________________

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11 Anticipation Guide

Areas of Polygons and Circles

Step 1 ***Before you begin Chapter 11***

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

STEP 1 A, D, or NS	Statement	STEP 2 A or D
	1. The area of a parallelogram whose sides measure 5 cm and 9 cm is 5 cm × 9 cm or 45 cm^2.	D
	2. The area of a triangle is one-half its base times its height.	A
	3. Given the coordinates of the vertices of any quadrilateral *ABCD*, its area can be found by multiplying the length of side *AB* by the length of side *BC*.	D
	4. The area of a rhombus equals half the product of the lengths of its diagonals.	A
	5. A segment drawn from the center to a vertex of a regular polygon is called an apothem.	D
	6. The formula for the area of a circle is $A = \pi r^2$.	A
	7. The area of an irregular figure can be found by separating the figure into shapes with known area formulas.	A
	8. If an irregular figure is in the shape of a pentagon, then the formula for the area of a regular pentagon can be used to find its area.	D
	9. A sector of a circle with a central angle of 35° will have an area of $\frac{35}{360}\pi r^2$.	A
	10. A segment of a circle is the triangular region of a circle bound by a chord and two radii.	D

Step 2 ***After you complete Chapter 11***

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

NAME ____________ DATE ________ PERIOD ____

11-1 Lesson Reading Guide

Areas of Parallelograms

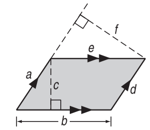
Get Ready for the Lesson

Read the introduction to Lesson 11-1 in your textbook.

How many 22-yard squares could fit in an acre?
10 squares.

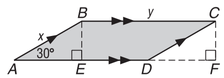
Read the Lesson

1. Which expression gives the area of the parallelogram? (Hint: There can be more than one correct response.) **B, D, E, G**



A. *ab* **B.** *cb* **C.** *ed*
D. *af* **E.** *ce* **F.** *cd*
G. *df* **H.** *bf* **I.** *cf*

2. Refer to the figure. Determine whether each statement is *true* or *false*. If the statement is false, explain why.



a. $\overline{AB}$ is an altitude of the parallelogram. **False; $\overline{AB}$ is not perpendicular to any side of the parallelogram.**

b. $\overline{CD}$ is a base of parallelogram *ABCD*.
true

c. The perimeter of *ABCD* is $(2x + 2y)$ $units^2$. **False; perimeter is measured in linear units, not square units. The perimeter is $(2x + 2y)$ units.**

d. $BE = CF$
true

e. $BE = \frac{\sqrt{3}}{2}x$ **False; $\overline{BE}$ is opposite the 30° angle in a 30°-60°-90° triangle, so $BE = \frac{1}{2}x$ or $\frac{x}{2}$.**

f. The area of *ABCD* is $2xy$ $units^2$. **False; since $BE = \frac{x}{2}$, the area of *ABCD* is $\frac{xy}{2}$ $units^2$.**

Remember What You Learned

3. A good way to remember a new formula in geometry is to relate it to a formula you already know. How can you use the formula for the area of a rectangle to help you remember the formula for the area of a parallelogram?
Sample answer: To find the area of a rectangle, you multiply the lengths of two segments that are perpendicular to each other. To find the area of a parallelogram, you do the same thing, but the height is not necessarily one of the sides.

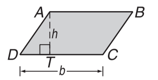
NAME ______________ DATE ________ PERIOD _____

11-1 Study Guide and Intervention

Areas of Parallelograms

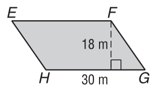
Areas of Parallelograms A parallelogram is a quadrilateral with both pairs of opposite sides parallel. Any side of a parallelogram can be called a **base**. Each base has a corresponding **altitude**, and the length of the altitude is the **height** of the parallelogram. The area of a parallelogram is the product of the base and the height.

Area of a Parallelogram	If a parallelogram has an area of A square units, a base of b units, and a height of h units, then $A = bh$.



The area of parallelogram $ABCD$ is $CD \cdot AT$.

Example **Find the area of parallelogram *EFGH*.**

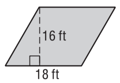


$A = bh$ Area of a parallelogram
$= 30(18)$ $b = 30, h = 18$
$= 540$ Multiply.

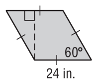
The area is 540 square meters.

Exercises

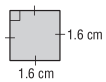
Find the area of each parallelogram.

1. 

288 ft^2

2. 

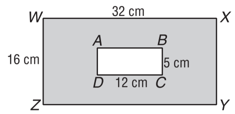
$288\sqrt{3} \text{ in}^2$

3. 

2.56 cm^2

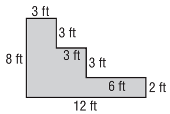
Find the area of each shaded region.

4. *WXYZ* and *ABCD* are rectangles.



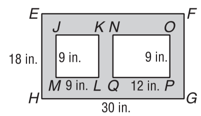
452 cm^2

5. All angles are right angles.



51 ft^2

6. *EFGH* and *NOPQ* are rectangles; *JKLM* is a square.



351 in^2

7. The area of a parallelogram is 3.36 square feet. The base is 2.8 feet. If the measures of the base and height are each doubled, find the area of the resulting parallelogram. 13.44 ft^2

8. A rectangle is 4 meters longer than it is wide. The area of the rectangle is 252 square meters. Find the length. 18 m

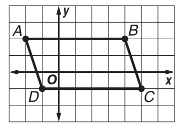
NAME ______________ DATE ________ PERIOD _____

11-1 Study Guide and Intervention *(continued)*

Areas of Parallelograms

Parallelograms on the Coordinate Plane To find the area of a quadrilateral on the coordinate plane, use the Slope Formula, the Distance Formula, and properties of parallelograms, rectangles, squares, and rhombi.

Example **The vertices of a quadrilateral are $A(-2, 2)$, $B(4, 2)$, $C(5, -1)$, and $D(-1, -1)$.**



a. Determine whether the quadrilateral is a *square*, a *rectangle*, or a *parallelogram*.

Graph the quadrilateral. Then determine the slope of each side.

slope of $\overline{AB} = \frac{2 - 2}{4 - (-2)}$ or 0

slope of $\overline{CD} = \frac{-1 - (-1)}{-1 - 5}$ or 0

slope $\overline{AD} = \frac{2 - (-1)}{-2 - (-1)}$ or -3

slope $\overline{BC} = \frac{-1 - 2}{5 - 4}$ or -3

Opposite sides have the same slope. The slopes of consecutive sides are not negative reciprocals of each other, so consecutive sides are not perpendicular. *ABCD* is a parallelogram; it is not a rectangle or a square.

b. Find the area of *ABCD*.

From the graph, the height of the parallelogram is 3 units and $AB = |4 - (-2)| = 6$.

$A = bh$ Area of a parallelogram
$= 6(3)$ $b = 6, h = 3$
$= 18 \text{ units}^2$ Multiply.

Exercises

Given the coordinates of the vertices of a quadrilateral, determine whether the quadrilateral is a *square*, a *rectangle*, or a *parallelogram*. Then find the area.

1. $A(-1, 2), B(3, 2), C(3, -2)$, and $D(-1, -2)$ square; 16 units^2

2. $R(-1, 2), S(5, 0), T(4, -3)$, and $U(-2, -1)$ rectangle; 20 units^2

3. $C(-2, 3), D(3, 3), E(5, 0)$, and $F(0, 0)$ parallelogram; 15 units^2

4. $A(-2, -2), B(0, 2), C(4, 0)$, and $D(2, -4)$ square; 20 units^2

5. $M(2, 3), N(4, -1), P(-2, -1)$, and $R(-4, 3)$ parallelogram; 24 units^2

6. $D(2, 1), E(2, -4), F(-1, -4)$, and $G(-1, 1)$ rectangle; 15 units^2

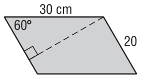
Lesson 11-1

NAME ______________________ DATE __________ PERIOD _____

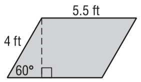
11-1 Skills Practice

Areas of Parallelograms

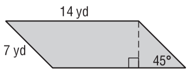
Find the perimeter and area of each parallelogram. Round to the nearest tenth if necessary.

1.


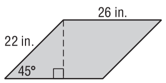
100 cm, 519.6 cm²

2.


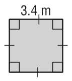
19 ft, 19.1 ft²

3.


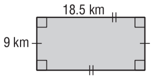
42 yd, 69.3 yd²

4.


96 in., 404.5 in²

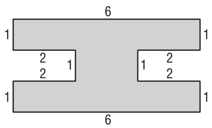
5.


13.6 m, 11.6 m²

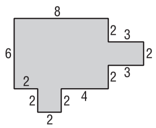
6.


55 km, 166.5 km²

Find the area of each figure.

7.


14 units²

8.


58 units²

COORDINATE GEOMETRY Given the coordinates of the vertices of a quadrilateral, determine whether it is a *square*, a *rectangle*, or a *parallelogram*. Then find the area of the quadrilateral.

9. $A(-4, 2), B(-1, 2), C(-1, -1), D(-4, -1)$
square, 9 units²

10. $P(-3, 3), Q(1, 3), R(1, -3), S(-3, -3)$
rectangle, 24 units²

11. $D(-5, 1), E(7, 1), F(4, -4), G(-8, -4)$
parallelogram, 60 units²

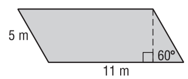
12. $R(2, 3), S(4, 10), T(12, 10), U(10, 3)$
parallelogram, 56 units²

NAME ______________________ DATE __________ PERIOD _____

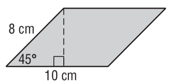
11-1 Practice

Areas of Parallelograms

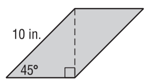
Find the perimeter and area of each parallelogram. Round to the nearest tenth if necessary.

1.


32 m, 47.6 m²

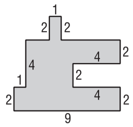
2.


36 cm, 56.6 cm²

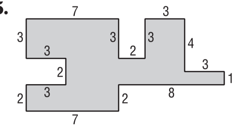
3.


34.1 in., 50 in²

Find the area of each figure.

4.


44 units²

5.


65 units²

COORDINATE GEOMETRY Given the coordinates of the vertices of a quadrilateral, determine whether it is a *square*, a *rectangle*, or a *parallelogram*. Then find the area of the quadrilateral.

