Glencoe Mathematics

# Geometry

## Chapter 8
## Resource Masters

McGraw-Hill Glencoe

New York, New York Columbus, Ohio Chicago, Illinois Peoria, Illinois Woodland Hills, California

**Consumable Workbooks** Many of the worksheets contained in the Chapter Resource Masters are available as consumable workbooks in both English and Spanish.

| | ISBN10 | ISBN13 |
|---|---|---|
| *Study Guide and Intervention Workbook* | 0-07-877344-X | 978-0-07-877344-0 |
| *Skills Practice Workbook* | 0-07-877346-6 | 978-0-07-877346-4 |
| *Practice Workbook* | 0-07-877347-4 | 978-0-07-877347-1 |
| *Word Problem Practice Workbook* | 0-07-877349-0 | 978-0-07-877349-5 |
| ***Spanish Versions*** | | |
| *Study Guide and Intervention Workbook* | 0-07-877345-8 | 978-0-07-877345-7 |
| *Practice Workbook* | 0-07-877348-2 | 978-0-07-877348-8 |

**Answers for Workbooks** The answers for Chapter 8 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

**StudentWorks Plus™** This CD-ROM includes the entire Student Edition test along with the English workbooks listed above.

**TeacherWorks Plus™** All of the materials found in this booklet are included for viewing, printing, and editing in this CD-ROM.

**Spanish Assessment Masters** (ISBN10: 0-07-877350-4, ISBN13: 978-0-07-877350-1) These masters contain a Spanish version of Chapter 8 Test Form 2A and Form 2C.

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# CONTENTS

# Teacher's Guide to Using the *Chapter 8 Resource Masters*

The *Chapter 8 Resource Masters* includes the core materials needed for Chapter 8. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing on the *TeacherWorks Plus*™ CD-ROM.

## Chapter Resources

***Student-Built Glossary*** (pages 1–2) These masters are a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar. Give this to students before beginning Lesson 8–1. Encourage them to add these pages to their mathematics study notebooks. Remind them to complete the appropriate words as they study each lesson.

***Anticipation Guide*** (pages 7–8) This master, presented in both English and Spanish, is a survey used before beginning the chapter to pinpoint what students may or may not know about the concepts in the chapter. Students will revisit this survey after they complete the chapter to see if their perceptions have changed.

## Lesson Resources

***Lesson Reading Guide*** Get Ready for the Lesson extends the discussion from the beginning of the Student Edition lesson. Read the Lesson asks students to interpret the context of and relationships among terms in the lesson. Finally, Remember What You Learned asks students to summarize what they have learned using various representation techniques. Use as a study tool for note taking or as an informal reading assignment. It is also a helpful tool for ELL (English Language Learners).

***Study Guide and Intervention*** These masters provide vocabulary, key concepts, additional worked-out examples and Check Your Progress exercises to use as a reteaching activity. It can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

***Skills Practice*** This master focuses more on the computational nature of the lesson. Use as an additional practice option or as homework for second-day teaching of the lesson.

***Practice*** This master closely follows the types of problems found in the Exercises section of the Student Edition and includes word problems. Use as an additional practice option or as homework for second-day teaching of the lesson.

***Word Problem Practice*** This master includes additional practice in solving word problems that apply the concepts of the lesson. Use as an additional practice or as homework for second-day teaching of the lesson.

***Enrichment*** These activities may extend the concepts of the lesson, offer a historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. They are written for use with all levels of students.

***Graphing Calculator, Scientific Calculator, or Spreadsheet Activities*** These activities present ways in which technology can be used with the concepts in some lessons of this chapter. Use as an alternative approach to some concepts or as an integral part of your lesson presentation.

## Assessment Options

The assessment masters in the *Chapter 8 Resource Masters* offer a wide range of assessment tools for formative (monitoring) assessment and summative (final) assessment.

***Student Recording Sheet*** This master corresponds with the standardized test practice at the end of the chapter.

***Pre-AP Rubric*** This master provides information for teachers and students on how to assess performance on open-ended questions.

***Quizzes*** Four free-response quizzes offer assessment at appropriate intervals in the chapter.

***Mid-Chapter Test*** This 1-page test provides an option to assess the first half of the chapter. It parallels the timing of the Mid-Chapter Quiz in the Student Edition and includes both multiple-choice and free-response questions.

***Vocabulary Test*** This test is suitable for all students. It includes a list of vocabulary words and 10 questions to assess students' knowledge of those words. This can also be used in conjunction with one of the leveled chapter tests.

***Leveled Chapter Tests***
- *Form 1* contains multiple-choice questions and is intended for use with below grade level students.
- *Forms 2A and 2B* contain multiple-choice questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- *Forms 2C and 2D* contain free-response questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- *Form 3* is a free-response test for use with above grade level students.

All of the above mentioned tests include a free-response Bonus question.

***Extended-Response Test*** Performance assessment tasks are suitable for all students. Sample answers and a scoring rubric are included for evaluation.

***Standardized Test Practice*** These three pages are cumulative in nature. It includes three parts: multiple-choice questions with bubble-in answer format, griddable questions with answer grids, and short-answer free-response questions.

## Answers

- The answers for the Anticipation Guide and Lesson Resources are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment masters.

# 8 Student-Built Glossary



**This is an alphabetical list of the key vocabulary terms you will learn in Chapter 8. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Geometry Study Notebook to review vocabulary at the end of the chapter.**

| Vocabulary Term | Found on Page | Definition/Description/Example |
|---|---|---|
| angle of depression | | |
| angle of elevation | | |
| cosine | | |
| geometric mean | | |
| Law of Cosines | | |
| Law of Sines | | |

*(continued on the next page)*

# 8 Student-Built Glossary *(continued)*

| Vocabulary Term | Found on Page | Definition/Description/Example |
|---|---|---|
| Pythagorean triple | | |
| sine | | |
| solving a triangle | | |
| tangent | | |
| trigonometric ratio | | |
| trigonometry | | |

NAME ______________________________________________ DATE ____________ PERIOD _____

# 8 Anticipation Guide

## *Right Triangles and Trigonometry*

Chapter Resources

**Step 1** ***Before you begin Chapter 8***

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

| STEP 1 A, D, or NS | Statement | STEP 2 A or D |
|---|---|---|
| | **1.** The geometric mean between two numbers is the positive square root of their product. | |
| | **2.** An altitude drawn from the right angle of a right triangle to its hypotenuse separates the triangle into two congruent triangles. | |
| | **3.** In a right triangle, the length of the hypotenuse is equal to the sum of the lengths of the legs. | |
| | **4.** If any triangle has sides with lengths 3, 4, and 5, then that triangle is a right triangle. | |
| | **5.** If the two acute angles of a right triangle are 45°, then the length of the hypotenuse is $\sqrt{2}$ times the length of either leg. | |
| | **6.** In any triangle whose angle measures are 30°, 60°, and 90°, the hypotenuse is $\sqrt{3}$ times as long as the shorter leg. | |
| | **7.** The sine ratio of an angle of a right triangle is equal to the length of the adjacent side divided by the length of the hypotenuse. | |
| | **8.** The tangent of an angle of a right triangle whose sides have lengths 3, 4, and 5 will be smaller than the tangent of an angle of a right triangle whose sides have lengths 6, 8, and 10. | |
| | **9.** Trigonometric ratios can be used to solve problems involving angles of elevation and angles of depression. | |
| | **10.** The Law of Sines can only be used in right triangles. | |

**Step 2** ***After you complete Chapter 8***

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

NOMBRE ______________________________ FECHA ____________ PERÍODO ___

# 8 Ejercicios preparatorios

## *Triángulos rectángulos y trigonometría*

**PASO 1** ***Antes de comenzar el Capítulo 8***

- Lee cada enunciado.
- Decide si estás de acuerdo (A) o en desacuerdo (D) con el enunciado.
- Escribe A o D en la primera columna O si no estás seguro(a) de la respuesta, escribe NS (No estoy seguro(a).

| PASO 1 A, D o NS | Enunciado | PASO 2 A o D |
|---|---|---|
| | **1.** La media geométrica de dos números es la raíz cuadrada positiva de su producto. | |
| | **2.** Una altitud que se dibuja desde el ángulo recto de un triángulo rectángulo hasta su hipotenusa divide el triángulo en dos triángulos congruentes. | |
| | **3.** En un triángulo rectángulo, la longitud de la hipotenusa es igual a la suma de las longitudes de los catetos. | |
| | **4.** Si un triángulo rectángulo tiene lados con longitudes 3, 4 y 5, entonces cualquier triángulo es un triángulo rectángulo. | |
| | **5.** Si los dos ángulos agudos de un triángulo rectángulo son de 45°, entonces la longitud de la hipotenusa es $\sqrt{2}$ veces la longitud de cualquiera de los catetos. | |
| | **6.** En cualquier triángulo cuyas medidas de ángulos sean de 30°, 60° y 90°, la hipotenusa es $\sqrt{3}$ veces tan larga como el cateto más corto. | |
| | **7.** La razón del seno para un ángulo en un triángulo rectángulo es igual a la longitud del lado adyacente dividido entre la longitud de la hipotenusa. | |
| | **8.** La tangente para un ángulo de un triángulo rectángulo cuyos lados tienen longitudes 3, 4 y 5 será menor que la tangente de un ángulo en un triángulo rectángulo cuyos lados tienen longitudes 6, 8 y 10. | |
| | **9.** Las razones trigonométricas se pueden usar para resolver problemas de ángulos de elevación y ángulos de depresión. | |
| | **10.** La ley de los senos sólo se puede usar con triángulos rectángulos. | |

**PASO 2** ***Después de completar el Capítulo 8***

- Vuelve a leer cada enunciado y completa la última columna con una A o una D.
- ¿Cambió cualquiera de tus opiniones sobre los enunciados de la primera columna?
- En una hoja de papel aparte, escribe un ejemplo de por qué estás en desacuerdo con los enunciados que marcaste con una D.

# 8-1 Reading to Learn Mathematics

## *Geometric Mean*



## Get Ready for the Lesson

**Read the introduction to Lesson 8-1 in your textbook.**

If your eye level is halfway between the top and bottom of a painting, what additional information do you need to know to calculate the distance that creates the best view?

## Read the Lesson

**1.** In the past, when you have seen the word *mean* in mathematics, it referred to the *average* or *arithmetic mean* of the two numbers.

**a.** Complete the following by writing an algebraic expression in each blank.

If $a$ and $b$ are two positive numbers, then the geometric mean between $a$ and $b$ is

_________ and their arithmetic mean is _________.

**b.** Explain in words, without using any mathematical symbols, the difference between the geometric mean and the algebraic mean.

**2.** Let $r$ and $s$ be two positive numbers. In which of the following equations is $z$ equal to the geometric mean between $r$ and $s$?

**A.** $\frac{s}{z} = \frac{z}{r}$ **B.** $\frac{r}{z} = \frac{s}{z}$ **C.** $s:z = z:r$ **D.** $\frac{r}{z} = \frac{z}{s}$ **E.** $\frac{z}{r} = \frac{z}{s}$ **F.** $\frac{z}{s} = \frac{r}{z}$

**3.** Supply the missing words or phrases to complete the statement of each theorem.

**a.** The measure of the altitude drawn from the vertex of the right angle of a right triangle to its hypotenuse is the ____________________ between the measures of the two segments of the ____________________.

**b.** If the altitude is drawn from the vertex of the ____________________ angle of a right triangle to its hypotenuse, then the measure of a ____________________ of the triangle is the ____________________ between the measure of the hypotenuse and the segment of the ____________________ adjacent to that leg.

**c.** If the altitude is drawn from the ____________________ of the right angle of a right triangle to its ____________________, then the two triangles formed are ____________________ to the given triangle and to each other.

## Remember What You Learned

**4.** A good way to remember a new mathematical concept is to relate it to something you already know. How can the meaning of *mean* in a proportion help you to remember how to find the geometric mean between two numbers?

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# 8-1 Study Guide and Intervention

## *Geometric Mean*

**Geometric Mean** The **geometric mean** between two numbers is the positive square root of their product. For two positive numbers $a$ and $b$, the geometric mean of $a$ and $b$ is the positive number $x$ in the proportion $\frac{a}{x} = \frac{x}{b}$. Cross multiplying gives $x^2 = ab$, so $x = \sqrt{ab}$.

**Example** **Find the geometric mean between each pair of numbers.**

**a. 12 and 3**

Let $x$ represent the geometric mean.

$\frac{12}{x} = \frac{x}{3}$ Definition of geometric mean

$x^2 = 36$ Cross multiply.

$x = \sqrt{36}$ or 6 Take the square root of each side.

**b. 8 and 4**

Let $x$ represent the geometric mean.

$\frac{8}{x} = \frac{x}{4}$

$x^2 = 32$

$x = \sqrt{32}$

$\approx 5.7$

**Exercises**

**Find the geometric mean between each pair of numbers.**

**1.** 4 and 4

**2.** 4 and 6

**3.** 6 and 9

**4.** $\frac{1}{2}$ and 2

**5.** $2\sqrt{3}$ and $3\sqrt{3}$

**6.** 4 and 25

**7.** $\sqrt{3}$ and $\sqrt{6}$

**8.** 10 and 100

**9.** $\frac{1}{2}$ and $\frac{1}{4}$

**10.** $\frac{2\sqrt{2}}{5}$ and $\frac{3\sqrt{2}}{5}$

**11.** 4 and 16

**12.** 3 and 24

**The geometric mean and one extreme are given. Find the other extreme.**

**13.** $\sqrt{24}$ is the geometric mean between $a$ and $b$. Find $b$ if $a = 2$.

**14.** $\sqrt{12}$ is the geometric mean between $a$ and $b$. Find $b$ if $a = 3$.

**Determine whether each statement is *always*, *sometimes*, or *never* true.**

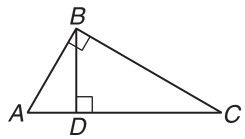
**15.** The geometric mean of two positive numbers is greater than the average of the two numbers.

**16.** If the geometric mean of two positive numbers is less than 1, then both of the numbers are less than 1.

# 8-1 Study Guide and Intervention *(continued)*

## *Geometric Mean*

**Altitude of a Triangle** In the diagram, $\triangle ABC \sim \triangle ADB \sim \triangle BDC$. An altitude to the hypotenuse of a right triangle forms two right triangles. The two triangles are similar and each is similar to the original triangle.




**Example 1** **Use right $\triangle ABC$ with $\overline{BD} \perp \overline{AC}$. Describe two geometric means.**

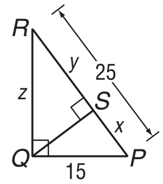
**a.** $\triangle ADB \sim \triangle BDC$ so $\frac{AD}{BD} = \frac{BD}{CD}$.

In $\triangle ABC$, the altitude is the geometric mean between the two segments of the hypotenuse.

**b.** $\triangle ABC \sim \triangle ADB$ and $\triangle ABC \sim \triangle BDC$, so $\frac{AC}{AB} = \frac{AB}{AD}$ and $\frac{AC}{BC} = \frac{BC}{DC}$.

In $\triangle ABC$, each leg is the geometric mean between the hypotenuse and the segment of the hypotenuse adjacent to that leg.

**Example 2** **Find $x$, $y$, and $z$.**




$\frac{PR}{PQ} = \frac{PQ}{PS}$

$\frac{25}{15} = \frac{15}{x}$    $PR = 25$, $PQ = 15$, $PS = x$

$25x = 225$    Cross multiply.

$x = 9$    Divide each side by 25.

Then

$y = PR - SP$

$= 25 - 9$

$= 16$

$\frac{PR}{QR} = \frac{QR}{RS}$

$\frac{25}{z} = \frac{z}{y}$    $PR = 25$, $QR = z$, $RS = y$

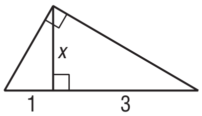
$\frac{25}{z} = \frac{z}{16}$    $y = 16$

$z^2 = 400$    Cross multiply.

$z = 20$    Take the square root of each side.

**Exercises**

**Find $x$, $y$, and $z$ to the nearest tenth.**

**1.**
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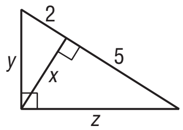


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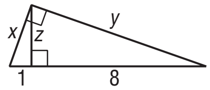


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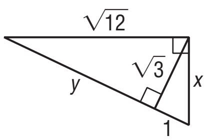
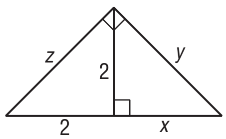
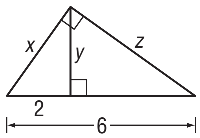



**5.**



**6.**


NAME ______________________________ DATE ____________ PERIOD _____

# 8-1 Skills Practice

## *Geometric Mean*

**Find the geometric mean between each pair of numbers. State exact answers and answers to the nearest tenth.**

**1.** 2 and 8

**2.** 9 and 36

**3.** 4 and 7

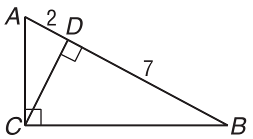
**4.** 5 and 10

**5.** $2\sqrt{2}$ and $5\sqrt{2}$

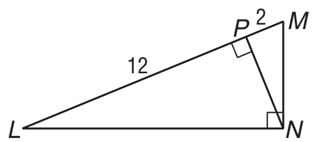
**6.** $3\sqrt{5}$ and $5\sqrt{5}$

**Find the measure of the altitude drawn to the hypotenuse. State exact answers and answers to the nearest tenth.**

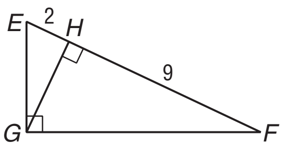
**7.**



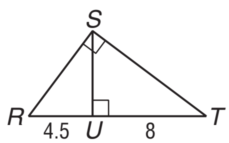

**8.**




**9.**

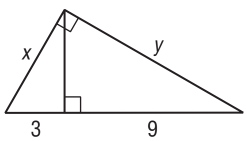



**10.**

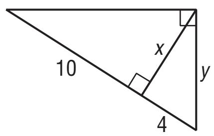



**Find *x* and *y*.**

**11.**




**12.**

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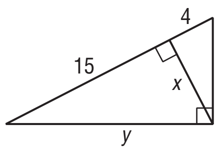


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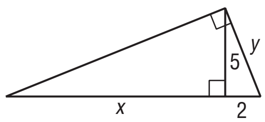


NAME _______________ DATE _______ PERIOD _____

# 8-1 Practice

## *Geometric Mean*



**Find the geometric mean between each pair of numbers to the nearest tenth.**

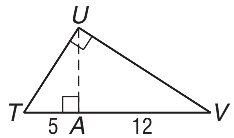
**1.** 8 and 12

**2.** $3\sqrt{7}$ and $6\sqrt{7}$

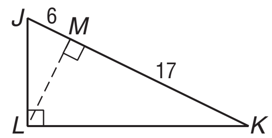
**3.** $\frac{4}{5}$ and 2

**Find the measure of the altitude drawn to the hypotenuse. State exact answers and answers to the nearest tenth.**

**4.**

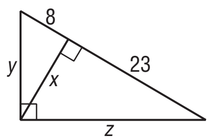



**5.**

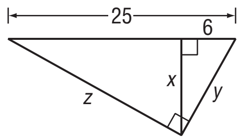



**Find *x*, *y*, and *z*.**

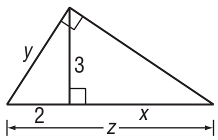
**6.**




**7.**




**8.**

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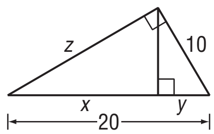



**10. CIVIL ENGINEERING** An airport, a factory, and a shopping center are at the vertices of a right triangle formed by three highways. The airport and factory are 6.0 miles apart. Their distances from the shopping center are 3.6 miles and 4.8 miles, respectively. A service road will be constructed from the shopping center to the highway that connects the airport and factory. What is the shortest possible length for the service road? Round to the nearest hundredth.

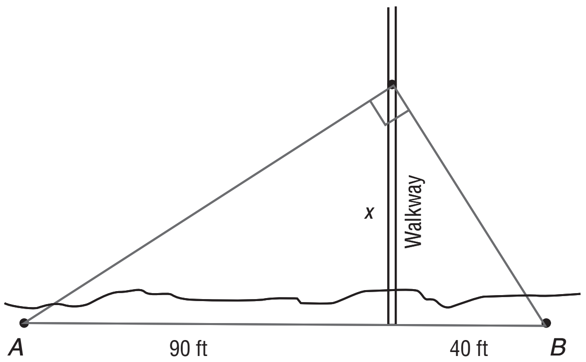
# 8-1 Word Problem Practice

## Geometric Mean

1. **SQUARES** Wilma has a rectangle of dimensions $\ell$ by $w$. She would like to replace it with a square that has the same area. What is the side length of the square with the same area as Wilma's rectangle?

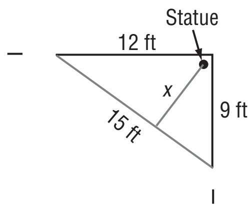
2. **EQUALITY** Gretchen computed the geometric mean of two numbers. One of the numbers was 7 and the geometric mean turned out to be 7 as well. What was the other number?

3. **VIEWING ANGLE** A photographer wants to take a picture of a beach front. His camera has a viewing angle of 90° and he wants to make sure two palm trees located at points $A$ and $B$ in the figure are just inside the edges of the photograph.



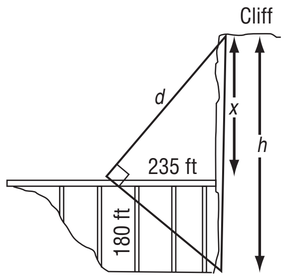

He walks out on a walkway that goes over the ocean to get the shot. If his camera has a viewing angle of 90º, at what distance down the walkway should he stop to take his photograph?

4. **EXHIBITIONS** A museum has a famous statue on display. The curator places the statue in the corner of a rectangular room and builds 15-foot-long railing in front of the statue. Use the information below to find how close visitors will be able to get to the statue.




**CLIFFS For Exercises 5–7, use the following information.**

A bridge connects to a tunnel as shown in the figure. The bridge is 180 feet above the ground. At a distance of 235 feet along the bridge out of the tunnel, the angle to the base and summit of the cliff is a right angle.




5. What is the height of the cliff? Round to the nearest whole number.

6. How high is the cliff from base to summit? Round to the nearest whole number.

7. What is $d$? Round to the nearest whole number.

# 8-1 Enrichment



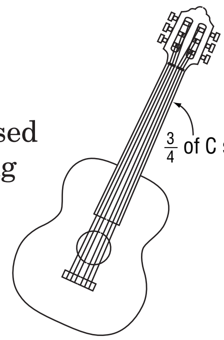
## *Mathematics and Music*

Pythagoras, a Greek philosopher who lived during the sixth century B.C., believed that all nature, beauty, and harmony could be expressed by whole-number relationships. Most people remember Pythagoras for his teachings about right triangles. (The sum of the squares of the legs equals the square of the hypotenuse.) But Pythagoras also discovered relationships between the musical notes of a scale. These relationships can be expressed as ratios.

| C | D | E | F | G | A | B | C′ |
|---|---|---|---|---|---|---|---|
| $\frac{1}{1}$ | $\frac{8}{9}$ | $\frac{4}{5}$ | $\frac{3}{4}$ | $\frac{2}{3}$ | $\frac{3}{5}$ | $\frac{8}{15}$ | $\frac{1}{2}$ |

When you play a stringed instrument, you produce different notes by placing your finger on different places on a string. This is the result of changing the length of the vibrating part of the string.

The C string can be used to produce F by placing a finger $\frac{3}{4}$ of the way along the string.




**Suppose a C string has a length of 16 inches. Write and solve proportions to determine what length of string would have to vibrate to produce the remaining notes of the scale.**

**1.** D

**2.** E

**3.** F

**4.** G

**5.** A

**6.** B

**7.** C′

**8.** Complete to show the distance between finger positions on the 16-inch C string for each note. For example, $C(16) - D\left(14\frac{2}{9}\right) = 1\frac{7}{9}$.

C $\underline{1\frac{7}{9}\text{ in.}}$ D ________ E ________ F ________ G ________ A ________ B ________ C′

**9.** Between two consecutive musical notes, there is either a whole step or a half step. Using the distances you found in Exercise 8, determine what two pairs of notes have a half step between them.

# 8-2 Lesson Reading Guide

## *The Pythagorean Theorem and Its Converse*

## Get Ready for the Lesson

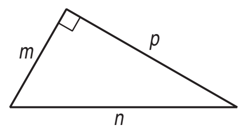
**Read the introduction to Lesson 8-2 in your textbook.**

Do the two right triangles shown in the drawing appear to be similar? Explain your reasoning.

## Read the Lesson

**1.** Explain in your own words the difference between how the Pythagorean Theorem is used and how the Converse of the Pythagorean Theorem is used.

**2.** Refer to the figure. For this figure, which statements are true?




**A.** $m^2 + n^2 = p^2$ **B.** $n^2 = m^2 + p^2$

**C.** $m^2 = n^2 + p^2$ **D.** $m^2 = p^2 - n^2$

**E.** $p^2 = n^2 - m^2$ **F.** $n^2 - p^2 = m^2$

**G.** $n = \sqrt{m^2 + p^2}$ **H.** $p = \sqrt{m^2 - n^2}$

**3.** Is the following statement true or false?
A Pythagorean triple is any group of three numbers for which the sum of the squares of the smaller two numbers is equal to the square of the largest number. Explain your reasoning.

**4.** If $x$, $y$, and $z$ form a Pythagorean triple and $k$ is a positive integer, which of the following groups of numbers are also Pythagorean triples?

**A.** $3x, 4y, 5z$ **B.** $3x, 3y, 3z$ **C.** $-3x, -3y, -3z$ **D.** $kx, ky, kz$

## Remember What You Learned

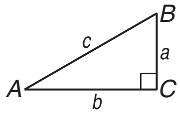
**5.** Many students who studied geometry long ago remember the Pythagorean Theorem as the equation $a^2 + b^2 = c^2$, but cannot tell you what this equation means. A formula is useless if you don't know what it means and how to use it. How could you help someone who has forgotten the Pythagorean Theorem remember the meaning of the equation $a^2 + b^2 = c^2$?

NAME _______________________________________________ DATE ____________ PERIOD _____

# 8-2 Study Guide and Intervention

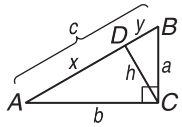
## *The Pythagorean Theorem and Its Converse*

**The Pythagorean Theorem** In a right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.




$\triangle ABC$ is a right triangle, so $a^2 + b^2 = c^2$.

**Example 1** **Prove the Pythagorean Theorem.**




With altitude $\overline{CD}$, each leg $a$ and $b$ is a geometric mean between hypotenuse $c$ and the segment of the hypotenuse adjacent to that leg.

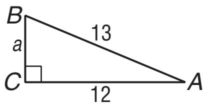
$\frac{c}{a} = \frac{a}{y}$ and $\frac{c}{b} = \frac{b}{x}$, so $a^2 = cy$ and $b^2 = cx$.

Add the two equations and substitute $c = y + x$ to get

$a^2 + b^2 = cy + cx = c(y + x) = c^2$.

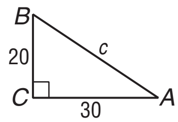
**Example 2**

**a. Find *a*.**




| | |
|---|---|
| $a^2 + b^2 = c^2$ | Pythagorean Theorem |
| $a^2 + 12^2 = 13^2$ | $b = 12$, $c = 13$ |
| $a^2 + 144 = 169$ | Simplify. |
| $a^2 = 25$ | Subtract. |
| $a = 5$ | Take the positive square root of each side. |

**b. Find *c*.**

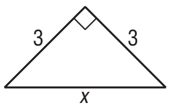



| | |
|---|---|
| $a^2 + b^2 = c^2$ | Pythagorean Theorem |
| $20^2 + 30^2 = c^2$ | $a = 20$, $b = 30$ |
| $400 + 900 = c^2$ | Simplify. |
| $1300 = c^2$ | Add. |
| $\sqrt{1300} = c$ | Take the positive square root of each side. |
| $36.1 \approx c$ | Use a calculator. |

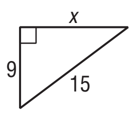
**Exercises**

**Find *x*.**

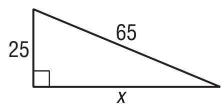
**1.**



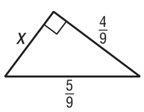

**2.**



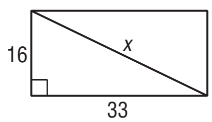

**3.**



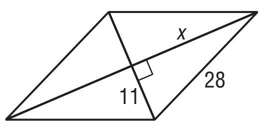

**4.**




**5.**




**6.**

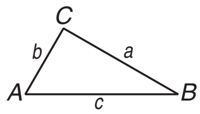



Lesson 8-2

# 8-2 Study Guide and Intervention *(continued)*

## *The Pythagorean Theorem and Its Converse*

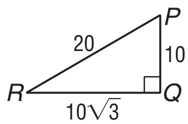
**Converse of the Pythagorean Theorem** If the sum of the squares of the measures of the two shorter sides of a triangle equals the square of the measure of the longest side, then the triangle is a right triangle.




If $a^2 + b^2 = c^2$, then $\triangle ABC$ is a right triangle.

If the three whole numbers $a$, $b$, and $c$ satisfy the equation $a^2 + b^2 = c^2$, then the numbers $a$, $b$, and $c$ form a **Pythagorean triple**.

**Example** **Determine whether $\triangle PQR$ is a right triangle.**




$a^2 + b^2 \stackrel{?}{=} c^2$ Pythagorean Theorem

$10^2 + (10\sqrt{3})^2 \stackrel{?}{=} 20^2$ $a = 10$, $b = 10\sqrt{3}$, $c = 20$

$100 + 300 \stackrel{?}{=} 400$ Simplify.

$400 = 400$ ✓ Add.

The sum of the squares of the two shorter sides equals the square of the longest side, so the triangle is a right triangle.

**Exercises**

**Determine whether each set of measures can be the measures of the sides of a right triangle. Then state whether they form a Pythagorean triple.**

**1.** 30, 40, 50

**2.** 20, 30, 40

**3.** 18, 24, 30

**4.** 6, 8, 9

**5.** $\frac{3}{7}, \frac{4}{7}, \frac{5}{7}$

**6.** 10, 15, 20

**7.** $\sqrt{5}, \sqrt{12}, \sqrt{13}$

**8.** $2, \sqrt{8}, \sqrt{12}$

**9.** 9, 40, 41

**A *family* of Pythagorean triples consists of multiples of known triples. For each Pythagorean triple, find two triples in the same family.**

**10.** 3, 4, 5

**11.** 5, 12, 13

**12.** 7, 24, 25

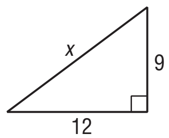
NAME ______________________ DATE __________ PERIOD _____

# 8-2 Skills Practice

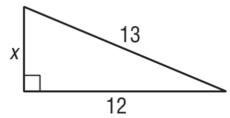
## *The Pythagorean Theorem and Its Converse*

**Find $x$.**

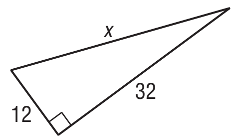
**1.**



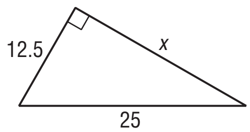

**2.**



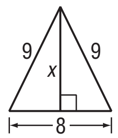

**3.**



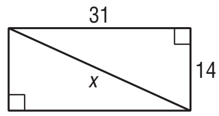

**4.**




**5.**




**6.**




**Determine whether $\triangle STU$ is a right triangle for the given vertices. Explain.**

**7.** $S(5, 5)$, $T(7, 3)$, $U(3, 2)$

**8.** $S(3, 3)$, $T(5, 5)$, $U(6, 0)$

**9.** $S(4, 6)$, $T(9, 1)$, $U(1, 3)$

**10.** $S(0, 3)$, $T(-2, 5)$, $U(4, 7)$

**11.** $S(-3, 2)$, $T(2, 7)$, $U(-1, 1)$

**12.** $S(2, -1)$, $T(5, 4)$, $U(6, -3)$

**Determine whether each set of measures can be the measures of the sides of a right triangle. Then state whether they form a Pythagorean triple.**

**13.** 12, 16, 20

**14.** 16, 30, 32

**15.** 14, 48, 50

**16.** $\frac{2}{5}, \frac{4}{5}, \frac{6}{5}$

**17.** $2\sqrt{6}, 5, 7$

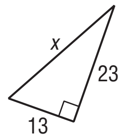
**18.** $2\sqrt{2}, 2\sqrt{7}, 6$

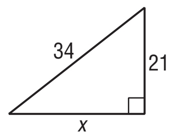
Lesson 8-2

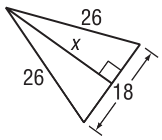
# 8-2 Practice

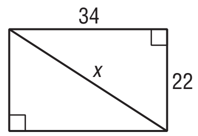
## *The Pythagorean Theorem and Its Converse*

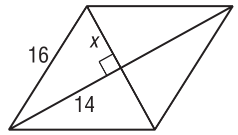
**Find *x*.**

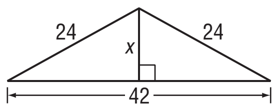
**1.**



**2.**



**3.**



**4.**



**5.**



**6.**



**Determine whether $\triangle GHI$ is a right triangle for the given vertices. Explain.**

**7.** $G(2, 7), H(3, 6), I(-4, -1)$

**8.** $G(-6, 2), H(1, 12), I(-2, 1)$

**9.** $G(-2, 1), H(3, -1), I(-4, -4)$

**10.** $G(-2, 4), H(4, 1), I(-1, -9)$

**Determine whether each set of measures can be the measures of the sides of a right triangle. Then state whether they form a Pythagorean triple.**

**11.** 9, 40, 41

**12.** 7, 28, 29

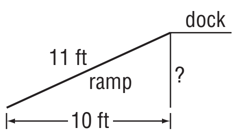
**13.** 24, 32, 40

**14.** $\frac{9}{5}, \frac{12}{5}, 3$

**15.** $\frac{1}{3}, \frac{2\sqrt{2}}{3}, 1$

**16.** $\frac{\sqrt{4}}{7}, \frac{2\sqrt{3}}{7}, \frac{4}{7}$

**17. CONSTRUCTION** The bottom end of a ramp at a warehouse is 10 feet from the base of the main dock and is 11 feet long. How high is the dock?



# 8-2 Word Problem Practice

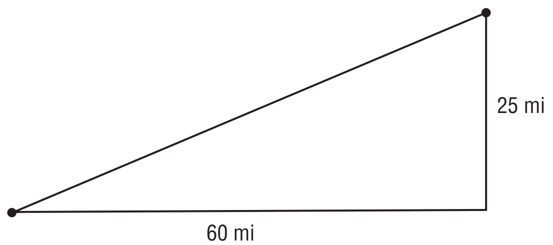
## *The Pythagorean Theorem and Its Converse*

1. **SIDEWALKS** Construction workers are building a marble sidewalk around a park that is shaped like a right triangle. Each marble slab adds 2 feet to the length of the sidewalk. The workers find that exactly 1071 and 1840 slabs are required to make the sidewalks along the short sides of the park. How many slabs are required to make the sidewalk that runs along the long side of the park?

2. **RIGHT ANGLES** Clyde makes a triangle using three sticks of lengths 20 inches, 21 inches, and 28 inches. Is the triangle a right triangle? Explain.

3. **TETHERS** To help support a flag pole, a 50-foot-long tether is tied to the pole at a point 40 feet above the ground. The tether is pulled taut and tied to an anchor in the ground. How far away from the base of the pole is the anchor?

4. **FLIGHT** An airplane lands at an airport 60 miles east and 25 miles north of where it took off.




How far apart are the two airports?

**PYTHAGOREAN TRIPLES For Exercises 5–7, use the following information.**

Ms. Jones assigned her fifth-period geometry class the following problem.

Let $m$ and $n$ be two positive integers with $m > n$. Let $a = m^2 - n^2$, $b = 2mn$, and $c = m^2 + n^2$.

5. Show that there is a right triangle with side lengths $a$, $b$, and $c$.

6. Complete the following table.

| $m$ | $n$ | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| 2 | 1 | 3 | 4 | 5 |
| 3 | 1 | | | |
| 3 | 2 | | | |
| 4 | 1 | | | |
| 4 | 2 | | | |
| 4 | 3 | | | |
| 5 | 1 | | | |

7. Find a Pythagorean triple that corresponds to a right triangle with a hypotenuse $25^2 = 625$ units long. (*Hint:* Use the table you completed for Exercise 6 to find two positive integers $m$ and $n$ with $m > n$ and $m^2 + n^2 = 625$.)

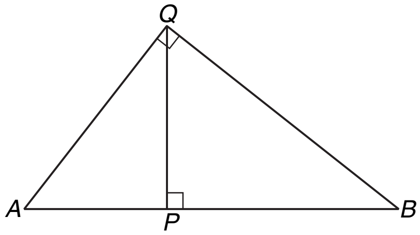
Lesson 8-2

# 8-2 Enrichment

## *Converse of a Right Triangle Theorem*

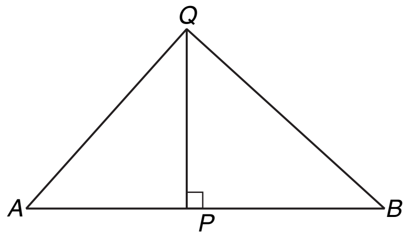
You have learned that the measure of the altitude from the vertex of the right angle of a right triangle to its hypotenuse is the geometric mean between the measures of the two segments of the hypotenuse. Is the converse of this theorem true? In order to find out, it will help to rewrite the original theorem in if-then form as follows.

If $\triangle ABQ$ is a right triangle with right angle at $Q$, then $QP$ is the geometric mean between $AP$ and $PB$, where $P$ is between $A$ and $B$ and $\overline{QP}$ is perpendicular to $\overline{AB}$.




