Glencoe Mathematics

Algebra 2

Chapter 4 Resource Masters

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Consumable Workbooks Many of the worksheets contained in the Chapter Resource Masters are available as consumable workbooks in both English and Spanish.

	ISBN10	ISBN13
Study Guide and Intervention Workbook	0-07-877355-5	978-0-07-877355-6
Skills Practice Workbook	0-07-877357-1	978-0-07-877357-0
Practice Workbook	0-07-877358-X	978-0-07-877358-7
Word Problem Practice Workbook	0-07-877360-1	978-0-07-877360-0
Spanish Versions		
Study Guide and Intervention Workbook	0-07-877356-3	978-0-07-877356-3
Practice Workbook	0-07-877359-8	978-0-07-877359-4

Answers for Workbooks The answers for Chapter 4 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

StudentWorks Plus™ This CD-ROM includes the entire Student Edition test along with the English workbooks listed above.

TeacherWorks Plus™ All of the materials found in this booklet are included for viewing, printing, and editing in this CD-ROM.

Spanish Assessment Masters (ISBN10: 0-07-877361-X, ISBN13: 978-0-07-877361-7) These masters contain a Spanish version of Chapter 4 Test Form 2A and Form 2C.

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Contents

Teacher's Guide to Using the *Chapter 4 Resource Masters*

The *Chapter 4 Resource Masters* includes the core materials needed for Chapter 4. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing on the *TeacherWorks Plus*™ CD-ROM.

Chapter Resources

Student-Built Glossary (pages 1–2) These masters are a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar. Give this to students before beginning Lesson 4-1. Encourage them to add these pages to their mathematics study notebooks. Remind them to complete the appropriate words as they study each lesson.

Anticipation Guide (pages 3–4) This master, presented in both English and Spanish, is a survey used before beginning the chapter to pinpoint what students may or may not know about the concepts in the chapter. Students will revisit this survey after they complete the chapter to see if their perceptions have changed.

Lesson Resources

Lesson Reading Guide Get Ready for the Lesson extends the discussion from the beginning of the Student Edition lesson. Read the Lesson asks students to interpret the context of and relationships among terms in the lesson. Finally, Remember What You Learned asks students to summarize what they have learned using various representation techniques. Use as a study tool for note taking or as an informal reading assignment. It is also a helpful tool for ELL (English Language Learners).

Study Guide and Intervention These masters provide vocabulary, key concepts, additional worked-out examples and Check Your Progress exercises to use as a reteaching activity. It can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice This master focuses more on the computational nature of the lesson. Use as an additional practice option or as homework for second-day teaching of the lesson.

Practice This master closely follows the types of problems found in the Exercises section of the Student Edition and includes word problems. Use as an additional practice option or as homework for second-day teaching of the lesson.

Word Problem Practice This master includes additional practice in solving word problems that apply the concepts of the lesson. Use as an additional practice or as homework for second-day teaching of the lesson.

Enrichment These activities may extend the concepts of the lesson, offer an historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. They are written for use with all levels of students.

Graphing Calculator, Scientific Calculator*, or *Spreadsheet Activities These activities present ways in which technology can be used with the concepts in some lessons of this chapter. Use as an alternative approach to some concepts or as an integral part of your lesson presentation.

Assessment Options

The assessment masters in the *Chapter 4 Resource Masters* offer a wide range of assessment tools for formative (monitoring) assessment and summative (final) assessment.

Student Recording Sheet This master corresponds with the standardized test practice at the end of the chapter.

Pre-AP Rubric This master provides information for teachers and students on how to assess performance on open-ended questions.

Quizzes Four free-response quizzes offer assessment at appropriate intervals in the chapter.

Mid-Chapter Test This 1-page test provides an option to assess the first half of the chapter. It parallels the timing of the Mid-Chapter Quiz in the Student Edition and includes both multiple-choice and free-response questions.

Vocabulary Test This test is suitable for all students. It includes a list of vocabulary words and 10 questions to assess students' knowledge of those words. This can also be used in conjunction with one of the leveled chapter tests.

Leveled Chapter Tests

- *Form 1* contains multiple-choice questions and is intended for use with below grade level students.
- *Forms 2A and 2B* contain multiple-choice questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- *Forms 2C and 2D* contain free-response questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- *Form 3* is a free-response test for use with above grade level students.