6. $C(-4, -1), D(-4, 2), F(1, 2), G(1, -1)$
rectangle, 15 units²

7. $W(2, 2), X(1, -2), Y(-2, -2), Z(-1, 2)$
parallelogram, 12 units²

8. $M(0, 4), N(4, 6), O(6, 2), P(2, 0)$
square, 20 units²

9. $P(-5, 2), Q(4, 2), R(5, 5), S(-4, 5)$
parallelogram, 27 units²

FRAMING For Exercises 10–12, use the following information.

A rectangular poster measures 42 inches by 26 inches. A frame shop fitted the poster with a half-inch mat border.

10. Find the area of the poster. **1092 in²**

11. Find the area of the mat border. **69 in²**

12. Suppose the wall is marked where the poster will hang. The marked area includes an additional 12-inch space around the poster and frame. Find the total wall area that has been marked for the poster. **3417 in²**

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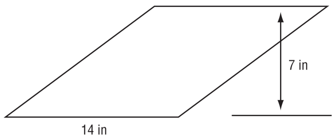
11-1 Word Problem Practice

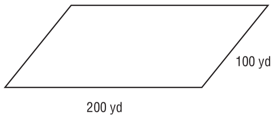
Areas of Parallelograms

1. **PACKAGING** A box with a square opening is squashed into the rhombus shown below.



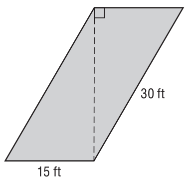
What is the area of the opening?
98 in^2

2. **RUNNING** Jason jogs once around a city block shaped like a parallelogram.



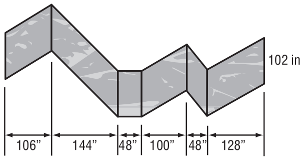
How far did Jason jog?
600 yd

3. **SHADOWS** A rectangular billboard casts a shadow on the ground in the shape of a parallelogram. What is the area of the ground covered by the shadow? Round your answer to the nearest tenth.



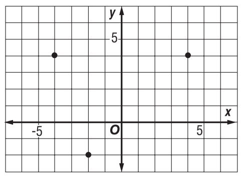
389.7 ft^2

4. **PATHS** A concrete path shown below is made by joining several parallelograms.



What is the total area of the path?
58,548 in^2

HIGHWAY SUPPORTS For Exercises 5 and 6, use the following information.
Four columns are being placed at the vertices of a parallelogram to support a highway. Three of the columns are marked on the coordinate plane shown.



5. What are the coordinates of the three possible locations of the fourth column?
(6, –2), (2, 10), and (–10, –2)

6. What is the area in square units of each of the three parallelograms that result from the possibilities you found in Exercise 5? Explain.
**Sample answer:
All of the parallelograms are congruent. The width of each is 8 units and the height of each is 6 units, so the area of the parallelograms is 8 x 6 or 48 units^2.**

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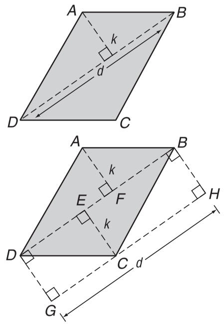
11-1 Enrichment

Area of a Parallelogram

You can prove some interesting results using the formula you have proved for the area of a parallelogram by drawing auxiliary lines to form congruent regions. Consider the top parallelogram shown at the right. In the figure, d is the length of the diagonal $\overline{BD}$, and k is the length of the perpendicular segment from A to $\overline{BD}$. Now consider the second figure, which shows the same parallelogram with a number of auxiliary perpendiculars added. Use what you know about perpendicular lines, parallel lines, and congruent triangles to answer the following.



1. What kind of figure is $DBHG$?
rectangle

2. If you moved $\triangle AFB$ to the lower-left end of figure $DBHG$, would it fit perfectly on top of $\triangle DGC$? Explain your answer.
Yes; $\triangle AFB \cong \triangle CED$ (by HA) and $\triangle CED \cong \triangle DGC$ (since $\overline{DC}$ is a diagonal of rectangle $DECD$). So $\triangle AFB \cong \triangle DGC$.

3. Which two triangular pieces of $\square ABCD$ are congruent to $\triangle CBH$?
$\triangle DAF$ and $\triangle BCE$

4. The area of $\square ABCD$ is the same as that of figure $DBHG$, since the pieces of $\square ABCD$ can be rearranged to form $DBHG$. Express the area of $\square ABCD$ in terms of the measurements k and d.
Area of $\square ABCD = dk$

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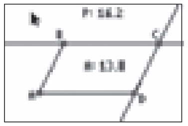
11-1 Graphing Calculator Activity

Cabri Junior: Areas of Parallelograms

Cabri Junior can be used to find the perimeters and areas of parallelograms.

Step 1 Draw a parallelogram.
- Select **F2 Segment** to draw a segment.
- Select **F5 Alph-num** to label the endpoints of the segment *A* and *B*.
- Draw segment *AD*.
- Select **F3 Parallel** to draw a line parallel to segment *AB* through *D*. Select point *D*, and then segment *AB*.
- Draw a line parallel to segment *AD* through *B*.
- Select **F2 Point, Intersection** to place a point at the intersection of the two lines drawn. Label the point *C*.
- Select **F2 Quad** and draw a quadrilateral by selecting points *A, B, C,* and *D*.

Step 2 Find the measure of the area of parallelogram *ABCD*.
- Select **F5 Measure, Area.**
- Place the cursor on any segment of parallelogram *ABCD*. Then press ENTER .
- The area appears with the hand attached. Move the number to an appropriate place.



Step 3 Find the measure of the perimeter of parallelogram *ABCD*.
- Select **F5 Measure, D. & Length.**
- Place the cursor on any segment of parallelogram *ABCD*. Then press ENTER .
- The area appears with the hand attached. Move the number to an appropriate place.

The perimeter of the parallelogram is 16.2 units and the area is 13.8 square units.

Exercises

Analyze your drawing.

1. Find the lengths of all four sides of the parallelogram. **See students' work.**
2. Using the information from Exercise 1, what is the perimeter of the parallelogram? Does this measurement match that found by Cabri Jr.? **See students' work; yes.**
3. Construct the altitude of the parallelogram. What is the length of the altitude? **See students' work.**
4. What is the measure of the base? **See students' work.**
5. Using the information from Exercises 3 and 4, what is the area of the parallelogram? Does this measurement match the one found by Cabri Jr.? **See students' work; yes.**
6. Select one of the vertices and drag it to change the dimensions of the parallelogram. (Press CLEAR so the pointer becomes a black arrow. Move the pointer close to a vertex until the arrow becomes transparent and the vertex is blinking. Press ALPHA to change the arrow to a hand. Then move the vertex.) Do you see any patterns or relationships? **See students' work.**

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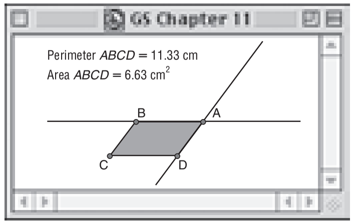
11-1 Geometer's Sketchpad Activity

Areas of Parallelograms

The Geometer's Sketchpad can be used to find the perimeters and areas of parallelograms.

Step 1 Use The Geometer's Sketchpad to draw a parallelogram.
- Construct a segment by selecting the Segment tool from the toolbar. First, click the first point. Then click on a second point to draw the segment.
- Next, use one of the endpoints of the original segment as the first point for the new segment and click on a second point to construct the new segment.
- Construct a parallel line to the original segment by first highlighting the original segment and the endpoint not on that segment. Then select **Parallel Line** from the **Construct** menu.
- Construct a parallel line to the second segment by highlighting the second segment and the point not on it. Then select **Parallel Line** from the **Construct** menu.
- Next, construct a point on the intersection of the two lines. Use the Point tool from the toolbar to select the point where the two lines intersect.
- Construct the interior of the parallelogram by highlighting all four points and selecting **Quadrilateral Interior** under the **Construct** menu.

Step 2 Use The Geometer's Sketchpad to find the perimeter of the parallelogram.
- Highlight the interior of the parallelogram using the Selection Arrow tool from the toolbar.
- Next, find the perimeter by selecting **Perimeter** under the **Measure** menu.



Step 3 Use The Geometer's Sketchpad to find the area of the parallelogram.
- Highlight the interior of the parallelogram using the Selection Arrow tool from the toolbar.
- Next, find the area by selecting **Area** under the **Measure** menu.

The perimeter of the parallelogram is 11.33 cm and the area is 6.63 cm^2.

Exercises

Analyze your drawing.

1. Find the lengths of all four sides of the parallelogram. **See students' work.**
2. Using the information from Exercise 1, what is the perimeter of the parallelogram? Does this measurement match that found by The Geometer's Sketchpad? **See students' work; yes.**
3. Construct the altitude of the parallelogram. What is the length of the altitude? **See students' work.**
4. What is the measure of the base? **See students' work.**
5. Using the information from Exercises 3 and 4, what is the area of the parallelogram? Does this measurement match the one found by The Geometer's Sketchpad? **See students' work; yes.**
6. Select one of the vertices and drag it to change the dimensions of the parallelogram. Do you see any patterns or relationships? **See students' work.**

Lesson 11-1

Answers

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11-2 Lesson Reading Guide

Areas of Triangles, Trapezoids, and Rhombi

Get Ready for the Lesson

Read the introduction to Lesson 11-2 in your textbook.