**1.** Write the converse of the if-then form of the theorem.

**2.** Is the converse of the original theorem true? Refer to the figure at the right to explain your answer.




You may find it interesting to examine the other theorems in Chapter 7 to see whether their converses are true or false. You will need to restate the theorems carefully in order to write their converses.

# 8-2 Spreadsheet Activity

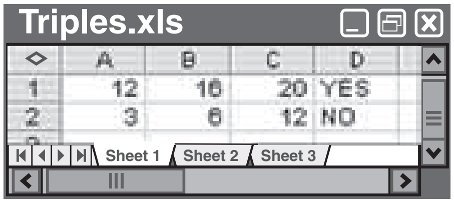
## *Pythagorean Triples*

You can use a spreadsheet to determine whether three whole numbers form a Pythagorean triple.

**Example 1** **Use a spreadsheet to determine whether the numbers 12, 16, and 20 form a Pythagorean triple.**

**Step 1** In cell A1, enter 12. In cell B1, enter 16 and in cell C1, enter 20. *The longest side should be entered in column C.*

**Step 2** In cell D1, enter an equals sign followed by IF(A1^2+B1^2=C1^2,"YES","NO"). This will return "YES" if the set of numbers is a Pythagorean triple and will return "NO" if it is not.




The numbers 12, 16, and 20 form a Pythagorean triple.

**Example 2** **Use a spreadsheet to determine whether the numbers 3, 6, and 12 form a Pythagorean triple.**

**Step 1** In cell A2, enter 3, in cell B2, enter 6, and in cell C2, enter 12.

**Step 2** Click on the bottom right corner of cell D1 and drag it to D2. This will determine whether or not the set of numbers is a Pythagorean triple.

The numbers 3, 6, and 12 do not form a Pythagorean triple.



**Exercises**

**Use a spreadsheet to determine whether each set of numbers forms a Pythagorean triple.**

**1.** 14, 48, 50 **2.** 16, 30, 34 **3.** 5, 5, 9

**4.** 4, 5, 7 **5.** 18, 24, 30 **6.** 10, 24, 26

**7.** 25, 60, 65 **8.** 2, 4, 5 **9.** 19, 21, 22

**10.** 18, 80, 82 **11.** 5, 12, 13 **12.** 20, 48, 52

# 8-3 Lesson Reading Guide

## *Special Right Triangles*

### Get Ready for the Lesson

**Read the introduction to Lesson 8-3 in your textbook.**

Suppose you have the four half square triangles that are each pinwheel pattern. If there are 9 squares total in each pattern, how many additional squares of material do you need to complete the pattern?

### Read the Lesson

**1.** Supply the correct number or numbers to complete each statement.

**a.** In a 45°-45°-90° triangle, to find the length of the hypotenuse, multiply the length of a leg by ______.

**b.** In a 30°-60°-90° triangle, to find the length of the hypotenuse, multiply the length of the shorter leg by ______.

**c.** In a 30°-60°-90° triangle, the longer leg is opposite the angle with a measure of ______.

**d.** In a 30°-60°-90° triangle, to find the length of the longer leg, multiply the length of the shorter leg by ______.

**e.** In an isosceles right triangle, each leg is opposite an angle with a measure of ______.

**f.** In a 30°-60°-90° triangle, to find the length of the shorter leg, divide the length of the longer leg by ______.

**g.** In 30°-60°-90° triangle, to find the length of the longer leg, divide the length of the hypotenuse by ______ and multiply the result by ______.

**h.** To find the length of a side of a square, divide the length of the diagonal by ______.

**2.** Indicate whether each statement is *always*, *sometimes*, or *never* true.

**a.** The lengths of the three sides of an isosceles triangle satisfy the Pythagorean Theorem.

**b.** The lengths of the sides of a 30°-60°-90° triangle form a Pythagorean triple.

**c.** The lengths of all three sides of a 30°-60°-90° triangle are positive integers.

### Remember What You Learned

**3.** Some students find it easier to remember mathematical concepts in terms of specific numbers rather than variables. How can you use specific numbers to help you remember the relationship between the lengths of the three sides in a 30°-60°-90° triangle?

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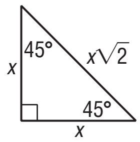
# 8-3 Study Guide and Intervention

## *Special Right Triangles*

**Properties of 45°-45°-90° Triangles** The sides of a 45°-45°-90° right triangle have a special relationship.

**Example 1** **If the leg of a 45°-45°-90° right triangle is $x$ units, show that the hypotenuse is $x\sqrt{2}$ units.**




Using the Pythagorean Theorem with $a = b = x$, then

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= x^2 + x^2 \\ &= 2x^2 \\ c &= \sqrt{2x^2} \\ &= x\sqrt{2} \end{aligned}$$

**Example 2** **In a 45°-45°-90° right triangle the hypotenuse is $\sqrt{2}$ times the leg. If the hypotenuse is 6 units, find the length of each leg.**

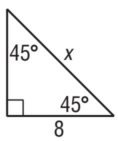
The hypotenuse is $\sqrt{2}$ times the leg, so divide the length of the hypotenuse by $\sqrt{2}$.

$$\begin{aligned} a &= \frac{6}{\sqrt{2}} \\ &= \frac{6\sqrt{2}}{\sqrt{2}\sqrt{2}} \\ &= \frac{6\sqrt{2}}{2} \\ &= 3\sqrt{2} \text{ units} \end{aligned}$$

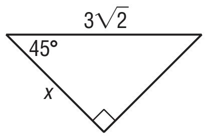
**Exercises**

**Find $x$.**

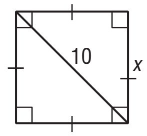
**1.**



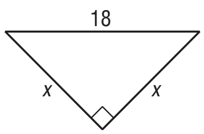

**2.**



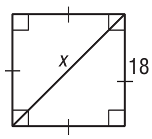

**3.**



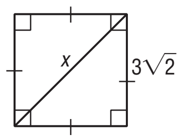

**4.**




**5.**




**6.**




**7.** Find the perimeter of a square with diagonal 12 centimeters.

**8.** Find the diagonal of a square with perimeter 20 inches.

**9.** Find the diagonal of a square with perimeter 28 meters.

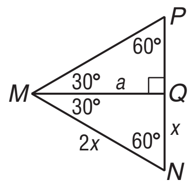
Lesson 8-3

# 8-3 Study Guide and Intervention *(continued)*

## *Special Right Triangles*

**Properties of 30°-60°-90° Triangles** The sides of a 30°-60°-90° right triangle also have a special relationship.

**Example 1** **In a 30°-60°-90° right triangle, show that the hypotenuse is twice the shorter leg and the longer leg is $\sqrt{3}$ times the shorter leg.**




$\triangle MNQ$ is a 30°-60°-90° right triangle, and the length of the hypotenuse $\overline{MN}$ is two times the length of the shorter side $\overline{NQ}$.
Using the Pythagorean Theorem,

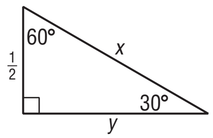
$$\begin{aligned} a^2 &= (2x)^2 - x^2 \\ &= 4x^2 - x^2 \\ &= 3x^2 \\ a &= \sqrt{3x^2} \\ &= x\sqrt{3} \end{aligned}$$

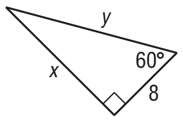
**Example 2** **In a 30°-60°-90° right triangle, the hypotenuse is 5 centimeters. Find the lengths of the other two sides of the triangle.**

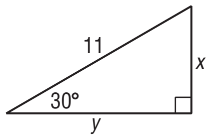
If the hypotenuse of a 30°-60°-90° right triangle is 5 centimeters, then the length of the shorter leg is half of 5 or 2.5 centimeters. The length of the longer leg is $\sqrt{3}$ times the length of the shorter leg, or $(2.5)(\sqrt{3})$ centimeters.

**Exercises**

**Find *x* and *y*.**

**1.**



**2.**



**3.**
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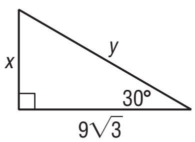
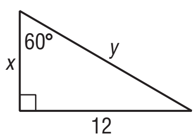
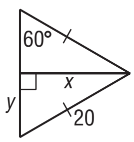



**5.**



**6.**



**7.** The perimeter of an equilateral triangle is 32 centimeters. Find the length of an altitude of the triangle to the nearest tenth of a centimeter.

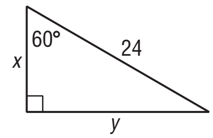
**8.** An altitude of an equilateral triangle is 8.3 meters. Find the perimeter of the triangle to the nearest tenth of a meter.

# 8-3 Skills Practice

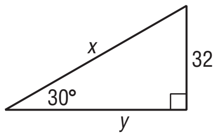
## Special Right Triangles

**Find the exact values of *x* and *y*.**

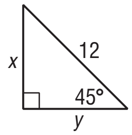
**1.**



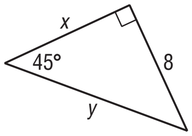

**2.**



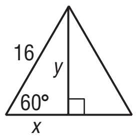

**3.**



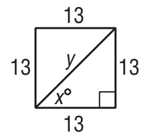

**4.**



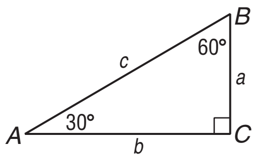

**5.**




**6.**




**For Exercises 7–9, use the figure at the right.**

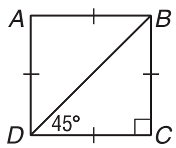



**7.** If $a = 11$, find $b$ and $c$.

**8.** If $b = 15$, find $a$ and $c$.

**9.** If $c = 9$, find $a$ and $b$.

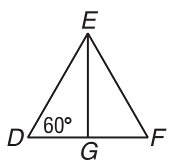
**For Exercises 10 and 11, use the figure at the right.**




**10.** The perimeter of the square is 30 inches. Find the length of $\overline{BC}$.

**11.** Find the length of the diagonal $\overline{BD}$.

**12.** The perimeter of the equilateral triangle is 60 meters. Find the length of an altitude.




**13.** $\triangle GEC$ is a 30°-60°-90° triangle with right angle at $E$, and $\overline{EC}$ is the longer leg. Find the coordinates of $G$ in Quadrant I for $E(1, 1)$ and $C(4, 1)$.

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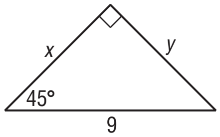
# 8-3 Practice

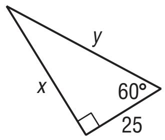
## *Special Right Triangles*

**Find *x* and *y*.**

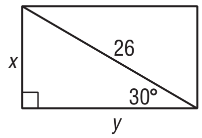
**1.**



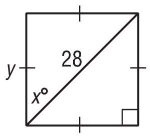

**2.**



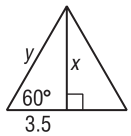

**3.**



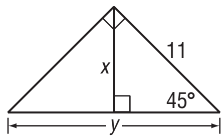

**4.**



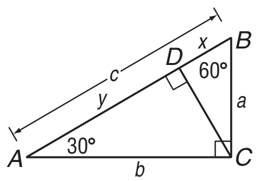

**5.**




**6.**




**For Exercises 7–9, use the figure at the right.**




**7.** If $a = 4\sqrt{3}$, find $b$ and $c$.

**8.** If $x = 3\sqrt{3}$, find $a$ and $CD$.

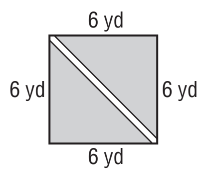
**9.** If $a = 4$, find $CD$, $b$, and $y$.

**10.** The perimeter of an equilateral triangle is 39 centimeters. Find the length of an altitude of the triangle.

**11.** $\triangle MIP$ is a 30°-60°-90° triangle with right angle at $I$, and $\overline{IP}$ the longer leg. Find the coordinates of $M$ in Quadrant I for $I(3, 3)$ and $P(12, 3)$.

**12.** $\triangle TJK$ is a 45°-45°-90° triangle with right angle at $J$. Find the coordinates of $T$ in Quadrant II for $J(-2, -3)$ and $K(3, -3)$.

**13. BOTANICAL GARDENS** One of the displays at a botanical garden is an herb garden planted in the shape of a square. The square measures 6 yards on each side. Visitors can view the herbs from a diagonal pathway through the garden. How long is the pathway?

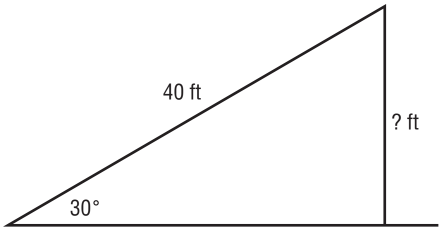


# 8-3 Word Problem Practice

## *Special Right Triangles*

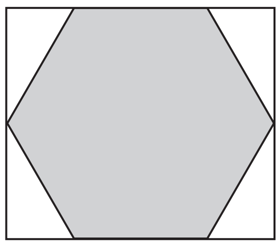
1. **ORIGAMI** A square piece of paper 150 millimeters on a side is folded in half along a diagonal. The result is a 45°-45°-90° triangle. What is the length of the hypotenuse of this triangle?

2. **ESCALATORS** A 40-foot-long escalator rises from the first floor to the second floor of a shopping mall. The escalator makes a 30° angle with the horizontal.



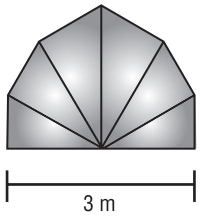

How high above the first floor is the second floor?

3. **HEXAGONS** A box of chocolates shaped like a regular hexagon is placed snugly inside of a rectangular box as shown in the figure.



If the side length of the hexagon is 3 inches, what are the dimensions of the rectangular box?

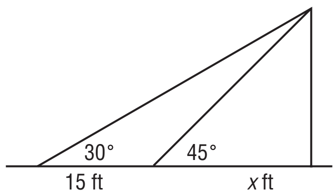
4. **WINDOWS** A large stained glass window is constructed from six 30°-60°-90° triangles as shown in the figure.




What is the height of the window?

**MOVIES For Exercises 5–7, use the following information.**

Kim and Yolanda are watching a movie in a movie theater. Yolanda is sitting $x$ feet from the screen and Kim is 15 feet behind Yolanda.




The angle that Kim's line of sight to the top of the screen makes with the horizontal is 30°. The angle that Yolanda's line of sight to the top of the screen makes with the horizontal is 45°.

5. How high is the top of the screen in terms of x?

6. What is $\frac{x+15}{x}$ ?

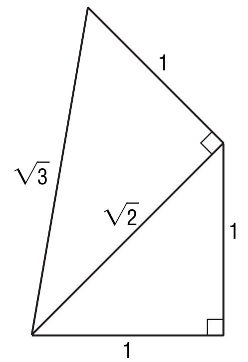
7. How far is Yolanda from the screen? Round your answer to the nearest tenth.

Lesson 8-3

# 8-3 Enrichment

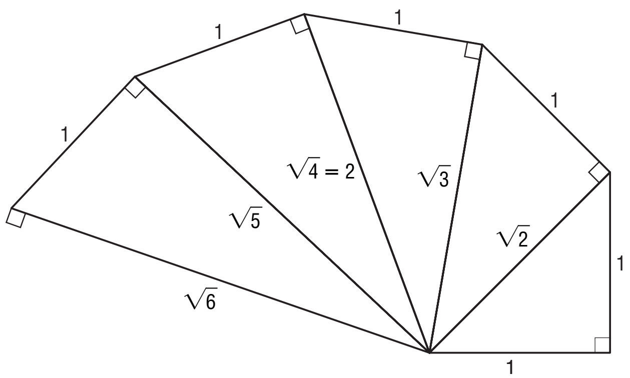
## *Constructing Values of Square Roots*

The diagram at the right shows a right isosceles triangle with two legs of length 1 inch. By the Pythagorean Theorem, the length of the hypotenuse is $\sqrt{2}$ inches. By constructing an adjacent right triangle with legs of $\sqrt{2}$ inches and 1 inch, you can create a segment of length $\sqrt{3}$.




By continuing this process as shown below, you can construct a "wheel" of square roots. This wheel is called the "Wheel of Theodorus" after a Greek philosopher who lived about 400 B.C.

Continue constructing the wheel until you make a segment of length $\sqrt{18}$.



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# 8-4 Lesson Reading Guide

## *Trigonometry*

### Get Ready for the Lesson

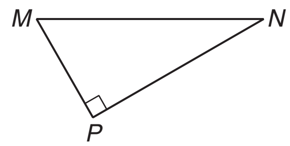
**Read the introduction to Lesson 8-4 in your textbook.**

- Why is it important to determine the relative positions accurately in navigation? (Give two possible reasons.)

- What does *calibrated* mean?

### Read the Lesson

**1.** Refer to the figure. Write a ratio using the side lengths in the figure to represent each of the following trigonometric ratios.




**A.** sin *N* 　　　　　**B.** cos *N*

**C.** tan *N* 　　　　　**D.** tan *M*

**E.** sin *M* 　　　　　**F.** cos *M*

**2.** Assume that you enter each of the expressions in the list on the left into your calculator. Match each of these expressions with a description from the list on the right to tell what you are finding when you enter this expression.

| | |
|---|---|
| **a.** sin 20 | **i.** the degree measure of an acute angle whose cosine is 0.8 |
| **b.** cos 20 | **ii.** the ratio of the length of the leg adjacent to the 20° angle to the length of hypotenuse in a 20°-70°-90° triangle |
| **c.** $\sin^{-1} 0.8$ | **iii.** the degree measure of an acute angle in a right triangle for which the ratio of the length of the opposite leg to the length of the adjacent leg is 0.8 |
| **d.** $\tan^{-1} 0.8$ | **iv.** the ratio of the length of the leg opposite the 20° angle to the length of the leg adjacent to it in a 20°-70°-90° triangle |
| **e.** tan 20 | **v.** the ratio of the length of the leg opposite the 20° angle to the length of hypotenuse in a 20°-70°-90° triangle |
| **f.** $\cos^{-1} 0.8$ | **vi.** the degree measure of an acute angle in a right triangle for which the ratio of the length of the opposite leg to the length of the hypotenuse is 0.8 |

### Remember What You Learned

**3.** How can the *co* in *cosine* help you to remember the relationship between the sines and cosines of the two acute angles of a right triangle?

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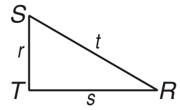
# 8-4 Study Guide and Intervention

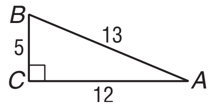
## *Trigonometry*

**Trigonometric Ratios** The ratio of the lengths of two sides of a right triangle is called a **trigonometric ratio**. The three most common ratios are **sine**, **cosine**, and **tangent**, which are abbreviated *sin*, *cos*, and *tan*, respectively.




$\sin R = \dfrac{\text{leg opposite } \angle R}{\text{hypotenuse}} = \dfrac{r}{t}$ $\quad\cos R = \dfrac{\text{leg adjacent to } \angle R}{\text{hypotenuse}} = \dfrac{s}{t}$ $\quad\tan R = \dfrac{\text{leg opposite } \angle R}{\text{leg adjacent to } \angle R} = \dfrac{r}{s}$

**Example** **Find sin *A*, cos *A*, and tan *A*. Express each ratio as a decimal to the nearest thousandth.**



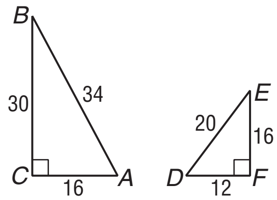

$\sin A = \dfrac{\text{opposite leg}}{\text{hypotenuse}} = \dfrac{BC}{AB} = \dfrac{5}{13} \approx 0.385$

$\cos A = \dfrac{\text{adjacent leg}}{\text{hypotenuse}} = \dfrac{AC}{AB} = \dfrac{12}{13} \approx 0.923$

$\tan A = \dfrac{\text{opposite leg}}{\text{adjacent leg}} = \dfrac{BC}{AC} = \dfrac{5}{12} \approx 0.417$

**Exercises**

**Find the indicated trigonometric ratio as a fraction and as a decimal. If necessary, round to the nearest ten-thousandth.**




**1.** sin *A* **2.** tan *B*

**3.** cos *A* **4.** cos *B*

**5.** sin *D* **6.** tan *E*

**7.** cos *E* **8.** cos *D*

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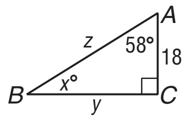
# 8-4 Study Guide and Intervention *(continued)*

## *Trigonometry*

**Use Trigonometric Ratios** In a right triangle, if you know the measures of two sides or if you know the measures of one side and an acute angle, then you can use trigonometric ratios to find the measures of the missing sides or angles of the triangle.

**Example** **Find $x$, $y$, and $z$. Round each measure to the nearest whole number.**




**a. Find $x$.**

$$x + 58 = 90$$
$$x = 32$$

**b. Find $y$.**

$$\tan A = \frac{y}{18}$$
$$\tan 58° = \frac{y}{18}$$
$$y = 18 \tan 58°$$
$$y \approx 29$$

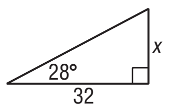
**c. Find $z$.**

$$\cos A = \frac{18}{z}$$
$$\cos 58° = \frac{18}{z}$$
$$z \cos 58° = 18$$
$$z = \frac{18}{\cos 58°}$$
$$z \approx 34$$

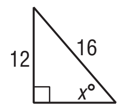
**Exercises**

**Find $x$. Round to the nearest tenth.**

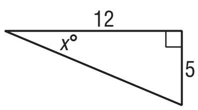
**1.**




**2.**




**3.**

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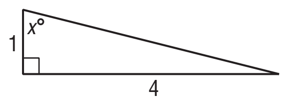
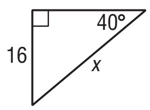
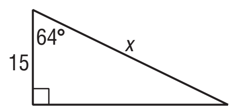



**5.**




**6.**

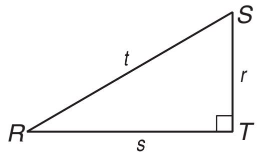


NAME ______________________________ DATE ____________ PERIOD _____

# 8-4 Skills Practice

## *Trigonometry*

**Use $\triangle RST$ to find sin $R$, cos $R$, tan $R$, sin $S$, cos $S$, and tan $S$. Express each ratio as a fraction and as a decimal to the nearest hundredth.**




**1.** $r = 16, s = 30, t = 34$

**2.** $r = 10, s = 24, t = 26$

**Use a calculator to find each value. Round to the nearest ten-thousandth.**

**3.** sin 5

**4.** tan 23

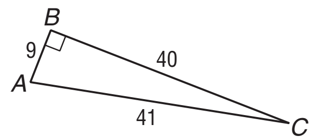
**5.** cos 61

**6.** sin 75.8

**7.** tan 17.3

**8.** cos 52.9

**Use the figure to find each trigonometric ratio. Express answers as a fraction and as a decimal rounded to the nearest ten-thousandth.**




**9.** tan $C$

**10.** sin $A$

**11.** cos $C$

**Find the measure of each acute angle to the nearest tenth of a degree.**

**12.** sin $B = 0.2985$

**13.** tan $A = 0.4168$

**14.** cos $R = 0.8443$

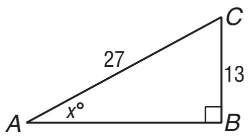
**15.** tan $C = 0.3894$

**16.** cos $B = 0.7329$

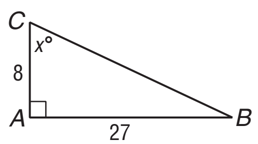
**17.** sin $A = 0.1176$

**Find $x$. Round to the nearest tenth.**

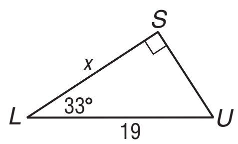
**18.**




**19.**




**20.**

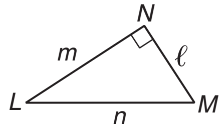


NAME ______________________________ DATE __________ PERIOD _____

# 8-4 Practice

## *Trigonometry*

**Use △*LMN* to find sin *L*, cos *L*, tan *L*, sin *M*, cos *M*, and tan *M*. Express each ratio as a fraction and as a decimal to the nearest hundredth.**




**1.** $\ell = 15, m = 36, n = 39$

**2.** $\ell = 12, m = 12\sqrt{3}, n = 24$

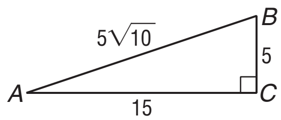
**Use a calculator to find each value. Round to the nearest ten-thousandth.**

**3.** sin 72.5

**4.** tan 27.5

**5.** cos 64.8

**Use the figure to find each trigonometric ratio. Express answers as a fraction and as a decimal rounded to the nearest ten-thousandth.**




**6.** cos *A*

**7.** tan *B*

**8.** sin *A*

**Find the measure of each acute angle to the nearest tenth of a degree.**

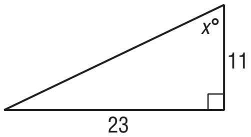
**9.** $\sin B = 0.7823$

**10.** $\tan A = 0.2356$

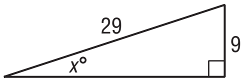
**11.** $\cos R = 0.6401$

**Find *x*. Round to the nearest tenth.**

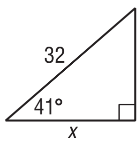
**12.**



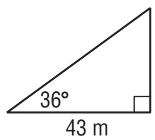

**13.**




**14.**




**15. GEOGRAPHY** Diego used a theodolite to map a region of land for his class in geomorphology. To determine the elevation of a vertical rock formation, he measured the distance from the base of the formation to his position and the angle between the ground and the line of sight to the top of the formation. The distance was 43 meters and the angle was 36 degrees. What is the height of the formation to the nearest meter?

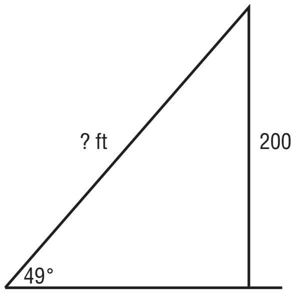


NAME ______________________________ DATE ____________ PERIOD _____

# 8-4 Word Problem Practice

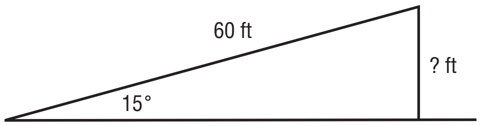
## *Trigonometry*

1. **RADIO TOWERS** Kay is standing near a 200-foot-high radio tower.




Use the information in the figure to determine how far Kay is from the top of the tower. Express your answer as a trigonometric function.

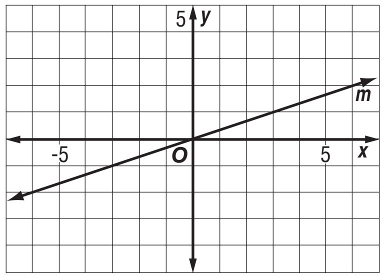
2. **RAMPS** A 60-foot ramp rises from the first floor to the second floor of a parking garage. The ramp makes a 15° angle with the ground.




How high above the first floor is the second floor? Express your answer as a trigonometric function.

3. **TRIGONOMETRY** Melinda and Walter were both solving the same trigonometry problem. However, after they finished their computations, Melinda said the answer was 52 sin 27° and Walter said the answer was 52 cos 63°. Could they both be correct? Explain.

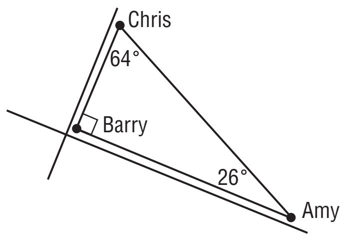
4. **LINES** Jasmine draws line $m$ on a coordinate plane.




What angle does $m$ make with the $x$-axis? Round your answer to the nearest degree.

**NEIGHBORS For Exercises 5–7, use the following information.**

Amy, Barry, and Chris live on the same block. Chris lives up the street and around the corner from Amy, and Barry lives at the corner between Amy and Chris. The three homes make a right triangle.




5. Give two trigonometric expressions for the ratio of Barry's distance from Amy to Chris' distance from Amy.

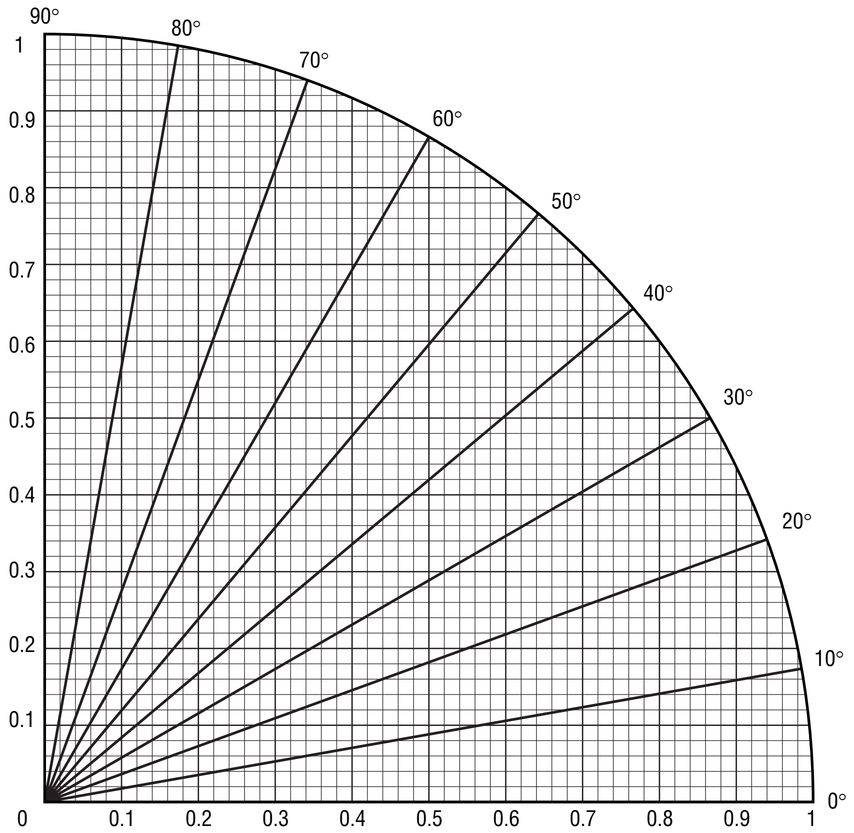
6. Give two trigonometric expressions for the ratio of Barry's distance from Chris to Amy's distance from Chris.

7. Give a trigonometric expression for the ratio of Amy's distance from Barry to Chris' distance from Barry.

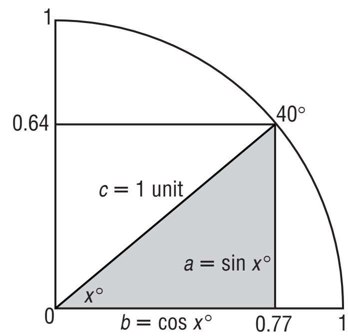
# 8-4 Enrichment

## *Sine and Cosine of Angles*

The following diagram can be used to obtain approximate values for the sine and cosine of angles from 0° to 90°. The radius of the circle is 1. So, the sine and cosine values can be read directly from the vertical and horizontal axes.




**Example** **Find approximate values for sin 40° and cos 40°. Consider the triangle formed by the segment marked 40°, as illustrated by the shaded triangle at right.**




$\sin 40° = \frac{a}{c} \approx \frac{0.64}{1}$ or 0.64 $\qquad \cos 40° = \frac{b}{c} \approx \frac{0.77}{1}$ or 0.77

**1.** Use the diagram above to complete the chart of values.

| $x°$ | 0° | 10° | 20° | 30° | 40° | 50° | 60° | 70° | 80° | 90° |
|---|---|---|---|---|---|---|---|---|---|---|
| sin $x°$ | | | | | 0.64 | | | | | |
| cos $x°$ | | | | | 0.77 | | | | | |

**2.** Compare the sine and cosine of two complementary angles (angles with a sum is 90°). What do you notice?



Lesson 8-4

# 8-5 Lesson Reading Guide

## *Angles of Elevation and Depression*

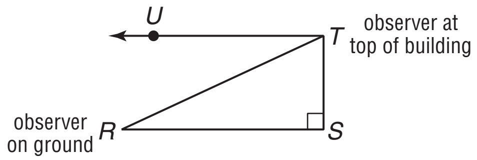
## Get Ready for the Lesson

**Read the introduction to Lesson 8-5 in your textbook.**

What does the angle measure tell the pilot?

## Read the Lesson

**1.** Refer to the figure. The two observers are looking at one another. Select the correct choice for each question.




**a.** What is the line of sight?
(i) line *RS* (ii) line *ST* (iii) line *RT* (iv) line *TU*

**b.** What is the angle of elevation?
(i) ∠*RST* (ii) ∠*SRT* (iii) ∠*RTS* (iv) ∠*UTR*

**c.** What is the angle of depression?
(i) ∠*RST* (ii) ∠*SRT* (iii) ∠*RTS* (iv) ∠*UTR*

**d.** How are the angle of elevation and the angle of depression related?
(i) They are complementary.
(ii) They are congruent.
(iii) They are supplementary.
(iv) The angle of elevation is larger than the angle of depression.

**e.** Which postulate or theorem that you learned in Chapter 3 supports your answer for part c?
(i) Corresponding Angles Postulate
(ii) Alternate Exterior Angles Theorem
(iii) Consecutive Interior Angles Theorem
(iv) Alternate Interior Angles Theorem

**2.** A student says that the angle of elevation from his eye to the top of a flagpole is 135°. What is wrong with the student's statement?

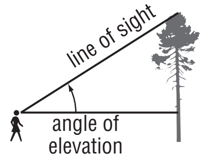
## Remember What You Learned

**3.** A good way to remember something is to explain it to someone else. Suppose a classmate finds it difficult to distinguish between angles of elevation and angles of depression. What are some hints you can give her to help her get it right every time?

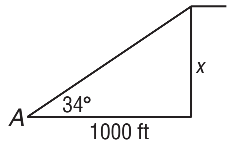
# 8-5 Study Guide and Intervention

## *Angles of Elevation and Depression*

**Angles of Elevation** Many real-world problems that involve looking up to an object can be described in terms of an **angle of elevation**, which is the angle between an observer's line of sight and a horizontal line.




**Example** **The angle of elevation from point $A$ to the top of a cliff is 34°. If point $A$ is 1000 feet from the base of the cliff, how high is the cliff?**




Let $x$ = the height of the cliff.

$\tan 34° = \frac{x}{1000}$    $\tan = \frac{\text{opposite}}{\text{adjacent}}$

$1000(\tan 34°) = x$    Multiply each side by 1000.

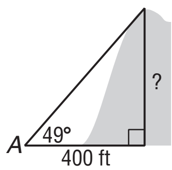
$674.5 = x$    Use a calculator.

The height of the cliff is about 674.5 feet.

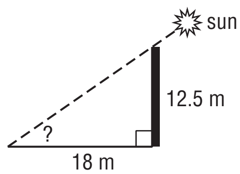
**Exercises**

**Solve each problem. Round measures of segments to the nearest whole number and angles to the nearest degree.**

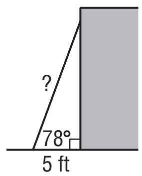
**1.** The angle of elevation from point $A$ to the top of a hill is 49°. If point $A$ is 400 feet from the base of the hill, how high is the hill?



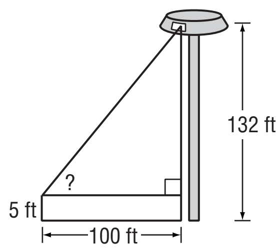

**2.** Find the angle of elevation of the sun when a 12.5-meter-tall telephone pole casts an 18-meter-long shadow.




**3.** A ladder leaning against a building makes an angle of 78° with the ground. The foot of the ladder is 5 feet from the building. How long is the ladder?



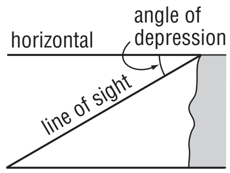

**4.** A person whose eyes are 5 feet above the ground is standing on the runway of an airport 100 feet from the control tower. That person observes an air traffic controller at the window of the 132-foot tower. What is the angle of elevation?



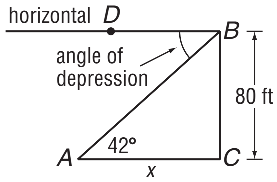
# 8-5 Study Guide and Intervention *(continued)*

## *Angles of Elevation and Depression*

**Angles of Depression** When an observer is looking down, the **angle of depression** is the angle between the observer's line of sight and a horizontal line.




**Example** **The angle of depression from the top of an 80-foot building to point $A$ on the ground is 42°. How far is the foot of the building from point $A$?**




Let $x$ = the distance from point $A$ to the foot of the building. Since the horizontal line is parallel to the ground, the angle of depression $\angle DBA$ is congruent to $\angle BAC$.

$\tan 42° = \frac{80}{x}$ — $\tan = \frac{\text{opposite}}{\text{adjacent}}$

$x(\tan 42°) = 80$ — Multiply each side by $x$.

$x = \frac{80}{\tan 42°}$ — Divide each side by tan 42°.

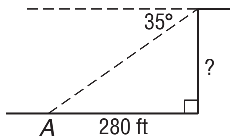
$x \approx 88.8$ — Use a calculator.

Point $A$ is about 89 feet from the base of the building.

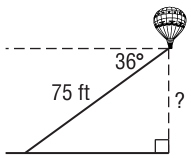
**Exercises**

**Solve each problem. Round measures of segments to the nearest whole number and angles to the nearest degree.**

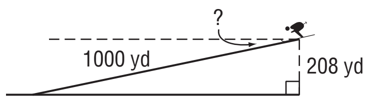
**1.** The angle of depression from the top of a sheer cliff to point $A$ on the ground is 35°. If point $A$ is 280 feet from the base of the cliff, how tall is the cliff?