All of the above mentioned tests include a free-response Bonus question.

Extended-Response Test Performance assessment tasks are suitable for all students. Sample answers and a scoring rubric are included for evaluation.

Standardized Test Practice These three pages are cumulative in nature. It includes three parts: multiple-choice questions with bubble-in answer format, griddable questions with answer grids, and short-answer free-response questions.

Answers

- The answers for the Anticipation Guide and Lesson Resources are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment masters.

NAME ____ DATE______________ PERIOD _____

4 Student-Built Glossary

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 4. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Algebra Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
Cramer's (KRAY·muhrs) Rule		
determinant		
dimension		
dilation dy·LAY·shuhn		
element		
equal matrices		
expansion by minors		
identity matrix		
image		
inverse		
isometry eye·SAH·muh·tree		
matrix MAY·trihks		

(continued on the next page)

4 Student-Built Glossary

Vocabulary Term	Found on Page	Definition/Description/Example
matrix equation		
minor		
preimage		
reflection		
rotation		
scalar (SKAY·luhr) multiplication		
square matrix		
transformation		
translation		
vertex matrix		
zero matrix		

4 Anticipation Guide

Matrices

STEP 1 ***Before you begin Chapter 4***

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

STEP 1 A, D, or NS	Statement	STEP 2 A or D
	1. A matrix contains constants or variables in horizontal rows and vertical columns.	
	2. Each value in a matrix is called a term.	
	3. If two matrices contain the same numbers but have a different number of rows or columns, then they are not equal.	
	4. Two matrices with different dimensions can be added or subtracted by adding zeros so that both matrices have the same dimensions.	
	5. The product of a matrix and a constant can be found by multiplying each element of the matrix by that constant.	
	6. The associative, commutative, and distributive properties of multiplication are all true for matrices.	
	7. A translation is a transformation in which a figure is turned around a single point.	
	8. A vertex matrix is a matrix containing the coordinates of the vertices of a figure.	
	9. A third-order determinant of a matrix contains three columns and any number of rows.	
	10. Each element of an identity matrix for multiplication is 1.	
	11. Two matrices are inverses of each other if their product is the identity matrix.	
	12. To solve a system of equations using matrices, you must write a matrix for the coefficients, one for the variables, and one for the constants.	

STEP 2 ***After you complete Chapter 4***

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

4 Ejercicios preparatorios

Matrices

PASO 1 ***Antes de comenzar el Capítulo 4***

- Lee cada enunciado.
- Decide si estás de acuerdo (A) o en desacuerdo (D) con el enunciado.
- Escribe A o D en la primera columna O si no estás seguro(a) de la respuesta, escribe NS (No estoy seguro(a).

PASO 1 A, D o NS	Enunciado	PASO 2 A o D
	1. Una matriz contiene constantes o variables en filas horizontales y columnas verticales.	
	2. Cada valor en una matriz se llama término.	
	3. Si dos matrices contienen los mismos números, pero tienen distinto número de filas o columnas, entonces no son iguales.	
	4. Dos matrices con dimensiones diferentes se pueden sumar o restar al añadir ceros para que así ambas tengan las mismas dimensiones.	
	5. Se puede calcular el producto de una matriz y una constante al multiplicar cada elemento de la matriz por dicha constante.	
	6. Las propiedades asociativa, conmutativa y distributiva de la multiplicación son todas verdaderas para las matrices.	
	7. Una traslación es una transformación en la cual una figura se hace girar alrededor de un punto fijo.	
	8. Una matriz vértice es una matriz que contiene las coordenadas de los vértices de una figura.	
	9. Un determinante de tercer orden de una matriz contiene tres columnas y cualquier número de filas.	
	10. Cada elemento de una matriz identidad para multiplicación es 1.	
	11. Dos matrices son inversas entre sí si su producto es la matriz identidad.	
	12. Para resolver un sistema de ecuaciones usando matrices, debes escribir una matriz para los coeficientes, una para las variables y otra para las constantes.	