Classify the polygons in the panels of the beach umbrella.
Isosceles triangles and isosceles trapezoids

Read the Lesson

1. Match each area formula from the first column with the corresponding polygon in the second column.

a. $A = \ell w$ **vi**	**i.** triangle
b. $A = \frac{1}{2}d_1d_2$ **iv**	**ii.** parallelogram
c. $A = s^2$ **v**	**iii.** trapezoid
d. $A = \frac{1}{2}h(b_1 + b_2)$ **iii**	**iv.** rhombus
e. $A = \frac{1}{2}bh$ **i**	**v.** square
f. $A = bh$ **ii**	**vi.** rectangle

2. Determine whether each statement is *always*, *sometimes*, or *never* true. In each case, explain your reasoning. **For explanations, sample answers are given.**

a. The area of a square is half the product of its diagonals. **Always; a square is a rhombus, so you can use the rhombus formula.**

b. The area of a triangle is half the product of two of its sides. **Sometimes; this is true only for a right triangle.**

c. You can find the area of a rectangle by multiplying base times height. **Always; a rectangle is a parallelogram, so you can use the parallelogram formula. If the length of a rectangle is used as the base, then the width is the height.**

d. You can find the area of a rectangle by multiplying the lengths of any two of its sides. **Sometimes; this is true only for a square. Otherwise, you must use two *consecutive* sides, not any two sides.**

e. The area of a trapezoid is the product of its height and the sum of the bases. **Never; the area is one-half the product of its height and the sum of the bases.**

f. The square of the length of a side of a square is equal to half the product of its diagonals. **Always; a square is a rhombus, so the formulas for a square and a rhombus must give the same answer whenever the rhombus is a square.**

Remember What You Learned

3. A good way to remember a new geometric formula is to state it in words. Write a short sentence that tells how to find the area of a trapezoid in a way that is easy to remember. **Sample answer: Average the lengths of the bases and multiply by the height.**

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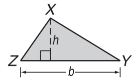
11-2 Study Guide and Intervention

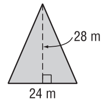
Areas of Triangles, Trapezoids, and Rhombi

Areas of Triangles The area of a triangle is half the area of a rectangle with the same base and height as the triangle.

If a triangle has an area of A square units, a base of b units, and a corresponding height of h units, then $A = \frac{1}{2}bh$.



Example **Find the area of the triangle.**



$A = \frac{1}{2}bh$ Area of a triangle

$= \frac{1}{2}(24)(28)$ $b = 24, h = 28$

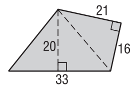
$= 336$ Multiply.

The area is 336 square meters.

Exercises

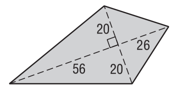
Find the area of each figure.

1.



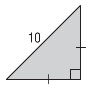
498 units^2

2.



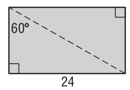
1640 units^2

3.



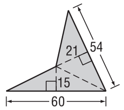
25 units^2

4.



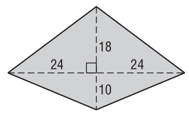
332.6 units^2

5.



1017 units^2

6.



672 units^2

7. The area of a triangle is 72 square inches. If the height is 8 inches, find the length of the base. **18 in.**

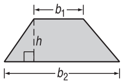
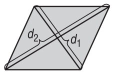
8. A right triangle has a perimeter of 36 meters, a hypotenuse of 15 meters, and a leg of 9 meters. Find the area of the triangle. **54 m^2**

NAME ______________________ DATE ________ PERIOD _____

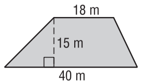
11-2 Study Guide and Intervention *(continued)*

Areas of Triangles, Trapezoids, and Rhombi

Areas of Trapezoids and Rhombi The area of a trapezoid is the product of half the height and the sum of the lengths of the bases. The area of a rhombus is half the product of the diagonals.

If a trapezoid has an area of A square units, bases of b_1 and b_2 units, and a height of h units, then $A = \frac{1}{2}h(b_1 + b_2)$. 	If a rhombus has an area of A square units and diagonals of d_1 and d_2 units, then $A = \frac{1}{2}d_1d_2$. 

Example **Find the area of the trapezoid.**



$A = \frac{1}{2}h(b_1 + b_2)$ — Area of a trapezoid

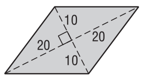
$= \frac{1}{2}(15)(18 + 40)$ — $h = 15$, $b_1 = 18$, $b_2 = 40$

$= 435$ — Simplify.

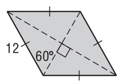
The area is 435 square meters.

Exercises

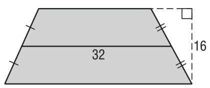
Find the area of each quadrilateral.

1. 

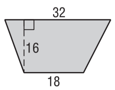
400 units²

2. 

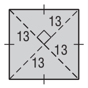
$72\sqrt{3}$ units²

3. 

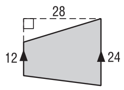
512 units²

4. 

400 units²

5. 

338 units²

6. 

504 units²

7. The area of a trapezoid is 144 square inches. If the height is 12 inches, find the length of the median. **12 in.**

8. A rhombus has a perimeter of 80 meters and the length of one diagonal is 24 meters. Find the area of the rhombus. **384 m²**

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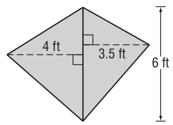
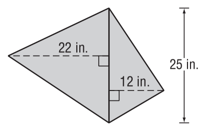
11-2 Skills Practice

Areas of Triangles, Trapezoids, and Rhombi

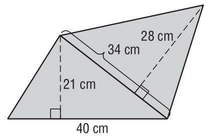
Find the area of each figure. Round to the nearest tenth if necessary.

1. 

22.5 ft²

2. 

425 in²

3. 

896 cm²

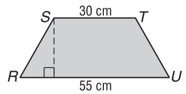
Find the area of each quadrilateral given the coordinates of the vertices.

4. trapezoid $WXYZ$
$W(-5, 3), X(3, 3), Y(6, -3), Z(-8, -3)$
66 units²

5. rhombus $HIJK$
$H(4, -3), I(2, -7), J(0, -3), K(2, 1)$
16 units²

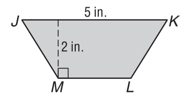
Find the missing measure for each figure.

6. Trapezoid $RSTU$ has an area of 935 square centimeters. Find the height of $RSTU$.



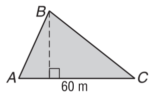
22 cm

7. Trapezoid $JKLM$ has an area of 7.5 square inches. Find ML.



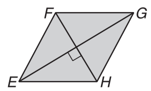
2.5 in.

8. Triangle ABC has an area of 1050 square meters. Find the height of $\triangle ABC$.



35 m

9. Rhombus $EFGH$ has an area of 750 square feet. If EG is 50 feet, find FH.



30 ft

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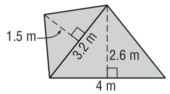
11-2 Practice

Areas of Triangles, Trapezoids, and Rhombi

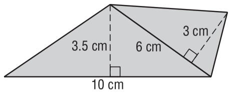
Find the area of each figure. Round to the nearest tenth if necessary.

1.



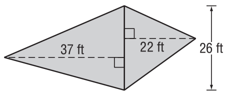
7.6 m^2

2.



26.5 cm^2

3.



767 ft^2

Find the area of each quadrilateral given the coordinates of the vertices.

4. trapezoid *ABCD*
$A(-7, 1), B(-4, 4), C(-4, -6), D(-7, -3)$

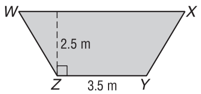
21 $units^2$

5. rhombus *LMNO*
$L(6, 8), M(14, 4), N(6, 0), O(-2, 4)$

64 $units^2$

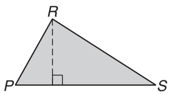
Find the missing measure for each figure.

6. Trapezoid *WXYZ* has an area of 13.75 square meters. Find *WX*.



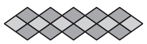
7.5 m

7. Triangle *PRS* has an area of 68 square yards. If the height of $\triangle PRS$ is 8 yards, find the base.



17 yd

DESIGN For Exercises 8 and 9, use the following information.
Mr. Hagarty used 16 congruent rhombi-shaped tiles to design the midsection of the backsplash area above a kitchen sink. The length of the design is 27 inches and the total area is 108 square inches.



8. Find the area of one rhombus.

$6\frac{3}{4}$ in^2

9. Find the length of each diagonal.

$4\frac{1}{2}$ in., 3 in.

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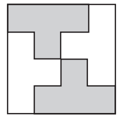
11-2 Word Problem Practice

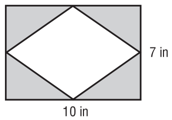
Areas of Triangles, Trapezoids, and Rhombi

1. INTERIOR DESIGN The 10 by 10 square shows an office floor plan composed of four congruent 8-sided cubicles. What is the area of one of these irregular 8-sided cubicles?



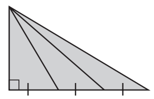
25 $units^2$

2. CUTOUTS Jeremy cut a rhombus out of a 10-inch by 7-inch rectangle. The diagonals of the rhombus are parallel and perpendicular to the sides of the rectangle and are congruent to the length and width of the rectangle, respectively. What is the total area of the four shaded triangles?



35 in^2

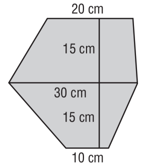
3. SHARING Bernard has a piece of cake that is shaped like a right triangle. He needs to cut it into three pieces to share it with two friends. He divides one of the legs into thirds and connects the division points to the opposite vertex of the triangle as shown in the figure.



Which piece is the largest?

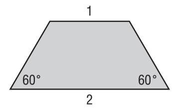
Since they all have the same base and height, all three triangles have the same area.

4. HEXAGONS Heather makes a hexagon by attaching two trapezoids together as shown. What is the area of the hexagon?

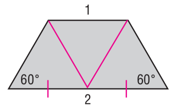


675 cm^2

TILINGS For Exercises 5 and 6, use the following information.
Tile making often requires an artist to find clever ways of dividing a shape into several smaller, congruent shapes. Consider the isosceles trapezoid shown below.

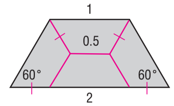


5. Show how to divide the trapezoid into 3 congruent triangles. What is the area of each triangle?



$\frac{\sqrt{3}}{4}$ $units^2$

6. Show how to divide the trapezoid into 4 congruent trapezoids. What is the area of each of the smaller trapezoids?



$\frac{3\sqrt{3}}{16}$ $units^2$

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11-2 Enrichment

Areas of Similar Triangles

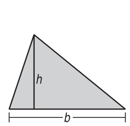
You have learned that if two triangles are similar, the ratio of the lengths of corresponding altitudes is equal to the ratio of the lengths of a pair of corresponding sides. However, there is a different relationship between the areas of the two triangles.

Theorem If two triangles are similar, the ratio of their areas is the square of the ratio of the lengths of a pair of corresponding sides.

Triangle II is k times larger than Triangle I. Thus, its base is k times as large as that of Triangle I and its height is k times as large as that of Triangle I.