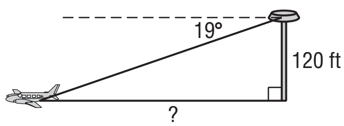

**2.** The angle of depression from a balloon on a 75-foot string to a person on the ground is 36°. How high is the balloon?




**3.** A ski run is 1000 yards long with a vertical drop of 208 yards. Find the angle of depression from the top of the ski run to the bottom.




**4.** From the top of a 120-foot-high tower, an air traffic controller observes an airplane on the runway at an angle of depression of 19°. How far from the base of the tower is the airplane?



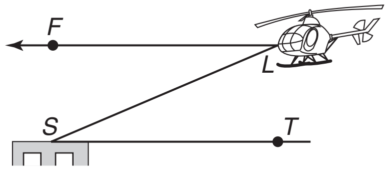
NAME ______________________________________________ DATE ____________ PERIOD _____

# 8-5 Skills Practice

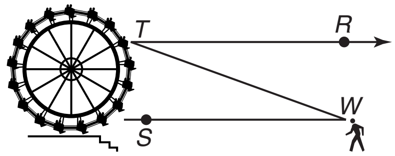
## *Angles of Elevation and Depression*

**Name the angle of depression or angle of elevation in each figure.**

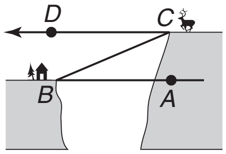
**1.**



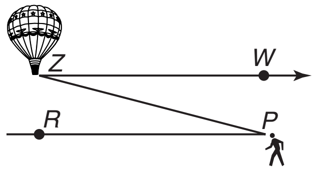

**2.**




**3.**

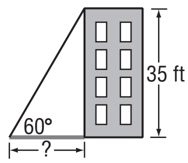



**4.**



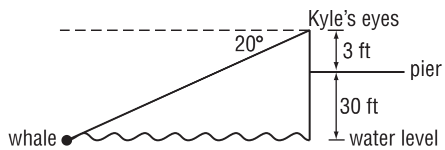

**5. MOUNTAIN BIKING** On a mountain bike trip along the Gemini Bridges Trail in Moab, Utah, Nabuko stopped on the canyon floor to get a good view of the twin sandstone bridges. Nabuko is standing about 60 meters from the base of the canyon cliff, and the natural arch bridges are about 100 meters up the canyon wall. If her line of sight is five feet above the ground, what is the angle of elevation to the top of the bridges? Round to the nearest tenth degree.

**6. SHADOWS** Suppose the sun casts a shadow off a 35-foot building. If the angle of elevation to the sun is 60°, how long is the shadow to the nearest tenth of a foot?




**7. BALLOONING** From her position in a hot-air balloon, Angie can see her car parked in a field. If the angle of depression is 8° and Angie is 38 meters above the ground, what is the straight-line distance from Angie to her car? Round to the nearest whole meter.

**8. INDIRECT MEASUREMENT** Kyle is at the end of a pier 30 feet above the ocean. His eye level is 3 feet above the pier. He is using binoculars to watch a whale surface. If the angle of depression of the whale is 20°, how far is the whale from Kyle's binoculars? Round to the nearest tenth foot.

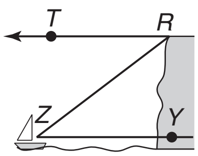


# 8-5 Practice

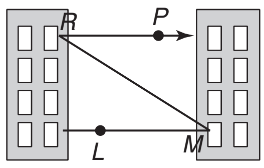
## *Angles of Elevation and Depression*

**Name the angle of depression or angle of elevation in each figure.**

**1.**



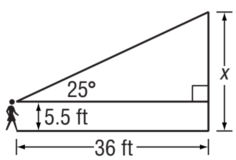

**2.**




**3. WATER TOWERS** A student can see a water tower from the closest point of the soccer field at San Lobos High School. The edge of the soccer field is about 110 feet from the water tower and the water tower stands at a height of 32.5 feet. What is the angle of elevation if the eye level of the student viewing the tower from the edge of the soccer field is 6 feet above the ground? Round to the nearest tenth degree.

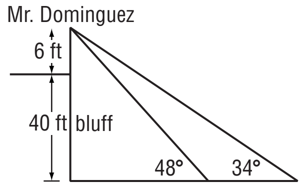
**4. CONSTRUCTION** A roofer props a ladder against a wall so that the top of the ladder reaches a 30-foot roof that needs repair. If the angle of elevation from the bottom of the ladder to the roof is 55°, how far is the ladder from the base of the wall? Round your answer to the nearest foot.

**5. TOWN ORDINANCES** The town of Belmont restricts the height of flagpoles to 25 feet on any property. Lindsay wants to determine whether her school is in compliance with the regulation. Her eye level is 5.5 feet from the ground and she stands 36 feet from the flagpole. If the angle of elevation is about 25°, what is the height of the flagpole to the nearest tenth foot?




**6. GEOGRAPHY** Stephan is standing on a mesa at the Painted Desert. The elevation of the mesa is about 1380 meters and Stephan's eye level is 1.8 meters above ground. If Stephan can see a band of multicolored shale at the bottom and the angle of depression is 29°, about how far is the band of shale from his eyes? Round to the nearest meter.

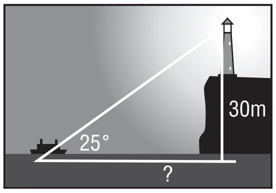
**7. INDIRECT MEASUREMENT** Mr. Dominguez is standing on a 40-foot ocean bluff near his home. He can see his two dogs on the beach below. If his line of sight is 6 feet above the ground and the angles of depression to his dogs are 34° and 48°, how far apart are the dogs to the nearest foot?



# 8-5 Word Problem Practice

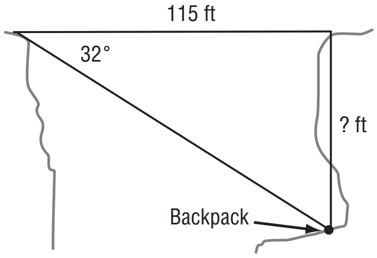
## *Angles of Elevation and Depression*

1. **LIGHTHOUSES** Sailors on a ship at sea spot the light from a lighthouse. The angle of elevation to the light is 25°.




The light of the lighthouse is 30 meters above sea level. How far from the shore is the ship? Round your answer to the nearest meter.

2. **RESCUE** A hiker dropped his backpack over one side of a canyon onto a ledge below. Because of the shape of the cliff, he could not see exactly where it landed.



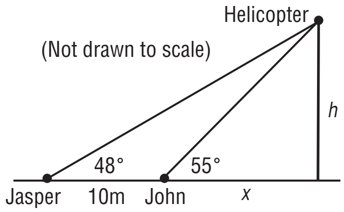

From the other side, the park ranger reports that the angle of depression to the backpack is 32°. If the width of the canyon is 115 feet, how far down did the backpack fall? Round your answer to the nearest whole number.

3. **AIRPLANES** The angle of elevation to an airplane viewed from the control tower at an airport is 7°. The tower is 200 feet high and the pilot reports that the altitude is 5200 feet. How far away from the control tower is the airplane? Round your answer to the nearest foot.

4. **PEAK TRAM** The Peak Tram in Hong Kong connects two terminals, one at the base of a mountain, and the other at the summit. The angle of elevation of the upper terminal from the lower terminal is about 15.5°. The distance between the two terminals is about 1365 meters. About how much higher above sea level is the upper terminal compared to the lower terminal? Round your answer to the nearest meter.

**HELICOPTERS For Exercises 5–7, use the following information.**

Jermaine and John are watching a helicopter hover above the ground.




Jermaine and John are standing 10 meters apart.

5. Find two different expressions that can be used to find the $h$, height of the helicopter.

6. Equate the two expressions you found for Exercise 5 to solve for $x$. Round your answer to the nearest hundredth.

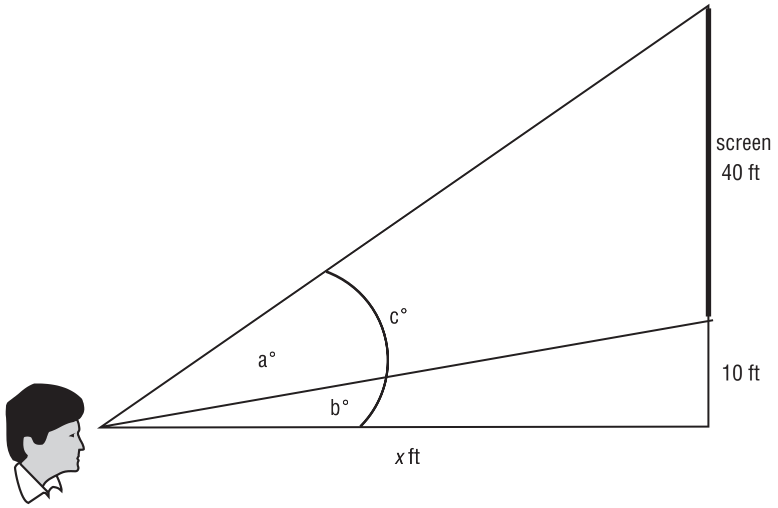
7. How high above the ground is the helicopter? Round your answer to the nearest hundredth.

Lesson 8-5

# 8-5 Enrichment

## *Best Seat in the House*

Most people want to sit in the best seat in the movie theater. The best seat could be defined as the seat that allows you to see the maximum amount of screen. The picture below represents this situation.




To determine the best seat in the house, you want to find what value of $x$ allows you to see the maximum amount of screen. The value of $x$ is how far from the screen you should sit.

**1.** To maximize the amount of screen viewed, which angle value needs to be maximized? Why?

**2.** What is the value of $a$ if $x = 10$ feet?

**3.** What is the value of $a$ if $x = 20$ feet?

**4.** What is the value of $a$ if $x = 25$ feet?

**5.** What is the value of $a$ if $x = 35$ feet?

**6.** What is the value of $a$ if $x = 55$ feet?

**7.** Which value of $x$ gives the greatest value of $a$? So, where is the best seat in the movie theater?

# 8-6 Lesson Reading Guide

## *The Law of Sines*



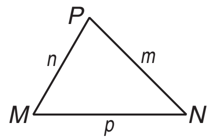
## Get Ready for the Lesson

**Read the introduction to Lesson 8-6 in your textbook.**

- If a triangle is a right triangle, what theorem can be used to determine the lengths of the sides?
- If a triangle is not a right triangle, can this theorem still be used to determine the lengths of the sides?

## Read the Lesson

**1.** Refer to the figure. According to the Law of Sines, which of the following are correct statements?




**A.** $\frac{m}{\sin M} = \frac{n}{\sin N} = \frac{p}{\sin P}$

**B.** $\frac{\sin m}{M} = \frac{\sin n}{N} = \frac{\sin p}{P}$

**C.** $\frac{\cos M}{m} = \frac{\cos N}{n} = \frac{\cos P}{p}$

**D.** $\frac{\sin M}{m} + \frac{\sin N}{n} = \frac{\sin P}{p}$

**E.** $(\sin M)^2 + (\sin N)^2 = (\sin P)^2$

**F.** $\frac{\sin P}{p} = \frac{\sin M}{m} = \frac{\sin N}{n}$

**2.** State whether each of the following statements is *true* or *false*. If the statement is false, explain why.

**a.** The Law of Sines applies to all triangles.

**b.** The Pythagorean Theorem applies to all triangles.

**c.** If you are given the length of one side of a triangle and the measures of any two angles, you can use the Law of Sines to find the lengths of the other two sides.

**d.** If you know the measures of two angles of a triangle, you should use the Law of Sines to find the measure of the third angle.

**e.** A friend tells you that in triangle $RST$, $m\angle R = 132$, $r = 24$ centimeters, and $s = 31$ centimeters. Can you use the Law of Sines to solve the triangle? Explain.

## Remember What You Learned

**3.** Many students remember mathematical equations and formulas better if they can state them in words. State the Law of Sines in your own words without using variables or mathematical symbols.

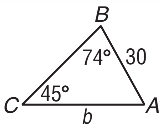
# 8-6 Study Guide and Intervention

## *The Law of Sines*

**The Law of Sines** In any triangle, there is a special relationship between the angles of the triangle and the lengths of the sides opposite the angles.

| | |
|---|---|
| **Law of Sines** | $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ |

**Example 1** **In $\triangle ABC$, find $b$.**




$\frac{\sin C}{c} = \frac{\sin B}{b}$ Law of Sines

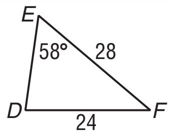
$\frac{\sin 45°}{30} = \frac{\sin 74°}{b}$ $m\angle C = 45$, $c = 30$, $m\angle B = 74$

$b \sin 45° = 30 \sin 74°$ Cross multiply.

$b = \frac{30 \sin 74°}{\sin 45°}$ Divide each side by sin 45°.

$b \approx 40.8$ Use a calculator.

**Example 2** **In $\triangle DEF$, find $m\angle D$.**




$\frac{\sin D}{d} = \frac{\sin E}{e}$ Law of Sines

$\frac{\sin D}{28} = \frac{\sin 58°}{24}$ $d = 28$, $m\angle E = 58$, $e = 24$

$24 \sin D = 28 \sin 58°$ Cross multiply.

$\sin D = \frac{28 \sin 58°}{24}$ Divide each side by 24.

$D = \sin^{-1} \frac{28 \sin 58°}{24}$ Use the inverse sine.

$D \approx 81.6°$ Use a calculator.

**Exercises**

**Find each measure using the given measures of $\triangle ABC$. Round angle measures to the nearest degree and side measures to the nearest tenth.**

**1.** If $c = 12$, $m\angle A = 80$, and $m\angle C = 40$, find $a$.

**2.** If $b = 20$, $c = 26$, and $m\angle C = 52$, find $m\angle B$.

**3.** If $a = 18$, $c = 16$, and $m\angle A = 84$, find $m\angle C$.

**4.** If $a = 25$, $m\angle A = 72$, and $m\angle B = 17$, find $b$.

**5.** If $b = 12$, $m\angle A = 89$, and $m\angle B = 80$, find $a$.

**6.** If $a = 30$, $c = 20$, and $m\angle A = 60$, find $m\angle C$.

# 8-6 Study Guide and Intervention *(continued)*

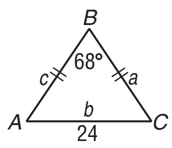
## *The Law of Sines*



**Use the Law of Sines to Solve Problems** You can use the **Law of Sines** to solve some problems that involve triangles.

| | |
|---|---|
| **Law of Sines** | Let $\triangle ABC$ be any triangle with $a$, $b$, and $c$ representing the measures of the sides opposite the angles with measures $A$, $B$, and $C$, respectively. Then $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$. |

**Example** **Isosceles $\triangle ABC$ has a base of 24 centimeters and a vertex angle of 68°. Find the perimeter of the triangle.**




The vertex angle is 68°, so the sum of the measures of the base angles is 112 and $m\angle A = m\angle C = 56$.

$\frac{\sin B}{b} = \frac{\sin A}{a}$ Law of Sines

$\frac{\sin 68°}{24} = \frac{\sin 56°}{a}$ $m\angle B = 68$, $b = 24$, $m\angle A = 56$

$a \sin 68° = 24 \sin 56°$ Cross multiply.

$a = \frac{24 \sin 56°}{\sin 68°}$ Divide each side by sin 68°.

$\approx 21.5$ Use a calculator.

The triangle is isosceles, so $c = 21.5$.
The perimeter is 24 + 21.5 + 21.5 or about 67 centimeters.

**Exercises**

**Draw a triangle to go with each exercise and mark it with the given information. Then solve the problem. Round angle measures to the nearest degree and side measures to the nearest tenth.**

**1.** One side of a triangular garden is 42.0 feet. The angles on each end of this side measure 66° and 82°. Find the length of fence needed to enclose the garden.

**2.** Two radar stations $A$ and $B$ are 32 miles apart. They locate an airplane $X$ at the same time. The three points form $\angle XAB$, which measures 46°, and $\angle XBA$, which measures 52°. How far is the airplane from each station?

**3.** A civil engineer wants to determine the distances from points $A$ and $B$ to an inaccessible point $C$ in a river. $\angle BAC$ measures 67° and $\angle ABC$ measures 52°. If points $A$ and $B$ are 82.0 feet apart, find the distance from $C$ to each point.

**4.** A ranger tower at point $A$ is 42 kilometers north of a ranger tower at point $B$. A fire at point $C$ is observed from both towers. If $\angle BAC$ measures 43° and $\angle ABC$ measures 68°, which ranger tower is closer to the fire? How much closer?

# 8-6 Skills Practice

## *The Law of Sines*

**Find each measure using the given measures from $\triangle ABC$. Round angle measures to the nearest tenth degree and side measures to the nearest tenth.**

**1.** If $m\angle A = 35$, $m\angle B = 48$, and $b = 28$, find $a$.

**2.** If $m\angle B = 17$, $m\angle C = 46$, and $c = 18$, find $b$.

**3.** If $m\angle C = 86$, $m\angle A = 51$, and $a = 38$, find $c$.

**4.** If $a = 17$, $b = 8$, and $m\angle A = 73$, find $m\angle B$.

**5.** If $c = 38$, $b = 34$, and $m\angle B = 36$, find $m\angle C$.

**6.** If $a = 12$, $c = 20$, and $m\angle C = 83$, find $m\angle A$.

**7.** If $m\angle A = 22$, $a = 18$, and $m\angle B = 104$, find $b$.

**Solve each $\triangle PQR$ described below. Round measures to the nearest tenth.**

**8.** $p = 27$, $q = 40$, $m\angle P = 33$

**9.** $q = 12$, $r = 11$, $m\angle R = 16$

**10.** $p = 29$, $q = 34$, $m\angle Q = 111$

**11.** If $m\angle P = 89$, $p = 16$, $r = 12$

**12.** If $m\angle Q = 103$, $m\angle P = 63$, $p = 13$

**13.** If $m\angle P = 96$, $m\angle R = 82$, $r = 35$

**14.** If $m\angle R = 49$, $m\angle Q = 76$, $r = 26$

**15.** If $m\angle Q = 31$, $m\angle P = 52$, $p = 20$

**16.** If $q = 8$, $m\angle Q = 28$, $m\angle R = 72$

**17.** If $r = 15$, $p = 21$, $m\angle P = 128$

# 8-6 Practice

## *The Law of Sines*



**Find each measure using the given measures from $\triangle EFG$. Round angle measures to the nearest tenth degree and side measures to the nearest tenth.**

**1.** If $m\angle G = 14$, $m\angle E = 67$, and $e = 14$, find $g$.

**2.** If $e = 12.7$, $m\angle E = 42$, and $m\angle F = 61$, find $f$.

**3.** If $g = 14$, $f = 5.8$, and $m\angle G = 83$, find $m\angle F$.

**4.** If $e = 19.1$, $m\angle G = 34$, and $m\angle E = 56$, find $g$.

**5.** If $f = 9.6$, $g = 27.4$, and $m\angle G = 43$, find $m\angle F$.

**Solve each $\triangle STU$ described below. Round measures to the nearest tenth.**

**6.** $m\angle T = 85$, $s = 4.3$, $t = 8.2$

**7.** $s = 40$, $u = 12$, $m\angle S = 37$

**8.** $m\angle U = 37$, $t = 2.3$, $m\angle T = 17$

**9.** $m\angle S = 62$, $m\angle U = 59$, $s = 17.8$

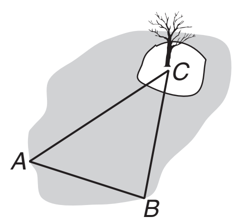
**10.** $t = 28.4$, $u = 21.7$, $m\angle T = 66$

**11.** $m\angle S = 89$, $s = 15.3$, $t = 14$

**12.** $m\angle T = 98$, $m\angle U = 74$, $u = 9.6$

**13.** $t = 11.8$, $m\angle S = 84$, $m\angle T = 47$

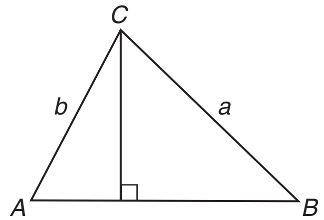
**14. INDIRECT MEASUREMENT** To find the distance from the edge of the lake to the tree on the island in the lake, Hannah set up a triangular configuration as shown in the diagram. The distance from location $A$ to location $B$ is 85 meters. The measures of the angles at $A$ and $B$ are 51° and 83°, respectively. What is the distance from the edge of the lake at $B$ to the tree on the island at $C$?



# 8-6 Word Problem Practice

## *The Law of Sines*

1. **ALTITUDES** In triangle $ABC$, the altitude to side $AB$ is drawn.



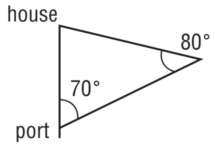

Give two expressions for the length of the altitude in terms of $a$, $b$, and the sine of the angles $A$ and $B$.

2. **MAPS** Three cities form the vertices of a triangle. The angles of the triangle are 40°, 60°, and 80°. The two most distant cities are 40 miles apart. How close are the two closest cities? Round your answer to the nearest tenth of a mile.

3. **PHOTOS** Greg took a photograph of the view from his city apartment. The building on the left is the Rocket Tower and the building on the right is the Cloud Scratcher.

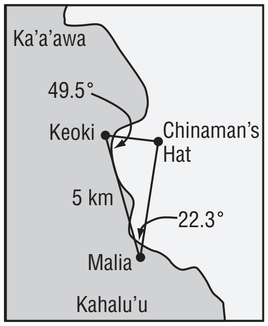
Greg's camera has a 60° viewing angle. Greg knows that he is 2 miles from the Cloud Scratcher and that the Rocket Tower is 3 miles from the Cloud Scatcher. How far is Greg from the Rocket Tower? Round your answer to the nearest hundredth.

4. **BOATING** A boat heads out to sea from a port that sits along a straight shoreline. The boat heads in a direction that makes a 70° angle with the shoreline. After sailing for 3 miles, the skipper looks back at the shore and sees his house. The house, like the port, also sits on the shore. The lines of sight to the port and to his home make an 80° angle. How far is the skipper's home from the port? Round your answer to the nearest tenth of a mile.




**ISLANDS For Exercises 5 and 6, use the following information.**

Oahu is a Hawaiian Island. Off of the coast of Oahu, there is a very tiny island known as Chinaman's Hat. Keoki and Malia are observing Chinaman's Hat from locations 5 kilometers apart. Use the information in the figure to answer the following questions.




5. How far is Keoki from Chinaman's Hat? Round your answer to the nearest tenth of a kilometer.

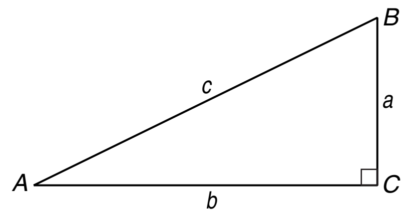
6. How far is Malia from Chinaman's Hat? Round your answer to the nearest tenth of a kilometer.

# 8-6 Enrichment



## *Identities*

An **identity** is an equation that is true for all values of the variable for which both sides are defined. One way to verify an identity is to use a right triangle and the definitions for trigonometric functions.




**Example 1** **Verify that $(\sin A)^2 + (\cos A)^2 = 1$ is an identity.**

$$(\sin A)^2 + (\cos A)^2 = \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2$$
$$= \frac{a^2 + b^2}{c^2} = \frac{c^2}{c^2} = 1$$

To check whether an equation *may* be an identity, you can test several values. However, since you cannot test all values, you cannot be *certain* that the equation is an identity.

**Example 2** **Test $\sin 2x = 2 \sin x \cos x$ to see if it could be an identity.**

Try $x = 20$. Use a calculator to evaluate each expression.

$\sin 2x = \sin 40$
$\approx 0.643$

$2 \sin x \cos x = 2 (\sin 20)(\cos 20)$
$\approx 2(0.342)(0.940)$
$\approx 0.643$

Since the left and right sides seem equal, the equation may be an identity.

**Exercises**

**Use triangle *ABC* shown above. Verify that each equation is an identity.**

**1.** $\frac{\cos A}{\sin A} = \frac{1}{\tan A}$

**2.** $\frac{\tan B}{\sin B} = \frac{1}{\cos B}$

**3.** $\tan B \cos B = \sin B$

**4.** $1 - (\cos B)^2 = (\sin B)^2$

**Try several values for $x$ to test whether each equation could be an identity.**

**5.** $\cos 2x = (\cos x)^2 - (\sin x)^2$

**6.** $\cos (90 - x) = \sin x$

# 8-6 Graphing Calculator Activity

## *Solving Triangles Using the Law of Sines*

You can use a calculator to solve triangles using the Law of Sines.

**Example** **Solve $\triangle ABC$ if $a = 6$, $b = 2$, and $m\angle A = 35$.**

Use the Law of Sines.

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin 35°}{6} = \frac{\sin B}{2} = \frac{\sin C}{c}$

Use your calculator to solve the proportion $\frac{\sin 35°}{6} = \frac{\sin B}{2}$.

**Keystrokes:** [SIN] 35 [)] [×] 2 [÷] 6 [ENTER] .1911921455

[2nd] [SIN$^{-1}$] [2nd] [ANS] [ENTER] 11.02236462

So $m\angle B \approx 11$ and $m\angle C \approx 180 - (35 + 11) = 134$.

Use your calculator and the value 134 for $m\angle C$ and solve the proportion $\frac{\sin 35°}{6} = \frac{\sin 134°}{c}$.

**Keystrokes:** [SIN] 134 [)] [×] 6 [÷] [SIN] 35 [ENTER] 7.524784019

**Exercises**

**Solve each $\triangle ABC$. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.**

**1.** $a = 9$, $m\angle A = 36$, $c = 12$

**2.** $m\angle C = 80$, $c = 9$, $m\angle A = 40$

**3.** $m\angle B = 45$, $m\angle C = 56$, $a = 2$

**4.** $b = 6$, $c = 5$, $m\angle B = 68$

**5.** $a = 11$, $b = 15$, $m\angle B = 42$

# 8-7 Lesson Reading Guide

## *The Law of Cosines*



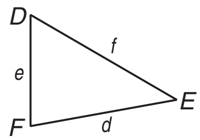
## Get Ready for the Lesson

**Read the introduction to Lesson 8-7 in your textbook.**

If a triangular room and a square room have the same floor area, which room has a greater perimeter?

## Read the Lesson

**1.** Refer to the figure. According to the Law of Cosines, which statements are correct for $\triangle DEF$?




**A.** $d^2 = e^2 + f^2 - ef \cos D$

**B.** $e^2 = d^2 + f^2 - 2df \cos E$

**C.** $d^2 = e^2 + f^2 + 2ef \cos D$

**D.** $f^2 = d^2 + e^2 - 2ef \cos F$

**E.** $f^2 = d^2 + e^2 - 2de \cos \text{F}$

**F.** $d^2 = e^2 + f^2$

**G.** $\frac{\sin D}{d} = \frac{\sin E}{e} = \frac{\sin F}{f}$

**H.** $d = \sqrt{e^2 + f^2 - 2ef \cos D}$

**2.** Each of the following describes three given parts of a triangle. In each case, indicate whether you would use the Law of Sines or the Law of Cosines first in solving a triangle with those given parts. (In some cases, only one of the two laws would be used in solving the triangle.)

**a.** SSS

**b.** ASA

**c.** AAS

**d.** SAS

**e.** SSA

**3.** Indicate whether each statement is *true* or *false*. If the statement is false, explain why.

**a.** The Law of Cosines applies to right triangles.

**b.** The Pythagorean Theorem applies to acute triangles.

**c.** The Law of Cosines is used to find the third side of a triangle when you are given the measures of two sides and the nonincluded angle.

**d.** The Law of Cosines can be used to solve a triangle in which the measures of the three sides are 5 centimeters, 8 centimeters, and 15 centimeters.

## Remember What You Learned

**4.** A good way to remember a new mathematical formula is to relate it to one you already know. The Law of Cosines looks somewhat like the Pythagorean Theorem. Both formulas must be true for a right triangle. How can that be?

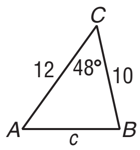
# 8-7 Study Guide and Intervention

## *The Law of Cosines*

**The Law of Cosines** Another relationship between the sides and angles of any triangle is called the **Law of Cosines**. You can use the Law of Cosines if you know three sides of a triangle or if you know two sides and the included angle of a triangle.

| | |
|---|---|
| **Law of Cosines** | Let $\triangle ABC$ be any triangle with $a$, $b$, and $c$ representing the measures of the sides opposite the angles with measures $A$, $B$, and $C$, respectively. Then the following equations are true. $a^2 = b^2 + c^2 - 2bc \cos A$ $\quad b^2 = a^2 + c^2 - 2ac \cos B$ $\quad c^2 = a^2 + b^2 - 2ab \cos C$ |

**Example 1** **In $\triangle ABC$, find $c$.**



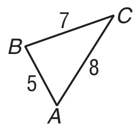

$c^2 = a^2 + b^2 - 2ab \cos C$ — Law of Cosines

$c^2 = 12^2 + 10^2 - 2(12)(10)\cos 48°$ — $a = 12$, $b = 10$, $m\angle C = 48$

$c = \sqrt{12^2 + 10^2 - 2(12)(10)\cos 48°}$ — Take the square root of each side.

$c \approx 9.1$ — Use a calculator.

**Example 2** **In $\triangle ABC$, find $m\angle A$.**




$a^2 = b^2 + c^2 - 2bc \cos A$ — Law of Cosines

$7^2 = 5^2 + 8^2 - 2(5)(8) \cos A$ — $a = 7$, $b = 5$, $c = 8$

$49 = 25 + 64 - 80 \cos A$ — Multiply.

$-40 = -80 \cos A$ — Subtract 89 from each side.

$\frac{1}{2} = \cos A$ — Divide each side by $-80$.

$\cos^{-1}\frac{1}{2} = A$ — Use the inverse cosine.

$60° = A$ — Use a calculator.

**Exercises**

**Find each measure using the given measures from $\triangle ABC$. Round angle measures to the nearest degree and side measures to the nearest tenth.**

**1.** If $b = 14$, $c = 12$, and $m\angle A = 62$, find $a$.

**2.** If $a = 11$, $b = 10$, and $c = 12$, find $m\angle B$.

**3.** If $a = 24$, $b = 18$, and $c = 16$, find $m\angle C$.

**4.** If $a = 20$, $c = 25$, and $m\angle B = 82$, find $b$.

**5.** If $b = 18$, $c = 28$, and $m\angle A = 59$, find $a$.

**6.** If $a = 15$, $b = 19$, and $c = 15$, find $m\angle C$.

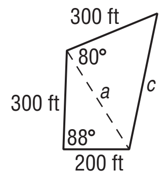
# 8-7 Study Guide and Intervention *(continued)*

## *The Law of Cosines*

**Use the Law of Cosines to Solve Problems** You can use the **Law of Cosines** to solve some problems involving triangles.

| | |
|---|---|
| **Law of Cosines** | Let $\triangle ABC$ be any triangle with $a$, $b$, and $c$ representing the measures of the sides opposite the angles with measures $A$, $B$, and $C$, respectively. Then the following equations are true.<br>$a^2 = b^2 + c^2 - 2bc \cos A$   $b^2 = a^2 + c^2 - 2ac \cos B$   $c^2 = a^2 + b^2 - 2ab \cos C$ |

**Example** **Ms. Jones wants to purchase a piece of land with the shape shown. Find the perimeter of the property.**




Use the Law of Cosines to find the value of $a$.

$a^2 = b^2 + c^2 - 2bc \cos A$ Law of Cosines

$a^2 = 300^2 + 200^2 - 2(300)(200) \cos 88°$ $b = 300$, $c = 200$, $m\angle A = 88$

$a = \sqrt{130{,}000 - 120{,}000 \cos 88°}$ Take the square root of each side.

$\approx 354.7$ Use a calculator.

Use the Law of Cosines again to find the value of $c$.

$c^2 = a^2 + b^2 - 2ab \cos C$ Law of Cosines

$c^2 = 354.7^2 + 300^2 - 2(354.7)(300) \cos 80°$ $a = 354.7$, $b = 300$, $m\angle C = 80$

$c = \sqrt{215{,}812.09 - 212{,}820 \cos 80°}$ Take the square root of each side.

$\approx 422.9$ Use a calculator.

The perimeter of the land is 300 + 200 + 422.9 + 200 or about 1223 feet.

**Exercises**

**Draw a figure or diagram to go with each exercise and mark it with the given information. Then solve the problem. Round angle measures to the nearest degree and side measures to the nearest tenth.**

**1.** A triangular garden has dimensions 54 feet, 48 feet, and 62 feet. Find the angles at each corner of the garden.

**2.** A parallelogram has a 68° angle and sides 8 and 12. Find the lengths of the diagonals.

**3.** An airplane is sighted from two locations, and its position forms an acute triangle with them. The distance to the airplane is 20 miles from one location with an angle of elevation 48.0°, and 40 miles from the other location with an angle of elevation of 21.8°. How far apart are the two locations?

**4.** A ranger tower at point $A$ is directly north of a ranger tower at point $B$. A fire at point $C$ is observed from both towers. The distance from the fire to tower $A$ is 60 miles, and the distance from the fire to tower $B$ is 50 miles. If $m\angle ACB = 62$, find the distance between the towers.

Lesson 8-7

# 8-7 Skills Practice

## *The Law of Cosines*

**In △*RST*, given the following measures, find the measure of the missing side.**

**1.** $r = 5, s = 8, m\angle T = 39$

**2.** $r = 6, t = 11, m\angle S = 87$

**3.** $r = 9, t = 15, m\angle S = 103$

**4.** $s = 12, t = 10, m\angle R = 58$

**In △*HIJ*, given the lengths of the sides, find the measure of the stated angle to the nearest tenth.**

**5.** $h = 12, i = 18, j = 7; m\angle H$

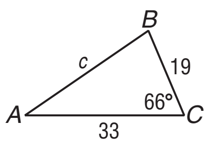
**6.** $h = 15, i = 16, j = 22; m\angle I$

**7.** $h = 23, i = 27, j = 29; m\angle J$

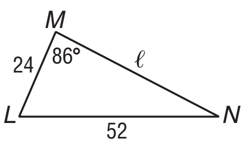
**8.** $h = 37, i = 21, j = 30; m\angle H$

**Determine whether the Law of *Sines* or the Law of *Cosines* should be used first to solve each triangle. Then solve each triangle. Round angle measures to the nearest degree and side measures to the nearest tenth.**

**9.**




**10.**




**11.** $a = 10, b = 14, c = 19$

**12.** $a = 12, b = 10, m\angle C = 27$

**Solve each △*RST* described below. Round measures to the nearest tenth.**

**13.** $r = 12, s = 32, t = 34$

**14.** $r = 30, s = 25, m\angle T = 42$

**15.** $r = 15, s = 11, m\angle R = 67$

**16.** $r = 21, s = 28, t = 30$

# 8-7 Practice

## *The Law of Cosines*



**In △*JKL*, given the following measures, find the measure of the missing side. Round to the nearest tenth.**

**1.** $j = 1.3, k = 10, m\angle L = 77$

**2.** $j = 9.6, \ell = 1.7, m\angle K = 43$

**3.** $j = 11, k = 7, m\angle L = 63$

**4.** $k = 4.7, \ell = 5.2, m\angle J = 112$

**In △*MNQ*, given the lengths of the sides, find the measure of the stated angle to the nearest tenth.**

**5.** $m = 17, n = 23, q = 25; m\angle Q$

**6.** $m = 24, n = 28, q = 34; m\angle M$

**7.** $m = 12.9, n = 18, q = 20.5; m\angle N$

**8.** $m = 23, n = 30.1, q = 42; m\angle Q$

**Determine whether the Law of Sines or the Law of Cosines should be used first to solve △*ABC*. Then solve each triangle. Round angle measures to the nearest degree and side measure to the nearest tenth.**

**9.** $a = 13, b = 18, c = 19$

**10.** $a = 6, b = 19, m\angle C = 38$

**11.** $a = 17, b = 22, m\angle B = 49$

**12.** $a = 15.5, b = 18, m\angle C = 72$

**Solve each △*FGH* described below. Round measures to the nearest tenth.**

**13.** $m\angle F = 54, f = 12.5, g = 11$

**14.** $f = 20, g = 23, m\angle H = 47$

**15.** $f = 15.8, g = 11, h = 14$

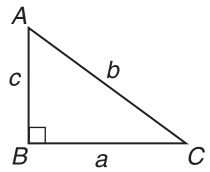
**16.** $f = 36, h = 30, m\angle G = 54$

**17. REAL ESTATE** The Esposito family purchased a triangular plot of land on which they plan to build a barn and corral. The lengths of the sides of the plot are 320 feet, 286 feet, and 305 feet. What are the measures of the angles formed on each side of the property?

# 8-7 Word Problem Practice

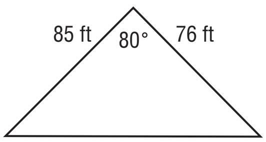
## *The Law of Cosines*

1. **RIGHT TRIANGLES** Triangle $ABC$ is a right triangle with right angle at $B$. Let $a$ be the length of the side opposite $A$, $b$ be the length of the side opposite $B$, and $c$ be the length of the side opposite $C$.

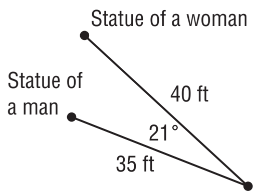



Rewrite the Law of Cosines with respect to the right angle $B$ in simplest form.

2. **LANDSCAPING** Hanna wants to fence a triangular lot as shown. What is the length of the missing side? Round your answer to the nearest foot.