PASO 2 ***Después de completar el Capítulo 4***

- Vuelve a leer cada enunciado y completa la última columna con una A o una D.
- ¿Cambió cualquiera de tus opiniones sobre los enunciados de la primera columna?
- En una hoja de papel aparte, escribe un ejemplo de por qué estás en desacuerdo con los enunciados que marcaste con una D.

4-1 Lesson Reading Guide

Introduction to Matrices

Get Ready for the Lesson

Read the introduction to Lesson 4-1 in your textbook.

a. What is the base price of a Mid-Size SUV?
b. What is the exterior length of a Compact SUV?

Read the Lesson

1. Give the dimensions of each matrix.

a. $\begin{bmatrix} 3 & 2 & 5 \\ -1 & 0 & 6 \end{bmatrix}$ **b.** $\begin{bmatrix} 1 & 4 & 0 & -8 & 2 \end{bmatrix}$

2. Identify each matrix with as many of the following descriptions that apply: *row matrix*, *column matrix*, *square matrix*, *zero matrix*.

a. $\begin{bmatrix} 6 & 5 & 4 & 3 \end{bmatrix}$

b. $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

c. $\begin{bmatrix} 0 \end{bmatrix}$

3. Write a system of equations that you could use to solve the following matrix equation for x, y, and z. (Do not actually solve the system.)

$$\begin{bmatrix} 3x \\ x + y \\ y - z \end{bmatrix} = \begin{bmatrix} -9 \\ 5 \\ 6 \end{bmatrix}$$

Remember What You Learned

4. Some students have trouble remembering which number comes first in writing the dimensions of a matrix. Think of an easy way to remember this.

Lesson 4-1

4-1 Study Guide and Intervention

Introduction to Matrices

Organize Data

Matrix	a rectangular array of variables or constants in horizontal rows and vertical columns, usually enclosed in brackets.

A matrix can be described by its **dimensions**. A matrix with m rows and n columns is an $m \times n$ matrix.

Example 1 **Owls' eggs incubate for 30 days and their fledgling period is also 30 days. Swifts' eggs incubate for 20 days and their fledgling period is 44 days. Pigeon eggs incubate for 15 days, and their fledgling period is 17 days. Eggs of the king penguin incubate for 53 days, and the fledgling time for a king penguin is 360 days. Write a 2 × 4 matrix to organize this information.** **Source:** *The Cambridge Factfinder*

$$\begin{array}{r c} & \begin{array}{cccc} \text{Owl} & \text{Swift} & \text{Pigeon} & \text{King Penguin} \end{array} \\ \begin{array}{r} \text{Incubation} \\ \text{Fledgling} \end{array} & \begin{bmatrix} 30 & 20 & 15 & 53 \\ 30 & 44 & 17 & 360 \end{bmatrix} \end{array}$$

Example 2 **What are the dimensions of matrix A if $A = \begin{bmatrix} 13 & 10 & -3 & 45 \\ 2 & 8 & 15 & 80 \end{bmatrix}$?**

Since matrix A has 2 rows and 4 columns, the dimensions of A are 2×4.

Exercises

State the dimensions of each matrix.

1. $\begin{bmatrix} 15 & 5 & 27 & -4 \\ 23 & 6 & 0 & 5 \\ 14 & 70 & 24 & -3 \\ 63 & 3 & 42 & 90 \end{bmatrix}$

2. $[16 \quad 12 \quad 0]$

3. $\begin{bmatrix} 71 & 44 \\ 39 & 27 \\ 45 & 16 \\ 92 & 53 \\ 78 & 65 \end{bmatrix}$

4. A travel agent provides for potential travelers the normal high temperatures for the months of January, April, July, and October for various cities. In Boston these figures are 36°, 56°, 82°, and 63°. In Dallas they are 54°, 76°, 97°, and 79°. In Los Angeles they are 68°, 72°, 84°, and 79°. In Seattle they are 46°, 58°, 74°, and 60°, and in St. Louis they are 38°, 67°, 89°, and 69°. Organize this information in a 4 × 5 matrix. **Source:** *The New York Times Almanac*

4-1 Study Guide and Intervention *(continued)*

Introduction to Matrices

Equations Involving Matrices

Equal Matrices	Two matrices are equal if they have the same dimensions and each element of one matrix is equal to the corresponding element of the other matrix.