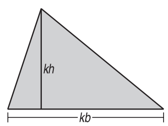
$\frac{\text{side of } \triangle II}{\text{side of } \triangle I} = \frac{kb}{b}$ or $\frac{k}{1}$

$\frac{\text{area of } \triangle II}{\text{area of } \triangle I} = \frac{\frac{1}{2}k^2bh}{\frac{1}{2}bh}$ or $\frac{k^2}{1}$



Triangle I

area $\triangle I = \frac{1}{2}bh$



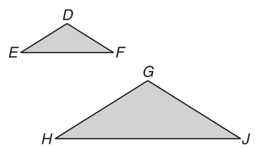
Triangle II

area $\triangle II = \frac{1}{2}(kb)(kh)$

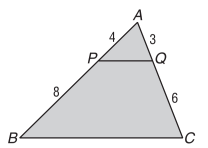
$= \frac{1}{2}k^2bh$

Solve.

1. $\triangle DEF \sim \triangle GHJ$, $HJ = 16$, and $EF = 8$. The area of $\triangle GHJ$ is 40. Find the area of $\triangle DEF$. **10**



2. In the figure below, $\overline{PQ} \parallel \overline{BC}$. The area of $\triangle ABC$ is 72. Find the area of $\triangle APQ$. **8**



3. Two similar triangles have areas of 16 and 36. The length of a side of the smaller triangle is 10 feet. Find the length of the corresponding side of the larger triangle.

15 ft

4. Find the ratio of the areas of two similar triangles if the lengths of two corresponding sides of the triangles are 3 centimeters and 5 centimeters.

$\frac{9}{25}$

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11-3 Lesson Reading Guide

Areas of Regular Polygons and Circles

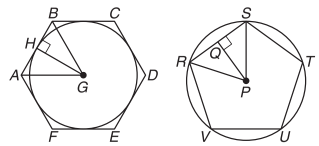
Get Ready for the Lesson

Read the introduction to Lesson 11-3 in your textbook.

Describe what type of triangles you could form by drawing the radii from the center of the octagon. **8 congruent isosceles triangles with angle measures 45, 67.5, and 67.5.**

Read the Lesson

1. *ABCDEF* and *RSTUV* are regular polygons. Name each of the following in one of the figures.



a. a circumscribed polygon **hexagon *ABCDEF***

b. an inscribed polygon **pentagon *RSTUV***

c. an apothem of a regular hexagon $\overline{GH}$

d. an isosceles triangle **$\triangle PRS$ or $\triangle GAB$**

e. a 30°-60°-90° triangle **$\triangle GAH$ or $\triangle GBH$**

f. a central angle with a measure of 72 **$\angle RPS$**

2. Refer to the figures in Exercise 1. Match each item in the first column with an expression in the second column.

a. perimeter of *ABCDEF* **v**	**i.** $\pi(PS)^2$
b. circumference of circle *G* **viii**	**ii.** $2\pi(PR)$
c. perimeter of *RSTUV* **vii**	**iii.** $\frac{5}{2}(RS)(PQ)$
d. area of circle *G* **vi**	**iv.** $3(AB)(HG)$
e. area of *RSTUV* **iii**	**v.** $6(CD)$
f. area of *ABCDEF* **iv**	**vi.** $\pi(GH)^2$
g. area of circle *P* **i**	**vii.** $5(UV)$
h. circumference of circle *P* **ii**	**viii.** $2\pi(GH)$

3. Explain in your own words how to find the area of a circle if you know the circumference. **Sample answer: Divide the circumference by 2π to find the radius. Then square the radius and multiply by π to find the area.**

Remember What You Learned

4. A good way to remember something is to explain it to someone else. Suppose your classmate Joelle is having trouble remembering which formula is for circumference and which is for area. How can you help her? **Sample answer: Circumference is measured in linear units, while area is measured in square units, so the formula containing r^2 must be the one for area.**

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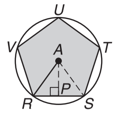
11-3 Study Guide and Intervention

Areas of Regular Polygons and Circles

Areas of Regular Polygons In a regular polygon, the segment drawn from the center of the polygon perpendicular to the opposite side is called the **apothem**. In the figure at the right, $\overline{AP}$ is the apothem and $\overline{AR}$ is the radius of the circumscribed circle.



Area of a Regular Polygon	If a regular polygon has an area of A square units, a perimeter of P units, and an apothem of a units, then $A = \frac{1}{2}Pa$.

Example 1 **Verify the formula $A = \frac{1}{2}Pa$ for the regular pentagon above.**

For $\triangle RAS$, the area is $A = \frac{1}{2}bh = \frac{1}{2}(RS)(AP)$. So the area of the pentagon is $A = 5\left(\frac{1}{2}\right)(RS)(AP)$. Substituting P for $5RS$ and substituting a for AP, then $A = \frac{1}{2}Pa$.

Example 2 **Find the area of regular pentagon $RSTUV$ above if its perimeter is 60 centimeters.**

First find the apothem.

The measure of central angle RAS is $\frac{360}{5}$ or 72. Therefore, $m\angle RAP = 36$. The perimeter is 60, so $RS = 12$ and $RP = 6$.

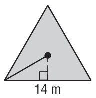
$$\tan \angle RAP = \frac{RP}{AP}$$
$$\tan 36° = \frac{6}{AP}$$
$$AP = \frac{6}{\tan 36°}$$
$$\approx 8.26$$

So, $A = \frac{1}{2}Pa = \frac{1}{2}60(8.26)$ or 247.8.

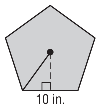
The area is about 248 square centimeters.

Exercises

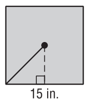
Find the area of each regular polygon. Round to the nearest tenth.

1. 

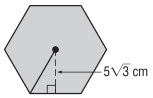
84.9 m²

2. 

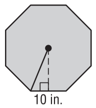
172.0 in²

3. 

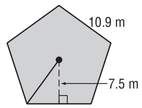
225 in²

4. 

259.8 cm²

5. 

482.8 in²

6. 

204.4 m²

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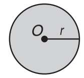
11-3 Study Guide and Intervention *(continued)*

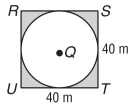
Areas of Regular Polygons and Circles

Areas of Circles As the number of sides of a regular polygon increases, the polygon gets closer and closer to a circle and the area of the polygon gets closer to the area of a circle.

Area of a Circle	If a circle has an area of A square units and a radius of r units, then $A = \pi r^2$.



Example **Circle Q is inscribed in square $RSTU$. Find the area of the shaded region.**



A side of the square is 40 meters, so the radius of the circle is 20 meters.

The shaded area is

Area of $RSTU$ − Area of circle Q

$= 40^2 - \pi r^2$

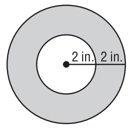
$= 1600 - 400\pi$

$\approx 1600 - 1256.6$

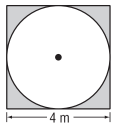
$= 343.4\ \text{m}^2$

Exercises

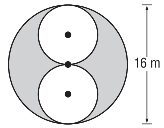
Find the area of each shaded region. Assume that all polygons are regular. Round to the nearest tenth.

1. 

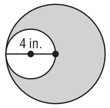
37.7 in²

2. 

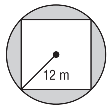
3.4 m²

3. 

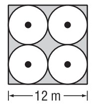
100.5 m²

4. 

37.7 in²

5. 

164.4 m²

6. 

30.9 m²

NAME ______________ DATE ________ PERIOD _____

11-3 Skills Practice

Areas of Regular Polygons and Circles

Find the area of each regular polygon. Round to the nearest tenth.

1. a pentagon with a perimeter of 45 feet
139.4 ft^2

2. a hexagon with a side length of 4 inches
41.6 in^2

3. a nonagon with a side length of 8 meters
395.6 m^2

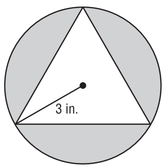
4. a triangle with a perimeter of 54 centimeters
140.3 cm^2

Find the area of each circle. Round to the nearest tenth.

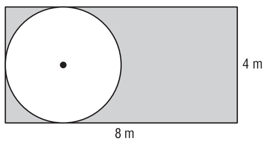
5. a circle with a radius of 6 yards
113.1 yd^2

6. a circle with a diameter of 18 millimeters
254.5 mm^2

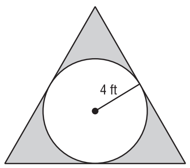
Find the area of each shaded region. Assume that all polygons are regular. Round to the nearest tenth.

7.


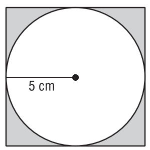
16.6 in^2

8.


19.4 m^2

9.


32.9 ft^2

10.


21.5 cm^2

NAME ______________ DATE ________ PERIOD _____

11-3 Practice

Areas of Regular Polygons and Circles

Find the area of each regular polygon. Round to the nearest tenth.

1. a nonagon with a perimeter of 117 millimeters
1044.7 mm^2

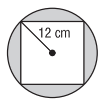
2. an octagon with a perimeter of 96 yards
695.3 yd^2

Find the area of each circle. Round to the nearest tenth.

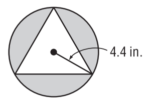
3. a circle with a diameter of 26 feet
530.9 ft^2

4. a circle with a circumference of 88 kilometers
616.2 km^2

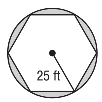
Find the area of each shaded region. Assume that all polygons are regular. Round to the nearest tenth.

5.


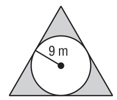
164.4 cm^2

6.


35.7 in^2

7.


339.7 ft^2

8.


166.4 m^2

DISPLAYS For Exercises 9 and 10, use the following information.
A display case in a jewelry store has a base in the shape of a regular octagon. The length of each side of the base is 10 inches. The owners of the store plan to cover the base in black velvet.

9. Find the area of the base of the display case.
about 482.8 in^2

10. Find the number of square yards of fabric needed to cover the base.
about 0.37 yd^2

Lesson 11-3

Answers

NAME ______________ DATE ________ PERIOD _____

11-3 Word Problem Practice

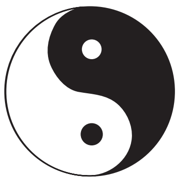
Areas of Regular Polygons and Circles

1. **LOBBY** The lobby of a bank features a large marble regular octagon. Each side of the octagon is 15 feet long.

What is the area of the octagon? Round your answer to the nearest tenth.
1086.4 ft²

2. **PORTHOLES** A circular window on a ship has a radius of 8 inches. What is the area of the window? Round your answer to the nearest hundredth.
201.06 in²

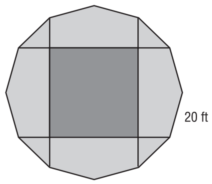
3. **YIN-YANG SYMBOL** A well-known symbol from Chinese culture is the yin-yang symbol, shown below.