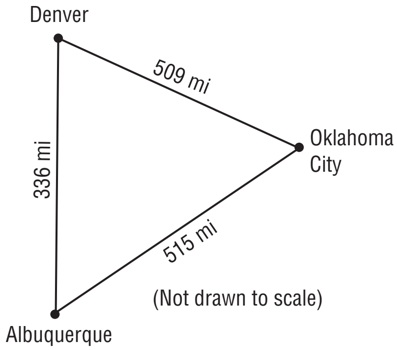

3. **STATUES** Gail was visiting an art gallery. In one room, she stood so that she had a view of two statues, one of a man, and the other of a woman. She was 40 feet from the statue of the woman, and 35 feet from the statue of the man. The angle created by the lines of sight to the two statues was 21°. What is the distance between the two statues? Round your answer to the nearest tenth.




4. **CARS** Two cars start moving from the same location. They head straight, but in different directions. The angle between where they are heading is 43°. The first car travels 20 miles and the second car travels 37 miles. How far apart are the two cars? Round your answer to the nearest tenth.

**CITIES For Exercises 5–7, use the following information.**

The cities of Denver, Oklahoma City, and Albuquerque form the vertices of a triangle.




Use the information in the figure and round your answers to the nearest tenth of a degree.

5. What is the measure of the angle at Albuquerque?

6. What is the measure of the angle at Oklahoma City?

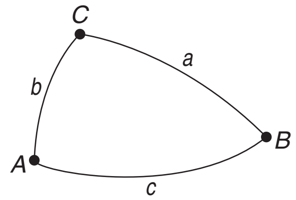
7. What is the measure of the angle at Denver?

# 8-7 Enrichment



## *Spherical Triangles*

Spherical trigonometry is an extension of plane trigonometry. Figures are drawn on the surface of a sphere. Arcs of great circles correspond to line segments in the plane. The arcs of three great circles intersecting on a sphere form a spherical triangle. Angles have the same measure as the tangent lines drawn to each great circle at the vertex. Since the sides are arcs, they too can be measured in degrees.




The sum of the sides of a spherical triangle is less than 360°.
The sum of the angles is greater than 180° and less than 540°.
The Law of Sines for spherical triangles is as follows.

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

There is also a Law of Cosines for spherical triangles.

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$
$$\cos b = \cos a \cos c + \sin a \sin c \cos B$$
$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

**Example** **Solve the spherical triangle given $a = 72°$, $b = 105°$, and $c = 61°$.**

Use the Law of Cosines.

$$0.3090 = (-0.2588)(0.4848) + (0.9659)(0.8746) \cos A$$
$$\cos A = 0.5143$$
$$A = 59°$$

$$-0.2588 = (0.3090)(0.4848) + (0.9511)(0.8746) \cos B$$
$$\cos B = -0.4912$$
$$B = 119°$$

$$0.4848 = (0.3090)(-0.2588) + (0.9511)(0.9659) \cos C$$
$$\cos C = 0.6148$$
$$C = 52°$$

Check by using the Law of Sines.

$$\frac{\sin 72°}{\sin 59°} = \frac{\sin 105°}{\sin 119°} = \frac{\sin 61°}{\sin 52°} = 1.1$$

**Exercises**

**Solve each spherical triangle.**

**1.** $a = 56°, b = 53°, c = 94°$

**2.** $a = 110°, b = 33°, c = 97°$

**3.** $a = 76°, b = 110°, C = 49°$

**4.** $b = 94°, c = 55°, A = 48°$

NAME ______________________________ DATE ____________ PERIOD _____

# 8 Student Recording Sheet

*Use this recording sheet with pages 492–493 of the Student Edition.*

**Read each question. Then fill in the correct answer.**

1. Ⓐ Ⓑ Ⓒ Ⓓ
2. Ⓕ Ⓖ Ⓗ Ⓙ
3. Ⓐ Ⓑ Ⓒ Ⓓ
4. Ⓕ Ⓖ Ⓗ Ⓙ
5. Ⓐ Ⓑ Ⓒ Ⓓ
6. Ⓕ Ⓖ Ⓗ Ⓙ
7. Ⓐ Ⓑ Ⓒ Ⓓ
8. Record your answer and fill in the bubbles in the grid below. Be sure to use the correct place value.

| | | | | . | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | | 1 | 1 | 1 |
| 2 | 2 | 2 | 2 | | 2 | 2 | 2 |
| 3 | 3 | 3 | 3 | | 3 | 3 | 3 |
| 4 | 4 | 4 | 4 | | 4 | 4 | 4 |
| 5 | 5 | 5 | 5 | | 5 | 5 | 5 |
| 6 | 6 | 6 | 6 | | 6 | 6 | 6 |
| 7 | 7 | 7 | 7 | | 7 | 7 | 7 |
| 8 | 8 | 8 | 8 | | 8 | 8 | 8 |
| 9 | 9 | 9 | 9 | | 9 | 9 | 9 |

9. Ⓕ Ⓖ Ⓗ Ⓙ
10. Ⓐ Ⓑ Ⓒ Ⓓ
11. Ⓕ Ⓖ Ⓗ Ⓙ

**Pre-AP**

**Record your answers for Question 12 on the back of this paper.**

# 8 Rubric for Scoring Pre-AP

*(Use to score the Pre-AP question on page 493 of the Student Edition.)*

**General Scoring Guidelines**

- If a student gives only a correct numerical answer to a problem but does not show how he or she arrived at the answer, the student will be awarded only 1 credit. All extended-response questions require the student to show work.
- A fully correct answer for a multiple-part question requires correct responses for all parts of the question. For example, if a question has three parts, the correct response to one or two parts of the question that required work to be shown is not considered a fully correct response.
- Students who use trial and error to solve a problem must show their method. Merely showing that the answer checks or is correct is not considered a complete response for full credit.

**Exercise 12 Rubric**

| Score | Specific Criteria |
|---|---|
| 4 | The value is determined by setting up the expression, $\cos 49° = \frac{x}{500}$ which is approximately equal to 328 feet. **OR** The value is determined by setting up the expression, $\sin 41° = \frac{x}{500}$ which is approximately equal to 328 feet. |
| 3 | A generally correct solution, but may contain minor flaws in reasoning or computation. |
| 2 | A partially correct interpretation and/or solution to the problem. |
| 1 | A correct solution with no evidence or explanation. |
| 0 | An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given. |

NAME ______________________________ DATE ____________ PERIOD _____

# 8 Chapter 8 Quiz 1

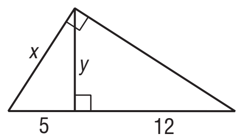
SCORE _____

*(Lessons 8–1 and 8–2)*

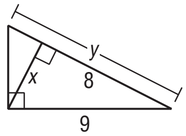
**1.** Find the geometric mean between 12 and 16. **1.** ____________

**For Questions 2 and 3, find $x$ and $y$.**

**2.**



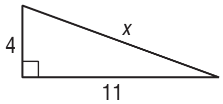

**3.**




**2.** ____________

**3.** ____________

**4.** Find $x$.




**4.** ____________

**5.** State whether 19, 15, and 13 form a Pythagorean triple. Explain. **5.** ____________

Assessment

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NAME ______________________________ DATE ____________ PERIOD _____

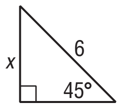
# 8 Chapter 8 Quiz 2

SCORE _____

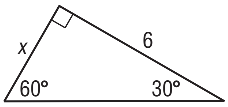
*(Lesson 8–3 and 8–4.)*

**For Questions 1 and 2, find $x$.**

**1.**




**2.**

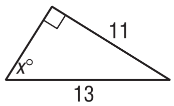



**1.** ____________

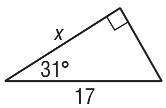
**2.** ____________

**For Questions 3 and 4, find $x$ to the nearest tenth.**

**3.**




**4.**




**3.** ____________

**4.** ____________

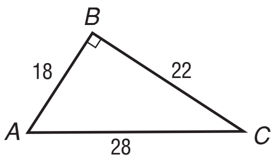
**5.** A rectangle has a diagonal 20 inches long that forms angles of 60° and 30° with the sides. Find the perimeter of the rectangle. **5.** ____________

**6.** Find sin 52°. Round to the nearest ten-thousandth. **6.** ____________

**7.** If $\cos A = 0.8945$, find $m\angle A$ to the nearest degree. **7.** ____________

**8.** The distance along a hill is 24 feet. If the land slopes uphill at an angle of 8°, find the vertical distance from the top to the bottom of the hill. **8.** ____________

**For Questions 9-10, use the figure.**




**9.** Find $\sin A$ to the nearest hundredth. **9.** ____________

**10.** Find $\tan C$ to the nearest hundredth. **10.** ____________

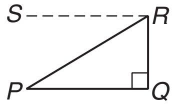
NAME ________________________ DATE ____________ PERIOD _____

# 8 Chapter 8 Quiz 3

SCORE _____

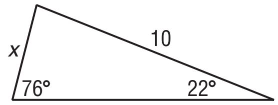
*(Lessons 8–5 and 8–6)*

**1.** Name the angle of elevation in the figure.



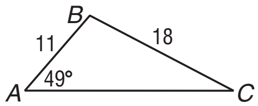

**1.** ____________________

**2.** Find $x$ to the nearest tenth.




**2.** ____________________

**3.** Solve $\triangle ABC$. Round angle measures to the nearest degree and side measures to the nearest tenth.




**3.** ____________________

**4.** A triangular lot has 500 feet of frontage along a river. The other two sides make angles of 48° and 75° with the riverfront side. Find the length of the shortest side to the nearest foot.

**4.** ____________________

**5.** A squirrel 37 feet up in a tree sees a dog 29 feet from the base of the tree. Find the angle of depression to the nearest degree.

**5.** ____________________

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NAME ________________________ DATE ____________ PERIOD _____

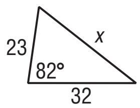
# 8 Chapter 8 Quiz 4

SCORE _____

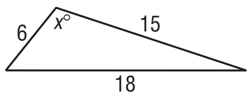
*(Lesson 8–7)*

**For Questions 1 and 2, find $x$. Round to the nearest tenth.**

**1.**



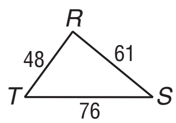

**2.**




**1.** ____________________

**2.** ____________________

**3.** Solve $\triangle RST$. Round your answers to the nearest degree.




**3.** ____________________

**4.** A hiker is 6 miles from a tower and 8 miles from the lodge. She estimates that the angle between her path to the tower and her path to the lodge is 42°. Find the distance from the tower to the lodge to the nearest tenth of a mile.

**4.** ____________________

**5.** **MULTIPLE CHOICE** In $\triangle ABC$ $m\angle A = 96$, $b = 41$, and $c = 50$. Find $a$ to the nearest tenth.

**A.** 66.3 **B.** 67.9 **C.** 4395.3 **D.** 4609.6

**5.** ____________________

NAME _______________________________________________ DATE ____________ PERIOD _____

# 8 Chapter 8 Mid-Chapter Test

SCORE _____

*(Lessons 8–1 through 8–4)*

**Part I** ***Write the letter for the correct answer in the blank at the right of each question.***

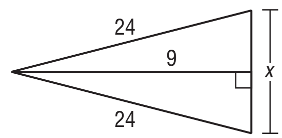
**1.** Find the geometric mean between 7 and 9. **1.** ______

**A.** $3\sqrt{7}$ **B.** 16 **C.** 8 **D.** 2

**2.** Find $x$. **2.** ______

**F.** $\sqrt{216}$ **H.** $6\sqrt{55}$

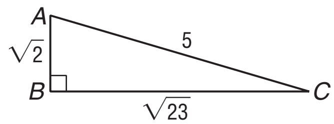
**G.** $\frac{\sqrt{2}}{5}$ **J.** $\frac{\sqrt{23}}{5}$




**3.** Find sin $C$. **3.** ______

**A.** $\sqrt{2}$ **C.** $\frac{\sqrt{23}}{\sqrt{2}}$

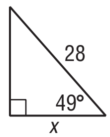
**B.** $\frac{\sqrt{2}}{5}$ **D.** $\frac{\sqrt{23}}{5}$




**4.** Find $x$ to the nearest tenth. **4.** ______

**F.** 14 **H.** 18.4

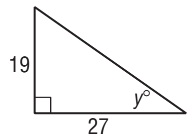
**G.** 21.1 **J.** 32.2




**5.** Find $y$ to the nearest degree. **5.** ______

**A.** 145 **C.** 45

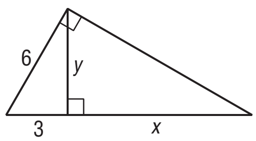
**B.** 60 **D.** 35



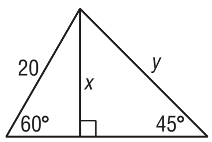

**Part II**

**For Questions 6–8, find $x$ and $y$.**

**6.**



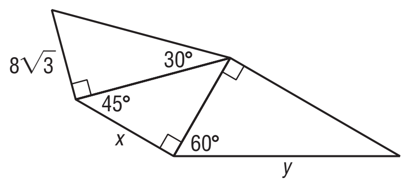

**7.**




**6.** ____________________

**7.** ____________________

**8.**




**8.** ____________________

**9.** State whether 56, 90, and 106 form a Pythagorean triple. Explain. **9.** ____________________

**10.** Guy wires 80 feet long support a 65-foot tall telephone pole. To the nearest degree, what angle will the wires make with the ground? **10.** ____________________

Assessment

NAME _______________ DATE _______ PERIOD ____

# 8 Chapter 8 Vocabulary Test

SCORE ____

| | | |
|---|---|---|
| angle of depression | Pythagorean triple | tangent |
| angle of elevation | sine | trigonometric ratio |
| cosine | solving a triangle | trigonometry |
| geometric mean | | |

**Chose the correct word to complete the sentence.**

**1.** The square root of the product of two numbers is the (*geometric mean* or *Pythagorean triple*) of the numbers. **1.** ____________

**2.** A group of three whole numbers that satisfy the equation $a^2 + b^2 = c^2$, where $c$ is the greatest number, is called a(n) (*trigonometric ratio* or *Pythagorean triple*). **2.** ____________

**State whether each sentence is *true* or *false*. If false, replace the underlined word or number to make a true sentence.**

**3.** The ratio of the lengths of any two sides of a right triangle is called a(n) <u>geometric mean</u>. **3.** ____________

**4.** An angle between the line of sight and the horizontal when an observer looks upward is called a(n) <u>angle of elevation</u>. **4.** ____________

**Choose from the terms above to complete each sentence.**

**5.** An angle between the line of sight and the horizontal when an observer looks downward is called a(n) ___?___. **5.** ____________

**6.** The word ___?___ is derived from the Greek terms for triangle and measure. **6.** ____________

**7.** In a right triangle, the ___?___ of an angle can be found by dividing the length of the opposite leg by the length of the triangle's hypotenuse. **7.** ____________

**8.** In a right triangle, the ___?___ of an angle can be found by dividing the length of the opposite leg by the length of the adjacent leg. **8.** ____________

**Define each term in your own words.**

**9.** solving a triangle **9.** ____________

**10.** Pythagorean Theorem **10.** ____________

NAME ______________________________ DATE ____________ PERIOD _____

# 8 Chapter 8 Test, Form 1

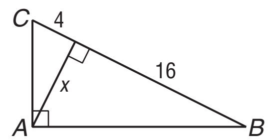
SCORE _____

***Write the letter for the correct answer in the blank at the right of each question.***

**1.** Find the geometric mean between 20 and 5. **1.** ______

**A.** 100 **B.** 50 **C.** 12.5 **D.** 10

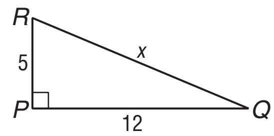
**2.** Find $x$ in $\triangle ABC$. **2.** ______




**F.** 8 **H.** $\sqrt{20}$

**G.** 10 **J.** 64

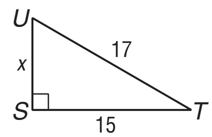
**3.** Find $x$ in $\triangle PQR$. **3.** ______




**A.** 13 **C.** 16

**B.** 15 **D.** $\sqrt{60}$

**4.** Find $x$ in $\triangle STU$. **4.** ______




**F.** 2 **H.** $\sqrt{32}$

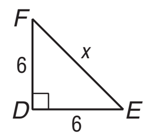
**G.** 8 **J.** $\sqrt{514}$

**5.** Which set of measures could represent the sides of a right triangle? **5.** ______

**A.** 2, 3, 4 **C.** 8, 10, 12

**B.** 7, 11, 14 **D.** 9, 12, 15

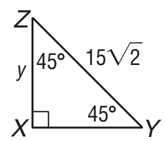
**6.** Find $x$ in $\triangle DEF$. **6.** ______




**F.** 6 **H.** $6\sqrt{3}$

**G.** $6\sqrt{2}$ **J.** 12

**7.** Find $y$ in $\triangle XYZ$. **7.** ______




**A.** $7.5\sqrt{3}$ **C.** 15

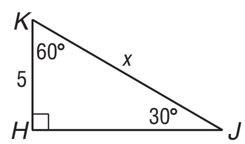
**B.** $15\sqrt{3}$ **D.** 30

**8.** The length of the sides of a square is 10 meters. Find the length of the diagonal of the square. **8.** ______

**F.** 10 m **H.** $10\sqrt{3}$ m

**G.** $10\sqrt{2}$ m **J.** 20 m

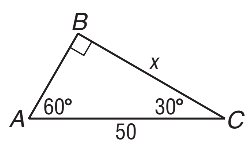
**9.** Find $x$ in $\triangle HJK$. **9.** ______




**A.** $5\sqrt{2}$ **C.** 10

**B.** $5\sqrt{3}$ **D.** 15

**10.** Find $x$ in $\triangle ABC$. **10.** ______




**F.** 25 **H.** $25\sqrt{3}$

**G.** $25\sqrt{2}$ **J.** 100

Assessment

NAME ________________________ DATE __________ PERIOD _____

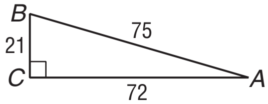
# 8 Chapter 8 Test, Form 1 *(continued)*

**11.** In $\triangle QRS$, $\angle R$ is a right angle. Which is the ratio for the tangent of $\angle S$? **11.** _____

**A.** $\frac{\text{measure of leg adjacent to } \angle S}{\text{measure of hypotenuse}}$ **C.** $\frac{\text{measure of leg opposite } \angle S}{\text{measure of hypotenuse}}$

**B.** $\frac{\text{measure of hypotenuse}}{\text{measure of leg opposite } \angle S}$ **D.** $\frac{\text{measure of leg opposite } \angle S}{\text{measure of leg adjacent to } \angle S}$

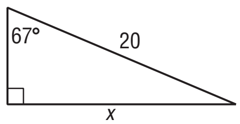
**12.** Find cos $A$ in $\triangle ABC$. **12.** _____




**F.** $\frac{7}{24}$ **H.** $\frac{25}{24}$

**G.** $\frac{7}{25}$ **J.** $\frac{24}{25}$

**13.** Find $x$ to the nearest tenth. **13.** _____



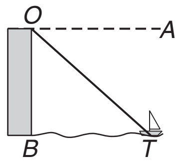

**A.** 7.3 **C.** 18.4

**B.** 17.3 **D.** 47.1

**14.** Find the angle of elevation of the sun when a pole 25 feet tall casts a shadow 42 feet long. **14.** _____

**F.** 30.8° **G.** 36.5° **H.** 53.5° **J.** 59.2°

**15.** Which is the angle of depression in the figure at the right? **15.** _____



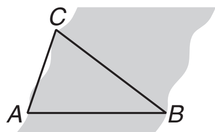

**A.** $\angle AOT$ **C.** $\angle TOB$

**B.** $\angle AOB$ **D.** $\angle BTO$

**16.** Find $y$ in $\triangle XYZ$ if $m\angle Y = 36$, $m\angle X = 49$, and $x = 12$. Round to the nearest hundreth. **16.** _____

**F.** 0.04 **G.** 9.35 **H.** 14.80 **J.** 15.41

**17.** To find the distance between two points $A$ and $B$ on opposite sides of a river, a surveyor measures the distance from $A$ to $C$ as 200 feet, $m\angle A = 72$, and $m\angle B = 37$. Find the distance from $A$ to $B$. Round your answer to the nearest tenth. **17.** _____




**A.** 77.4 ft **B.** 201.2 ft **C.** 250.4 ft **D.** 314.2 ft

**18.** In $\triangle ABC$, $a = 12$, $b = 8$, and $m\angle A = 40$. Find $m\angle B$ to the nearest degree. **18.** _____

**F.** 25 **G.** 56 **H.** 59 **J.** 75

**19.** Find the third side of a triangular garden if two sides measure 8 feet and 12 feet and the included angle measures 50°. **19.** _____

**A.** 7.8 ft **B.** 9.2 ft **C.** 14.4 ft **D.** 146.3 ft

**20.** In $\triangle DEF$, $d = 20$, $e = 25$, and $f = 30$. Find $m\angle F$ to the nearest degree. **20.** _____

**F.** 83 **G.** 76 **H.** 56 **J.** 47

**Bonus** In $\triangle ABC$, $a = 50$, $b = 48$, and $c = 40$. Find $m\angle A$ to the nearest degree. **B:** ____________

NAME ______________________________ DATE ____________ PERIOD _____

# 8 Chapter 8 Test, Form 2A

SCORE _____

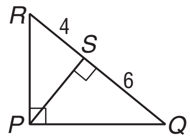
***Write the letter for the correct answer in the blank at the right of each question.***

**1.** Find the geometric mean between 7 and 12. **1.** _____

**A.** 5
**B.** 9.5
**C.** $\sqrt{19}$
**D.** $2\sqrt{21}$

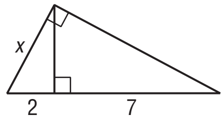
**2.** In $\triangle PQR$, $RS = 4$ and $QS = 6$. Find $PS$. **2.** _____

**F.** 2
**G.** 5
**H.** $\sqrt{10}$
**J.** $2\sqrt{6}$



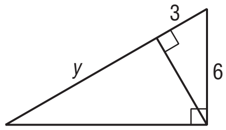

**3.** Find $x$. **3.** _____

**A.** $3\sqrt{2}$
**B.** $\sqrt{14}$
**C.** 4.5
**D.** 3




**4.** Find $y$. **4.** _____

**F.** 12
**G.** 11
**H.** 9
**J.** 2

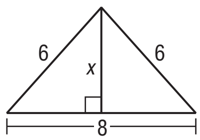



**5.** Find the length of the hypotenuse of a right triangle with legs that measure 5 and 7. **5.** _____

**A.** 12
**B.** $\sqrt{24}$
**C.** $\sqrt{35}$
**D.** $\sqrt{74}$

**6.** Find $x$. **6.** _____

**F.** 3
**G.** 4
**H.** $4\sqrt{3}$
**J.** $2\sqrt{5}$

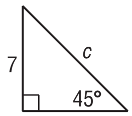



**7.** Which set of measures could represent the sides of a right triangle? **7.** _____

**A.** 9, 40, 41
**B.** 8, 30, 31
**C.** 7, 8, 15
**D.** $\sqrt{2}, \sqrt{3}, \sqrt{6}$

**8.** Find $c$. **8.** _____

**F.** 7
**G.** $7\sqrt{2}$
**H.** $7\sqrt{3}$
**J.** 14

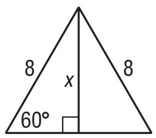



**9.** Find the perimeter of a square if the length of its diagonal is 12 inches. Round to the nearest tenth. **9.** _____

**A.** 8.5 in.
**B.** 33.9 in.
**C.** 48 in.
**D.** 67.9 in.

**10.** Find $x$. **10.** _____

**F.** 4
**G.** $4\sqrt{2}$
**H.** $4\sqrt{3}$
**J.** $8\sqrt{3}$



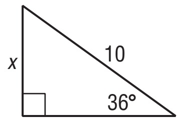

Assessment

# 8 Chapter 8 Test, Form 2A *(continued)*

**11.** Find $x$ to the nearest tenth. **11.** ______

**A.** 5.8 **C.** 8.1

**B.** 5.9 **D.** 17.3



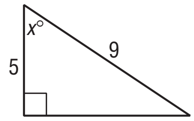

**12.** In right triangle $ABC$, $a = 12$, $b = 9$, and $c = 15$. Find $\tan \angle B$. **12.** ______

**F.** $\frac{4}{3}$ **G.** $\frac{5}{4}$ **H.** $\frac{3}{4}$ **J.** $\frac{3}{5}$

**13.** Find $x$ to the nearest degree. **13.** ______

**A.** 56 **C.** 34

**B.** 45 **D.** 29




**14.** If a 20-foot ladder makes a 65° angle with the ground, how many feet up a wall will it reach? Round your answer to the nearest tenth. **14.** ______

**F.** 8.5 ft **G.** 10 ft **H.** 18.1 ft **J.** 42.9 ft

**15.** A ship's sonar finds that the angle of depression to a wreck on the bottom of the ocean is 12.5°. If a point on the ocean floor is 60 meters directly below the ship, how many meters is it from that point on the ocean floor to the wreck? Round your answer to the nearest tenth. **15.** ______

**A.** 277.2 m **B.** 270.6 m **C.** 61.5 m **D.** 13.3 m

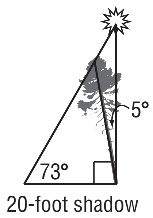
**16.** Find the angle of elevation of the sun if a building 100 feet tall casts a shadow 150 feet long. Round to the nearest degree. **16.** ______

**F.** 60° **G.** 48° **H.** 42° **J.** 34°

**17.** When the sun's angle of elevation is 73°, a tree tilted at an angle of 5° from the vertical casts a 20-foot shadow on the ground. Find the length of the tree to the nearest tenth of a foot. **17.** ______

**A.** 6.3 ft **C.** 51.1 ft

**B.** 19.2 ft **D.** 219.4 ft




**18.** In $\triangle CDE$, $m\angle C = 52$, $m\angle D = 17$, and $e = 28.6$. Find $c$ to the nearest tenth. **18.** ______

**F.** 77.1 **G.** 49.1 **H.** 24.1 **J.** 18.4

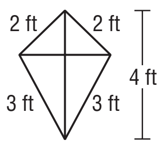
**19.** In $\triangle PQR$, $p = 56$, $r = 17$, and $m\angle Q = 110$. Find $q$ to the nearest tenth. **19.** ______

**A.** 4076.2 **B.** 63.8 **C.** 52.6 **D.** 3.1

**20.** Pete is building a kite using the dimensions given in the figure at the right. Find the measure of the angle the 2-foot edge makes with the 3-foot edge. **20.** ______

**F.** 104.5 **H.** 60

**G.** 85.2 **J.** 14.5




**Bonus** From a window 20 feet above the ground, the angle of elevation to the top of another building is 35°. The distance between the buildings is 52 feet. Find the height of the building to the nearest tenth of a foot. **B:** ____________

NAME ________________________________ DATE ____________ PERIOD _____

# 8 Chapter 8 Test, Form 2B

SCORE _____

***Write the letter for the correct answer in the blank at the right of each question.***

**1.** Find the geometric mean between 9 and 11. **1.** ______

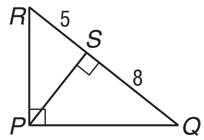
**A.** $3\sqrt{11}$ **C.** 10

**B.** $2\sqrt{5}$ **D.** 2

**2.** In $\triangle PQR$, $RS = 5$ and $QS = 8$. Find $PS$. **2.** ______

**F.** 3 **H.** $\sqrt{13}$

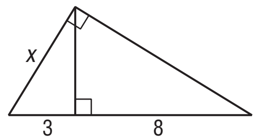
**G.** 6.5 **J.** $2\sqrt{10}$




**3.** Find $x$. **3.** ______

**A.** 5.5 **C.** $\sqrt{24}$

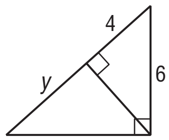
**B.** $\sqrt{11}$ **D.** $\sqrt{33}$




**4.** Find $y$. **4.** ______

**F.** 4 **H.** 8

**G.** 5 **J.** 9




**5.** Find the length of the hypotenuse of a right triangle whose legs measure 6 and 5. **5.** ______

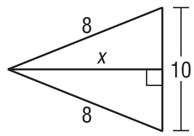
**A.** 11 **C.** $\sqrt{30}$

**B.** $\sqrt{11}$ **D.** $\sqrt{61}$

**6.** Find $x$. **6.** ______

**F.** $\sqrt{39}$ **H.** $5\sqrt{3}$

**G.** 6 **J.** 5




**7.** Which set of measures could represent the sides of a right triangle? **7.** ______

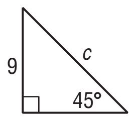
**A.** $\frac{3}{4}, 1, \frac{5}{4}$ **C.** 7, 17, 24

**B.** $\sqrt{3}, \sqrt{5}, \sqrt{15}$ **D.** 8, 15, 16

**8.** Find $c$. **8.** ______

**F.** 18 **H.** $9\sqrt{2}$

**G.** $9\sqrt{3}$ **J.** 9




**9.** Find the perimeter of a square if the length of its diagonal is 16 millimeters. Round to the nearest tenth. **9.** ______

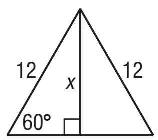
**A.** 11.3 mm **C.** 90.5 mm

**B.** 45.3 mm **D.** 128.0 mm

**10.** Find $x$. **10.** ______

**F.** 6 **H.** $6\sqrt{3}$

**G.** $6\sqrt{2}$ **J.** $12\sqrt{3}$




Assessment

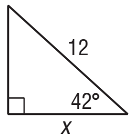
NAME ______________________________ DATE ____________ PERIOD _____

# 8 Chapter 8 Test, Form 2B *(continued)*

**11.** Find $x$. **11.** ______

**A.** 8.0 **C.** 10.4

**B.** 8.9 **D.** 10.8



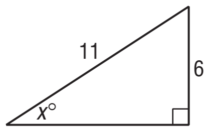

**12.** In right triangle $ABC$, $a = 14$, $b = 48$, and $c = 50$. Find tan $A$. **12.** ______

**F.** $\frac{7}{24}$ **G.** $\frac{7}{25}$ **H.** $\frac{24}{25}$ **J.** $\frac{24}{7}$

**13.** Find $x$ to the nearest degree. **13.** ______

**A.** 57 **C.** 33

**B.** 55 **D.** 29




**14.** If a 24-foot ladder makes a 58° angle with the ground, how many feet up a wall will it reach? Round your answer to the nearest tenth. **14.** ______

**F.** 38.4 ft **G.** 20.8 ft **H.** 20.4 ft **J.** 12.7 ft

**15.** A ship's sonar finds that the angle of depression to a wreck on the bottom of the ocean is 13.2°. If a point on the ocean floor is 75 meters directly below the ship, how many meters is it from that point on the ocean floor to the wreck? Round to the nearest tenth. **15.** ______

**A.** 328.4 m **B.** 319.8 m **C.** 77.0 m **D.** 17.6 m

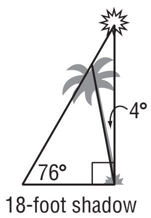
**16.** Find the angle of elevation of the sun if a building 125 feet tall casts a shadow 196 feet long. Round to the nearest degree. **16.** ______

**F.** 64° **G.** 50° **H.** 40° **J.** 33°

**17.** When the sun's angle of elevation is 76°, a tree tilted at an angle of 4° from the vertical casts an 18-foot shadow on the ground. Find the length of the tree to the nearest tenth of a foot. **17.** ______

**A.** 250.4 ft **C.** 17.7 ft

**B.** 56.5 ft **D.** 4.6 ft




**18.** In $\triangle ABC$, $m\angle A = 46$, $m\angle B = 105$, and $c = 19.8$. Find $a$ to the nearest tenth. **18.** ______

**F.** 29.4 **G.** 28.5 **H.** 15.7 **J.** 14.7

**19.** In $\triangle LMN$, $l = 42$, $m = 61$, and $m\angle N = 108$. Find $n$ to the nearest tenth. **19.** ______

**A.** 7068.4 **B.** 84.1 **C.** 79.2 **D.** 24.7

**20.** A triangular garden has sides measuring 50 feet, 80 feet, and 100 feet. To the nearest degree, what is the measure of the largest angle? **20.** ______

**F.** 8° **G.** 82° **H.** 90° **J.** 98°

**Bonus** From a window 24 feet above the ground, the angle of elevation to the top of another building is 38°. The distance between the buildings is 63 feet. Find the height of the building to the nearest tenth of a foot. **B:** ____________________

NAME ____________________ DATE __________ PERIOD _____

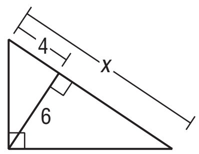
# 8 Chapter 8 Test, Form 2C

SCORE _____

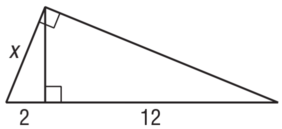
**1.** Find the geometric mean between $2\sqrt{5}$ and $5\sqrt{2}$. **1.** __________

**For Questions 2–5, find *x*.**

**2.**



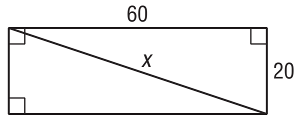

**3.**



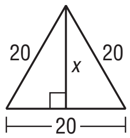

**2.** __________

**3.** __________

**4.**



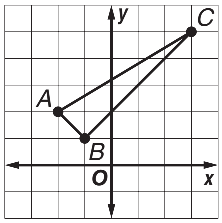

**5.**




**4.** __________

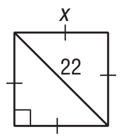
**5.** __________

**6.** Determine whether $\triangle ABC$ is a right triangle. Explain your answer.



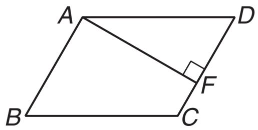

**6.** __________

**7.** Find $x$.



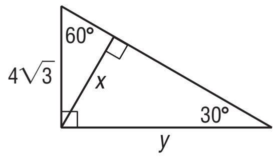

**7.** __________

**8.** In parallelogram $ABCD$, $AD = 4$ and $m\angle D = 60$. Find $AF$.



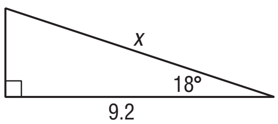

**8.** __________

**9.** Find $x$ and $y$.



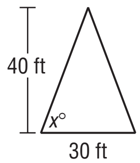

**9.** __________

**10.** Find $x$ to the nearest tenth.




**10.** __________

**11.** An A-frame house is 40 feet high and 30 feet wide. Find the angle that the roof makes with the floor. Round to the nearest degree.




**11.** __________

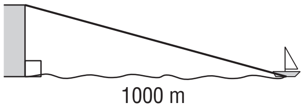
**12.** A 30-foot tree casts a 12-foot shadow. Find the angle of elevation of the sun to the nearest degree. **12.** __________

Assessment

NAME ______________________ DATE ____________ PERIOD _____

# 8 Chapter 8 Test, Form 2C *(continued)*

**13.** A boat is 1000 meters from a cliff. If the angle of depression from the top of the cliff to the boat is 15°, how tall is the cliff? Round your answer to the nearest tenth.

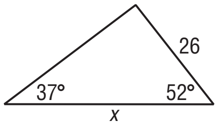



**13.** ____________

**14.** A plane flying at an altitude of 10,000 feet begins descending when the end of the runway is below a point 50,000 feet away. Find the angle of descent (depression) to the nearest degree.

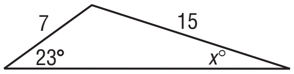
**14.** ____________

**15.** Find $x$ to the nearest tenth.



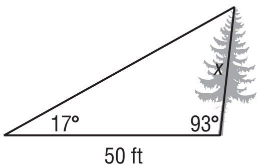

**15.** ____________

**16.** Find $x$ to the nearest degree.



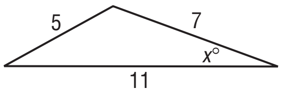

**16.** ____________

**17.** A tree grew at a 3° slant from the vertical. At a point 50 feet from the tree, the angle of elevation to the top of the tree is 17°. Find the length of the tree to the nearest tenth of a foot.




**17.** ____________

**18.** Find $x$ to the nearest degree.

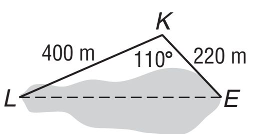



**18.** ____________

**19.** In $\triangle XYZ$, $m\angle X = 152$, $y = 15$, and $z = 19$. Find $x$ to the nearest tenth.

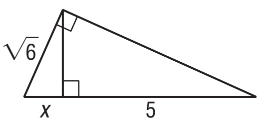
**19.** ____________

**20.** To approximate the length of a pond, a surveyor walks 400 meters from point $L$ to point $K$, then turns and walks 220 meters from point $K$ to point $E$. If $m\angle LKE = 110$, find the length $LE$ of the pond to the nearest tenth of a meter.




**20.** ____________

**Bonus** Find $x$.




**B:** ____________

NAME ______________________________ DATE ____________ PERIOD _____

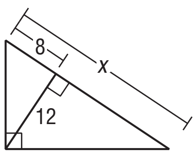
# 8 Chapter 8 Test, Form 2D

SCORE _____

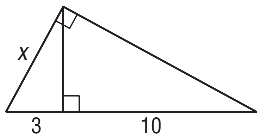
**1.** Find the geometric mean between $3\sqrt{6}$ and $5\sqrt{6}$. **1.** ____________

**For Questions 2–5, find *x*.**

**2.**



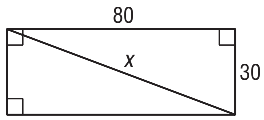

**3.**



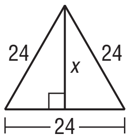

**2.** ____________

**3.** ____________

**4.**



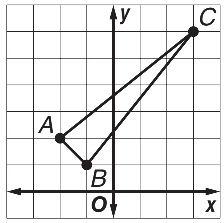

**5.**




**4.** ____________

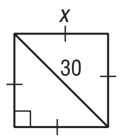
**5.** ____________

**6.** Determine whether $\triangle ABC$ is a right triangle. Explain your answer.



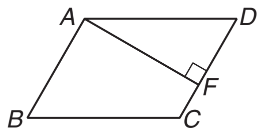

**6.** ____________

**7.** Find $x$.



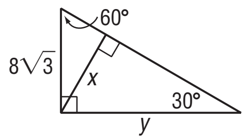

**7.** ____________

**8.** In parallelogram $ABCD$, $AD = 14$ and $m\angle D = 60$. Find $AF$.



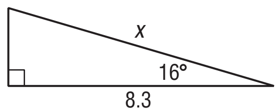

**8.** ____________

**9.** Find $x$ and $y$.



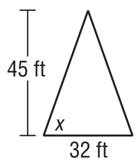

**9.** ____________

**10.** Find $x$ to the nearest tenth.




**10.** ____________

**11.** An A-frame house is 45 feet high and 32 feet wide. Find the measure of the angle that the roof makes with the floor. Round to the nearest degree.