You can use the definition of equal matrices to solve matrix equations.

Example **Solve** $\begin{bmatrix} 4x \\ y \end{bmatrix} = \begin{bmatrix} -2y + 2 \\ x - 8 \end{bmatrix}$ **for** x **and** y**.**

Since the matrices are equal, the corresponding elements are equal. When you write the sentences to show the equality, two linear equations are formed.

$4x = -2y + 2$
$y = x - 8$

This system can be solved using substitution.

$4x = -2y + 2$	First equation
$4x = -2(x - 8) + 2$	Substitute $x - 8$ for y.
$4x = -2x + 16 + 2$	Distributive Property
$6x = 18$	Add $2x$ to each side.
$x = 3$	Divide each side by 6.

To find the value of y, substitute 3 for x in either equation.

$y = x - 8$	Second equation
$y = 3 - 8$	Substitute 3 for x.
$y = -5$	Subtract.

The solution is $(3, -5)$.

Exercises

Solve each equation.

1. $[5x \;\; 4y] = [20 \;\; 20]$

2. $\begin{bmatrix} 3x \\ y \end{bmatrix} = \begin{bmatrix} 28 + 4y \\ -3x - 2 \end{bmatrix}$

3. $\begin{bmatrix} -2y \\ x \end{bmatrix} = \begin{bmatrix} 4 - 5x \\ y + 5 \end{bmatrix}$

4. $\begin{bmatrix} x - 2y \\ 3x - 4y \end{bmatrix} = \begin{bmatrix} -1 \\ 22 \end{bmatrix}$

5. $\begin{bmatrix} 2x + 3y \\ x - 2y \end{bmatrix} = \begin{bmatrix} 3 \\ 12 \end{bmatrix}$

6. $\begin{bmatrix} 5x + 3y \\ 2x - y \end{bmatrix} = \begin{bmatrix} -1 \\ -18 \end{bmatrix}$

7. $\begin{bmatrix} 8x - y & 16x \\ 12 & y - 4x \end{bmatrix} = \begin{bmatrix} 18 & 20 \\ 12 & -13 \end{bmatrix}$

8. $\begin{bmatrix} 8x - 6y \\ 12x + 4y \end{bmatrix} = \begin{bmatrix} -3 \\ -11 \end{bmatrix}$

9. $\begin{bmatrix} \frac{x}{3} + \frac{y}{7} \\ \frac{x}{2} + 2y \end{bmatrix} = \begin{bmatrix} 9 \\ 51 \end{bmatrix}$

10. $\begin{bmatrix} 3x + 1.5 \\ 2y - 2.4 \end{bmatrix} = \begin{bmatrix} 7.5 \\ 8.0 \end{bmatrix}$

11. $\begin{bmatrix} 2x + 3y \\ -4x + 0.5y \end{bmatrix} = \begin{bmatrix} 17 \\ -8 \end{bmatrix}$

12. $\begin{bmatrix} x - y \\ x + y \end{bmatrix} = \begin{bmatrix} 0 \\ -25 \end{bmatrix}$

Lesson 4-1

NAME ____ DATE______________ PERIOD _____

4-1 Skills Practice

Introduction to Matrices

State the dimensions of each matrix.

1. $\begin{bmatrix} 3 & 2 & 4 \\ -1 & 4 & 0 \end{bmatrix}$

2. $[0 \quad 15]$

3. $\begin{bmatrix} 3 & 2 \\ 1 & 8 \end{bmatrix}$

4. $\begin{bmatrix} 6 & 1 & 2 \\ -3 & 4 & 5 \\ -2 & 7 & 9 \end{bmatrix}$

5. $\begin{bmatrix} 9 & 3 & -3 & -6 \\ 3 & 4 & -4 & 5 \end{bmatrix}$

6. $\begin{bmatrix} -1 \\ -1 \\ -1 \\ -3 \end{bmatrix}$

Solve each equation.