Suppose the large circle has radius r, the small circles have radius $\frac{r}{8}$, and the S-curve is two semicircles each with radius $\frac{r}{2}$. In terms of r, what is the area of the black region?
$\frac{\pi r^2}{2}$

4. **PYRAMIDS** Martha's clubhouse is shaped like a square pyramid with four congruent equilateral triangles for its sides. All of the edges are 6 feet long. What is the total surface area of the clubhouse including the floor? Round your answer to the nearest hundredth.
98.35 ft²

POOL DECKS For Exercises 5-7, use the following information.
Ricardo designs a square pool with surrounding pool deck according to the plan shown. The outer edge of the deck is a regular dodecagon with side length 20 feet.



5. What is the length of the apothem of the dodecagonal deck?
37.32 ft

6. What is the length of the diagonal of the square pool?
54.64 ft

7. What is the area of the deck?
2985.64 ft²

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11-3 Enrichment

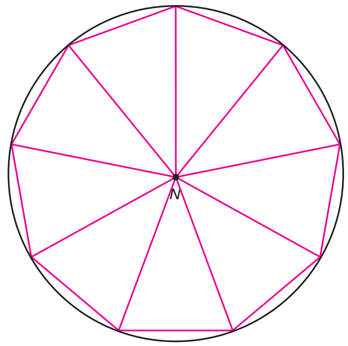
Areas of Inscribed Polygons

A protractor can be used to inscribe a regular polygon in a circle. Follow the steps below to inscribe a regular nonagon in ⊙*N*.

Step 1 Find the degree measure of each of the nine congruent arcs. **40**

Step 2 Draw 9 radii to form 9 angles with the measure you found in Step 1. The radii will intersect the circle in 9 points.

Step 3 Connect the nine points to form the nonagon.



1. Find the length of one side of the nonagon to the nearest tenth of a centimeter. What is the perimeter of the nonagon? **2.5 cm, *P* = 22.5 cm**

2. Measure the distance from the center perpendicular to one of the sides of the nonagon. **3.3 cm**

3. What is the area of one of the nine triangles formed? **4.125 cm²**

4. What is the area of the nonagon? **37.125 cm²**

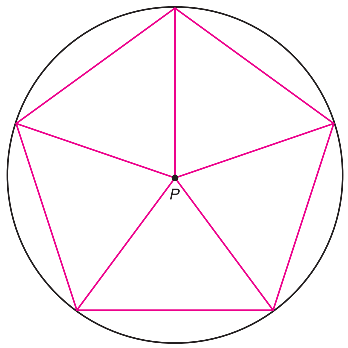
Make the appropriate changes in Steps 1–3 above to inscribe a regular pentagon in ⊙*P*. Answer each of the following.

5. Use a protractor to inscribe a regular pentagon in ⊙*P*.

6. What is the measure of each of the five congruent arcs? **72**

7. What is the perimeter of the pentagon to the nearest tenth of a centimeter? **21 cm**

8. What is the area of the pentagon to the nearest tenth of a centimeter? **30.5 cm²**



Lesson 11-3

NAME ______________________ DATE ________ PERIOD _____

11-4 Lesson Reading Guide

Areas of Composite Figures

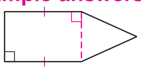
Get Ready for the Lesson

Read the introduction to Lesson 11-4 in your textbook.

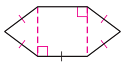
How do you think the areas of the figures outlined in the picture of the sail are related? **Sample answer: The areas get smaller as you move further up the sail. The area of the triangle is smaller than the area of any of the trapezoids.**

Read the Lesson

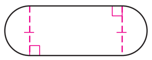
1. Use dashed segments to show how each figure can be subdivided into figures for which you have learned area formulas. Name the smaller figures that you have formed as specifically as possible and indicate whether any of them are congruent to each other. **Sample answers are given.**

a. 

rectangle and isosceles triangle

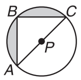
b. 

square and two congruent isosceles triangles

c. 

rectangle and two congruent semicircles

2. In the figure, B is the midpoint of $\overparen{ABC}$. Complete the following steps to derive a formula for the area of the shaded region in terms of the radius r of the circle.



The area of circle P is **πr^2**.

$m\angle ABC$ = **90** because

Sample answer: It is an inscribed angle that intercepts a semicircle.

$m\overparen{AB} = m\overparen{BC}$ because

Sample answer: B is the midpoint of $\overparen{ABC}$ (definition of midpoint).

$\overline{AB} \cong \overline{BC}$ because **Sample answer: If two minor arcs of a circle are congruent, their corresponding chords are congruent**.

Therefore, $\triangle ABC$ is a(n) **isosceles right or 45°-45°-90°** triangle.

AC = **$2r$**, so AB = **$\frac{2r}{\sqrt{2}}$ or $r\sqrt{2}$** and BC = **$\frac{2r}{\sqrt{2}}$ or $r\sqrt{2}$**.

The area of $\triangle ABC$ is $\frac{1}{2} \cdot$ **$r\sqrt{2}$** $\cdot$ **$r\sqrt{2}$** = **r^2**.

Therefore, the area of the shaded region is given by

A = **$\frac{1}{2}\pi r^2$** − **r^2** = **$\left(\frac{\pi}{2} - 1\right)r^2$**.

Remember What You Learned

3. Rolando is having trouble remembering when to subtract an area when finding the area of a composite figure. How can you help him remember? **Sample answer: Subtract when there is an indentation, or a hole in the figure.**

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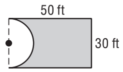
11-4 Study Guide and Intervention

Areas of Composite Figures

Composite Figures A composite figure is a figure that can be seprated into regions that are basic figures. To find the area of a composite figure separate the figure into basic figures of which we can find the area. The sum of the areas of the basic figures is the area of the figure.

Example 1 **Find the area of the composite figure.**



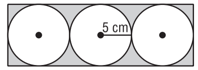
The figure is a rectangle minus one half of a circle. The radius of the circle is one half of 30 or 15.

$A = lw - \frac{1}{2}\pi r^2$

$= 50(30) - 0.5\pi(15)^2$

≈ 1146.6 or about 1147 ft^2

Example 2 **Find the area of the shaded region.**



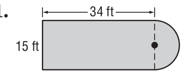
The dimensions of the rectangle are 10 centimeters and 30 centimeters. The area of the shaded region is

$(10)(30) - 3\pi(5^2) = 300 - 75\pi$

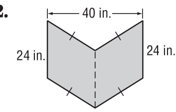
≈ 64.4 cm^2

Exercises

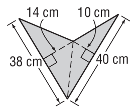
Find the area of each figure. Round to the nearest tenth if necessary.

1. 

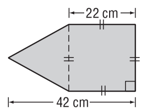
598.4 ft^2

2. 

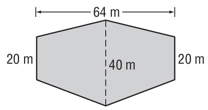
960 in^2

3. 

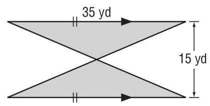
466 cm^2

4. 

704 cm^2

5. 

1920 m^2

6. 

262.5 yd^2

7. Refer to Example 2 above. Draw the largest possible square inside each of the three circles. What is the total area of the three squares? **150 cm^2**

Lesson 11-4

Answers

NAME ______________________ DATE __________ PERIOD _____

11-4 Study Guide and Intervention *(continued)*

Areas of Composite Figures

Composite Figures on the Coordinate Plane To find the area of a composite figure on the coordinate plane, separate the figure into basic figures.

Example **Find the area of pentagon *ABCDE*.**

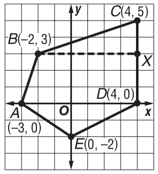
Draw $\overline{BX}$ between $B(-2, 3)$ and $X(4, 3)$ and draw $\overline{AD}$. The area of $ABCDE$ is the sum of the areas of $\triangle BCX$, trapezoid $BXDA$, and $\triangle ADE$.

A = area of $\triangle BCX$ + area of $BXDA$ + area of $\triangle ADE$

$= \frac{1}{2}(2)(6) + \frac{1}{2}(3)(6 + 7) + \frac{1}{2}(2)(7)$

$= 6 + \frac{39}{2} + 7$

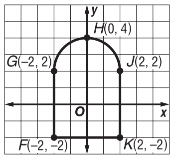
= 32.5 square units



Exercises

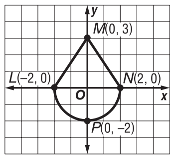
Find the area of each figure. Round to the nearest tenth.

1.



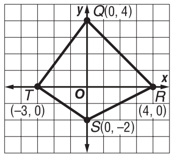
22.3 units^2

2.



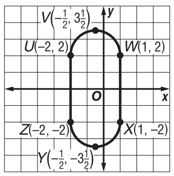
12.3 units^2

3.



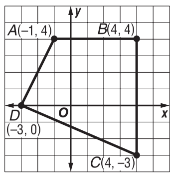
21 units^2

4.



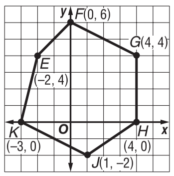
19.1 units^2

5.



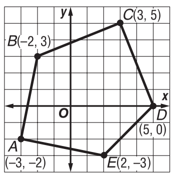
34.5 units^2

6.

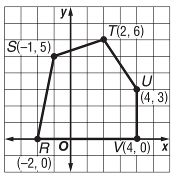


39 units^2

7. pentagon *ABCDE* 42.5 units^2



8. pentagon *RSTUV* 28 units^2



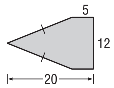
NAME ______________________ DATE __________ PERIOD _____

11-4 Skills Practice

Areas of Composite Figures

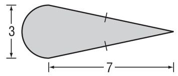
Find the area of each figure. Round to the nearest tenth if necessary.

1.



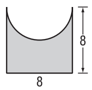
150 units^2

2.



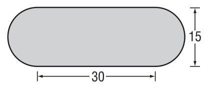
14.0 units^2

3.



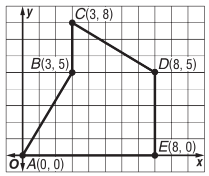
38.9 units^2

4.



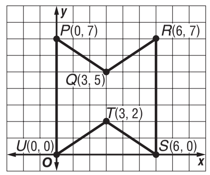
626.7 units^2

5.