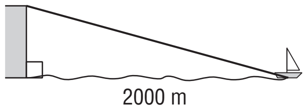

**11.** ____________

**12.** A 38-foot tree casts a 16-foot shadow. Find the angle of elevation of the sun to the nearest degree. **12.** ____________

Assessment

# 8 Chapter 8 Test, Form 2D *(continued)*

**13.** A boat is 2000 meters from a cliff. If the angle of depression from the top of the cliff to the boat is 10°, how tall is the cliff? Round your answer to the nearest tenth.

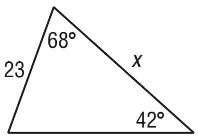



**13.** ____________________

**14.** A plane flying at an altitude of 10,000 feet begins descending when the end of the runway is below a point 60,000 feet away. Find the angle of descent (depression) to the nearest degree.

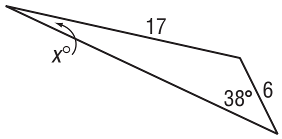
**14.** ____________________

**15.** Find $x$ to the nearest tenth.



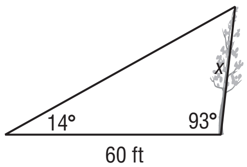

**15.** ____________________

**16.** Find $x$ to the nearest degree.



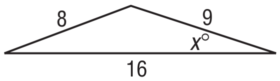

**16.** ____________________

**17.** A tree grew at a 3° slant from the vertical. At a point 60 feet from the tree, the angle of elevation to the top of the tree is 14°. Find the length of the tree to the nearest tenth of a foot.




**17.** ____________________

**18.** Find $x$ to the nearest degree.

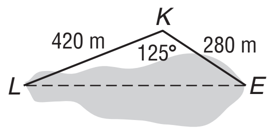



**18.** ____________________

**19.** In $\triangle XYZ$, $m\angle X = 156$, $y = 18$, and $z = 21$. Find $x$ to the nearest tenth.

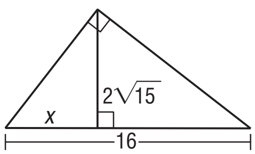
**19.** ____________________

**20.** To approximate the length of a pond, a surveyor walks 420 meters from point $L$ to point $K$, then turns and walks 280 meters from point $K$ to point $E$. If $m\angle LKE = 125$, find the length $LE$ of the pond to the nearest tenth of a meter.




**20.** ____________________

**Bonus** Find $x$.




**B:** ____________________

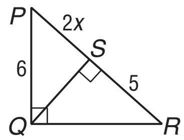
NAME _______________ DATE ________ PERIOD _____

SCORE _____

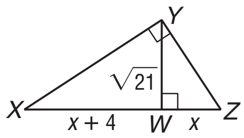
# 8 Chapter 8 Test, Form 3

**1.** Find the geometric mean between $\frac{2}{9}$ and $\frac{3}{9}$. **1.** ____________

**2.** Find $x$ in $\triangle PQR$. **2.** ____________



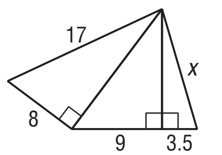

**3.** Find $x$ in $\triangle XYZ$. **3.** ____________




**4.** If the length of one leg of a right triangle is three times the length of the other and the hypotenuse is 20, find the length of the shorter leg. **4.** ____________

**5.** Find the measure of the altitude drawn to the hypotenuse of a right triangle whose legs measure 3 and 4. **5.** ____________

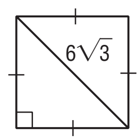
**6.** Find $x$. **6.** ____________



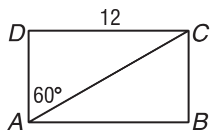

**7.** Richmond is 200 kilometers due east of Teratown and Hamilton is 150 kilometers directly north of Teratown. Find the shortest distance in kilometers between Hamilton and Richmond. **7.** ____________

**8.** State whether 48, 55, and 73 form a Pythagorean triple. Explain. **8.** ____________

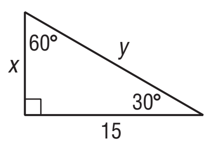
**9.** Find the perimeter of this square. **9.** ____________




**10.** Find the perimeter of rectangle $ABCD$. **10.** ____________




**11.** Find $x$ and $y$. **11.** ____________

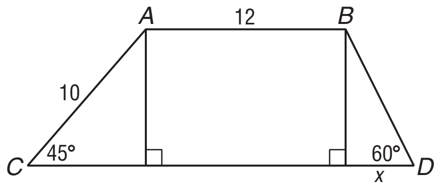



**12.** $\triangle ABC$ is a 30°-60°-90° triangle with right angle $A$ and with $\overline{AC}$ as the longer leg. If $A(-4, -2)$ and $B(-4, 6)$, find the coordinates of $C$. **12.** ____________

Assessment

# 8 Chapter 8 Test, Form 3 *(continued)*

**13.** If $\overline{AB} \parallel \overline{CD}$, find $x$ and the length of $\overline{CD}$.




**13.** ____________

**14.** The angle of elevation from a point on the street to the top of a building is 29°. The angle of elevation from another point on the street, 50 feet farther away from the building, to the top of the building is 25°. To the nearest foot, how tall is the building?

**14.** ____________

**15.** The angle of depression from the top of a flagpole on top of a lighthouse to a boat on the ocean is 2.9°. The angle of depression from the bottom of the flagpole to the boat is 2.6°. If the boat is 1 mile away from shore and the lighthouse is right on the edge of the shore, how tall is the flagpole? Round your answer to the nearest foot.

**15.** ____________

**16.** In $\triangle JKL$, $m\angle J = 26.8$, $m\angle K = 19$, and $k = 17$. Find $\ell$ to the nearest tenth.

**16.** ____________

**17.** Solve $\triangle PQR$ for $r = 22$, $p = 51$, and $m\angle Q = 96$. Round angle measures to the nearest degree and side measures to the nearest tenth.

**17.** ____________

**18.** Don hit a golf ball 250 yards toward the hole. However, due to the wind, his drive was 5° off course. If the angle between the hole and where the ball landed is 97°, how far is the hole from where Don hit the ball? Round to the nearest tenth of a yard.

**18.** ____________

**19.** In $\triangle HJK$, $m\angle H = 32$, $k = 8$, and $h = 7$. Find $m\angle K$. Round your answer(s) to the nearest degree.

**19.** ____________

**20.** The distance from Albany to Bethany is 75 miles and from Bethany to Celina is 105 miles. If the roads from Bethany to Albany and from Bethany to Celina make an 87° angle, what is the distance from Albany to Celina? Round to the nearest tenth.

**20.** ____________

**Bonus** A 50-foot vertical pole that stands on a hillside makes an angle of 10° with the horizontal. Two guy wires extend from the top of the pole to points on the hill 60 feet uphill and downhill from its base. Find the length of each guy wire to the nearest tenth of a foot.

**B:** ____________

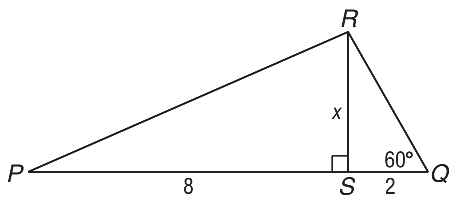
# 8 Chapter 8 Extended-Response Test

SCORE _____

**Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.**

**1.** If the geometric mean between 10 and $x$ is 6, what is $x$? Show how you obtained your answer.

**2.**




**a.** Max used the following equations to find $x$ in $\triangle PQR$. Is Max correct? Explain.

$$\frac{2}{x} = \frac{x}{8}$$
$$x^2 = 2 \cdot 8$$
$$x^2 = 16$$
$$x = 4$$

**b.** If $\angle PRQ$ is a right angle, what is the measure of $\overline{PS}$?

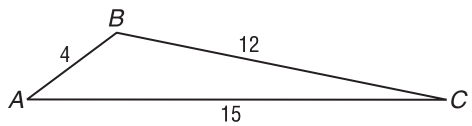
**c.** Is $\triangle PRS$ a 45°-45°-90° triangle? Explain.

**3.** To solve for $x$ in a triangle, when would you use *sin* and when would you use $sin^{-1}$? Give an example for each type of situation.

**4.** Draw a diagram showing the angles of elevation and depression and label each. How are the measures of these angles related?

**5.** Draw an obtuse triangle and label the vertices, the measures of two angles, and the length of one side. Explain how to solve the triangle.

**6.** Irina is solving $\triangle ABC$. She plans to first use the Law of Sines to find two of the angles. Is Irina's plan a good one? Explain.




Assessment

NAME ________________________________ DATE __________ PERIOD _____

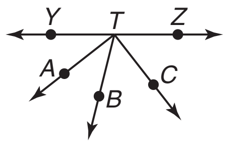
# 8 Standardized Test Practice *(Chapters 1–8)*

SCORE _____

## Part 1: Multiple Choice

**Instructions:** Fill in the appropriate circle for the best answer.

**1.** If $\overline{TA}$ bisects $\angle YTB$, $\overline{TC}$ bisects $\angle BTZ$, $m\angle YTA = 4y + 6$, and $m\angle BTC = 7y - 4$, find $m\angle CTZ$. (Lesson 1-4)




**A** 52 **B** 38 **C** 25 **D** 8

1. Ⓐ Ⓑ Ⓒ Ⓓ

**2.** Which statement is *always* true? (Lesson 2-5)

**F** If right triangle $QPR$ has sides $q$, $p$, and $r$, where $r$ is the hypotenuse, then $r^2 = p^2 + q^2$.

**G** If $\overline{EF} \parallel \overline{HJ}$, then $EF = HJ$.

**H** If lines $KL$ and $VT$ are cut by a transversal, then $\overline{KL} \parallel \overline{VT}$.

**J** If $\overline{DR}$ and $\overline{RH}$ are congruent, then $R$ bisects $\overline{DH}$.

2. Ⓕ Ⓖ Ⓗ Ⓙ

**3.** The equation for $\overleftrightarrow{PT}$ is $y - 2 = 8(x + 3)$. Determine an equation for a line perpendicular to $\overleftrightarrow{PT}$. (Lesson 3-4)

**A** $y = \frac{1}{8}x - 7$ **C** $y = -\frac{1}{8}x + 2$

**B** $y = 8x - 13$ **D** $y = -8x$

3. Ⓐ Ⓑ Ⓒ Ⓓ

**4.** Angle $Y$ in $\triangle XYZ$ measures 90°. $\overline{XY}$ and $\overline{YZ}$ each measure 16 meters. Classify $\triangle XYZ$. (Lesson 4-1)

**F** acute and isosceles **H** right and scalene

**G** equiangular and equilateral **J** right and isosceles

4. Ⓕ Ⓖ Ⓗ Ⓙ

**5.** Two sides of a triangle measure 4 inches and 9 inches. Determine which cannot be the perimeter of the triangle. (Lesson 5-4)

**A** 19 in. **B** 21 in. **C** 23 in. **D** 26 in.

5. Ⓐ Ⓑ Ⓒ Ⓓ

**6.** $\triangle ABC \sim \triangle STR$, so $\frac{AB}{CA} =$ ___?___. (Lesson 6-2)

**F** $\frac{AB}{BC}$ **G** $\frac{ST}{RS}$ **H** $\frac{TR}{RS}$ **J** $\frac{RS}{ST}$

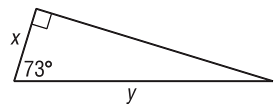
6. Ⓕ Ⓖ Ⓗ Ⓙ

**7.** The Petronas Towers in Kuala Lumpur, Malaysia, are 452 meters tall. A woman who is 1.75 meters tall stands 120 meters from the base of one tower. Find the angle of elevation between the woman's hat and the top of the tower. Round to the nearest tenth. (Lesson 7-5)

**A** 14.8° **B** 15.4° **C** 74.5° **D** 75.1°

7. Ⓐ Ⓑ Ⓒ Ⓓ

**8.** Which equation can be used to find $x$? (Lesson 7-4)

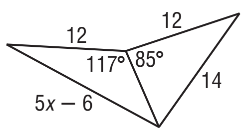



**F** $x = y \sin 73°$ **H** $x = \frac{y}{\cos 73°}$

**G** $x = y \cos 73°$ **J** $x = \frac{y}{\sin 73°}$

8. Ⓕ Ⓖ Ⓗ Ⓙ

**9.** Which inequality describes the possible values of $x$? (Lesson 5-5)




**A** $x<4$ **C** $x>2$

**B** $x<2$ **D** $x>4$

9. Ⓐ Ⓑ Ⓒ Ⓓ

# 8 Standardized Test Practice *(continued)*

**10.** Stan invests \$1875 in a certificate of deposit that earns 4.5% interest compounded annually. Find his balance after 4 years. (Lesson 7-6)

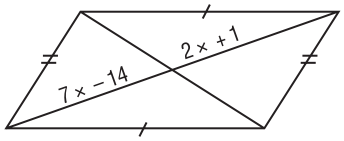
**F** \$2212.50 **G** \$2139.68 **H** \$2212.50 **J** \$2235.97

**10.** Ⓕ Ⓖ Ⓗ Ⓙ

**11.** Find $x$.

**A** –3 **C** 5

**B** 3 **D.** 9



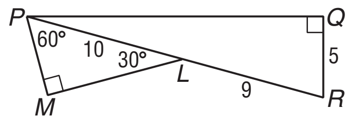

**11.** Ⓐ Ⓑ Ⓒ Ⓓ

**12.** Trapezoid ABCD has vertices $A(1, 6)$, $B(-2, 6)$, $C(-10, -10)$, and $D(20, -10)$. Find the measure of $ABCD$'s median to the nearest tenth.

**F** 3 **G** 5.3 **H** 7.2 **J** 16.5

**12.** Ⓕ Ⓖ Ⓗ Ⓙ

**For Questions 11 and 12, use the figure to the right.**




**13.** Find QP to the nearest tenth. (Lesson 8-2)

**A.** 7.5 **B.** 12 **C.** 18.3 **D.** 19.6

**13.** Ⓐ Ⓑ Ⓒ Ⓓ

**14.** Find LM. (Lesson 8-3)

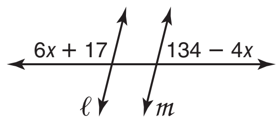
**F.** 5 **G.** $5\sqrt{3}$ **H.** 9 **J.** $10\sqrt{3}$

**14.** Ⓕ Ⓖ Ⓗ Ⓙ

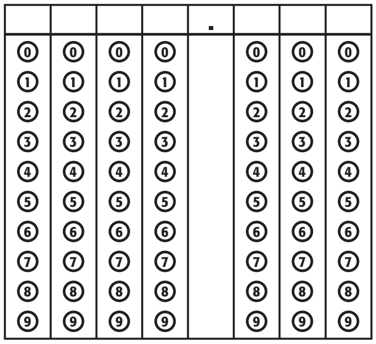
## Part 2: Griddable

**Instructions:** Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate circle that corresponds to that entry.

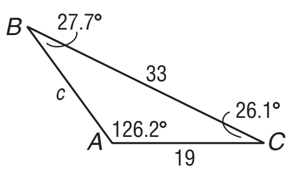
**15.** Find $x$ so that $\ell \parallel m$. (Lesson 3-5)



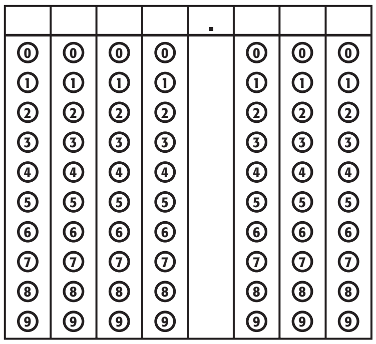

**15.**



**16.** Find $c$ to the nearest tenth. (Lesson 7-6)




**16.**



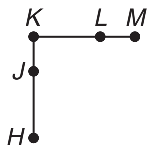
Assessment

# 8 Standardized Test Practice *(continued)*

## Part 3: Short Response

**Instructions:** Write your answer in the space provided.

**For Questions 17 and 18, complete the following proof.** (Lesson 2-7)




**Given:** $\overline{JK} \cong \overline{LM}$
$\overline{HJ} \cong \overline{KL}$

**Prove:** $\overline{HK} \cong \overline{KM}$

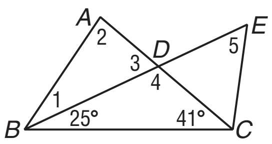
**Proof:**

| Statements | Reasons |
|---|---|
| **1.** $\overline{JK} \cong \overline{LM}$, $\overline{HJ} \cong \overline{KL}$ | **1.** Given |
| **2.** $JK = LM$, $HJ = KL$ | **2.** (Question 17) |
| **3.** (Question 18) | **3.** Segment Addition Post. |
| **4.** $HJ + JK = KL + LM$ | **4.** Substitution Prop. |
| **5.** $HK = KM$ | **5.** Substitution Prop. |
| **6.** $\overline{HK} \cong \overline{KM}$ | **6.** Def. of ≅ segments |

**17.** ____________________

**18.** ____________________

**For Questions 19 and 20, use the figure at the right.**




**19.** Find the measure of the numbered angles if $m\angle ABC = 57$ and $m\angle BCE = 98$. (Lesson 4-2)

**19.** ____________________

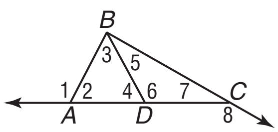
**20.** If $\overline{BD}$ is a median, $AD = 2x - 6$, and $DC = 22.5 - 4x$, find $AC$. (Lesson 5-1)

**20.** ____________________

**21.** If $\triangle DEF \cong \triangle HJK$, $m\angle D = 26$, $m\angle J = 3x + 5$, and $m\angle F = 92$, find $x$. (Lesson 4-3)

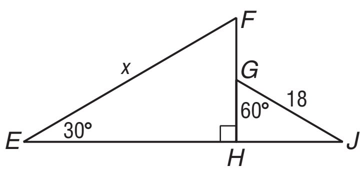
**21.** ____________________

**22.** Use the Exterior Angle Inequality Theorem to list all of the angles with measures that are less than $m\angle 1$. (Lesson 5-2)




**22.** ____________________

**23. a.** Determine whether $\triangle EFH \sim \triangle JGH$. Justify your answer. (Lesson 6-3)




**23. a.** ____________________

**b.** If $G$ is the midpoint of $FH$, find $x$. (Lesson 7-3)

**23. b.** ____________________

**c.** What is the scale factor of $\triangle EFH$ to $\triangle JGH$?

**23. c.** ____________________

NAME ______________________ DATE __________ PERIOD _____

# 8 Anticipation Guide

## *Right Triangles and Trigonometry*



**Step 1** ***Before you begin Chapter 8***

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

| STEP 1 A, D, or NS | Statement | STEP 2 A or D |
|---|---|---|
| | **1.** The geometric mean between two numbers is the positive square root of their product. | A |
| | **2.** An altitude drawn from the right angle of a right triangle to its hypotenuse separates the triangle into two congruent triangles. | D |
| | **3.** In a right triangle, the length of the hypotenuse is equal to the sum of the lengths of the legs. | D |
| | **4.** If any triangle has sides with lengths 3, 4, and 5, then that triangle is a right triangle. | A |
| | **5.** If the two acute angles of a right triangle are 45°, then the length of the hypotenuse is $\sqrt{2}$ times the length of either leg. | A |
| | **6.** In any triangle whose angle measures are 30°, 60°, and 90°, the hypotenuse is $\sqrt{3}$ times as long as the shorter leg. | D |
| | **7.** The sine ratio of an angle of a right triangle is equal to the length of the adjacent side divided by the length of the hypotenuse. | D |
| | **8.** The tangent of an angle of a right triangle whose sides have lengths 3, 4, and 5 will be smaller than the tangent of an angle of a right triangle whose sides have lengths 6, 8, and 10. | D |
| | **9.** Trigonometric ratios can be used to solve problems involving angles of elevation and angles of depression. | A |
| | **10.** The Law of Sines can only be used in right triangles. | D |

**Step 2** ***After you complete Chapter 8***

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.



NAME ______________________ DATE __________ PERIOD _____

# 8-1 Reading to Learn Mathematics

## *Geometric Mean*



### Get Ready for the Lesson

**Read the introduction to Lesson 8-1 in your textbook.**

If your eye level is halfway between the top and bottom of a painting, what additional information do you need to know to calculate the distance that creates the best view? **Sample answer: You need to know the height of the painting.**

### Read the Lesson

**1.** In the past, when you have seen the word *mean* in mathematics, it referred to the *average* or *arithmetic mean* of the two numbers.

**a.** Complete the following by writing an algebraic expression in each blank.

If $a$ and $b$ are two positive numbers, then the geometric mean between $a$ and $b$ is $\sqrt{ab}$ and their arithmetic mean is $\frac{a+b}{2}$.

**b.** Explain in words, without using any mathematical symbols, the difference between the geometric mean and the algebraic mean. **Sample answer: The geometric mean between two numbers is the square root of their product. The arithmetic mean of two numbers is half their sum.**

**2.** Let $r$ and $s$ be two positive numbers. In which of the following equations is $z$ equal to the geometric mean between $r$ and $s$? **A, C, D, F**

**A.** $\frac{s}{z} = \frac{z}{r}$ **B.** $\frac{r}{z} = \frac{s}{z}$ **C.** $s:z = z:r$ **D.** $\frac{r}{z} = \frac{z}{s}$ **E.** $\frac{z}{r} = \frac{z}{s}$ **F.** $\frac{z}{s} = \frac{r}{z}$

**3.** Supply the missing words or phrases to complete the statement of each theorem.

**a.** The measure of the altitude drawn from the vertex of the right angle of a right triangle to its hypotenuse is the **geometric mean** between the measures of the two segments of the **hypotenuse**.

**b.** If the altitude is drawn from the vertex of the **right** angle of a right triangle to its hypotenuse, then the measure of a **leg** of the triangle is the **geometric mean** between the measure of the hypotenuse and the segment of the **hypotenuse** adjacent to that leg.

**c.** If the altitude is drawn from the **vertex** of the right angle of a right triangle to its **hypotenuse**, then the two triangles formed are **similar** to the given triangle and to each other.

### Remember What You Learned

**4.** A good way to remember a new mathematical concept is to relate it to something you already know. How can the meaning of *mean* in a proportion help you to remember how to find the geometric mean between two numbers? **Sample answer: Write a proportion in which the two means are equal. This common mean is the geometric mean between the two extremes.**

NAME ______________________ DATE ___________ PERIOD _____

# 8-1 Study Guide and Intervention

## *Geometric Mean*

**Geometric Mean** The **geometric mean** between two numbers is the positive square root of their product. For two positive numbers $a$ and $b$, the geometric mean of $a$ and $b$ is the positive number $x$ in the proportion $\frac{a}{x} = \frac{x}{b}$. Cross multiplying gives $x^2 = ab$, so $x = \sqrt{ab}$.

**Example** **Find the geometric mean between each pair of numbers.**

**a. 12 and 3**

Let $x$ represent the geometric mean.

$\frac{12}{x} = \frac{x}{3}$ Definition of geometric mean

$x^2 = 36$ Cross multiply.

$x = \sqrt{36}$ or 6 Take the square root of each side.

**b. 8 and 4**

Let $x$ represent the geometric mean.

$\frac{8}{x} = \frac{x}{4}$

$x^2 = 32$

$x = \sqrt{32}$

$\approx 5.7$

**Exercises**

**Find the geometric mean between each pair of numbers.**

**1.** 4 and 4 **4**

**2.** 4 and 6 $\sqrt{24} \approx 4.9$

**3.** 6 and 9 $\sqrt{54} \approx 7.3$

**4.** $\frac{1}{2}$ and 2 **1**

**5.** $2\sqrt{3}$ and $3\sqrt{3}$ $\sqrt{18} \approx 4.2$

**6.** 4 and 25 **10**

**7.** $\sqrt{3}$ and $\sqrt{6}$ $18^{\frac{1}{4}} \approx 2.1$

**8.** 10 and 100 $\sqrt{1000} \approx 31.6$

**9.** $\frac{1}{2}$ and $\frac{1}{4}$ $\sqrt{\frac{1}{8}} \approx 0.4$

**10.** $\frac{2\sqrt{2}}{5}$ and $\frac{3\sqrt{2}}{5}$ $\sqrt{\frac{12}{25}} \approx 0.7$

**11.** 4 and 16 **8**

**12.** 3 and 24 $\sqrt{72} \approx 8.5$

**The geometric mean and one extreme are given. Find the other extreme.**

**13.** $\sqrt{24}$ is the geometric mean between $a$ and $b$. Find $b$ if $a = 2$. **12**

**14.** $\sqrt{12}$ is the geometric mean between $a$ and $b$. Find $b$ if $a = 3$. **4**

**Determine whether each statement is *always*, *sometimes*, or *never* true.**

**15.** The geometric mean of two positive numbers is greater than the average of the two numbers. **never**

**16.** If the geometric mean of two positive numbers is less than 1, then both of the numbers are less than 1. **sometimes**

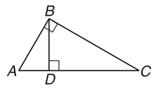


NAME ______________________ DATE ___________ PERIOD _____

# 8-1 Study Guide and Intervention *(continued)*

## *Geometric Mean*

**Altitude of a Triangle** In the diagram, $\triangle ABC \sim \triangle ADB \sim \triangle BDC$. An altitude to the hypotenuse of a right triangle forms two right triangles. The two triangles are similar and each is similar to the original triangle.




**Example 1** **Use right $\triangle ABC$ with $\overline{BD} \perp \overline{AC}$. Describe two geometric means.**

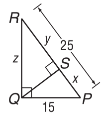
**a.** $\triangle ADB \sim \triangle BDC$ so $\frac{AD}{BD} = \frac{BD}{CD}$.

In $\triangle ABC$, the altitude is the geometric mean between the two segments of the hypotenuse.

**b.** $\triangle ABC \sim \triangle ADB$ and $\triangle ABC \sim \triangle BDC$, so $\frac{AC}{AB} = \frac{AB}{AD}$ and $\frac{AC}{BC} = \frac{BC}{DC}$.

In $\triangle ABC$, each leg is the geometric mean between the hypotenuse and the segment of the hypotenuse adjacent to that leg.

**Example 2** **Find $x$, $y$, and $z$.**




$\frac{PR}{PQ} = \frac{PQ}{PS}$

$\frac{25}{15} = \frac{15}{x}$ $PR = 25$, $PQ = 15$, $PS = x$

$25x = 225$ Cross multiply.

$x = 9$ Divide each side by 25.

Then

$y = PR - SP$

$= 25 - 9$

$= 16$

$\frac{PR}{QR} = \frac{QR}{RS}$

$\frac{25}{z} = \frac{z}{y}$ $PR = 25$, $QR = z$, $RS = y$

$\frac{25}{z} = \frac{z}{16}$ $y = 16$

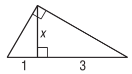
$z^2 = 400$ Cross multiply.

$z = 20$ Take the square root of each side.

**Exercises**

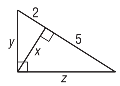
**Find $x$, $y$, and $z$ to the nearest tenth.**

**1.**



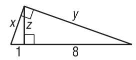

$\sqrt{3} \approx 1.7$

**2.**



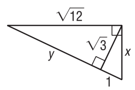

$\sqrt{10} \approx 3.2$; $\sqrt{14} \approx 3.7$; $\sqrt{35} \approx 5.9$

**3.**



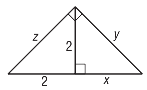

3; $\sqrt{72} \approx 8.5$; $\sqrt{8} \approx 2.8$

**4.**



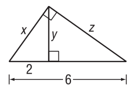

**2; 3**

**5.**




2; $\sqrt{8} \approx 2.8$; $\sqrt{8} \approx 2.8$

**6.**




$\sqrt{12} \approx 3.5$; $\sqrt{8} \approx 2.8$; $\sqrt{24} \approx 4.9$

NAME ______________________ DATE ________ PERIOD ____

## 8-1 Skills Practice

### Geometric Mean

**Find the geometric mean between each pair of numbers. State exact answers and answers to the nearest tenth.**

**1.** 2 and 8
4

**2.** 9 and 36
18

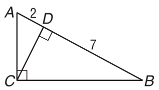
**3.** 4 and 7
$\sqrt{28} \approx 5.3$

**4.** 5 and 10
$\sqrt{50} \approx 7.1$

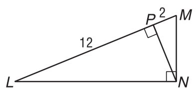
**5.** $2\sqrt{2}$ and $5\sqrt{2}$
$\sqrt{20} \approx 4.5$

**6.** $3\sqrt{5}$ and $5\sqrt{5}$
$\sqrt{75} \approx 8.7$

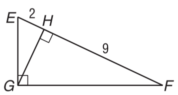
**Find the measure of the altitude drawn to the hypotenuse. State exact answers and answers to the nearest tenth.**

**7.**


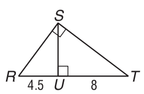
$\sqrt{14} \approx 3.7$

**8.**


$\sqrt{24} \approx 4.9$

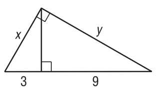
**9.**


$\sqrt{18} \approx 4.2$

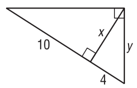
**10.**


6

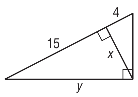
**Find $x$ and $y$.**

**11.**


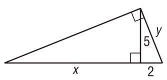
$6; \sqrt{108} \approx 10.4$

**12.**


$\sqrt{40} \approx 6.3; \sqrt{56} \approx 7.5$

**13.**


$\sqrt{60} \approx 7.7; \sqrt{285} \approx 16.9$

**14.**


$12.5; \sqrt{29} \approx 5.4$



NAME ______________________ DATE ________ PERIOD ____

## 8-1 Practice

### Geometric Mean

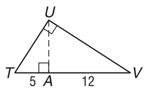
**Find the geometric mean between each pair of numbers to the nearest tenth.**

**1.** 8 and 12
$\sqrt{96} \approx 9.8$

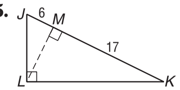
**2.** $3\sqrt{7}$ and $6\sqrt{7}$
$\sqrt{126} \approx 11.2$

**3.** $\frac{4}{5}$ and 2
$\sqrt{\frac{8}{5}} \approx 1.3$

**Find the measure of the altitude drawn to the hypotenuse. State exact answers and answers to the nearest tenth.**

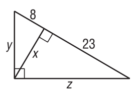
**4.**


$\sqrt{60} \approx 7.7$

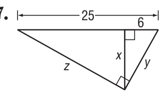
**5.**


$\sqrt{102} \approx 10.1$

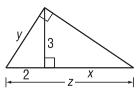
**Find $x$, $y$, and $z$.**

**6.**


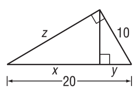
$\sqrt{184} \approx 13.6; \sqrt{248} \approx 15.7; \sqrt{713} \approx 26.7$

**7.**


$\sqrt{114} \approx 10.7; \sqrt{150} \approx 12.2; \sqrt{475} \approx 21.8$

**8.**


$4.5; \sqrt{13} \approx 3.6; 6.5$

**9.**


$15; 5; \sqrt{300} \approx 17.3$

**10. CIVIL ENGINEERING** An airport, a factory, and a shopping center are at the vertices of a right triangle formed by three highways. The airport and factory are 6.0 miles apart. Their distances from the shopping center are 3.6 miles and 4.8 miles, respectively. A service road will be constructed from the shopping center to the highway that connects the airport and factory. What is the shortest possible length for the service road? Round to the nearest hundredth. **2.88 mi**



Lesson 8-1

Answers

NAME ______________________ DATE __________ PERIOD _____

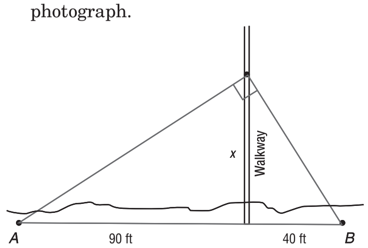
# 8-1 Word Problem Practice

## Geometric Mean

1. **SQUARES** Wilma has a rectangle of dimensions $\ell$ by $w$. She would like to replace it with a square that has the same area. What is the side length of the square with the same area as Wilma's rectangle?
**$\sqrt{\ell w}$, the geometric mean of $\ell$ and $w$**

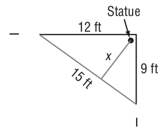
2. **EQUALITY** Gretchen computed the geometric mean of two numbers. One of the numbers was 7 and the geometric mean turned out to be 7 as well. What was the other number?
**7**

3. **VIEWING ANGLE** A photographer wants to take a picture of a beach front. His camera has a viewing angle of 90° and he wants to make sure two palm trees located at points $A$ and $B$ in the figure are just inside the edges of the photograph.



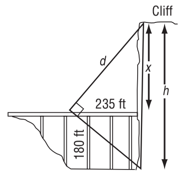

He walks out on a walkway that goes over the ocean to get the shot. If his camera has a viewing angle of 90°, at what distance down the walkway should he stop to take his photograph?
**60 ft**

4. **EXHIBITIONS** A museum has a famous statue on display. The curator places the statue in the corner of a rectangular room and builds 15-foot-long railing in front of the statue. Use the information below to find how close visitors will be able to get to the statue.
**7.2 ft**




**CLIFFS For Exercises 5–7, use the following information.**

A bridge connects to a tunnel as shown in the figure. The bridge is 180 feet above the ground. At a distance of 235 feet along the bridge out of the tunnel, the angle to the base and summit of the cliff is a right angle.




5. What is the height of the cliff? Round to the nearest whole number.
**307 ft**

6. How high is the cliff from base to summit? Round to the nearest whole number.
**487 ft**

7. What is $d$? Round to the nearest whole number.
**386 ft**



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# 8-1 Enrichment

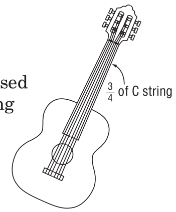
## Mathematics and Music

Pythagoras, a Greek philosopher who lived during the sixth century B.C., believed that all nature, beauty, and harmony could be expressed by whole-number relationships. Most people remember Pythagoras for his teachings about right triangles. (The sum of the squares of the legs equals the square of the hypotenuse.) But Pythagoras also discovered relationships between the musical notes of a scale. These relationships can be expressed as ratios.

| C | D | E | F | G | A | B | C′ |
|---|---|---|---|---|---|---|---|
| $\frac{1}{1}$ | $\frac{8}{9}$ | $\frac{4}{5}$ | $\frac{3}{4}$ | $\frac{2}{3}$ | $\frac{3}{5}$ | $\frac{8}{15}$ | $\frac{1}{2}$ |

When you play a stringed instrument, you produce different notes by placing your finger on different places on a string. This is the result of changing the length of the vibrating part of the string.

The C string can be used to produce F by placing a finger $\frac{3}{4}$ of the way along the string.




**Suppose a C string has a length of 16 inches. Write and solve proportions to determine what length of string would have to vibrate to produce the remaining notes of the scale.**

1. D **$14\frac{2}{9}$ in.**
2. E **$12\frac{4}{5}$ in.**
3. F **12 in.**
4. G **$10\frac{2}{3}$ in.**
5. A **$9\frac{3}{5}$ in.**
6. B **$8\frac{8}{15}$ in.**
7. C′ **8 in.**

8. Complete to show the distance between finger positions on the 16-inch C string for each note. For example, $C(16) - D\left(14\frac{2}{9}\right) = 1\frac{7}{9}$.

C $1\frac{7}{9}$ in. D **$3\frac{1}{5}$ in.** E **4 in.** F **$5\frac{1}{3}$ in.** G **$6\frac{2}{5}$ in.** A **$7\frac{7}{15}$ in.** B **8 in.** C′

9. Between two consecutive musical notes, there is either a whole step or a half step. Using the distances you found in Exercise 8, determine what two pairs of notes have a half step between them.
***E* and *F*, *B* and *C′***

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# 8-2 Lesson Reading Guide

## *The Pythagorean Theorem and Its Converse*

### Get Ready for the Lesson

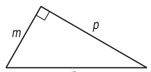
**Read the introduction to Lesson 8-2 in your textbook.**

Do the two right triangles shown in the drawing appear to be similar? Explain your reasoning. **Sample answer: No; their sides are not proportional. In the smaller triangle, the longer leg is more than twice the length of the shorter leg, while in the larger triangle, the longer leg is less than twice the length of the shorter leg.**

### Read the Lesson

**1.** Explain in your own words the difference between how the Pythagorean Theorem is used and how the Converse of the Pythagorean Theorem is used. **Sample answer: The Pythagorean Theorem is used to find the third side of a right triangle if you know the lengths of any two of the sides. The converse is used to tell whether a triangle with three given side lengths is a right triangle.**

**2.** Refer to the figure. For this figure, which statements are true? **B, E, F, G**




**A.** $m^2 + n^2 = p^2$ **B.** $n^2 = m^2 + p^2$

**C.** $m^2 = n^2 + p^2$ **D.** $m^2 = p^2 - n^2$

**E.** $p^2 = n^2 - m^2$ **F.** $n^2 - p^2 = m^2$

**G.** $n = \sqrt{m^2 + p^2}$ **H.** $p = \sqrt{m^2 - n^2}$

**3.** Is the following statement true or false?

A Pythagorean triple is any group of three numbers for which the sum of the squares of the smaller two numbers is equal to the square of the largest number. Explain your reasoning.
**Sample answer: The statement is false because in a Pythagorean triple, all three numbers must be *whole* numbers.**

**4.** If $x$, $y$, and $z$ form a Pythagorean triple and $k$ is a positive integer, which of the following groups of numbers are also Pythagorean triples? **B, D**

**A.** $3x, 4y, 5z$ **B.** $3x, 3y, 3z$ **C.** $-3x, -3y, -3z$ **D.** $kx, ky, kz$

### Remember What You Learned

**5.** Many students who studied geometry long ago remember the Pythagorean Theorem as the equation $a^2 + b^2 = c^2$, but cannot tell you what this equation means. A formula is useless if you don't know what it means and how to use it. How could you help someone who has forgotten the Pythagorean Theorem remember the meaning of the equation $a^2 + b^2 = c^2$?
**Sample answer: Draw a right triangle. Label the lengths of the two legs as *a* and *b* and the length of the hypotenuse as *c*.**



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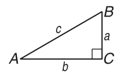
# 8-2 Study Guide and Intervention

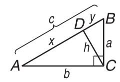
## *The Pythagorean Theorem and Its Converse*

**The Pythagorean Theorem** In a right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.




$\triangle ABC$ is a right triangle, so $a^2 + b^2 = c^2$.