7. $[5x \quad 3y] = [15 \quad 12]$

8. $[3x - 2] = [7]$

9. $\begin{bmatrix} 7x \\ 14 \end{bmatrix} = \begin{bmatrix} -14 \\ 2y \end{bmatrix}$

10. $[2x \quad -8y \quad z] = [10 \quad 16 \quad -1]$

11. $\begin{bmatrix} 8 - x \\ 2y - 8 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

12. $\begin{bmatrix} 20 \\ 56 - 6y \end{bmatrix} = \begin{bmatrix} 10x \\ 32 \end{bmatrix}$

13. $\begin{bmatrix} 5x \\ 24 \end{bmatrix} = \begin{bmatrix} -20 \\ 8y \end{bmatrix}$

14. $\begin{bmatrix} 3x + 2 \\ 7y - 2 \end{bmatrix} = \begin{bmatrix} 5x + 2 \\ 3y - 10 \end{bmatrix}$

15. $\begin{bmatrix} 4x - 1 \\ 9y + 5 \end{bmatrix} = \begin{bmatrix} 3x \\ y - 3 \end{bmatrix}$

16. $\begin{bmatrix} 3x + 1 & 18 \\ 12 & 4z \end{bmatrix} = \begin{bmatrix} 7 & 2y - 4 \\ 12 & 28 \end{bmatrix}$

17. $\begin{bmatrix} x \\ 16 \\ 3z \end{bmatrix} = \begin{bmatrix} 9 \\ 4y \\ 9 \end{bmatrix}$

18. $\begin{bmatrix} 5x \\ 4y - 3 \\ 8z \end{bmatrix} = \begin{bmatrix} 4x + 1 \\ 13 \\ 4z \end{bmatrix}$

19. $\begin{bmatrix} 2x \\ y + 2 \end{bmatrix} = \begin{bmatrix} 6y \\ x \end{bmatrix}$

20. $\begin{bmatrix} x \\ 3y \end{bmatrix} = \begin{bmatrix} 4y \\ x - 3 \end{bmatrix}$

NAME ____ DATE______________ PERIOD _____

4-1 Practice

Introduction to Matrices

State the dimensions of each matrix.

1. $\begin{bmatrix} -3 & -3 & 7 \end{bmatrix}$

2. $\begin{bmatrix} 5 & 8 & -1 \\ -2 & 1 & 8 \end{bmatrix}$

3. $\begin{bmatrix} -2 & 2 & -2 & 3 \\ 5 & 16 & 0 & 0 \\ 4 & 7 & -1 & 4 \end{bmatrix}$

Solve each equation.

4. $[4x \quad 42] = [24 \quad 6y]$

5. $[-2x \quad 22 \quad -3z] = [6x \quad -2y \quad 45]$

6. $\begin{bmatrix} 6x \\ 2y + 3 \end{bmatrix} = \begin{bmatrix} -36 \\ 17 \end{bmatrix}$

7. $\begin{bmatrix} 7x - 8 \\ 8y - 3 \end{bmatrix} = \begin{bmatrix} 20 \\ 2y + 3 \end{bmatrix}$

8. $\begin{bmatrix} -4x - 3 \\ 6y \end{bmatrix} = \begin{bmatrix} -3x \\ -2y + 16 \end{bmatrix}$

9. $\begin{bmatrix} 6x - 12 \\ -3y + 6 \end{bmatrix} = \begin{bmatrix} -3x - 21 \\ 8y - 5 \end{bmatrix}$

10. $\begin{bmatrix} -5 & 3x + 1 \\ 2y - 1 & 3z - 2 \end{bmatrix} = \begin{bmatrix} -5 & x - 1 \\ 3y & 5z - 4 \end{bmatrix}$

11. $\begin{bmatrix} 3x \\ y + 4 \end{bmatrix} = \begin{bmatrix} y + 8 \\ 17 \end{bmatrix}$

12. $\begin{bmatrix} 5x + 8y \\ 3x - 11 \end{bmatrix} = \begin{bmatrix} -1 \\ y \end{bmatrix}$

13. $\begin{bmatrix} 2x + y \\ 3x + 2y \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$

14. TICKET PRICES The table at the right gives ticket prices for a concert. Write a 2×3 matrix that represents the cost of a ticket.

	Child	Student	Adult
Cost Purchased in Advance	\$6	\$12	\$18
Cost Purchased at the Door	\$8	\$15	\$22

CONSTRUCTION For Exercises 15 and 16, use the following information.