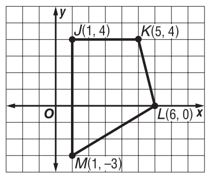
40 units^2

6.



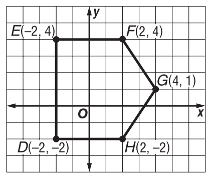
30 units^2

7.



25.5 units^2

8.



30 units^2

Lesson 11-4

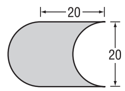
NAME ______________________ DATE ________ PERIOD _____

11-4 Practice

Areas of Composite Figures

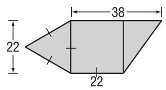
Find the area of each figure. Round to the nearest tenth if necessary.

1.



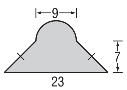
400 units^2

2.



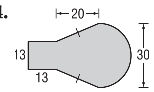
869.6 units^2

3.



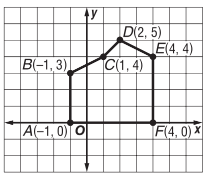
143.8 units^2

4.



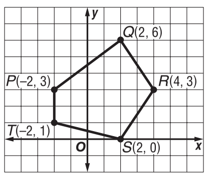
952.4 units^2

5.



20.5 units^2

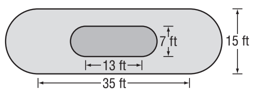
6.



22 units^2

LANDSCAPING For Exercises 7 and 8, use the following information.

One of the displays at a botanical garden is a koi pond with a walkway around it. The figure shows the dimensions of the pond and the walkway.



7. Find the area of the pond to the nearest tenth.

129.5 ft^2

8. Find the area of the walkway to the nearest tenth.

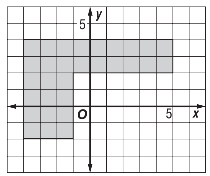
572.2 ft^2

NAME ______________________ DATE ________ PERIOD _____

11-4 Word Problem Practice

Areas of Composite Figures

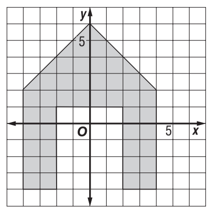
1. FLOOR PLANS The floor plan of an L-shaped building is shown in the coordinate plane. Each unit represents 5 meters.



What is the area of the building?

750 m^2

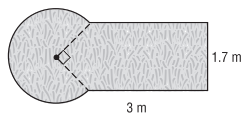
2. DOG HOUSES Miranda is building a dog house out of wood. The front view of the dog house is shown on the coordinate plane below.



If each unit corresponds to 5 inches, what is the area of the front?

1100 in^2

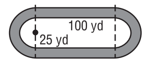
3. MINIATURE GOLF The plan for a miniature golf hole is shown below. The right angle in the drawing is a central angle.



What is the area of the playing surface? Round your answer to the nearest hundredth of a square meter.

9.23 m^2

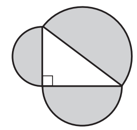
4. TRACK A running track has an inner and outer edge. Both the inner and outer edge consists of two semicircles joined by two straight line segments. The straight line segments are 100 yards long. The radii of the inner edge semicircles are 25 yards and the radii of the outer edge semicircles are 32 yards. What is the area of the track? Round your answer to the nearest hundredth of a yard.



2653.50 yd^2

SEMICIRCLES For Exercises 5 and 6, use the following information.

Bridget arranged three semicircles in the pattern shown.



The right triangle has side lengths 6, 8, and 10 inches.

5. What is the total area of the three semicircles? Round your answer to the nearest hundredth of a square inch.

78.54 in^2

6. If the right triangle had side lengths $\sqrt{21}$, $\sqrt{79}$, and 10 inches, what would the total area of the three semicircles be? Round your answer to the nearest hundredth of a square inch.

78.54 in^2

Lesson 11-4

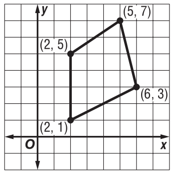
Answers

NAME ________________ DATE ________ PERIOD _____

11-4 Enrichment

Aerial Surveyors and Area

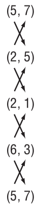
Many land regions have irregular shapes. Aerial surveyors often use coordinates when finding areas of such regions. The coordinate method described in the steps below can be used to find the area of *any* polygonal region. Study how this method is used to find the area of the region at the right.



Step 1 List the ordered pairs for the vertices in counter-clockwise order, repeating the first ordered pair at the bottom of the list.

Step 2 Find D, the sum of the downward diagonal products (from left to right).

$D = (5 \cdot 5) + (2 \cdot 1) + (2 \cdot 3) + (6 \cdot 7)$
$= 25 + 2 + 6 + 42$ or 75



Step 3 Find U, the sum of the upward diagonal products (from left to right).

$U = (2 \cdot 7) + (2 \cdot 5) + (6 \cdot 1) + (5 \cdot 3)$
$= 14 + 10 + 6 + 15$ or 45

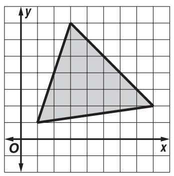
Step 4 Use the formula $A = \frac{1}{2}(D - U)$ to find the area.

$A = \frac{1}{2}(D - U)$
$= \frac{1}{2}(75 - 45)$
$= \frac{1}{2}(30)$ or 15

The area is 15 square units. Count the number of square units enclosed by the polygon. Does this result seem reasonable?

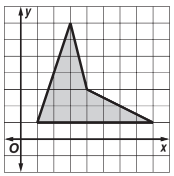
Use the coordinate method to find the area of each region in square units.

1.



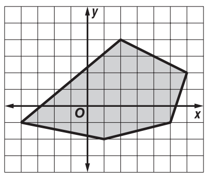
20 units^2

2.



14 units^2

3.



34 units^2

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11-5 Lesson Reading Guide

Geometric Probability and Areas of Sectors

Get Ready for the Lesson

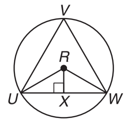
Read the introduction to Lesson 11-5 in your textbook.

To find the probability of winning at darts, would you use geometric probability to compare areas or lengths? **areas**

Read the Lesson

1. Explain the difference between a sector of a circle and a segment of a circle **Sample answer: A sector of a circle is bounded by a central angle and its intercept arc, while a segment is bounded by an arc and a chord.**

2. Suppose you are playing a game of darts with a target like the one shown at the right. If your dart lands inside equilateral $\triangle UVW$, you get a point. Assume that every dart will land on the target. The radius of the circle is 1. Complete the following steps to figure out the probability of getting a point.



The area of circle R is **π**.

$\triangle URW$ is a(n) **isosceles** triangle because $\overline{RU}$ and $\overline{RW}$ are **radii** of the same **circle**.

$\angle URW$ is a(n) **central** angle of the circle, and $m\angle URW =$ **120**.

$m\angle RUX =$ **30** and $m\angle RWX =$ **30**.

The angle measures in $\triangle RUX$ are **30**, **60**, and **90**.

$\overline{RU}$ is a **radius** of the circle, so $RU =$ **1**.

$\overline{RX}$ is the leg of $\triangle RUX$ opposite the **30°** angle, so $RX =$ **$\frac{1}{2}$**.

Also, $\overline{UX}$ is the leg of $\triangle RUX$ opposite the **60°** angle, so $UX =$ **$\frac{\sqrt{3}}{2}$**.

$UW =$ **$\sqrt{3}$**, so the area of $\triangle URW$ is $\frac{1}{2} \cdot$ **$\sqrt{3}$** $\cdot$ **$\frac{1}{2}$** $=$ **$\frac{\sqrt{3}}{4}$**.

Then, the area of $\triangle UVW = 3 \cdot$ **$\frac{\sqrt{3}}{4}$** $=$ **$\frac{3\sqrt{3}}{4}$**.

Therefore, the probability that the dart will fall inside the triangle is the ratio of **$\frac{3\sqrt{3}}{4}$** to **π**, which is approximately **0.413** (to the nearest thousandth).

Remember What You Learned

3. Many students find it difficult to remember a large number of geometric formulas. How can you use the formula for the area of a circle to find the area of a sector of a circle without having to learn a new formula? **Sample answer: First use $A = \pi r^2$ to find the area of the circle. Then use the measure of the central angle to find out what fraction of the circle the sector is. Multiply the area of the circle by this fraction and you will have the area of the sector.**

NAME ______________________ DATE ___________ PERIOD _____

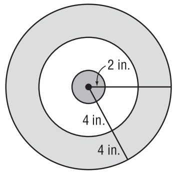
11-5 Study Guide and Intervention

Geometric Probability and Areas of Sectors

Geometric Probability The probability that a point in a figure will lie in a particular part of the figure can be calculated by dividing the area of the part of the figure by the area of the entire figure. The quotient is called the **geometric probability** for the part of the figure.

> If a point in region A is chosen at random, then the probability $P(B)$ that the point is in region B, which is in the interior of region A, is
> $P(B) = \frac{\text{area of region } B}{\text{area of region } A}$.

Example **Darts are thrown at a circular dartboard. If a dart hits the board, what is the probability that the dart lands in the bull's-eye?**



Area of bull's-eye: $A = \pi(2)^2$ or 4π

Area of entire dartboard: $A = \pi(10)^2$ or 100π

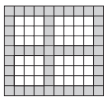
The probability of landing in the bull's-eye is

$\frac{\text{area of bull's-eye}}{\text{area of dartboard}} = \frac{4\pi}{100\pi}$

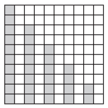
$= \frac{1}{25}$ or 0.04.

Exercises

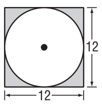
Find the probability that a point chosen at random lies in the shaded region. Round to the nearest hundredth if necessary.

1. 

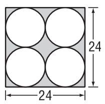
0.53

2. 

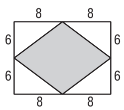
0.3

3. 

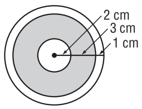
0.21

4. 

0.21

5. 

0.5

6. 

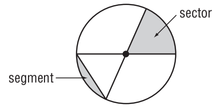
0.58

NAME ______________________ DATE ___________ PERIOD _____

11-5 Study Guide and Intervention *(continued)*

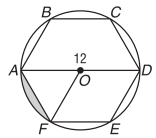
Geometric Probability and Areas of Sectors

Sectors and Segments of Circles A **sector of a circle** is a region of a circle bounded by a central angle and its intercepted arc. A **segment of a circle** is bounded by a chord and its arc. Geometric probability problems sometimes involve sectors or segments of circles.