**Example 1** **Prove the Pythagorean Theorem.**



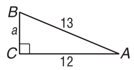

With altitude $\overline{CD}$, each leg $a$ and $b$ is a geometric mean between hypotenuse $c$ and the segment of the hypotenuse adjacent to that leg.

$\frac{c}{a} = \frac{a}{y}$ and $\frac{c}{b} = \frac{b}{x}$, so $a^2 = cy$ and $b^2 = cx$.

Add the two equations and substitute $c = y + x$ to get $a^2 + b^2 = cy + cx = c(y + x) = c^2$.

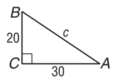
**Example 2**

**a. Find $a$.**




| | |
|---|---|
| $a^2 + b^2 = c^2$ | Pythagorean Theorem |
| $a^2 + 12^2 = 13^2$ | $b = 12$, $c = 13$ |
| $a^2 + 144 = 169$ | Simplify. |
| $a^2 = 25$ | Subtract. |
| $a = 5$ | Take the positive square root of each side. |

**b. Find $c$.**

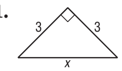



| | |
|---|---|
| $a^2 + b^2 = c^2$ | Pythagorean Theorem |
| $20^2 + 30^2 = c^2$ | $a = 20$, $b = 30$ |
| $400 + 900 = c^2$ | Simplify. |
| $1300 = c^2$ | Add. |
| $\sqrt{1300} = c$ | Take the positive square root of each side. |
| $36.1 \approx c$ | Use a calculator. |

**Exercises**

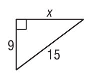
**Find $x$.**

**1.**



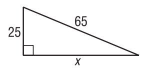

**$\sqrt{18} \approx 4.2$**

**2.**



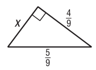

**12**

**3.**



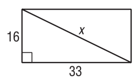

**60**

**4.**



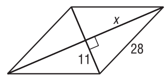

**$\frac{1}{3}$**

**5.**




**$\sqrt{1345} \approx 36.7$**

**6.**




**$\sqrt{663} \approx 25.7$**

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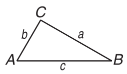
# 8-2 Study Guide and Intervention *(continued)*

## *The Pythagorean Theorem and Its Converse*

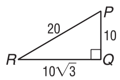
**Converse of the Pythagorean Theorem** If the sum of the squares of the measures of the two shorter sides of a triangle equals the square of the measure of the longest side, then the triangle is a right triangle.




If $a^2 + b^2 = c^2$, then $\triangle ABC$ is a right triangle.

If the three whole numbers $a$, $b$, and $c$ satisfy the equation $a^2 + b^2 = c^2$, then the numbers $a$, $b$, and $c$ form a **Pythagorean triple**.

**Example** **Determine whether $\triangle PQR$ is a right triangle.**




$a^2 + b^2 \stackrel{?}{=} c^2$ Pythagorean Theorem

$10^2 + (10\sqrt{3})^2 \stackrel{?}{=} 20^2$ $a = 10$, $b = 10\sqrt{3}$, $c = 20$

$100 + 300 \stackrel{?}{=} 400$ Simplify.

$400 = 400$✓ Add.

The sum of the squares of the two shorter sides equals the square of the longest side, so the triangle is a right triangle.

**Exercises**

**Determine whether each set of measures can be the measures of the sides of a right triangle. Then state whether they form a Pythagorean triple.**

**1.** 30, 40, 50 — **yes; yes**

**2.** 20, 30, 40 — **no; no**

**3.** 18, 24, 30 — **yes; yes**

**4.** 6, 8, 9 — **no; no**

**5.** $\frac{3}{7}, \frac{4}{7}, \frac{5}{7}$ — **yes; no**

**6.** 10, 15, 20 — **no; no**

**7.** $\sqrt{5}, \sqrt{12}, \sqrt{13}$ — **no; no**

**8.** $2, \sqrt{8}, \sqrt{12}$ — **yes; no**

**9.** 9, 40, 41 — **yes; yes**

**A *family* of Pythagorean triples consists of multiples of known triples. For each Pythagorean triple, find two triples in the same family.** **Sample answers are given.**

**10.** 3, 4, 5 — **30, 40, 50; 12, 16, 20**

**11.** 5, 12, 13 — **10, 24, 26; 15, 36, 39**

**12.** 7, 24, 25 — **14, 48, 50; 21, 72, 75**



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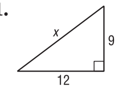
# 8-2 Skills Practice

## *The Pythagorean Theorem and Its Converse*

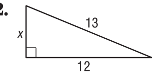
**Find $x$.**

**1.**



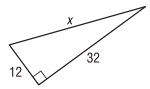

**15**

**2.**



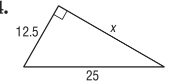

**5**

**3.**



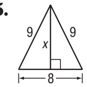

$\sqrt{1168} \approx 34.2$

**4.**



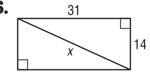

$\sqrt{468.75} \approx 21.7$

**5.**




$\sqrt{65} \approx 8.1$

**6.**




$\sqrt{1157} \approx 34.0$

**Determine whether $\triangle STU$ is a right triangle for the given vertices. Explain.**

**7.** $S(5, 5)$, $T(7, 3)$, $U(3, 2)$
**no; $ST = \sqrt{8}$, $TU = \sqrt{17}$, $US = \sqrt{13}$, $(\sqrt{8})^2 + (\sqrt{13})^2 \neq (\sqrt{17})^2$**

**8.** $S(3, 3)$, $T(5, 5)$, $U(6, 0)$
**yes; $ST = \sqrt{8}$, $TU = \sqrt{26}$, $US = \sqrt{18}$, $(\sqrt{8})^2 + (\sqrt{18})^2 = (\sqrt{26})^2$**

**9.** $S(4, 6)$, $T(9, 1)$, $U(1, 3)$
**yes; $ST = \sqrt{50}$, $TU = \sqrt{68}$, $US = \sqrt{18}$, $(\sqrt{18})^2 + (\sqrt{50})^2 = (\sqrt{68})^2$**

**10.** $S(0, 3)$, $T(-2, 5)$, $U(4, 7)$
**yes; $ST = \sqrt{8}$, $TU = \sqrt{40}$, $US = \sqrt{32}$, $(\sqrt{8})^2 + (\sqrt{32})^2 = (\sqrt{40})^2$**

**11.** $S(-3, 2)$, $T(2, 7)$, $U(-1, 1)$
**yes; $ST = \sqrt{50}$, $TU = \sqrt{45}$, $US = \sqrt{5}$, $(\sqrt{45})^2 + (\sqrt{5})^2 = (\sqrt{50})^2$**

**12.** $S(2, -1)$, $T(5, 4)$, $U(6, -3)$
**no; $ST = \sqrt{34}$, $TU = \sqrt{50}$, $US = \sqrt{20}$, $(\sqrt{34})^2 + (\sqrt{20})^2 \neq (\sqrt{50})^2$**

**Determine whether each set of measures can be the measures of the sides of a right triangle. Then state whether they form a Pythagorean triple.**

**13.** 12, 16, 20 — **yes, yes**

**14.** 16, 30, 32 — **no, no**

**15.** 14, 48, 50 — **yes, yes**

**16.** $\frac{2}{5}, \frac{4}{5}, \frac{6}{5}$ — **no, no**

**17.** $2\sqrt{6}, 5, 7$ — **yes, no**

**18.** $2\sqrt{2}, 2\sqrt{7}, 6$ — **yes, no**

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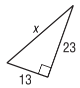
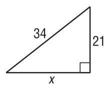
# 8-2 Practice

## The Pythagorean Theorem and Its Converse

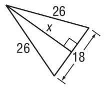
**Find $x$.**

**1.** 

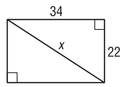

$\sqrt{698} \approx 26.4$

**2.** 

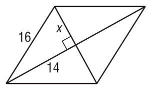

$\sqrt{715} \approx 26.7$

**3.** 

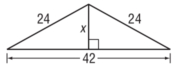

$\sqrt{595} \approx 24.4$

**4.** 


$\sqrt{1640} \approx 40.5$

**5.** 


$\sqrt{60} \approx 7.7$

**6.** 


$\sqrt{135} \approx 11.6$

**Determine whether $\triangle GHI$ is a right triangle for the given vertices. Explain.**

**7.** $G(2, 7), H(3, 6), I(-4, -1)$

**yes; $GH = \sqrt{2}$, $HI = \sqrt{98}$, $IG = \sqrt{100}$, $(\sqrt{2})^2 + (\sqrt{98})^2 = (\sqrt{100})^2$**

**8.** $G(-6, 2), H(1, 12), I(-2, 1)$

**no; $GH = \sqrt{149}$, $HI = \sqrt{130}$, $IG = \sqrt{17}$, $(\sqrt{130})^2 + (\sqrt{17})^2 \neq (\sqrt{149})^2$**

**9.** $G(-2, 1), H(3, -1), I(-4, -4)$

**yes; $GH = \sqrt{29}$, $HI = \sqrt{58}$, $IG = \sqrt{29}$, $(\sqrt{29})^2 + (\sqrt{29})^2 = (\sqrt{58})^2$**

**10.** $G(-2, 4), H(4, 1), I(-1, -9)$

**yes; $GH = \sqrt{45}$, $HI = \sqrt{125}$, $IG = \sqrt{170}$, $(\sqrt{45})^2 + (\sqrt{125})^2 = (\sqrt{170})^2$**

**Determine whether each set of measures can be the measures of the sides of a right triangle. Then state whether they form a Pythagorean triple.**

**11.** 9, 40, 41
**yes, yes**

**12.** 7, 28, 29
**no, no**

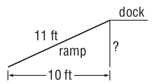
**13.** 24, 32, 40
**yes, yes**

**14.** $\frac{9}{5}, \frac{12}{5}, 3$
**yes, no**

**15.** $\frac{1}{3}, \frac{2\sqrt{2}}{3}, 1$
**yes, no**

**16.** $\frac{\sqrt{4}}{7}, \frac{2\sqrt{3}}{7}, \frac{4}{7}$
**yes, no**

**17. CONSTRUCTION** The bottom end of a ramp at a warehouse is 10 feet from the base of the main dock and is 11 feet long. How high is the dock? **about 4.6 ft high**



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# 8-2 Word Problem Practice

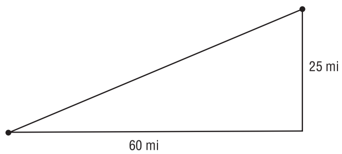
## The Pythagorean Theorem and Its Converse

**1. SIDEWALKS** Construction workers are building a marble sidewalk around a park that is shaped like a right triangle. Each marble slab adds 2 feet to the length of the sidewalk. The workers find that exactly 1071 and 1840 slabs are required to make the sidewalks along the short sides of the park. How many slabs are required to make the sidewalk that runs along the long side of the park?
**2129**

**2. RIGHT ANGLES** Clyde makes a triangle using three sticks of lengths 20 inches, 21 inches, and 28 inches. Is the triangle a right triangle? Explain.
**no; $20^2 + 21^2 \neq 28^2$**

**3. TETHERS** To help support a flag pole, a 50-foot-long tether is tied to the pole at a point 40 feet above the ground. The tether is pulled taut and tied to an anchor in the ground. How far away from the base of the pole is the anchor?
**30 ft**

**4. FLIGHT** An airplane lands at an airport 60 miles east and 25 miles north of where it took off.




How far apart are the two airports?
**65 mi**

**PYTHAGOREAN TRIPLES For Exercises 5–7, use the following information.**

Ms. Jones assigned her fifth-period geometry class the following problem.

Let $m$ and $n$ be two positive integers with $m > n$. Let $a = m^2 - n^2$, $b = 2mn$, and $c = m^2 + n^2$.

**5.** Show that there is a right triangle with side lengths $a$, $b$, and $c$.

**Sample answer: $a^2 + b^2 = (m^2 - n^2)^2 + (2mn)^2 = m^4 - 2m^2n^2 + n^4 + 4m^2n^2 = m^4 + 2m^2n^2 + n^4$ and $c^2 = (m^2 + n^2)^2 = m^4 + 2m^2n^2 + n^4$. This means that $a^2 + b^2 = c^2$, so that $a$, $b$, and $c$ do form the sides of a right triangle by the converse of the Pythagorean Theorem.**

**6.** Complete the following table.

| $m$ | $n$ | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| 2 | 1 | 3 | 4 | 5 |
| 3 | 1 | **8** | **6** | **10** |
| 3 | 2 | **5** | **12** | **13** |
| 4 | 1 | **15** | **8** | **17** |
| 4 | 2 | **12** | **16** | **20** |
| 4 | 3 | **7** | **24** | **25** |
| 5 | 1 | **24** | **10** | **26** |

**7.** Find a Pythagorean triple that corresponds to a right triangle with a hypotenuse $25^2 = 625$ units long. (*Hint:* Use the table you completed for Exercise 6 to find two positive integers $m$ and $n$ with $m > n$ and $m^2 + n^2 = 625$.)
**Sample answer: Take $m = 24$ and $n = 7$ to get $a = 527$, $b = 336$, and $c = 625$.**

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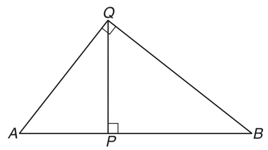
## 8-2 Enrichment

### *Converse of a Right Triangle Theorem*

You have learned that the measure of the altitude from the vertex of the right angle of a right triangle to its hypotenuse is the geometric mean between the measures of the two segments of the hypotenuse. Is the converse of this theorem true? In order to find out, it will help to rewrite the original theorem in if-then form as follows.

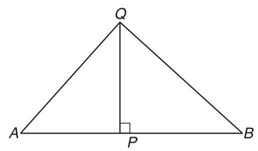
If $\triangle ABQ$ is a right triangle with right angle at $Q$, then $QP$ is the geometric mean between $AP$ and $PB$, where $P$ is between $A$ and $B$ and $\overline{QP}$ is perpendicular to $\overline{AB}$.




**1.** Write the converse of the if-then form of the theorem.

**If $QP$ is the geometric mean between $\overline{AP}$ and $\overline{PB}$, where $P$ is between $A$ and $B$ and $\overline{QP} \perp \overline{AB}$, then $\triangle ABQ$ is a right triangle with right angle at $Q$.**

**2.** Is the converse of the original theorem true? Refer to the figure at the right to explain your answer.




**Yes; $(PQ)^2 = (AP)(PB)$ implies that $\frac{PQ}{AP} = \frac{PB}{PQ}$. Since both $\angle APQ$ and $\angle QPB$ are right angles, they are congruent. Therefore $\triangle APQ \sim \triangle QPB$ by SAS similarity. So $\angle A \cong \angle PQB$ and $\angle AQP \cong \angle B$. But the acute angles of $\triangle AQP$ are complementary and $m\angle AQB = m\angle AQP + m\angle PQB$. Hence $m\angle AQB = 90$ and $\triangle AQB$ is a right triangle with right angle at $Q$.**

You may find it interesting to examine the other theorems in Chapter 7 to see whether their converses are true or false. You will need to restate the theorems carefully in order to write their converses.



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## 8-2 Spreadsheet Activity

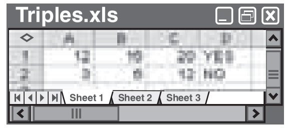
### *Pythagorean Triples*

You can use a spreadsheet to determine whether three whole numbers form a Pythagorean triple.

**Example 1** **Use a spreadsheet to determine whether the numbers 12, 16, and 20 form a Pythagorean triple.**

**Step 1** In cell A1, enter 12. In cell B1, enter 16 and in cell C1, enter 20. *The longest side should be entered in column C.*

**Step 2** In cell D1, enter an equals sign followed by IF(A1^2+B1^2=C1^2,"YES","NO"). This will return "YES" if the set of numbers is a Pythagorean triple and will return "NO" if it is not.




The numbers 12, 16, and 20 form a Pythagorean triple.

**Example 2** **Use a spreadsheet to determine whether the numbers 3, 6, and 12 form a Pythagorean triple.**

**Step 1** In cell A2, enter 3, in cell B2, enter 6, and in cell C2, enter 12.

**Step 2** Click on the bottom right corner of cell D1 and drag it to D2. This will determine whether or not the set of numbers is a Pythagorean triple.

The numbers 3, 6, and 12 do not form a Pythagorean triple.

**Exercises**

**Use a spreadsheet to determine whether each set of numbers forms a Pythagorean triple.**

**1.** 14, 48, 50 **yes**
**2.** 16, 30, 34 **yes**
**3.** 5, 5, 9 **no**
**4.** 4, 5, 7 **no**
**5.** 18, 24, 30 **yes**
**6.** 10, 24, 26 **yes**
**7.** 25, 60, 65 **yes**
**8.** 2, 4, 5 **no**
**9.** 19, 21, 22 **no**
**10.** 18, 80, 82 **yes**
**11.** 5, 12, 13 **yes**
**12.** 20, 48, 52 **yes**

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# 8-3 Lesson Reading Guide

## *Special Right Triangles*

### Get Ready for the Lesson

**Read the introduction to Lesson 8-3 in your textbook.**

Suppose you have the four half square triangles that are each pinwheel pattern. If there are 9 squares total in each pattern, how many additional squares of material do you need to complete the pattern? **Sample answer: 7 additional squares of material**

### Read the Lesson

**1.** Supply the correct number or numbers to complete each statement.

**a.** In a 45°-45°-90° triangle, to find the length of the hypotenuse, multiply the length of a leg by $\sqrt{2}$.

**b.** In a 30°-60°-90° triangle, to find the length of the hypotenuse, multiply the length of the shorter leg by 2.

**c.** In a 30°-60°-90° triangle, the longer leg is opposite the angle with a measure of 60.

**d.** In a 30°-60°-90° triangle, to find the length of the longer leg, multiply the length of the shorter leg by $\sqrt{3}$.

**e.** In an isosceles right triangle, each leg is opposite an angle with a measure of 45.

**f.** In a 30°-60°-90° triangle, to find the length of the shorter leg, divide the length of the longer leg by $\sqrt{3}$.

**g.** In 30°-60°-90° triangle, to find the length of the longer leg, divide the length of the hypotenuse by 2 and multiply the result by $\sqrt{3}$.

**h.** To find the length of a side of a square, divide the length of the diagonal by $\sqrt{2}$.

**2.** Indicate whether each statement is *always*, *sometimes*, or *never* true.

**a.** The lengths of the three sides of an isosceles triangle satisfy the Pythagorean Theorem. **sometimes**

**b.** The lengths of the sides of a 30°-60°-90° triangle form a Pythagorean triple. **never**

**c.** The lengths of all three sides of a 30°-60°-90° triangle are positive integers. **never**

### Remember What You Learned

**3.** Some students find it easier to remember mathematical concepts in terms of specific numbers rather than variables. How can you use specific numbers to help you remember the relationship between the lengths of the three sides in a 30°-60°-90° triangle?
**Sample answer: Draw a 30°-60°-90° triangle. Label the length of the shorter leg as 1. Then the length of the hypotenuse is 2, and the length of the longer leg is $\sqrt{3}$. Just remember: 1, 2, $\sqrt{3}$.**



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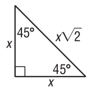
# 8-3 Study Guide and Intervention

## *Special Right Triangles*

**Properties of 45°-45°-90° Triangles** The sides of a 45°-45°-90° right triangle have a special relationship.

**Example 1** **If the leg of a 45°-45°-90° right triangle is $x$ units, show that the hypotenuse is $x\sqrt{2}$ units.**




Using the Pythagorean Theorem with $a = b = x$, then

$$c^2 = a^2 + b^2$$
$$= x^2 + x^2$$
$$= 2x^2$$
$$c = \sqrt{2x^2}$$
$$= x\sqrt{2}$$

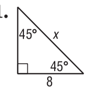
**Example 2** **In a 45°-45°-90° right triangle the hypotenuse is $\sqrt{2}$ times the leg. If the hypotenuse is 6 units, find the length of each leg.**

The hypotenuse is $\sqrt{2}$ times the leg, so divide the length of the hypotenuse by $\sqrt{2}$.

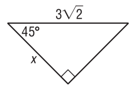
$$a = \frac{6}{\sqrt{2}}$$
$$= \frac{6\sqrt{2}}{\sqrt{2}\sqrt{2}}$$
$$= \frac{6\sqrt{2}}{2}$$
$$= 3\sqrt{2} \text{ units}$$

**Exercises**

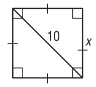
**Find $x$.**

**1.**


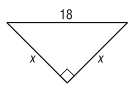

**$8\sqrt{2} \approx 11.3$**

**2.**


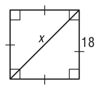

**3**

**3.**


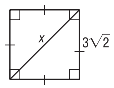

**$5\sqrt{2} \approx 7.1$**

**4.**



**$9\sqrt{2} \approx 12.7$**

**5.**



**$18\sqrt{2} \approx 25.5$**

**6.**



**6**

**7.** Find the perimeter of a square with diagonal 12 centimeters. **$24\sqrt{2} \approx 33.9$ cm**

**8.** Find the diagonal of a square with perimeter 20 inches. **$5\sqrt{2} \approx 7.1$ in.**

**9.** Find the diagonal of a square with perimeter 28 meters. **$7\sqrt{2} \approx 9.9$ m**

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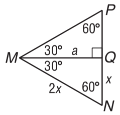
# 8-3 Study Guide and Intervention *(continued)*

## *Special Right Triangles*

**Properties of 30°-60°-90° Triangles** The sides of a 30°-60°-90° right triangle also have a special relationship.

**Example 1** **In a 30°-60°-90° right triangle, show that the hypotenuse is twice the shorter leg and the longer leg is $\sqrt{3}$ times the shorter leg.**




$\triangle MNQ$ is a 30°-60°-90° right triangle, and the length of the hypotenuse $\overline{MN}$ is two times the length of the shorter side $\overline{NQ}$.
Using the Pythagorean Theorem,

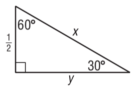
$$a^2 = (2x)^2 - x^2$$
$$= 4x^2 - x^2$$
$$= 3x^2$$
$$a = \sqrt{3x^2}$$
$$= x\sqrt{3}$$

**Example 2** **In a 30°-60°-90° right triangle, the hypotenuse is 5 centimeters. Find the lengths of the other two sides of the triangle.**

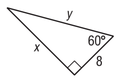
If the hypotenuse of a 30°-60°-90° right triangle is 5 centimeters, then the length of the shorter leg is half of 5 or 2.5 centimeters. The length of the longer leg is $\sqrt{3}$ times the length of the shorter leg, or $(2.5)(\sqrt{3})$ centimeters.

**Exercises**

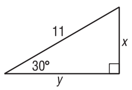
**Find *x* and *y*.**

**1.**


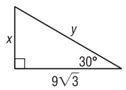

$1; 0.5\sqrt{3} \approx 0.9$

**2.**


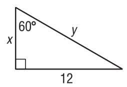

$8\sqrt{3} \approx 13.9; 16$

**3.**


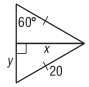

$5.5; 5.5\sqrt{3} \approx 9.5$

**4.**



$9; 18$

**5.**



$4\sqrt{3} \approx 6.9; 8\sqrt{3} \approx 13.9$

**6.**



$10\sqrt{3} \approx 17.3; 10$

**7.** The perimeter of an equilateral triangle is 32 centimeters. Find the length of an altitude of the triangle to the nearest tenth of a centimeter. **9.2 cm**

**8.** An altitude of an equilateral triangle is 8.3 meters. Find the perimeter of the triangle to the nearest tenth of a meter. **28.8 m**



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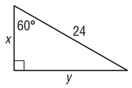
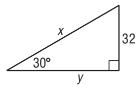
# 8-3 Skills Practice

## *Special Right Triangles*

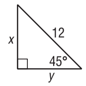
**Find the exact values of *x* and *y*.**

**1.**


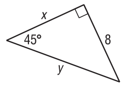

$12, 12\sqrt{3}$

**2.**


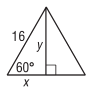

$64, 32\sqrt{3}$

**3.**


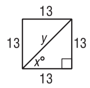

$6\sqrt{2}, 6\sqrt{2}$

**4.**



$8, 8\sqrt{2}$

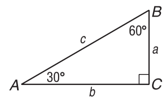
**5.**



$8, 8\sqrt{3}$

**6.**



$45, 13\sqrt{2}$

**For Exercises 7–9, use the figure at the right.**

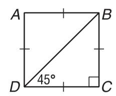



**7.** If $a = 11$, find $b$ and $c$.
$b = 11\sqrt{3}; c = 22$

**8.** If $b = 15$, find $a$ and $c$.
$a = 5\sqrt{3}; c = 10\sqrt{3}$

**9.** If $c = 9$, find $a$ and $b$.
$a = 4.5; b = 4.5\sqrt{3}$

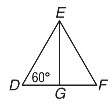
**For Exercises 10 and 11, use the figure at the right.**




**10.** The perimeter of the square is 30 inches. Find the length of $\overline{BC}$.
**7.5 in.**

**11.** Find the length of the diagonal $\overline{BD}$.
$7.5\sqrt{2}$ **in. or about 10.61 in.**

**12.** The perimeter of the equilateral triangle is 60 meters. Find the length of an altitude.
$10\sqrt{3}$ **m or about 17.32 m**




**13.** $\triangle GEC$ is a 30°-60°-90° triangle with right angle at $E$, and $\overline{EC}$ is the longer leg. Find the coordinates of $G$ in Quadrant I for $E(1, 1)$ and $C(4, 1)$.
$(1, 1 + \sqrt{3})$ **or about (1, 2.73)**

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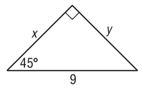
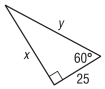
# 8-3 Practice

## *Special Right Triangles*

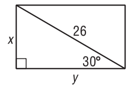
**Find $x$ and $y$.**

1. 

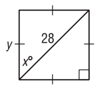

$\frac{9\sqrt{2}}{2}, \frac{9\sqrt{2}}{2}$

2. 

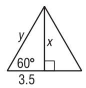

$25\sqrt{3}, 50$

3. 

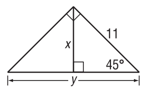

$13, 13\sqrt{3}$

4. 


$45, 14\sqrt{2}$

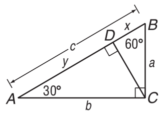
5. 


$3.5\sqrt{3}, 7$

6. 


$\frac{11\sqrt{2}}{2}; 11\sqrt{2}$

**For Exercises 7–9, use the figure at the right.**




7. If $a = 4\sqrt{3}$, find $b$ and $c$.
**$b = 12$, $c = 8\sqrt{3}$**

8. If $x = 3\sqrt{3}$, find $a$ and $CD$.
**$a = 6\sqrt{3}$, $CD = 9$**

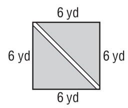
9. If $a = 4$, find $CD$, $b$, and $y$.
**$CD = 2\sqrt{3}$, $b = 4\sqrt{3}$, $y = 6$**

10. The perimeter of an equilateral triangle is 39 centimeters. Find the length of an altitude of the triangle.
**$6.5\sqrt{3}$ in. or about 11.26 in.**

11. $\triangle MIP$ is a 30°-60°-90° triangle with right angle at $I$, and $\overline{IP}$ the longer leg. Find the coordinates of $M$ in Quadrant I for $I(3, 3)$ and $P(12, 3)$.
**$(3, 3 + 3\sqrt{3})$ or about (3, 8.20)**

12. $\triangle TJK$ is a 45°-45°-90° triangle with right angle at $J$. Find the coordinates of $T$ in Quadrant II for $J(-2, -3)$ and $K(3, -3)$.
**$(-2, 2)$**

13. **BOTANICAL GARDENS** One of the displays at a botanical garden is an herb garden planted in the shape of a square. The square measures 6 yards on each side. Visitors can view the herbs from a diagonal pathway through the garden. How long is the pathway?




**$6\sqrt{2}$ yd or about 8.49 yd**



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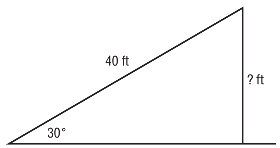
# 8-3 Word Problem Practice

## *Special Right Triangles*

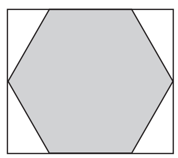
1. **ORIGAMI** A square piece of paper 150 millimeters on a side is folded in half along a diagonal. The result is a 45°-45°-90° triangle. What is the length of the hypotenuse of this triangle?
**$150\sqrt{2}$ mm**

2. **ESCALATORS** A 40-foot-long escalator rises from the first floor to the second floor of a shopping mall. The escalator makes a 30° angle with the horizontal.



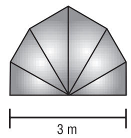

How high above the first floor is the second floor?
**20 ft**

3. **HEXAGONS** A box of chocolates shaped like a regular hexagon is placed snugly inside of a rectangular box as shown in the figure.



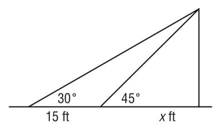
If the side length of the hexagon is 3 inches, what are the dimensions of the rectangular box?
**6 in. by $3\sqrt{3}$ in.**

4. **WINDOWS** A large stained glass window is constructed from six 30°-60°-90° triangles as shown in the figure.




What is the height of the window?
**$\frac{4\sqrt{3}}{3}$ m**

**MOVIES For Exercises 5–7, use the following information.**
Kim and Yolanda are watching a movie in a movie theater. Yolanda is sitting $x$ feet from the screen and Kim is 15 feet behind Yolanda.




The angle that Kim's line of sight to the top of the screen makes with the horizontal is 30°. The angle that Yolanda's line of sight to the top of the screen makes with the horizontal is 45°.

5. How high is the top of the screen in terms of x?
**$x$**

6. What is $\frac{x+15}{x}$?
**$\sqrt{3}$**

7. How far is Yolanda from the screen? Round your answer to the nearest tenth.
**20.5 ft**

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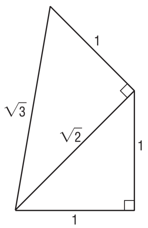
# 8-3 Enrichment

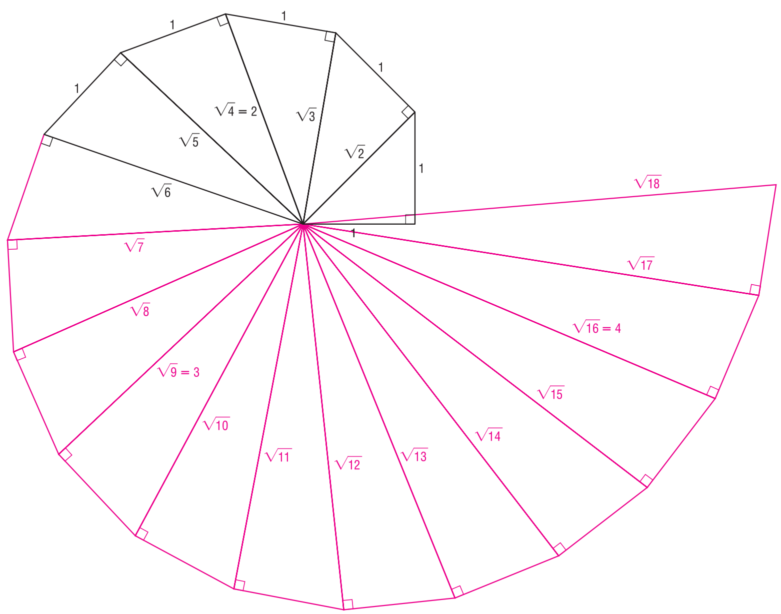
## *Constructing Values of Square Roots*

The diagram at the right shows a right isosceles triangle with two legs of length 1 inch. By the Pythagorean Theorem, the length of the hypotenuse is $\sqrt{2}$ inches. By constructing an adjacent right triangle with legs of $\sqrt{2}$ inches and 1 inch, you can create a segment of length $\sqrt{3}$.




By continuing this process as shown below, you can construct a "wheel" of square roots. This wheel is called the "Wheel of Theodorus" after a Greek philosopher who lived about 400 B.C.

Continue constructing the wheel until you make a segment of length $\sqrt{18}$.



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# 8-4 Lesson Reading Guide

## *Trigonometry*

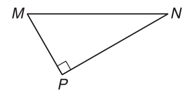
### Get Ready for the Lesson

**Read the introduction to Lesson 8-4 in your textbook.**

- Why is it important to determine the relative positions accurately in navigation? (Give two possible reasons.) **Sample answers: (1) To avoid collisions between ships, and (2) to prevent ships from losing their bearings and getting lost at sea.**
- What does *calibrated* mean? **Sample answer: marked precisely to permit accurate measurements to be made**

### Read the Lesson

1. Refer to the figure. Write a ratio using the side lengths in the figure to represent each of the following trigonometric ratios.




**A.** $\sin N$ $\frac{MP}{MN}$ **B.** $\cos N$ $\frac{NP}{MN}$

**C.** $\tan N$ $\frac{MP}{NP}$ **D.** $\tan M$ $\frac{NP}{MP}$

**E.** $\sin M$ $\frac{NP}{MN}$ **F.** $\cos M$ $\frac{MP}{MN}$

2. Assume that you enter each of the expressions in the list on the left into your calculator. Match each of these expressions with a description from the list on the right to tell what you are finding when you enter this expression.

| | |
|---|---|
| **a.** sin 20 **v** | **i.** the degree measure of an acute angle whose cosine is 0.8 |
| **b.** cos 20 **ii** | **ii.** the ratio of the length of the leg adjacent to the 20° angle to the length of hypotenuse in a 20°-70°-90° triangle |
| **c.** $\sin^{-1} 0.8$ **vi** | **iii.** the degree measure of an acute angle in a right triangle for which the ratio of the length of the opposite leg to the length of the adjacent leg is 0.8 |
| **d.** $\tan^{-1} 0.8$ **iii** | **iv.** the ratio of the length of the leg opposite the 20° angle to the length of the leg adjacent to it in a 20°-70°-90° triangle |
| **e.** tan 20 **iv** | **v.** the ratio of the length of the leg opposite the 20° angle to the length of hypotenuse in a 20°-70°-90° triangle |
| **f.** $\cos^{-1} 0.8$ **i** | **vi.** the degree measure of an acute angle in a right triangle for which the ratio of the length of the opposite leg to the length of the hypotenuse is 0.8 |

### Remember What You Learned

3. How can the *co* in *cosine* help you to remember the relationship between the sines and cosines of the two acute angles of a right triangle?
**Sample answer: The *co* in *cosine* comes from *complement*, as in *complementary* angles. The cosine of an acute angle is equal to the sine of its complement.**

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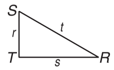
# 8-4 Study Guide and Intervention

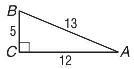
## *Trigonometry*

**Trigonometric Ratios** The ratio of the lengths of two sides of a right triangle is called a **trigonometric ratio**. The three most common ratios are **sine**, **cosine**, and **tangent**, which are abbreviated *sin*, *cos*, and *tan*, respectively.




$\sin R = \frac{\text{leg opposite } \angle R}{\text{hypotenuse}} = \frac{r}{t}$ $\cos R = \frac{\text{leg adjacent to } \angle R}{\text{hypotenuse}} = \frac{s}{t}$ $\tan R = \frac{\text{leg opposite } \angle R}{\text{leg adjacent to } \angle R} = \frac{r}{s}$

**Example** **Find sin *A*, cos *A*, and tan *A*. Express each ratio as a decimal to the nearest thousandth.**



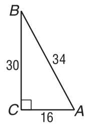

$\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{BC}{AB} = \frac{5}{13} \approx 0.385$

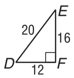
$\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{AC}{AB} = \frac{12}{13} \approx 0.923$

$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{BC}{AC} = \frac{5}{12} \approx 0.417$

**Exercises**

**Find the indicated trigonometric ratio as a fraction and as a decimal. If necessary, round to the nearest ten-thousandth.**




**1.** sin $A$ $\frac{15}{17}$; 0.8824

**2.** tan $B$ $\frac{8}{15}$; 0.5333

**3.** cos $A$ $\frac{8}{17}$; 0.4706

**4.** cos $B$ $\frac{15}{17}$; 0.8824

**5.** sin $D$ $\frac{4}{5}$; 0.8

**6.** tan $E$ $\frac{3}{4}$; 0.75

**7.** cos $E$ $\frac{4}{5}$; 0.8

**8.** cos $D$ $\frac{3}{5}$; 0.6



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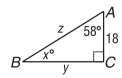
# 8-4 Study Guide and Intervention *(continued)*

## *Trigonometry*

**Use Trigonometric Ratios** In a right triangle, if you know the measures of two sides or if you know the measures of one side and an acute angle, then you can use trigonometric ratios to find the measures of the missing sides or angles of the triangle.