During each of the last three weeks, a road-building crew has used three truckloads of gravel. The table at the right shows the amount of gravel in each load.

Week 1	Week 2	Week 3
Load 1 40 tons	Load 1 40 tons	Load 1 32 tons
Load 2 32 tons	Load 2 40 tons	Load 2 24 tons
Load 3 24 tons	Load 3 32 tons	Load 3 24 tons

15. Write a matrix for the amount of gravel in each load.

16. What are the dimensions of the matrix?

Lesson 4-1

NAME ____ DATE______________ PERIOD _____

4-1 Word Problem Practice

Introduction to Matrices

1. HAWAII The table shows the population and area of some of the islands in Hawaii. What would be the dimensions of a matrix that represented this information?

Island	Population	Area
Hawaii	120,317	4,038
Maui	91,361	729
Oahu	836,231	594
Kauai	50,947	549
Lanai	2,426	140

Source: www.vthawaii.com

2. LAUNDRY Carl is looking for a Laundromat. SuperWash has 20 small washers, 10 large washers, and 20 dryers. QuickClean has 40 small washers, 5 large washers, and 50 dryers. ToughSuds has 15 small washers, 40 large washers, and 100 dryers. Write a matrix to organize this information.

3. CITY DISTANCES The incomplete matrix shown gives the approximate distances between Chicago, Los Angeles, and New York City. Complete the matrix.

$$\begin{array}{r} \\ \text{NYC} \\ \text{Chicago} \\ \text{Los Angeles} \end{array} \begin{array}{c} \begin{array}{ccc} \text{NYC} & \text{Chicago} & \text{Los Angeles} \end{array} \\ \left[\begin{array}{ccc} \mathbf{0} & & \mathbf{2790} \\ \mathbf{810} & & \mathbf{2050} \\ & & \end{array}\right] \end{array}$$

4. INVENTORY A store manager records the number of light bulbs in stock for 3 different brands over a five-day period. The manager decides to make a matrix of this information. Each row represents a different brand, and each column represents a different day. The entry in column N represents the inventories at the beginning of day N.

$$\begin{bmatrix} 25 & 24 & 22 & 20 & 19 \\ 30 & 27 & 25 & 22 & 21 \\ 28 & 25 & 21 & 19 & 19 \end{bmatrix}$$

Assuming that the inventories were never replenished, which brand holds the record for most light bulbs sold on a given day?

SHOE SALES For Exercises 5 and 6, use the following information.
A shoe store manager keeps track of the amount of money made by each of three salespeople for each day of a workweek. Monday through Friday, Carla made \$40, \$70, \$35, \$50, and \$20. John made \$30, \$60, \$20, \$45, and \$30. Mary made \$35, \$90, \$30, \$40, and \$30.

5. Organize this data in a 3 by 5 matrix.

6. Which salesperson made the most money that week?

NAME ____ DATE______________ PERIOD _____

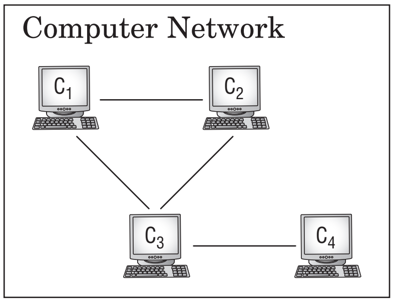
4-1 Enrichment

Lesson 4-1

Matrices and Networks

Graph theory is a branch of mathematics that explores situations represented by points called vertices and the segments that may connect them, called edges. For example, graphs can be used to represent computer networks or airline routes between major cities.

An incidence matrix is a matrix used to represent the vertices, edges, and relationships among the vertices of a graph. The vertices name each row and column. For example, the incidence matrix for the computer network shown in the figure is shown below. The numbers represent how many edges connect the vertices.