> If a sector of a circle has an area of A square units, a central angle measuring $N°$, and a radius of r units, then $A = \frac{N}{360}\pi r^2$.

Example **A regular hexagon is inscribed in a circle with diameter 12. Find the probability that a point chosen at random in the circle lies in the shaded region.**



The area of the shaded segment is the area of sector AOF − the area of $\triangle AOF$.

Area of sector $AOF = \frac{N}{360}\pi r^2$

$= \frac{60}{360}\pi(6^2)$

$= 6\pi$

Area of $\triangle AOF = \frac{1}{2}bh$

$= \frac{1}{2}(6)(3\sqrt{3})$

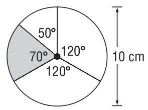
$= 9\sqrt{3}$

The shaded area is $6\pi - 9\sqrt{3}$ or about 3.26.

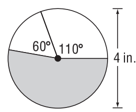
The probability is $\frac{\text{area of segment}}{\text{area of circle}} = \frac{3.26}{36\pi}$ or about 0.03.

Exercises

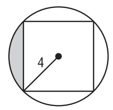
Find the probability that a point in the circle chosen at random lies in the shaded region. Round to the nearest hundredth.

1. 

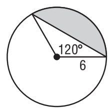
0.19

2. 

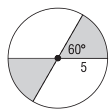
0.53

3. 

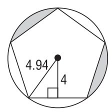
0.09

4. 

0.20

5. 

0.33

6. 

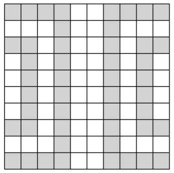
0.10

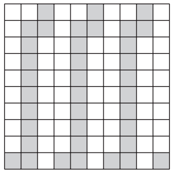
NAME ______________________ DATE __________ PERIOD _____

11-5 Skills Practice

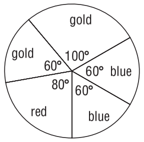
Geometric Probability and Areas of Sectors

Find the probability that a point chosen at random lies in the shaded region.

1.  **0.48**

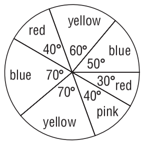
2.  **0.37**

Find the area of the indicated sector. Then find the probability of spinning the color indicated if the diameter of each spinner is 6 inches.



3. red **6.3 in^2, $0.\overline{2}$**

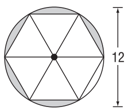
4. gold **12.6 in^2, $0.\overline{4}$**



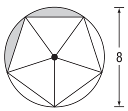
5. blue **9.4 in^2, $0.\overline{3}$**

6. yellow **10.2 in^2, $0.3\overline{61}$**

Find the area of the shaded region. Then find the probability that a point chosen at random is in the shaded region. Assume that all inscribed polygons are regular.

7. 

16.3 units^2, 0.14

8. 

4.9 units^2, 0.097

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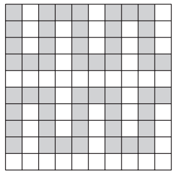
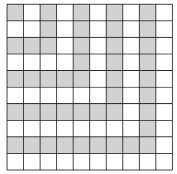
11-5 Practice

Geometric Probability and Areas of Sectors

Find the probability that a point chosen at random lies in the shaded region.

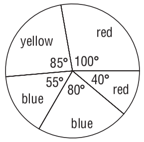
1. 

0.50

2. 

0.45

Find the area of the indicated sector. Then find the probability of spinning the color indicated if the diameter of the spinner is 9 meters.

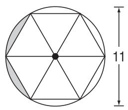


3. red **24.7 m^2, 0.39**

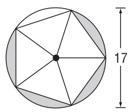
4. blue **23.9 m^2, 0.38**

5. yellow **15.0 m^2, 0.24**

Find the area of the shaded region. Then find the probability that a point chosen at random is in the shaded region. Assume that all inscribed polygons are regular.

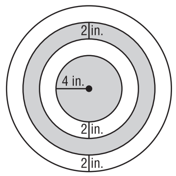
6. 

5.5 units^2, 0.058

7. 

44.2 units^2, 0.195

8. ARCHERY A target consists of four concentric rings. The radius of the center circle is 4 inches, and the circles are spaced 2 inches apart. Find the probability that an arrow shot at random by an inexperienced archer will land in a shaded region. **0.44**



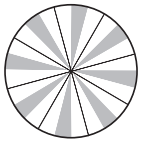
Lesson 11-5

NAME ______________ DATE ______ PERIOD ____

11-5 Word Problem Practice

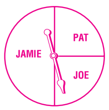
Geometric Probability and Areas of Sectors

1. **DARTS** A dart is thrown at the dartboard shown. Each sector has the same central angle. The dart has equal probability of hitting any point on the dartboard. What is the probability that the dart will land in a shaded sector?

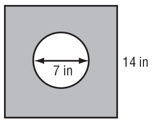


$\frac{1}{3}$

2. **SPINNERS** Jamie, Joe, and Pat celebrate the end of each work week by ordering spring rolls from a Chinese restaurant. The order comes with 4 spring rolls so somebody gets an extra roll. Because Jamie works full time and Joe and Pat work half time, they decide who gets the extra roll by using a spinner that has a 50% chance of coming up Jamie, and 25% chances of coming up either Joe or Pat. Design such a spinner.



3. **RAIN** A container has a square top with a hole as shown. What is the probability that a raindrop that hits the container falls into the hole? Round your answer to the nearest thousandth.



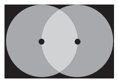
0.196

4. **ELECTRON MICROSCOPES** Crystal places a 7 millimeter by 10 millimeter rectangular plate into the sample chamber of an electron microscope. A black and white checkerboard pattern of 1-millimeter squares was painted over the plate to identify different treatments of the material. When she turns on the monitor, she has no idea at what point on the plate she is looking because the white and black contrast does not show up on the screen. If there are 2 more black squares than white squares, what is the probability that she is looking at a white square?

$\frac{17}{35}$

ENTERTAINMENT For Exercises 5 and 6, use the following information.

A rectangular dance stage is lit by two lights that light up circular regions of the stage. The circles have the same radius and each circle passes through the center of the other. The stage perfectly circumscribes the two circles. A spectator throws a bouquet of flowers onto the stage. Assume the bouquet has an equal chance of landing anywhere on the stage. (*Hint:* Use inscribed equilateral triangles.)



5. What is the probability that the flowers land on a lit part of the stage?

0.842

6. What is the probability that the flowers land on the part of the stage where the spotlights overlap?

0.205

NAME ______________ DATE ______ PERIOD ____

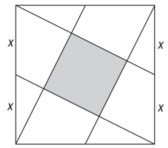
11-5 Enrichment

Polygon Probability

Each problem on this page involves one or more regular polygons. To find the probability of a point chosen at random being in the shaded region, you need to find the ratio of the shaded area to the total area. If you wish, you may substitute numbers for the variables.

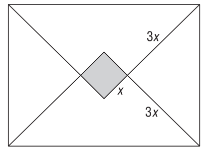
Find the probability that a point chosen at random in each figure is in the shaded region. Assume polygons that appear to be regular are regular. Round your answer to the nearest hundredth.

1.



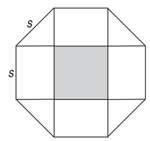
$\frac{1}{5}$ or 0.20

2.



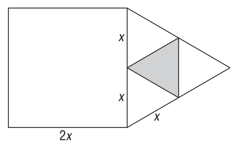
$\frac{1}{24} \approx 0.04$

3.



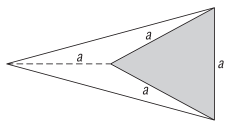
$\frac{1}{2 + 2\sqrt{2}} \approx 0.21$

4.



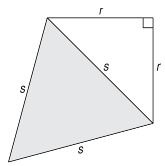
$\frac{\sqrt{3}}{16 + 4\sqrt{3}} \approx 0.08$

5.



$\frac{\sqrt{3}}{2 + \sqrt{3}} \approx 0.46$

6.



$\frac{\sqrt{3}}{\sqrt{3} + 1} \approx 0.63$

Chapter 11 Assessment Answer Key

Quiz 1 *(Lessons 11–1 and 11–2)*
Page 45

1. 33.8 in^2
2. 10 cm^2
3. 61.3 m^2
4. 336 ft^2
5. square

Quiz 2 *(Lesson 11–3)*
Page 45

1. 83.1 in^2
2. 86.6 cm^2
3. 482.8 m^2
4. 18.3 cm^2
5. D

Quiz 3 *(Lessons 11–4 and 11–5)*
Page 46

1. 198 cm^2
2. 26.3 in^2
3. 331.6 m^2
4. 40 $units^2$
5. 24 $units^2$

Quiz 4 *(Lesson 11–6)*
Page 46

1. 0.49
2. 0.11
3. 0.03
4. 0.28
5. 0.09

Mid-Chapter Test
Page 47

Part I

1. D
2. F
3. B
4. H

Part II

5. 39.6 in^2
6. 120 m^2
7. 83.1 ft^2
8. approx. 99 in^2
9. 19.1 m^2

Chapter 11 Assessment Answer Key

Vocabulary Test
Page 48

1. $A = \pi r^2$
2. $A = \frac{1}{2}bh$
3. $A = \frac{1}{2}h(b_1 + b_2)$
4. $A = \frac{1}{2}Pa$
5. $A = \frac{N}{360}\pi r^2$
6. sector
7. segment
8. apothem
9. a figure that cannot be classified into a specific shape such as a square or rectangle
10. the ratio of the area of a specified part of the figure to the total area

Form 1
Page 49

1. D
2. G
3. B
4. F
5. C
6. G
7. A
8. H
9. D

Page 50

10. F
11. C
12. G
13. C
14. G
15. B

B: 22 units

Answers

Chapter 11 Assessment Answer Key

Form 2A

Page 51

1. C
2. G
3. A
4. G
5. D
6. F
7. C
8. G

Page 52

9. D
10. H
11. B
12. F
13. D
14. G

B: about 227.3 in^2

Form 2B

Page 53

1. B
2. J
3. C
4. F
5. D
6. G
7. C
8. G

Page 54

9. C
10. F
11. D
12. F
13. B
14. G

B: 104.7 in^2

Chapter 11 Assessment Answer Key

Form 2C

Page 55

1. 173.2 cm^2
2. 30.3 in^2
3. 45 m and 50 m
4. quadrilateral, parallelogram, rectangle
5. 16.5 units^2
6. 16 units^2
7. 16 units^2
8. 32 in^2
9. 64.1 cm^2
10. 46.8 m^2