**Example** **Find *x*, *y*, and *z*. Round each measure to the nearest whole number.**




**a. Find *x*.**

$x + 58 = 90$

$x = 32$

**b. Find *y*.**

$\tan A = \frac{y}{18}$

$\tan 58° = \frac{y}{18}$

$y = 18 \tan 58°$

$y \approx 29$

**c. Find *z*.**

$\cos A = \frac{18}{z}$

$\cos 58° = \frac{18}{z}$

$z \cos 58° = 18$

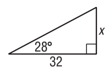
$z = \frac{18}{\cos 58°}$

$z \approx 34$

**Exercises**

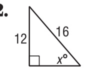
**Find *x*. Round to the nearest tenth.**

**1.**



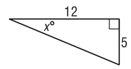

17.0

**2.**



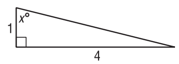

48.6

**3.**



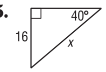

22.6

**4.**



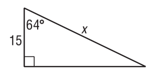

76.0

**5.**




24.9

**6.**

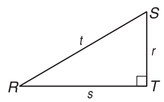



34.2

# 8-4 Skills Practice

## Trigonometry

**Use △$RST$ to find sin $R$, cos $R$, tan $R$, sin $S$, cos $S$, and tan $S$. Express each ratio as a fraction and as a decimal to the nearest hundredth.**




**1.** $r = 16, s = 30, t = 34$

$\sin R = \frac{16}{34} \approx 0.47$;
$\cos R = \frac{30}{34} \approx 0.88$;
$\tan R = \frac{16}{30} \approx 0.53$;
$\sin S = \frac{30}{34} \approx 0.88$;
$\cos S = \frac{16}{34} \approx 0.47$;
$\tan S = \frac{30}{16} \approx 1.88$

**2.** $r = 10, s = 24, t = 26$

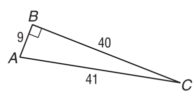
$\sin R = \frac{10}{26} \approx 0.38$;
$\cos R = \frac{24}{26} \approx 0.92$;
$\tan R = \frac{10}{24} \approx 0.42$;
$\sin S = \frac{24}{26} \approx 0.92$;
$\cos S = \frac{10}{26} \approx 0.38$;
$\tan S = \frac{24}{10} = 2.4$

**Use a calculator to find each value. Round to the nearest ten-thousandth.**

**3.** sin 5 **0.0872** **4.** tan 23 **0.4245** **5.** cos 61 **0.4848**

**6.** sin 75.8 **0.9694** **7.** tan 17.3 **0.3115** **8.** cos 52.9 **0.6032**

**Use the figure to find each trigonometric ratio. Express answers as a fraction and as a decimal rounded to the nearest ten-thousandth.**




**9.** tan $C$ $\frac{9}{40} \approx 0.2250$ **10.** sin $A$ $\frac{40}{41} \approx 0.9756$ **11.** cos $C$ $\frac{40}{41} \approx 0.9756$

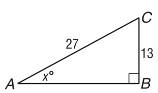
**Find the measure of each acute angle to the nearest tenth of a degree.**

**12.** $\sin B = 0.2985$ **17.4** **13.** $\tan A = 0.4168$ **22.6** **14.** $\cos R = 0.8443$ **32.4**

**15.** $\tan C = 0.3894$ **21.3** **16.** $\cos B = 0.7329$ **42.9** **17.** $\sin A = 0.1176$ **6.8**

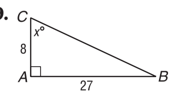
**Find $x$. Round to the nearest tenth.**

**18.**



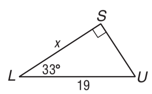

**28.8**

**19.**




**73.5**

**20.**




**15.9**

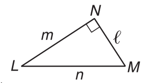




# 8-4 Practice

## Trigonometry

**Use △$LMN$ to find sin $L$, cos $L$, tan $L$, sin $M$, cos $M$, and tan $M$. Express each ratio as a fraction and as a decimal to the nearest hundredth.**




**1.** $\ell = 15, m = 36, n = 39$

$\sin L = \frac{15}{39} \approx 0.38$;
$\cos L = \frac{36}{39} \approx 0.92$;
$\tan L = \frac{15}{36} \approx 0.42$;
$\sin M = \frac{36}{39} \approx 0.92$;
$\cos M = \frac{15}{39} \approx 0.38$;
$\tan M = \frac{36}{15} = 2.4$

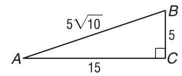
**2.** $\ell = 12, m = 12\sqrt{3}, n = 24$

$\sin L = \frac{12}{24} = 0.50$;
$\cos L = \frac{12\sqrt{3}}{24} \approx 0.87$;
$\tan L = \frac{12}{12\sqrt{3}} \approx 0.58$;
$\sin M = \frac{12\sqrt{3}}{24} \approx 0.87$;
$\cos M = \frac{12}{24} = 0.50$;
$\tan M = \frac{12\sqrt{3}}{12} \approx 1.73$

**Use a calculator to find each value. Round to the nearest ten-thousandth.**

**3.** sin 72.5 **0.9537** **4.** tan 27.5 **0.5206** **5.** cos 64.8 **0.4258**

**Use the figure to find each trigonometric ratio. Express answers as a fraction and as a decimal rounded to the nearest ten-thousandth.**



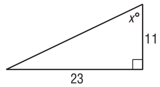

**6.** cos $A$ $\frac{3\sqrt{10}}{10} \approx 0.9487$ **7.** tan $B$ $\frac{3}{1} = 3.0000$ **8.** sin $A$ $\frac{\sqrt{10}}{10} \approx 0.3162$

**Find the measure of each acute angle to the nearest tenth of a degree.**

**9.** $\sin B = 0.7823$ **51.5** **10.** $\tan A = 0.2356$ **13.3** **11.** $\cos R = 0.6401$ **50.2**

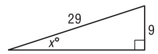
**Find $x$. Round to the nearest tenth.**

**12.**



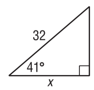

**64.4**

**13.**



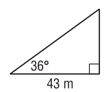

**18.1**

**14.**




**24.2**

**15. GEOGRAPHY** Diego used a theodolite to map a region of land for his class in geomorphology. To determine the elevation of a vertical rock formation, he measured the distance from the base of the formation to his position and the angle between the ground and the line of sight to the top of the formation. The distance was 43 meters and the angle was 36 degrees. What is the height of the formation to the nearest meter? **31 m**



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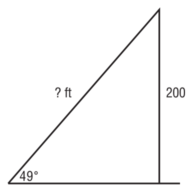
# 8-4 Word Problem Practice

## *Trigonometry*

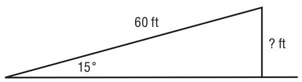
1. **RADIO TOWERS** Kay is standing near a 200-foot-high radio tower.




Use the information in the figure to determine how far Kay is from the top of the tower. Express your answer as a trigonometric function.

$\frac{200}{\sin 49°}$

2. **RAMPS** A 60-foot ramp rises from the first floor to the second floor of a parking garage. The ramp makes a 15° angle with the ground.



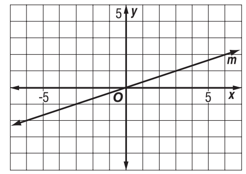

How high above the first floor is the second floor? Express your answer as a trigonometric function.

**60 sin 15°**

3. **TRIGONOMETRY** Melinda and Walter were both solving the same trigonometry problem. However, after they finished their computations, Melinda said the answer was 52 sin 27° and Walter said the answer was 52 cos 63°. Could they both be correct? Explain.

**Yes, they are both correct. Because 27 + 63 = 90, the sine of 27° is the same ratio as the cosine of 63°.**

4. **LINES** Jasmine draws line *m* on a coordinate plane.

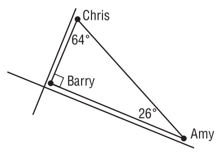



What angle does *m* make with the *x*-axis? Round your answer to the nearest degree.

**18°**

**NEIGHBORS For Exercises 5–7, use the following information.**

Amy, Barry, and Chris live on the same block. Chris lives up the street and around the corner from Amy, and Barry lives at the corner between Amy and Chris. The three homes make a right triangle.




5. Give two trigonometric expressions for the ratio of Barry's distance from Amy to Chris' distance from Amy.

**cos 26° or sin 64°**

6. Give two trigonometric expressions for the ratio of Barry's distance from Chris to Amy's distance from Chris.

**cos 64° or sin 26°**

7. Give a trigonometric expression for the ratio of Amy's distance from Barry to Chris' distance from Barry.

**tan 64°**



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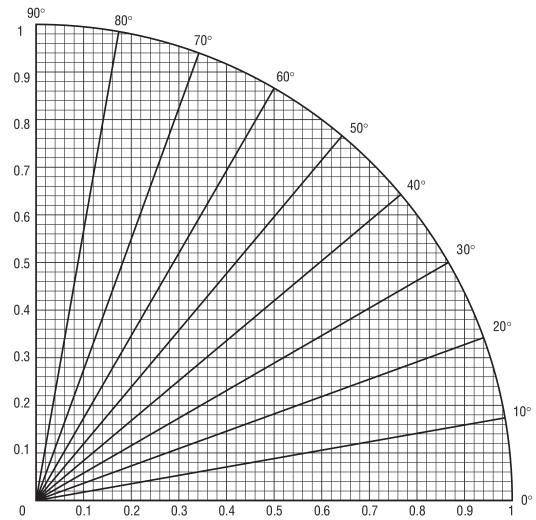
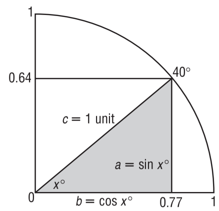
# 8-4 Enrichment

## *Sine and Cosine of Angles*

The following diagram can be used to obtain approximate values for the sine and cosine of angles from 0° to 90°. The radius of the circle is 1. So, the sine and cosine values can be read directly from the vertical and horizontal axes.




**Example** **Find approximate values for sin 40° and cos 40°. Consider the triangle formed by the segment marked 40°, as illustrated by the shaded triangle at right.**




$\sin 40° = \frac{a}{c} \approx \frac{0.64}{1}$ or 0.64 $\cos 40° = \frac{b}{c} \approx \frac{0.77}{1}$ or 0.77

1. Use the diagram above to complete the chart of values.

| x° | 0° | 10° | 20° | 30° | 40° | 50° | 60° | 70° | 80° | 90° |
|---|---|---|---|---|---|---|---|---|---|---|
| sin x° | **0** | **0.17** | **0.34** | **0.5** | 0.64 | **0.77** | **0.87** | **0.94** | **0.98** | **1** |
| cos x° | **1** | **0.98** | **0.94** | **0.87** | 0.77 | **0.64** | **0.5** | **0.34** | **0.17** | **0** |

2. Compare the sine and cosine of two complementary angles (angles with a sum is 90°). What do you notice?

**The sine of an angle is equal to the cosine of the complement of the angle.**



Lesson 8-4

Answers

NAME ______________ DATE ________ PERIOD ____

# 8-5 Lesson Reading Guide

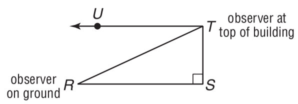
## *Angles of Elevation and Depression*

### Get Ready for the Lesson

**Read the introduction to Lesson 8-5 in your textbook.**

What does the angle measure tell the pilot? **Sample answer: how steep her ascent must be to clear the peak**

### Read the Lesson




**1.** Refer to the figure. The two observers are looking at one another. Select the correct choice for each question.

**a.** What is the line of sight? **iii**
(i) line $RS$ (ii) line $ST$ (iii) line $RT$ (iv) line $TU$

**b.** What is the angle of elevation? **ii**
(i) $\angle RST$ (ii) $\angle SRT$ (iii) $\angle RTS$ (iv) $\angle UTR$

**c.** What is the angle of depression? **iv**
(i) $\angle RST$ (ii) $\angle SRT$ (iii) $\angle RTS$ (iv) $\angle UTR$

**d.** How are the angle of elevation and the angle of depression related? **ii**
(i) They are complementary.
(ii) They are congruent.
(iii) They are supplementary.
(iv) The angle of elevation is larger than the angle of depression.

**e.** Which postulate or theorem that you learned in Chapter 3 supports your answer for part c? **iv**
(i) Corresponding Angles Postulate
(ii) Alternate Exterior Angles Theorem
(iii) Consecutive Interior Angles Theorem
(iv) Alternate Interior Angles Theorem

**2.** A student says that the angle of elevation from his eye to the top of a flagpole is 135°. What is wrong with the student's statement?
**An angle of elevation cannot be obtuse.**

### Remember What You Learned

**3.** A good way to remember something is to explain it to someone else. Suppose a classmate finds it difficult to distinguish between angles of elevation and angles of depression. What are some hints you can give her to help her get it right every time? **Sample answers: (1) The angle of depression and the angle of elevation are both measured between the horizontal and the line of sight. (2) The angle of depression is always congruent to the angle of elevation in the same diagram. (3) Associate the word *elevation* with the word *up* and the word *depression* with the word *down*.**

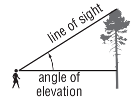


NAME ______________ DATE ________ PERIOD ____

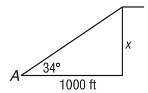
# 8-5 Study Guide and Intervention

## *Angles of Elevation and Depression*

**Angles of Elevation** Many real-world problems that involve looking up to an object can be described in terms of an **angle of elevation**, which is the angle between an observer's line of sight and a horizontal line.




**Example** **The angle of elevation from point $A$ to the top of a cliff is 34°. If point $A$ is 1000 feet from the base of the cliff, how high is the cliff?**




Let $x$ = the height of the cliff.

$\tan 34° = \frac{x}{1000}$    $\tan = \frac{\text{opposite}}{\text{adjacent}}$

$1000(\tan 34°) = x$    Multiply each side by 1000.

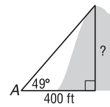
$674.5 = x$    Use a calculator.

The height of the cliff is about 674.5 feet.

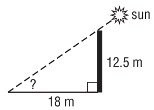
**Exercises**

**Solve each problem. Round measures of segments to the nearest whole number and angles to the nearest degree.**

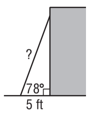
**1.** The angle of elevation from point $A$ to the top of a hill is 49°. If point $A$ is 400 feet from the base of the hill, how high is the hill?
**460 ft**



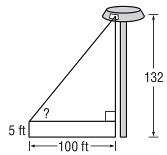

**2.** Find the angle of elevation of the sun when a 12.5-meter-tall telephone pole casts an 18-meter-long shadow.
**35°**




**3.** A ladder leaning against a building makes an angle of 78° with the ground. The foot of the ladder is 5 feet from the building. How long is the ladder?
**24 ft**




**4.** A person whose eyes are 5 feet above the ground is standing on the runway of an airport 100 feet from the control tower. That person observes an air traffic controller at the window of the 132-foot tower. What is the angle of elevation?
**52°**





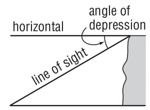

Lesson 8-5

NAME ________________ DATE ________ PERIOD _____

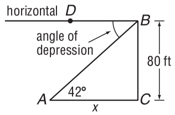
# 8-5 Study Guide and Intervention *(continued)*

## *Angles of Elevation and Depression*

**Angles of Depression** When an observer is looking down, the **angle of depression** is the angle between the observer's line of sight and a horizontal line.




**Example** **The angle of depression from the top of an 80-foot building to point *A* on the ground is 42°. How far is the foot of the building from point *A*?**




Let $x$ = the distance from point $A$ to the foot of the building. Since the horizontal line is parallel to the ground, the angle of depression $\angle DBA$ is congruent to $\angle BAC$.

$\tan 42° = \frac{80}{x}$ — $\tan = \frac{\text{opposite}}{\text{adjacent}}$

$x(\tan 42°) = 80$ — Multiply each side by $x$.

$x = \frac{80}{\tan 42°}$ — Divide each side by tan 42°.

$x \approx 88.8$ — Use a calculator.

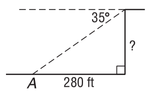
Point $A$ is about 89 feet from the base of the building.

**Exercises**

**Solve each problem. Round measures of segments to the nearest whole number and angles to the nearest degree.**

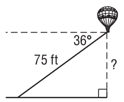
**1.** The angle of depression from the top of a sheer cliff to point $A$ on the ground is 35°. If point $A$ is 280 feet from the base of the cliff, how tall is the cliff?

**196 ft**



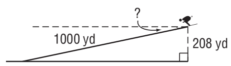

**2.** The angle of depression from a balloon on a 75-foot string to a person on the ground is 36°. How high is the balloon?

**44 ft**



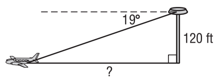

**3.** A ski run is 1000 yards long with a vertical drop of 208 yards. Find the angle of depression from the top of the ski run to the bottom.

**12°**




**4.** From the top of a 120-foot-high tower, an air traffic controller observes an airplane on the runway at an angle of depression of 19°. How far from the base of the tower is the airplane?

**349 ft**





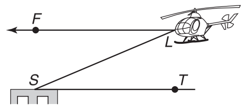

NAME ________________ DATE ________ PERIOD _____

# 8-5 Skills Practice

## *Angles of Elevation and Depression*

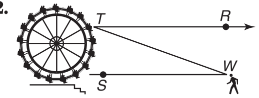
**Name the angle of depression or angle of elevation in each figure.**

**1.**



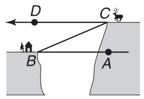

**∠FLS; ∠TSL**

**2.**



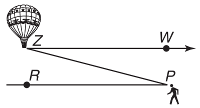

**∠RTW; ∠SWT**

**3.**




**∠DCB; ∠ABC**

**4.**



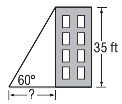

**∠WZP; ∠RPZ**

**5. MOUNTAIN BIKING** On a mountain bike trip along the Gemini Bridges Trail in Moab, Utah, Nabuko stopped on the canyon floor to get a good view of the twin sandstone bridges. Nabuko is standing about 60 meters from the base of the canyon cliff, and the natural arch bridges are about 100 meters up the canyon wall. If her line of sight is five feet above the ground, what is the angle of elevation to the top of the bridges? Round to the nearest tenth degree.

**about 57.7°**

**6. SHADOWS** Suppose the sun casts a shadow off a 35-foot building. If the angle of elevation to the sun is 60°, how long is the shadow to the nearest tenth of a foot?

**about 20.2 ft**

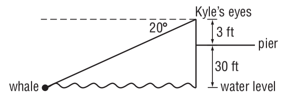



**7. BALLOONING** From her position in a hot-air balloon, Angie can see her car parked in a field. If the angle of depression is 8° and Angie is 38 meters above the ground, what is the straight-line distance from Angie to her car? Round to the nearest whole meter.

**about 273 m**

**8. INDIRECT MEASUREMENT** Kyle is at the end of a pier 30 feet above the ocean. His eye level is 3 feet above the pier. He is using binoculars to watch a whale surface. If the angle of depression of the whale is 20°, how far is the whale from Kyle's binoculars? Round to the nearest tenth foot.

**about 96.5 ft**






Lesson 8-5

Answers

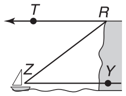
NAME ________________________________ DATE __________ PERIOD _____

# 8-5 Practice

## Angles of Elevation and Depression

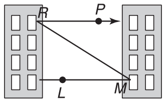
**Name the angle of depression or angle of elevation in each figure.**

**1.**




***∠TRZ; ∠YZR***

**2.**




***∠PRM; ∠LMR***

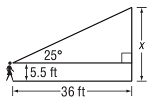
**3. WATER TOWERS** A student can see a water tower from the closest point of the soccer field at San Lobos High School. The edge of the soccer field is about 110 feet from the water tower and the water tower stands at a height of 32.5 feet. What is the angle of elevation if the eye level of the student viewing the tower from the edge of the soccer field is 6 feet above the ground? Round to the nearest tenth degree.

**about 13.5°**

**4. CONSTRUCTION** A roofer props a ladder against a wall so that the top of the ladder reaches a 30-foot roof that needs repair. If the angle of elevation from the bottom of the ladder to the roof is 55°, how far is the ladder from the base of the wall? Round your answer to the nearest foot.

**about 21 ft**

**5. TOWN ORDINANCES** The town of Belmont restricts the height of flagpoles to 25 feet on any property. Lindsay wants to determine whether her school is in compliance with the regulation. Her eye level is 5.5 feet from the ground and she stands 36 feet from the flagpole. If the angle of elevation is about 25°, what is the height of the flagpole to the nearest tenth foot?

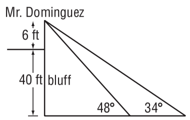



**about 22.3 ft**

**6. GEOGRAPHY** Stephan is standing on a mesa at the Painted Desert. The elevation of the mesa is about 1380 meters and Stephan's eye level is 1.8 meters above ground. If Stephan can see a band of multicolored shale at the bottom and the angle of depression is 29°, about how far is the band of shale from his eyes? Round to the nearest meter.

**about 2850 m**

**7. INDIRECT MEASUREMENT** Mr. Dominguez is standing on a 40-foot ocean bluff near his home. He can see his two dogs on the beach below. If his line of sight is 6 feet above the ground and the angles of depression to his dogs are 34° and 48°, how far apart are the dogs to the nearest foot?




**about 27 ft**

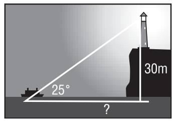


NAME ________________________________ DATE __________ PERIOD _____

# 8-5 Word Problem Practice

## Angles of Elevation and Depression

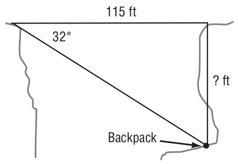
**1. LIGHTHOUSES** Sailors on a ship at sea spot the light from a lighthouse. The angle of elevation to the light is 25°.




The light of the lighthouse is 30 meters above sea level. How far from the shore is the ship? Round your answer to the nearest meter.

**64 m**

**2. RESCUE** A hiker dropped his backpack over one side of a canyon onto a ledge below. Because of the shape of the cliff, he could not see exactly where it landed.




From the other side, the park ranger reports that the angle of depression to the backpack is 32°. If the width of the canyon is 115 feet, how far down did the backpack fall? Round your answer to the nearest whole number.

**72 ft**

**3. AIRPLANES** The angle of elevation to an airplane viewed from the control tower at an airport is 7°. The tower is 200 feet high and the pilot reports that the altitude is 5200 feet. How far away from the control tower is the airplane? Round your answer to the nearest foot.

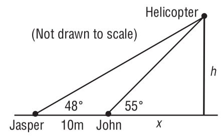
**41,028 ft**

**4. PEAK TRAM** The Peak Tram in Hong Kong connects two terminals, one at the base of a mountain, and the other at the summit. The angle of elevation of the upper terminal from the lower terminal is about 15.5°. The distance between the two terminals is about 1365 meters. About how much higher above sea level is the upper terminal compared to the lower terminal? Round your answer to the nearest meter.

**365 m**

**HELICOPTERS For Exercises 5–7, use the following information.**

Jermaine and John are watching a helicopter hover above the ground.




Jermaine and John are standing 10 meters apart.

**5.** Find two different expressions that can be used to find the $h$, height of the helicopter.

**$h = x \tan 55°$; $h = (x + 10)\tan 48°$**

**6.** Equate the two expressions you found for Exercise 5 to solve for $x$. Round your answer to the nearest hundredth.

**34.98 m**

**7.** How high above the ground is the helicopter? Round your answer to the nearest hundredth.

**49.95 m**

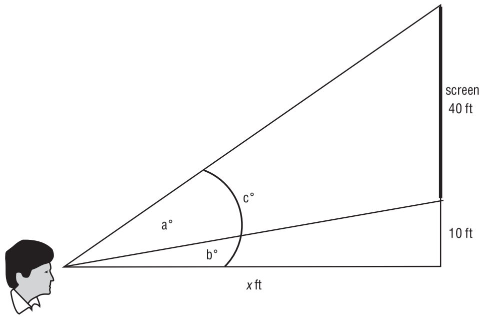
Lesson 8-5

NAME ______________________ DATE ____________ PERIOD _____

# 8-5 Enrichment

## *Best Seat in the House*

Most people want to sit in the best seat in the movie theater. The best seat could be defined as the seat that allows you to see the maximum amount of screen. The picture below represents this situation.




To determine the best seat in the house, you want to find what value of $x$ allows you to see the maximum amount of screen. The value of $x$ is how far from the screen you should sit.

1. To maximize the amount of screen viewed, which angle value needs to be maximized? Why?
   **Angle with measure *a* because this is how much of the screen can be viewed.**

2. What is the value of $a$ if $x = 10$ feet?
   **33.7**

3. What is the value of $a$ if $x = 20$ feet?
   **41.6**

4. What is the value of $a$ if $x = 25$ feet?
   **41.6**

5. What is the value of $a$ if $x = 35$ feet?
   **39.1**

6. What is the value of $a$ if $x = 55$ feet?
   **32**

7. Which value of $x$ gives the greatest value of $a$? So, where is the best seat in the movie theater?
   **The best seat in the house is around 20–25 feet away from the screen.**



NAME ______________________ DATE ____________ PERIOD _____

# 8-6 Lesson Reading Guide

## *The Law of Sines*



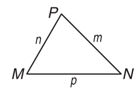
### Get Ready for the Lesson

**Read the introduction to Lesson 8-6 in your textbook.**

- If a triangle is a right triangle, what theorem can be used to determine the lengths of the sides? **Pythagorean Theorem**
- If a triangle is not a right triangle, can this theorem still be used to determine the lengths of the sides? **no**

### Read the Lesson

1. Refer to the figure. According to the Law of Sines, which of the following are correct statements? **A, F**




**A.** $\frac{m}{\sin M} = \frac{n}{\sin N} = \frac{p}{\sin P}$

**B.** $\frac{\sin m}{M} = \frac{\sin n}{N} = \frac{\sin p}{P}$

**C.** $\frac{\cos M}{m} = \frac{\cos N}{n} = \frac{\cos P}{p}$

**D.** $\frac{\sin M}{m} + \frac{\sin N}{n} = \frac{\sin P}{p}$

**E.** $(\sin M)^2 + (\sin N)^2 = (\sin P)^2$

**F.** $\frac{\sin P}{p} = \frac{\sin M}{m} = \frac{\sin N}{n}$

2. State whether each of the following statements is *true* or *false*. If the statement is false, explain why.
   a. The Law of Sines applies to all triangles. **true**
   b. The Pythagorean Theorem applies to all triangles. **False; sample answer: It only applies to right triangles.**
   c. If you are given the length of one side of a triangle and the measures of any two angles, you can use the Law of Sines to find the lengths of the other two sides. **true**
   d. If you know the measures of two angles of a triangle, you should use the Law of Sines to find the measure of the third angle. **False; sample answer: You should use the Angle Sum Theorem.**
   e. A friend tells you that in triangle $RST$, $m\angle R = 132$, $r = 24$ centimeters, and $s = 31$ centimeters. Can you use the Law of Sines to solve the triangle? Explain. **No; sample answer: In any triangle, the longest side is opposite the largest angle. Because a triangle can have only one obtuse angle, $\angle R$ must be the largest angle, but $s > r$, so it is impossible to have a triangle with the given measures.**

### Remember What You Learned

3. Many students remember mathematical equations and formulas better if they can state them in words. State the Law of Sines in your own words without using variables or mathematical symbols.
   **Sample answer: In any triangle, the ratio of the sine of an angle to the length of the opposite side is the same for all three angles.**

NAME ______________ DATE ________ PERIOD ____

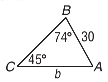
## 8-6 Study Guide and Intervention

### The Law of Sines

**The Law of Sines** In any triangle, there is a special relationship between the angles of the triangle and the lengths of the sides opposite the angles.

| Law of Sines | $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ |
|---|---|

**Example 1** **In $\triangle ABC$, find $b$.**




$\frac{\sin C}{c} = \frac{\sin B}{b}$ — Law of Sines

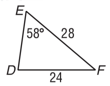
$\frac{\sin 45°}{30} = \frac{\sin 74°}{b}$ — $m\angle C = 45$, $c = 30$, $m\angle B = 74$

$b \sin 45° = 30 \sin 74°$ — Cross multiply.

$b = \frac{30 \sin 74°}{\sin 45°}$ — Divide each side by sin 45°.

$b \approx 40.8$ — Use a calculator.

**Example 2** **In $\triangle DEF$, find $m\angle D$.**




$\frac{\sin D}{d} = \frac{\sin E}{e}$ — Law of Sines

$\frac{\sin D}{28} = \frac{\sin 58°}{24}$ — $d = 28$, $m\angle E = 58$, $e = 24$

$24 \sin D = 28 \sin 58°$ — Cross multiply.

$\sin D = \frac{28 \sin 58°}{24}$ — Divide each side by 24.

$D = \sin^{-1} \frac{28 \sin 58°}{24}$ — Use the inverse sine.

$D \approx 81.6°$ — Use a calculator.

**Exercises**

**Find each measure using the given measures of $\triangle ABC$. Round angle measures to the nearest degree and side measures to the nearest tenth.**

**1.** If $c = 12$, $m\angle A = 80$, and $m\angle C = 40$, find $a$.
**18.4**

**2.** If $b = 20$, $c = 26$, and $m\angle C = 52$, find $m\angle B$.
**37**

**3.** If $a = 18$, $c = 16$, and $m\angle A = 84$, find $m\angle C$.
**62**

**4.** If $a = 25$, $m\angle A = 72$, and $m\angle B = 17$, find $b$.
**7.7**

**5.** If $b = 12$, $m\angle A = 89$, and $m\angle B = 80$, find $a$.
**12.2**

**6.** If $a = 30$, $c = 20$, and $m\angle A = 60$, find $m\angle C$.
**35**



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## 8-6 Study Guide and Intervention *(continued)*

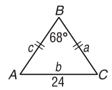
### The Law of Sines



**Use the Law of Sines to Solve Problems** You can use the **Law of Sines** to solve some problems that involve triangles.

| Law of Sines | Let $\triangle ABC$ be any triangle with $a$, $b$, and $c$ representing the measures of the sides opposite the angles with measures $A$, $B$, and $C$, respectively. Then $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$. |
|---|---|

**Example** **Isosceles $\triangle ABC$ has a base of 24 centimeters and a vertex angle of 68°. Find the perimeter of the triangle.**




The vertex angle is 68°, so the sum of the measures of the base angles is 112 and $m\angle A = m\angle C = 56$.

$\frac{\sin B}{b} = \frac{\sin A}{a}$ — Law of Sines

$\frac{\sin 68°}{24} = \frac{\sin 56°}{a}$ — $m\angle B = 68$, $b = 24$, $m\angle A = 56$

$a \sin 68° = 24 \sin 56°$ — Cross multiply.

$a = \frac{24 \sin 56°}{\sin 68°}$ — Divide each side by sin 68°.

$\approx 21.5$ — Use a calculator.

The triangle is isosceles, so $c = 21.5$.
The perimeter is $24 + 21.5 + 21.5$ or about 67 centimeters.

**Exercises**

**Draw a triangle to go with each exercise and mark it with the given information. Then solve the problem. Round angle measures to the nearest degree and side measures to the nearest tenth.**

**1.** One side of a triangular garden is 42.0 feet. The angles on each end of this side measure 66° and 82°. Find the length of fence needed to enclose the garden.
**192.9 ft**

**2.** Two radar stations $A$ and $B$ are 32 miles apart. They locate an airplane $X$ at the same time. The three points form $\angle XAB$, which measures 46°, and $\angle XBA$, which measures 52°. How far is the airplane from each station?
**25.5 mi from *A*; 23.2 mi from *B***

**3.** A civil engineer wants to determine the distances from points $A$ and $B$ to an inaccessible point $C$ in a river. $\angle BAC$ measures 67° and $\angle ABC$ measures 52°. If points $A$ and $B$ are 82.0 feet apart, find the distance from $C$ to each point.
**86.3 ft to point *B*; 73.9 ft to point *A***

**4.** A ranger tower at point $A$ is 42 kilometers north of a ranger tower at point $B$. A fire at point $C$ is observed from both towers. If $\angle BAC$ measures 43° and $\angle ABC$ measures 68°, which ranger tower is closer to the fire? How much closer?
**Tower *B* is 11.0 km closer than Tower *A*.**

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# 8-6 Skills Practice

## *The Law of Sines*

**Find each measure using the given measures from $\triangle ABC$. Round angle measures to the nearest tenth degree and side measures to the nearest tenth.**

**1.** If $m\angle A = 35$, $m\angle B = 48$, and $b = 28$, find $a$. **21.6**

**2.** If $m\angle B = 17$, $m\angle C = 46$, and $c = 18$, find $b$. **7.3**

**3.** If $m\angle C = 86$, $m\angle A = 51$, and $a = 38$, find $c$. **48.8**

**4.** If $a = 17$, $b = 8$, and $m\angle A = 73$, find $m\angle B$. **26.7**

**5.** If $c = 38$, $b = 34$, and $m\angle B = 36$, find $m\angle C$. **41.1 or 138.9**

**6.** If $a = 12$, $c = 20$, and $m\angle C = 83$, find $m\angle A$. **36.6**

**7.** If $m\angle A = 22$, $a = 18$, and $m\angle B = 104$, find $b$. **46.6**

**Solve each $\triangle PQR$ described below. Round measures to the nearest tenth.**

**8.** $p = 27$, $q = 40$, $m\angle P = 33$ **$m\angle Q \approx 53.8$, $m\angle R \approx 93.2$, $r \approx 49.5$; or $m\angle Q \approx 126.2$, $m\angle R \approx 20.8$, $r \approx 17.6$**

**9.** $q = 12$, $r = 11$, $m\angle R = 16$ **$m\angle P \approx 146.5$, $m\angle Q \approx 17.5$, $p \approx 22.0$; or $m\angle P \approx 1.5$, $m\angle Q \approx 162.5$, $p \approx 1.0$**

**10.** $p = 29$, $q = 34$, $m\angle Q = 111$ **$m\angle P \approx 52.8$, $m\angle R \approx 16.2$, $r \approx 10.2$**

**11.** If $m\angle P = 89$, $p = 16$, $r = 12$ **$m\angle Q \approx 42.4$, $m\angle R \approx 48.6$, $q \approx 10.8$**

**12.** If $m\angle Q = 103$, $m\angle P = 63$, $p = 13$ **$m\angle R = 14$, $q \approx 14.2$, $r \approx 3.5$**

**13.** If $m\angle P = 96$, $m\angle R = 82$, $r = 35$ **$m\angle Q = 2$, $p \approx 35.2$, $q \approx 1.2$**

**14.** If $m\angle R = 49$, $m\angle Q = 76$, $r = 26$ **$m\angle P = 55$, $p \approx 28.2$, $q \approx 33.4$**

**15.** If $m\angle Q = 31$, $m\angle P = 52$, $p = 20$ **$m\angle R = 97$, $q \approx 13.1$, $r \approx 25.2$**

**16.** If $q = 8$, $m\angle Q = 28$, $m\angle R = 72$ **$m\angle P = 80$, $p \approx 16.8$, $r \approx 16.2$**

**17.** If $r = 15$, $p = 21$, $m\angle P = 128$ **$m\angle Q \approx 17.7$, $m\angle R \approx 34.3$, $q \approx 8.1$**



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# 8-6 Practice

## *The Law of Sines*

Lesson 8-6

**Find each measure using the given measures from $\triangle EFG$. Round angle measures to the nearest tenth degree and side measures to the nearest tenth.**

**1.** If $m\angle G = 14$, $m\angle E = 67$, and $e = 14$, find $g$. **3.7**

**2.** If $e = 12.7$, $m\angle E = 42$, and $m\angle F = 61$, find $f$. **16.6**

**3.** If $g = 14$, $f = 5.8$, and $m\angle G = 83$, find $m\angle F$. **24.3**

**4.** If $e = 19.1$, $m\angle G = 34$, and $m\angle E = 56$, find $g$. **12.9**

**5.** If $f = 9.6$, $g = 27.4$, and $m\angle G = 43$, find $m\angle F$. **13.8**

**Solve each $\triangle STU$ described below. Round measures to the nearest tenth.**

**6.** $m\angle T = 85$, $s = 4.3$, $t = 8.2$ **$m\angle S \approx 31.5$, $m\angle U \approx 63.5$, $u \approx 7.4$**

**7.** $s = 40$, $u = 12$, $m\angle S = 37$ **$m\angle T \approx 132.6$, $m\angle U \approx 10.4$, $t \approx 48.9$**

**8.** $m\angle U = 37$, $t = 2.3$, $m\angle T = 17$ **$m\angle S = 126$, $s \approx 6.4$, $u \approx 4.7$**

**9.** $m\angle S = 62$, $m\angle U = 59$, $s = 17.8$ **$m\angle T = 59$, $t \approx 17.3$, $u \approx 17.3$**

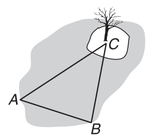
**10.** $t = 28.4$, $u = 21.7$, $m\angle T = 66$ **$m\angle S \approx 69.7$, $m\angle U \approx 44.3$, $s \approx 29.2$**

**11.** $m\angle S = 89$, $s = 15.3$, $t = 14$ **$m\angle T \approx 66.2$, $m\angle U \approx 24.8$, $u \approx 6.4$**

**12.** $m\angle T = 98$, $m\angle U = 74$, $u = 9.6$ **$m\angle S = 8$, $s \approx 1.4$, $t \approx 9.9$**

**13.** $t = 11.8$, $m\angle S = 84$, $m\angle T = 47$ **$m\angle U = 49$, $s \approx 16.0$, $u \approx 12.2$**

**14. INDIRECT MEASUREMENT** To find the distance from the edge of the lake to the tree on the island in the lake, Hannah set up a triangular configuration as shown in the diagram. The distance from location $A$ to location $B$ is 85 meters. The measures of the angles at $A$ and $B$ are 51° and 83°, respectively. What is the distance from the edge of the lake at $B$ to the tree on the island at $C$?




**about 91.8 m**



Answers

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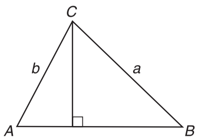
# 8-6 Word Problem Practice

## The Law of Sines

1. **ALTITUDES** In triangle $ABC$, the altitude to side $AB$ is drawn.



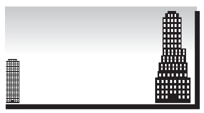

Give two expressions for the length of the altitude in terms of $a$, $b$, and the sine of the angles $A$ and $B$.

**$a \sin B = b \sin A$ = length of altitude**

2. **MAPS** Three cities form the vertices of a triangle. The angles of the triangle are 40°, 60°, and 80°. The two most distant cities are 40 miles apart. How close are the two closest cities? Round your answer to the nearest tenth of a mile.

**26.1 mi**

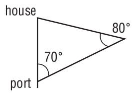
3. **PHOTOS** Greg took a photograph of the view from his city apartment. The building on the left is the Rocket Tower and the building on the right is the Cloud Scratcher.



Greg's camera has a 60° viewing angle. Greg knows that he is 2 miles from the Cloud Scratcher and that the Rocket Tower is 3 miles from the Cloud Scatcher. How far is Greg from the Rocket Tower? Round your answer to the nearest hundredth.

**3.45 mi**

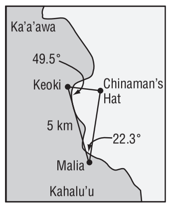
4. **BOATING** A boat heads out to sea from a port that sits along a straight shoreline. The boat heads in a direction that makes a 70° angle with the shoreline. After sailing for 3 miles, the skipper looks back at the shore and sees his house. The house, like the port, also sits on the shore. The lines of sight to the port and to his home make an 80° angle. How far is the skipper's home from the port? Round your answer to the nearest tenth of a mile.




**5.9 mi**

**ISLANDS For Exercises 5 and 6, use the following information.**

Oahu is a Hawaiian Island. Off of the coast of Oahu, there is a very tiny island known as Chinaman's Hat. Keoki and Malia are observing Chinaman's Hat from locations 5 kilometers apart. Use the information in the figure to answer the following questions.




5. How far is Keoki from Chinaman's Hat? Round your answer to the nearest tenth of a kilometer.

**2.0 km**

6. How far is Malia from Chinaman's Hat? Round your answer to the nearest tenth of a kilometer.

**4.0 km**



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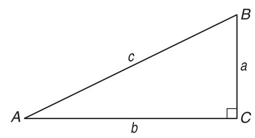
# 8-6 Enrichment



## *Identities*

An **identity** is an equation that is true for all values of the variable for which both sides are defined. One way to verify an identity is to use a right triangle and the definitions for trigonometric functions.




**Example 1** **Verify that $(\sin A)^2 + (\cos A)^2 = 1$ is an identity.**

$$(\sin A)^2 + (\cos A)^2 = \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = \frac{a^2 + b^2}{c^2} = \frac{c^2}{c^2} = 1$$

To check whether an equation *may* be an identity, you can test several values. However, since you cannot test all values, you cannot be *certain* that the equation is an identity.

**Example 2** **Test $\sin 2x = 2 \sin x \cos x$ to see if it could be an identity.**

Try $x = 20$. Use a calculator to evaluate each expression.

$\sin 2x = \sin 40 \approx 0.643$

$2 \sin x \cos x = 2 (\sin 20)(\cos 20) \approx 2(0.342)(0.940) \approx 0.643$

Since the left and right sides seem equal, the equation may be an identity.

**Exercises**

**Use triangle $ABC$ shown above. Verify that each equation is an identity.**

1. $\frac{\cos A}{\sin A} = \frac{1}{\tan A}$

**$\frac{\cos A}{\sin A} = \frac{b}{c} \div \frac{a}{c} = \frac{b}{a} = \frac{1}{\tan A}$**

2. $\frac{\tan B}{\sin B} = \frac{1}{\cos B}$

**$\frac{\tan B}{\sin B} = \frac{b}{a} \div \frac{b}{c} = \frac{c}{a} = \frac{1}{\cos B}$**

3. $\tan B \cos B = \sin B$

**$\tan B \cos B = \frac{b}{a} \cdot \frac{a}{c} = \frac{b}{c} = \sin B$**

4. $1 - (\cos B)^2 = (\sin B)^2$

**$1 - (\cos B)^2 = 1 - \left(\frac{a}{c}\right)^2 = \frac{c^2}{c^2} - \frac{a^2}{c^2} = \frac{c^2 - a^2}{c^2} = \frac{b^2}{c^2}$ or $(\sin B)^2$**

**Try several values for $x$ to test whether each equation could be an identity.**

5. $\cos 2x = (\cos x)^2 - (\sin x)^2$

**Yes; see students' work.**

6. $\cos (90 - x) = \sin x$

**Yes; see students' work.**

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# 8-6 Graphing Calculator Activity

## *Solving Triangles Using the Law of Sines*

You can use a calculator to solve triangles using the Law of Sines.

**Example** **Solve $\triangle ABC$ if $a = 6$, $b = 2$, and $m\angle A = 35$.**

Use the Law of Sines.

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin 35°}{6} = \frac{\sin B}{2} = \frac{\sin C}{c}$

Use your calculator to solve the proportion $\frac{\sin 35°}{6} = \frac{\sin B}{2}$.

**Keystrokes:** SIN 35 ) × 2 ÷ 6 ENTER .1911921455

2nd [SIN$^{-1}$] 2nd [ANS] ENTER 11.02236462

So $m\angle B \approx 11$ and $m\angle C \approx 180 - (35 + 11) = 134$.

Use your calculator and the value 134 for $m\angle C$ and solve the proportion $\frac{\sin 35°}{6} = \frac{\sin 134°}{c}$.

**Keystrokes:** SIN 134 ) × 6 ÷ SIN 35 ENTER 7.524784019

**Exercises**

**Solve each $\triangle ABC$. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.**

1. $a = 9$, $m\angle A = 36$, $c = 12$ **$m\angle C \approx 52$, $m\angle B \approx 92$, $b \approx 15.3$**

2. $m\angle C = 80$, $c = 9$, $m\angle A = 40$ **$m\angle B = 60$, $a \approx 5.9$, $b \approx 7.9$**

3. $m\angle B = 45$, $m\angle C = 56$, $a = 2$ **$m\angle A = 79$, $b \approx 1.4$, $c \approx 1.7$**

4. $b = 6$, $c = 5$, $m\angle B = 68$ **$m\angle C \approx 51$, $m\angle A \approx 61$, $a \approx 5.7$**

5. $a = 11$, $b = 15$, $m\angle B = 42$ **$m\angle A \approx 29$, $m\angle C \approx 109$, $c \approx 21.2$**



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# 8-7 Lesson Reading Guide

## *The Law of Cosines*

Lesson 8-7

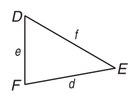
### Get Ready for the Lesson

**Read the introduction to Lesson 8-7 in your textbook.**

If a triangular room and a square room have the same floor area, which room has a greater perimeter? **The triangular room has a greater perimeter.**

### Read the Lesson

1. Refer to the figure. According to the Law of Cosines, which statements are correct for $\triangle DEF$? **B, E, H**




**A.** $d^2 = e^2 + f^2 - ef \cos D$ **B.** $e^2 = d^2 + f^2 - 2df \cos E$

**C.** $d^2 = e^2 + f^2 + 2ef \cos D$ **D.** $f^2 = d^2 + e^2 - 2ef \cos F$

**E.** $f^2 = d^2 + e^2 - 2de \cos F$ **F.** $d^2 = e^2 + f^2$

**G.** $\frac{\sin D}{d} = \frac{\sin E}{e} = \frac{\sin F}{f}$ **H.** $d = \sqrt{e^2 + f^2 - 2ef \cos D}$

2. Each of the following describes three given parts of a triangle. In each case, indicate whether you would use the Law of Sines or the Law of Cosines first in solving a triangle with those given parts. (In some cases, only one of the two laws would be used in solving the triangle.)

   a. SSS **Law of Cosines**
   b. ASA **Law of Sines**
   c. AAS **Law of Sines**
   d. SAS **Law of Cosines**
   e. SSA **Law of Sines**

3. Indicate whether each statement is *true* or *false*. If the statement is false, explain why.

   a. The Law of Cosines applies to right triangles. **true**

   b. The Pythagorean Theorem applies to acute triangles. **False; sample answer: It only applies to right triangles.**

   c. The Law of Cosines is used to find the third side of a triangle when you are given the measures of two sides and the nonincluded angle. **False; sample answer: It is used when you are given the measures of two sides and the included angle.**

   d. The Law of Cosines can be used to solve a triangle in which the measures of the three sides are 5 centimeters, 8 centimeters, and 15 centimeters. **False; sample answer: 5 + 8 < 15, so, by the Triangle Inequality Theorem, no such triangle exists.**

### Remember What You Learned

4. A good way to remember a new mathematical formula is to relate it to one you already know. The Law of Cosines looks somewhat like the Pythagorean Theorem. Both formulas must be true for a right triangle. How can that be? **cos 90 = 0, so in a right triangle, where the included angle is the right angle, the Law of Cosines becomes the Pythagorean Theorem.**

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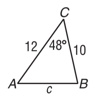
# 8-7 Study Guide and Intervention

## *The Law of Cosines*

**The Law of Cosines** Another relationship between the sides and angles of any triangle is called the **Law of Cosines**. You can use the Law of Cosines if you know three sides of a triangle or if you know two sides and the included angle of a triangle.

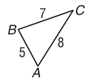
| | |
|---|---|
| **Law of Cosines** | Let $\triangle ABC$ be any triangle with $a$, $b$, and $c$ representing the measures of the sides opposite the angles with measures $A$, $B$, and $C$, respectively. Then the following equations are true. <br> $a^2 = b^2 + c^2 - 2bc \cos A$    $b^2 = a^2 + c^2 - 2ac \cos B$    $c^2 = a^2 + b^2 - 2ab \cos C$ |

**Example 1** **In $\triangle ABC$, find $c$.**




| | |
|---|---|
| $c^2 = a^2 + b^2 - 2ab \cos C$ | Law of Cosines |
| $c^2 = 12^2 + 10^2 - 2(12)(10)\cos 48°$ | $a = 12$, $b = 10$, $m\angle C = 48$ |
| $c = \sqrt{12^2 + 10^2 - 2(12)(10)\cos 48°}$ | Take the square root of each side. |
| $c \approx 9.1$ | Use a calculator. |

**Example 2** **In $\triangle ABC$, find $m\angle A$.**




| | |
|---|---|
| $a^2 = b^2 + c^2 - 2bc \cos A$ | Law of Cosines |
| $7^2 = 5^2 + 8^2 - 2(5)(8) \cos A$ | $a = 7$, $b = 5$, $c = 8$ |
| $49 = 25 + 64 - 80 \cos A$ | Multiply. |
| $-40 = -80 \cos A$ | Subtract 89 from each side. |
| $\frac{1}{2} = \cos A$ | Divide each side by $-80$. |
| $\cos^{-1}\frac{1}{2} = A$ | Use the inverse cosine. |
| $60° = A$ | Use a calculator. |

**Exercises**

**Find each measure using the given measures from $\triangle ABC$. Round angle measures to the nearest degree and side measures to the nearest tenth.**

**1.** If $b = 14$, $c = 12$, and $m\angle A = 62$, find $a$. **13.5**

**2.** If $a = 11$, $b = 10$, and $c = 12$, find $m\angle B$. **51**

**3.** If $a = 24$, $b = 18$, and $c = 16$, find $m\angle C$. **42**

**4.** If $a = 20$, $c = 25$, and $m\angle B = 82$, find $b$. **29.8**

**5.** If $b = 18$, $c = 28$, and $m\angle A = 59$, find $a$. **24.3**

**6.** If $a = 15$, $b = 19$, and $c = 15$, find $m\angle C$. **51**



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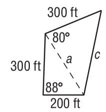
# 8-7 Study Guide and Intervention *(continued)*

## *The Law of Cosines*

**Use the Law of Cosines to Solve Problems** You can use the **Law of Cosines** to solve some problems involving triangles.

| | |
|---|---|
| **Law of Cosines** | Let $\triangle ABC$ be any triangle with $a$, $b$, and $c$ representing the measures of the sides opposite the angles with measures $A$, $B$, and $C$, respectively. Then the following equations are true. <br> $a^2 = b^2 + c^2 - 2bc \cos A$    $b^2 = a^2 + c^2 - 2ac \cos B$    $c^2 = a^2 + b^2 - 2ab \cos C$ |

**Example** **Ms. Jones wants to purchase a piece of land with the shape shown. Find the perimeter of the property.**




Use the Law of Cosines to find the value of $a$.

| | |
|---|---|
| $a^2 = b^2 + c^2 - 2bc \cos A$ | Law of Cosines |
| $a^2 = 300^2 + 200^2 - 2(300)(200) \cos 88°$ | $b = 300$, $c = 200$, $m\angle A = 88$ |
| $a = \sqrt{130{,}000 - 120{,}000 \cos 88°}$ | Take the square root of each side. |
| $\approx 354.7$ | Use a calculator. |

Use the Law of Cosines again to find the value of $c$.

| | |
|---|---|
| $c^2 = a^2 + b^2 - 2ab \cos C$ | Law of Cosines |
| $c^2 = 354.7^2 + 300^2 - 2(354.7)(300) \cos 80°$ | $a = 354.7$, $b = 300$, $m\angle C = 80$ |
| $c = \sqrt{215{,}812.09 - 212{,}820 \cos 80°}$ | Take the square root of each side. |
| $\approx 422.9$ | Use a calculator. |

The perimeter of the land is 300 + 200 + 422.9 + 200 or about 1223 feet.

**Exercises**

**Draw a figure or diagram to go with each exercise and mark it with the given information. Then solve the problem. Round angle measures to the nearest degree and side measures to the nearest tenth.**

**1.** A triangular garden has dimensions 54 feet, 48 feet, and 62 feet. Find the angles at each corner of the garden.
**75°; 48°; 57°**

**2.** A parallelogram has a 68° angle and sides 8 and 12. Find the lengths of the diagonals.
**11.7; 16.7**

**3.** An airplane is sighted from two locations, and its position forms an acute triangle with them. The distance to the airplane is 20 miles from one location with an angle of elevation 48.0°, and 40 miles from the other location with an angle of elevation of 21.8°. How far apart are the two locations?
**50.5 mi**

**4.** A ranger tower at point $A$ is directly north of a ranger tower at point $B$. A fire at point $C$ is observed from both towers. The distance from the fire to tower $A$ is 60 miles, and the distance from the fire to tower $B$ is 50 miles. If $m\angle ACB = 62$, find the distance between the towers.
**57.3 mi**

NAME ______________________ DATE ________ PERIOD _____

# 8-7 Skills Practice

## The Law of Cosines

**In $\triangle RST$, given the following measures, find the measure of the missing side.**

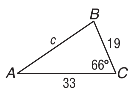
1. $r = 5, s = 8, m\angle T = 39$ **$t \approx 5.2$**
2. $r = 6, t = 11, m\angle S = 87$ **$s \approx 12.3$**
3. $r = 9, t = 15, m\angle S = 103$ **$s \approx 19.2$**
4. $s = 12, t = 10, m\angle R = 58$ **$r \approx 10.8$**

**In $\triangle HIJ$, given the lengths of the sides, find the measure of the stated angle to the nearest tenth.**

5. $h = 12, i = 18, j = 7; m\angle H$ **24.7**
6. $h = 15, i = 16, j = 22; m\angle I$ **46.7**
7. $h = 23, i = 27, j = 29; m\angle J$ **70.4**
8. $h = 37, i = 21, j = 30; m\angle H$ **91.3**

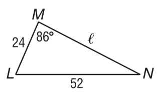
**Determine whether the Law of *Sines* or the Law of *Cosines* should be used first to solve each triangle. Then solve each triangle. Round angle measures to the nearest degree and side measures to the nearest tenth.**

9.




**Cosines; $m\angle A \approx 34$; $m\angle B \approx 80$; $c \approx 30.7$**

10.




**Sines; $m\angle L \approx 67$; $m\angle N \approx 27$; $\ell \approx 47.8$**

11. $a = 10, b = 14, c = 19$
**Cosines; $m\angle A \approx 31$; $m\angle B \approx 46$; $m\angle C \approx 103$**

12. $a = 12, b = 10, m\angle C = 27$
**Cosines; $m\angle A \approx 97$; $m\angle B \approx 56$; $c \approx 5.5$**

**Solve each $\triangle RST$ described below. Round measures to the nearest tenth.**

13. $r = 12, s = 32, t = 34$ **$m\angle R \approx 20.7$, $m\angle S \approx 70.2$, $m\angle T \approx 89.1$**
14. $r = 30, s = 25, m\angle T = 42$ **$m\angle R \approx 82.3$, $m\angle S \approx 55.7$, $t \approx 20.3$**
15. $r = 15, s = 11, m\angle R = 67$ **$m\angle S \approx 42.5$, $m\angle T \approx 70.5$, $t \approx 15.4$**
16. $r = 21, s = 28, t = 30$ **$m\angle R \approx 42.3$, $m\angle S \approx 63.8$, $m\angle T \approx 74.0$**



NAME ______________________ DATE ________ PERIOD _____

# 8-7 Practice

## The Law of Cosines

**In $\triangle JKL$, given the following measures, find the measure of the missing side. Round to the nearest tenth.**

1. $j = 1.3, k = 10, m\angle L = 77$ **$\ell \approx 9.8$**
2. $j = 9.6, \ell = 1.7, m\angle K = 43$ **$k \approx 8.4$**
3. $j = 11, k = 7, m\angle L = 63$ **$\ell \approx 10.0$**
4. $k = 4.7, \ell = 5.2, m\angle J = 112$ **$j \approx 8.2$**

**In $\triangle MNQ$, given the lengths of the sides, find the measure of the stated angle to the nearest tenth.**

5. $m = 17, n = 23, q = 25; m\angle Q$ **75.7**
6. $m = 24, n = 28, q = 34; m\angle M$ **44.2**
7. $m = 12.9, n = 18, q = 20.5; m\angle N$ **60.2**
8. $m = 23, n = 30.1, q = 42; m\angle Q$ **103.7**

**Determine whether the Law of Sines or the Law of Cosines should be used first to solve $\triangle ABC$. Then solve each triangle. Round angle measures to the nearest degree and side measure to the nearest tenth.**

9. $a = 13, b = 18, c = 19$
**Cosines; $m\angle A \approx 41$; $m\angle B \approx 65$; $m\angle C \approx 74$**

10. $a = 6, b = 19, m\angle C = 38$
**Cosines; $m\angle A \approx 15$; $m\angle B \approx 127$; $c \approx 14.7$**

11. $a = 17, b = 22, m\angle B = 49$
**Sines; $m\angle A \approx 36$; $m\angle C \approx 95$; $c \approx 29.0$**

12. $a = 15.5, b = 18, m\angle C = 72$
**Cosines; $m\angle A \approx 48$; $m\angle B \approx 60$; $c \approx 19.8$**

**Solve each $\triangle FGH$ described below. Round measures to the nearest tenth.**

13. $m\angle F = 54, f = 12.5, g = 11$ **$m\angle G \approx 45.4$, $m\angle H \approx 80.6$, $h \approx 15.2$**
14. $f = 20, g = 23, m\angle H = 47$ **$m\angle F \approx 57.4$, $m\angle G \approx 75.6$, $h \approx 17.4$**
15. $f = 15.8, g = 11, h = 14$ **$m\angle F \approx 77.4$, $m\angle G \approx 42.8$, $m\angle H \approx 59.8$**
16. $f = 36, h = 30, m\angle G = 54$ **$m\angle F \approx 73.1$, $m\angle H \approx 52.9$, $g \approx 30.4$**

17. **REAL ESTATE** The Esposito family purchased a triangular plot of land on which they plan to build a barn and corral. The lengths of the sides of the plot are 320 feet, 286 feet, and 305 feet. What are the measures of the angles formed on each side of the property?
**65.5, 54.4, 60.1**



Lesson 8-7

Answers

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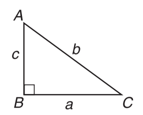
# 8-7 Word Problem Practice

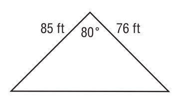
## *The Law of Cosines*

1. **RIGHT TRIANGLES** Triangle $ABC$ is a right triangle with right angle at $B$. Let $a$ be the length of the side opposite $A$, $b$ be the length of the side opposite $B$, and $c$ be the length of the side opposite $C$.

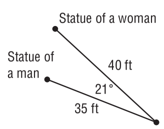



Rewrite the Law of Cosines with respect to the right angle $B$ in simplest form.
**$b^2 = a^2 + c^2$ (the Pythagorean Theorem)**

2. **LANDSCAPING** Hanna wants to fence a triangular lot as shown. What is the length of the missing side? Round your answer to the nearest foot.
**104 ft**

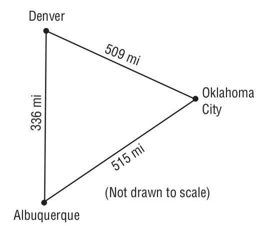



3. **STATUES** Gail was visiting an art gallery. In one room, she stood so that she had a view of two statues, one of a man, and the other of a woman. She was 40 feet from the statue of the woman, and 35 feet from the statue of the man. The angle created by the lines of sight to the two statues was 21°. What is the distance between the two statues? Round your answer to the nearest tenth.
**14.5 ft**




4. **CARS** Two cars start moving from the same location. They head straight, but in different directions. The angle between where they are heading is 43°. The first car travels 20 miles and the second car travels 37 miles. How far apart are the two cars? Round your answer to the nearest tenth.
**26.2 mi**

**CITIES For Exercises 5–7, use the following information.**
The cities of Denver, Oklahoma City, and Albuquerque form the vertices of a triangle.




Use the information in the figure and round your answers to the nearest tenth of a degree.

5. What is the measure of the angle at Albuquerque?
**69.9°**

6. What is the measure of the angle at Oklahoma City?
**38.3°**

7. What is the measure of the angle at Denver?
**71.8°**



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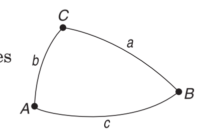
# 8-7 Enrichment

## *Spherical Triangles*

Spherical trigonometry is an extension of plane trigonometry. Figures are drawn on the surface of a sphere. Arcs of great circles correspond to line segments in the plane. The arcs of three great circles intersecting on a sphere form a spherical triangle. Angles have the same measure as the tangent lines drawn to each great circle at the vertex. Since the sides are arcs, they too can be measured in degrees.




The sum of the sides of a spherical triangle is less than 360°.
The sum of the angles is greater than 180° and less than 540°.
The Law of Sines for spherical triangles is as follows.

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

There is also a Law of Cosines for spherical triangles.

$\cos a = \cos b \cos c + \sin b \sin c \cos A$
$\cos b = \cos a \cos c + \sin a \sin c \cos B$
$\cos c = \cos a \cos b + \sin a \sin b \cos C$

**Example** **Solve the spherical triangle given $a = 72°$, $b = 105°$, and $c = 61°$.**

Use the Law of Cosines.

$0.3090 = (-0.2588)(0.4848) + (0.9659)(0.8746) \cos A$
$\cos A = 0.5143$
$A = 59°$

$-0.2588 = (0.3090)(0.4848) + (0.9511)(0.8746) \cos B$
$\cos B = -0.4912$
$B = 119°$

$0.4848 = (0.3090)(-0.2588) + (0.9511)(0.9659) \cos C$
$\cos C = 0.6148$
$C = 52°$

Check by using the Law of Sines.

$$\frac{\sin 72°}{\sin 59°} = \frac{\sin 105°}{\sin 119°} = \frac{\sin 61°}{\sin 52°} = 1.1$$

**Exercises**

**Solve each spherical triangle.**

1. $a = 56°, b = 53°, c = 94°$
**$A = 41°, B = 39°, C = 128°$**

2. $a = 110°, b = 33°, c = 97°$
**$A = 116°, B = 31°, C = 71°$**

3. $a = 76°, b = 110°, C = 49°$
**$A = 59°, B = 124°, c = 59°$**

4. $b = 94°, c = 55°, A = 48°$
**$a = 60°, B = 121°, C = 45°$**

# Chapter 8 Assessment Answer Key

**Quiz 1** *(Lessons 8–1 and 8–2)*
**Page 59**

1. $8\sqrt{3}$
2. $x = \sqrt{85}$, $y = 2\sqrt{15}$
3. $x = \sqrt{17}$, $y = \frac{81}{8}$
4. $\sqrt{137}$
5. no, $13^2 + 15^2 \neq 19^2$

**Quiz 2** *(Lessons 8–3 and 8–4)*
**Page 59**

1. $3\sqrt{2}$
2. $2\sqrt{3}$
3. 57.8°
4. 14.6
5. $20\sqrt{3} + 20$ in.
6. 0.7880
7. 27
8. 3.3 ft
9. 0.79
10. 0.82

**Quiz 3** *(Lessons 8–5 and 8–6)*
**Page 60**

1. $\angle QPR$
2. 3.9
3. $m\angle B = 104$, $m\angle C = 28$, $b = 23.2$
4. 443 ft
5. 52°

**Quiz 4** *(Lessons 8–7)*
**Page 60**

1. 36.7
2. 110.5°
3. $m\angle R = 88$, $m\angle T = 53$, $m\angle S = 39$
4. 5.4 mi
5. B

**Mid-Chapter Test**
**Page 61**

**Part I**

1. A
2. H
3. B
4. H
5. D

**Part II**

6. $x = 9$, $y = 3\sqrt{3}$
7. $x = 10\sqrt{3}$, $y = 10\sqrt{6}$
8. $x = 12\sqrt{2}$, $y = 24\sqrt{2}$
9. yes, $56^2 + 90^2 = 106^2$
10. 54°

Answers

# Chapter 8 Assessment Answer Key

**Vocabulary Test**
**Page 62**

1. geometric mean
2. Pythagorean triple
3. false, trigonometric ratio
4. true
5. angle of depression
6. trigonometry
7. sine
8. tangent
9. finding the measures of all of the angles and sides of a triangle
10. In a right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.

**Form 1**
**Page 63**

1. D
2. F
3. A
4. G
5. D
6. G
7. C
8. G
9. C
10. H

**Page 64**

11. D
12. J
13. C
14. F
15. A
16. G
17. D
18. F
19. B
20. F

B: 69

# Chapter 8 Assessment Answer Key

**Form 2A**

**Page 65**

1. D
2. J
3. A
4. H
5. D
6. J
7. A
8. G
9. B
10. H

**Page 66**

11. B
12. H
13. A
14. H
15. B
16. J
17. C
18. H
19. B
20. F

B: 56.4 ft

**Form 2B**

**Page 67**

1. A
2. J
3. D
4. G
5. D
6. F
7. A
8. H
9. B
10. H

**Page 68**

11. B
12. F
13. C
14. H
15. B
16. J
17. B
18. F
19. B
20. J

B: 73.2 ft

Answers

# Chapter 8 Assessment Answer Key

**Form 2C**

**Page 69**

1. $5\sqrt{2}$
2. 13
3. $2\sqrt{7}$
4. $\sqrt{4000}$ or $20\sqrt{10}$
5. $\sqrt{300}$ or $10\sqrt{3}$
6. yes, $(\sqrt{2})^2 + (\sqrt{32})^2 = (\sqrt{34})^2$
7. $11\sqrt{2}$
8. $2\sqrt{3}$
9. $x = 6$, $y = 12$
10. 9.7
11. 69°
12. 68°

**Page 70**

13. 267.9
14. 11°
15. 43.2
16. 11°
17. 15.6 ft
18. 20°
19. 33.0
20. 518.3m

B: 1

# Chapter 8 Assessment Answer Key

**Form 2D**

**Page 71**

1. $3\sqrt{10}$
2. 26
3. $\sqrt{39}$
4. $\sqrt{7300}$ or $10\sqrt{73}$
5. $\sqrt{432}$ or $12\sqrt{3}$
6. no, $(\sqrt{41})^2 + (\sqrt{2})^2 \neq (\sqrt{41})^2$
7. $15\sqrt{2}$
8. $7\sqrt{3}$
9. $x = 12$, $y = 24$
10. 8.6
11. 70°
12. 67°

**Page 72**

13. 352.7m
14. 9°
15. 32.3
16. 13°
17. 15.2 ft
18. 19°
19. 38.2
20. 624.3m

B: 6, 10

Answers

# Chapter 8 Assessment Answer Key

**Form 3**

**Page 73**

1. $\frac{\sqrt{6}}{9}$
2. 2
3. 3
4. $2\sqrt{10}$
5. $\frac{12}{5}$
6. 12.5
7. 250 km
8. yes, $48^2 + 55^2 = 73^2$
9. $12\sqrt{6}$
10. $24 + 8\sqrt{3}$
11. $x = 5\sqrt{3}$, $y = 10\sqrt{3}$
12. $(-4 + 8\sqrt{3}, -2)$ or $(-4 - 8\sqrt{3}, -2)$

**Page 74**

13. $x = \frac{5\sqrt{6}}{3}$, $CD = \frac{5\sqrt{6}}{3} + 5\sqrt{2} + 12$
14. 147 ft
15. 28 ft
16. 37.4
17. $m\angle R = 22$, $m\angle P = 62$, $q = 57.6$
18. 253.7 yd
19. 37 or 143
20. 125.8

B: 71.1 and 84.5

# Chapter 8 Assessment Answer Key

**Extended-Response Test, Page 75**
**Scoring Rubric**

| Score | General Description | Specific Criteria |
|---|---|---|
| 4 | **Superior**<br>A correct solution that is supported by well-developed, accurate explanations | • Shows thorough understanding of the concepts of *geometric mean*, *special right triangles*, *altitude to the hypotenuse theorems*, *Pythagorean Theorem*, *solving triangles*, *SOH CAH TOA*, *Law of Sines*, and *Law of Cosines*.<br>• Uses appropriate strategies to solve problems.<br>• Computations are correct.<br>• Written explanations are exemplary.<br>• Figures are accurate and appropriate.<br>• Goes beyond requirements of some or all problems. |
| 3 | **Satisfactory**<br>A generally correct solution, but may contain minor flaws in reasoning or computation | • Shows an understanding of the concepts of *geometric mean*, *special right triangles*, *altitude to the hypotenuse theorems*, *Pythagorean Theorem*, *solving triangles*, *SOH CAH TOA*, *Law of Sines*, and *Law of Cosines*.<br>• Uses appropriate strategies to solve problems.<br>• Computations are mostly correct.<br>• Written explanations are effective.<br>• Figures are mostly accurate and appropriate.<br>• Satisfies all requirements of problems. |
| 2 | **Nearly Satisfactory**<br>A partially correct interpretation and/or solution to the problem | • Shows an understanding of most of the concepts of *geometric mean*, *special right triangles*, *altitude to the hypotenuse theorems*, *Pythagorean Theorem*, *solving triangles*, *SOH CAH TOA*, *Law of Sines* and *Law of Cosines*.<br>• May not use appropriate strategies to solve problems.<br>• Computations are mostly correct.<br>• Written explanations are satisfactory.<br>• Figures are mostly accurate.<br>• Satisfies the requirements of most of the problems. |
| 1 | **Nearly Unsatisfactory**<br>A correct solution with no supporting evidence or explanation | • Final computation is correct.<br>• No written explanations or work shown to substantiate the final computation.<br>• Figures may be accurate but lack detail or explanation.<br>• Satisfies minimal requirements of some of the problems. |
| 0 | **Unsatisfactory**<br>An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given | • Shows little or no understanding of most of the concepts of *geometric mean*, *special right triangles*, *altitude to the hypotenuse theorems*, *Pythagorean Theorem*, *solving triangles*, *SOH CAH TOA*, *Law of Sines* and *Law of Cosines*.<br>• Does not use appropriate strategies to solve problems.<br>• Computations are incorrect.<br>• Written explanations are unsatisfactory.<br>• Figures are inaccurate or inappropriate.<br>• Does not satisfy requirements of problems.<br>• No answer given. |

# Chapter 8 Assessment Answer Key

**Extended-Response Test, Page 75**
**Sample Answers**

***In addition to the scoring rubric found on page A33, the following sample answers may be used as guidance in evaluating open-ended assessment items.***

**1.** $\frac{10}{6} = \frac{6}{x}$

$\sqrt{10x} = 6$

$10x = 36$

$x = \frac{18}{5}$

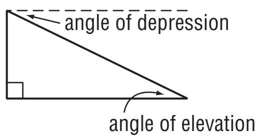
**2a.** No, his work is not correct. The triangle is not a right triangle so he cannot use the altitude to the hypotenuse theorem. Instead he must use the 30°-60°-90° triangle relationships and obtain $x = 2\sqrt{3}$.

**b.** Using the 30°-60°-90° triangle relationships, $RQ = 4$, and therefore $PQ$ would have to equal 8, so $PS = 8 - 2$ or 6.

**c.** No, the sides are not in a ratio of $1:1:\sqrt{2}$.

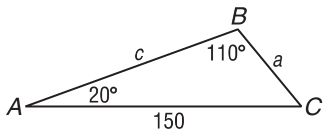
**3.** If you were finding the length of a side $x$ you would use *sin* of the given angle. If you are finding an angle $x$ you would use the $sin^{-1}$ of the ratio of two sides to find the angle. For example, if $\sin x = \frac{3}{4}$, use $\sin^{-1}\frac{3}{4}$ to find $x$. If $\sin 20° = \frac{x}{5}$ find the sine of 20° and multiply by 5 to find $x$, the length of the unknown side.

**4.**




Student should draw a right triangle and label the angle of elevation up from the horizontal and the angle of depression down from the horizontal. The angle of elevation has the same measure as the angle of depression.

**5.**




Let $m\angle B = 110$, $m\angle A = 20$, and $b = 150$. Use the Law of Sines to find the missing lengths. The length of $a$ is found using $\frac{\sin 110}{150} = \frac{\sin 20}{a}$. The third angle is found by evaluating $180 - (m\angle B + m\angle A)$. In this problem, $m\angle C = 50$. The length of $c$ is found by using $\frac{\sin 110}{150} = \frac{\sin 50}{c}$. In this problem, $c \approx 122.3$.

**6.** No, her plan is not a good one. Irina should use the Law of Cosines to find angle $B$. Then she can use the Law of Sines to find either angle $A$ or angle $C$ and subtract the two angles she finds from 180 degrees to get the measure of the third angle.

# Chapter 8 Assessment Answer Key

**Standardized Test Practice**

**Page 76**

1. A Ⓐ Ⓑ Ⓒ Ⓓ

2. F Ⓕ Ⓖ Ⓗ Ⓙ

3. C Ⓐ Ⓑ Ⓒ Ⓓ

4. J Ⓕ Ⓖ Ⓗ Ⓙ

5. D Ⓐ Ⓑ Ⓒ Ⓓ

6. G Ⓕ Ⓖ Ⓗ Ⓙ

7. D Ⓐ Ⓑ Ⓒ Ⓓ

8. G Ⓕ Ⓖ Ⓗ Ⓙ

**Page 77**

9. D Ⓐ Ⓑ Ⓒ Ⓓ

10. J Ⓕ Ⓖ Ⓗ Ⓙ

11. B Ⓐ Ⓑ Ⓒ Ⓓ

12. J Ⓕ Ⓖ Ⓗ Ⓙ

13. C Ⓐ Ⓑ Ⓒ Ⓓ

14. F Ⓕ Ⓖ Ⓗ Ⓙ

15.

| | | 1 | 4 | . | 5 | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | | 0 | 0 | 0 |
| 1 | 1 | **1** | 1 | | 1 | 1 | 1 |
| 2 | 2 | 2 | 2 | | 2 | 2 | 2 |
| 3 | 3 | 3 | 3 | | 3 | 3 | 3 |
| 4 | 4 | 4 | **4** | | 4 | 4 | 4 |
| 5 | 5 | 5 | 5 | | **5** | 5 | 5 |
| 6 | 6 | 6 | 6 | | 6 | 6 | 6 |
| 7 | 7 | 7 | 7 | | 7 | 7 | 7 |
| 8 | 8 | 8 | 8 | | 8 | 8 | 8 |
| 9 | 9 | 9 | 9 | | 9 | 9 | 9 |

16.

| | | 1 | 8 | . | 0 | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | | **0** | 0 | 0 |
| 1 | 1 | **1** | 1 | | 1 | 1 | 1 |
| 2 | 2 | 2 | 2 | | 2 | 2 | 2 |
| 3 | 3 | 3 | 3 | | 3 | 3 | 3 |
| 4 | 4 | 4 | 4 | | 4 | 4 | 4 |
| 5 | 5 | 5 | 5 | | 5 | 5 | 5 |
| 6 | 6 | 6 | 6 | | 6 | 6 | 6 |
| 7 | 7 | 7 | 7 | | 7 | 7 | 7 |
| 8 | 8 | 8 | **8** | | 8 | 8 | 8 |
| 9 | 9 | 9 | 9 | | 9 | 9 | 9 |

# Chapter 8 Assessment Answer Key

**Standardized Practice Test**
**Page 78**

17. Def. of ≅ segments

18. $HJ + JK = HK$; $KL + LM = KM$

19. $m\angle 1 = 32$, $m\angle 2 = 82$, $m\angle 3 = 66$, $m\angle 4 = 114$, $m\angle 5 = 57$

20. 7

21. 19

22. $\angle 3$, $\angle 4$, $\angle 5$, $\angle 7$, $\angle ABC$

23. a. yes, both are 30°, 60°, 90° triangles; AAA

23. b. 36

23. c. 2:1