Page 56

11. 184.3 in^2
12. 42 cm^2
13. 89.7 m^2
14. 30 units^2
15. 0.34
16. 0.19
17. 0.34

B: 20 m

Answers

Chapter 11 Assessment Answer Key

Form 2D
Page 57

1. $76.2\ cm^2$
2. $78\ in^2$
3. 30 m and 19 m
4. quadrilateral, parallelogram, rectangle, and square
5. $19.9\ units^2$
6. $16\ units^2$
7. $24\ units^2$
8. $36\ in^2$
9. $584.6\ cm^2$
10. $84.9\ m^2$

Page 58

11. $34.8\ in^2$
12. $189\ cm^2$
13. $2.3\ m^2$
14. $14.5\ units^2$
15. 0.41
16. 0.14
17. 0.09

B: 21 m

Chapter 11 Assessment Answer Key

Form 3
Page 59

1. 41.6 cm^2
2. 49 in^2
3. 20.2 in^2
4. quadrilateral, parallelogram, and rhombus
5. 129 in^2
6. 51 $units^2$
7. 336 m^2
8. 695.3 m^2
9. 90.8 in^2
10. 890.2 cm^2

Page 60

11. 90.3 m^2
12. 23.7 in^2
13. 24.3 cm^2
14. 23.0 $units^2$
15. 30 squares
16. 0.11
17. 0.12

B: 17.4 cm^2

Answers

Chapter 11 Assessment Answer Key

Extended-Response Test, Page 61
Scoring Rubric

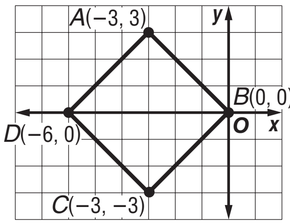
Score	General Description	Specific Criteria
4	**Superior** A correct solution that is supported by well-developed, accurate explanations	• Shows thorough understanding of *using formulas to find the areas of parallelograms, triangles, rhombi, trapezoids, regular polygons, circles, irregular figures, figures graphed on a coordinate plane, segments and sectors of circles, using the slope and/or distance formulas to determine whether a quadrilateral is a parallelogram, rhombus, square, or rectangle, and solving problems involving geometric probability, segments, and sectors*. • Uses appropriate strategies to solve problems. • Written explanations are exemplary. • Graphs are accurate and appropriate. • Goes beyond requirements of some or all problems.
3	**Satisfactory** A generally correct solution, but may contain minor flaws in reasoning or computation	• Shows understanding of *using formulas to find the areas of parallelograms, triangles, rhombi, trapezoids, regular polygons, circles, irregular figures, figures graphed on a coordinate plane, segments and sectors of circles, using the slope and/or distance formulas to determine whether a quadrilateral is a parallelogram, rhombus, square, or rectangle, and solving problems involving geometric probability, segments, and sectors*. • Uses appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are effective. • Graphs are mostly accurate and appropriate. • Satisfies all requirements of all problems.
2	**Nearly Satisfactory** A partially correct interpretation and/or solution to the problem	• Shows partial understanding of most of *using formulas to find the areas of parallelograms, triangles, rhombi, trapezoids, regular polygons, circles, irregular figures, figures graphed on a coordinate plane, segments and sectors of circles, using the slope and/or distance formulas to determine whether a quadrilateral is a parallelogram, rhombus, square, or rectangle, and solving problems involving geometric probability, segments, and sectors*. • May not use appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are satisfactory. • Graphs are mostly accurate. • Satisfies the requirements of most of the problems.
1	**Nearly Unsatisfactory** A correct solution with no supporting evidence or explanation	• Final computation is correct. • No written explanations or work is shown to substantiate the final computation. • Graphs may be accurate but lack detail or explanation. • Satisfies minimal requirements of some of the problems.
0	**Unsatisfactory** An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given	• Shows little or no understanding of *using formulas to find the areas of parallelograms, triangles, rhombi, trapezoids, regular polygons, circles, irregular figures, figures graphed on a coordinate plane, segments and sectors of circles, using the slope and/or distance formulas to determine whether a quadrilateral is a parallelogram, rhombus, square, or rectangle, and solving problems involving geometric probability, segments, and sectors*. • Does not use appropriate strategies to solve problems. • Computations are incorrect. • Written explanations are unsatisfactory. • Graphs are inaccurate or inappropriate. • Does not satisfy the requirements of the problems. • No answer may be given.

Chapter 11 Assessment Answer Key

Extended-Response Test, Page 61
Sample Answers

In addition to the scoring rubric found on page A26, the following sample answers may be used as guidance in evaluating open-ended assessment items.

1. a.



Slope of $\overline{AB} = -1$; slope of $\overline{CD} = -1$; $AB = 3\sqrt{2}$; $CD = 3\sqrt{2}$; slope of $\overline{BC} = 1$; slope of $\overline{DA} = 1$; $BC = 3\sqrt{2}$; and $DA = 3\sqrt{2}$.

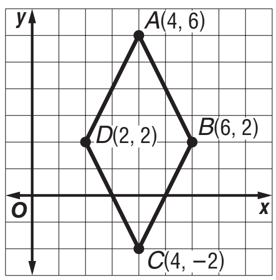
Since the slopes of opposite sides are the same, the figure is a parallelogram. Since the slopes of adjacent sides are negative reciprocals, indicating perpendicular lines, the figure is either a square or rectangle. Since the lengths of all the sides are congruent, this figure is a square.

b. $(3\sqrt{2})(3\sqrt{2}) = 18$ units^2

2. a. A 30°-60°-90° triangle can be used to find the altitude, $x\sqrt{3}$, and $\overline{PT}$, x. The hypotenuse of ΔPQT is $2x$ or 14. So, $x = 7$, the altitude is $7\sqrt{3}$, and base $PT = 7$. Since the height and base of parallelogram $PQRS$ are known, the area can be calculated.

b. The base is 7 + 14 or 21 and the height is $7\sqrt{3}$. $A = (21)(7\sqrt{3}) \approx 254.6$ in^2.

3. a.

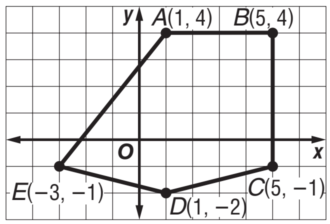


$DB = |2 - 6| = 4$ and $AC = |6 - (-2)| = 8$. $DB = d_1$ and $AC = d_2$, so $A = \frac{1}{2}d_1d_2$, or $\frac{1}{2}(4)(8) = 16$ units^2.

b. By using the ratios of a 30°-60°-90° triangle, the altitude of the trapezoid is $4\sqrt{3}$ cm and the base of ΔMNQ is 4 cm. The longer base of the trapezoid is 4 + 11 or 15 cm. Since $A = \frac{1}{2}h(b_1 + b_2)$, the area is $\frac{1}{2}(4\sqrt{3})(11 + 15)$ or 90.1 cm^2.

c. The area of the circle is 36π. The area of the hexagon is $A = \frac{1}{2}aP$. Since a regular hexagon can be divided into six equilateral triangles, use the 30°-60°-90° triangle ratios to find a and the length of one side of the hexagon. So, $a = 3\sqrt{3}$ in., each side is 6 in., and $P = 36$ in. By substitution, the area of the hexagon is 93.5 in^2. The area of the shaded region is $36\pi - 93.5 \approx 19.6$ in^2.

d.



The shape of this figure contains trapezoid $ABCD$ and $\triangle ADE$. The area of the trapezoid is $(4)\left(\frac{11}{2}\right)$ or 22 units^2. The area of the triangle is $\frac{1}{2}(4)(6)$ or 12 units^2. The total area is 22 + 12 or 34 units^2.

4. a. The area of this segment is $\frac{50}{360} \cdot \pi \cdot 5^2 - \frac{1}{2}(10 \sin 25)(5 \cos 25) \approx$ 1.3 cm^2. The probability that a point chosen at random lies in the shaded region is $\frac{\text{area of the segment}}{\text{area of the circle}}$.

b. 0.02

Answers

Chapter 11 Assessment Answer Key

Standardized Test Practice
Page 62

1. Ⓐ ● Ⓒ Ⓓ

2. Ⓕ ● Ⓗ Ⓙ

3. Ⓐ Ⓑ ● Ⓓ

4. Ⓕ ● Ⓗ Ⓙ

5. Ⓐ Ⓑ Ⓒ ●

6. Ⓕ ● Ⓗ Ⓙ

7. Ⓐ Ⓑ ● Ⓓ

8. Ⓕ Ⓖ ● Ⓙ

Page 63

9. Ⓐ Ⓑ Ⓒ ●

10. Ⓕ Ⓖ ● Ⓙ

11. Ⓐ Ⓑ ● Ⓓ

12. ● Ⓖ Ⓗ Ⓙ

13.

	1	2	5	.			
0	0	0	0		0	0	0
1	1	1	1		1	1	1
2	2	2	2		2	2	2
3	3	3	3		3	3	3
4	4	4	4		4	4	4
5	5	5	5		5	5	5
6	6	6	6		6	6	6
7	7	7	7		7	7	7
8	8	8	8		8	8	8
9	9	9	9		9	9	9

14.

		4	0	.			
0	0	0	0		0	0	0
1	1	1	1		1	1	1
2	2	2	2		2	2	2
3	3	3	3		3	3	3
4	4	4	4		4	4	4
5	5	5	5		5	5	5
6	6	6	6		6	6	6
7	7	7	7		7	7	7
8	8	8	8		8	8	8
9	9	9	9		9	9	9

Chapter 11 Assessment Answer Key

Standardized Test Practice
Page 64

15. 25.4, 16.8

16. 109, 71, 4

17. 312.6 mph, 7.4° west of due south

18. 36, 36, 25, 84.5

19. $X'(0, -3)$, $Y'(-4, -7)$, $Z'(5, -4)$

20. $m\angle TRU = 94$, $m\angle URV = 39$, $m\angle VRW = 47$

21. 15 in.

22a. 720

22b. $16\sqrt{3}$ or 27.7

22c. 139.1 unit^2

Answers