Computer Network

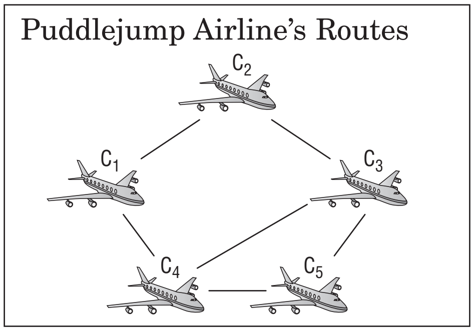
Indicates one edge from C_1 to C_2.

$$\begin{array}{c|cccc} & C_1 & C_2 & C_3 & C_4 \\ \hline C_1 & 0 & 1 & 1 & 0 \\ C_2 & 1 & 0 & 1 & 0 \\ C_3 & 1 & 1 & 0 & 1 \\ C_4 & 0 & 0 & 1 & 0 \end{array}$$

Indicates no edge from C_4 to C_1.

Complete the incidence matrix for each pictured network.

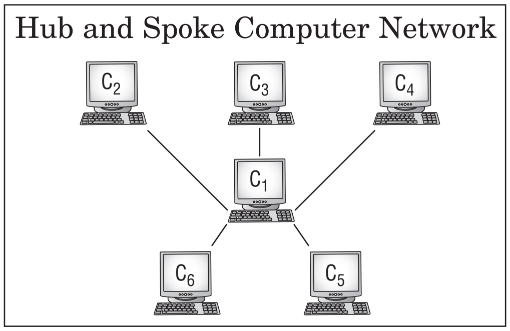
1. Puddlejump Airlines daily flights between cities 1–5.



Puddlejump Airline's Routes

	c_1	c_2	c_3	c_4	c_5
c_1					
c_2					
c_3					
c_4					
c_5					

2. Computers in a Hub and Spoke network.



Hub and Spoke Computer Network

	c_1	c_2	c_3	c_4	c_5	c_6
c_1						
c_2						
c_3						
c_4						
c_5						
c_6						

4-2 Lesson Reading Guide

Operations with Matrices

Get Ready for the Lesson

Read the introduction to Lesson 4-2 in your textbook.

- Write a sum that represents the total number of Calories in the patient's diet for Day 2. (Do not actually calculate the sum.)
- Write the sum that represents the total fat content in the patient's diet for Day 3. (Do not actually calculate the sum.)

Read the Lesson

1. For each pair of matrices, give the dimensions of the indicated sum, difference, or scalar product. If the indicated sum, difference, or scalar product does not exist, write *impossible*.

$A = \begin{bmatrix} 3 & 5 & 6 \\ -2 & 8 & 1 \end{bmatrix}$ $\quad B = \begin{bmatrix} -4 & 0 \\ 0 & -5 \end{bmatrix}$

$C = \begin{bmatrix} 5 & 10 \\ -3 & 6 \\ 4 & 12 \end{bmatrix}$ $\quad D = \begin{bmatrix} -3 & 6 & 0 \\ -8 & 4 & 0 \end{bmatrix}$

$A + D$: ____________ $C + D$: ____________ $5B$: ____________

$-4C$: ____________ $2D - 3A$: ____________

2. Suppose that M, N, and P are nonzero 2×4 matrices and k is a negative real number. Indicate whether each of the following statements is *true* or *false*.

a. $M + (N + P) = M + (P + N)$

b. $M - N = N - M$

c. $M - (N - P) = (M - N) - P$

d. $k(M - N) = kM - kN$

Remember What You Learned

3. The mathematical term *scalar* may be unfamiliar, but its meaning is related to the word *scale* as used in a *scale of miles* on a map. How can this usage of the word *scale* help you remember the meaning of *scalar*?

4-2 Study Guide and Intervention

Operations with Matrices

Add and Subtract Matrices

Addition of Matrices	$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} + \begin{bmatrix} j & k & l \\ m & n & o \\ p & q & r \end{bmatrix} = \begin{bmatrix} a+j & b+k & c+l \\ d+m & e+n & f+o \\ g+p & h+q & i+r \end{bmatrix}$
Subtraction of Matrices	$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} - \begin{bmatrix} j & k & l \\ m & n & o \\ p & q & r \end{bmatrix} = \begin{bmatrix} a-j & b-k & c-l \\ d-m & e-n & f-o \\ g-p & h-q & i-r \end{bmatrix}$

Example 1 **Find $A + B$ if $A = \begin{bmatrix} 6 & -7 \\ 2 & -12 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 2 \\ -5 & -6 \end{bmatrix}$.**

$$A + B = \begin{bmatrix} 6 & -7 \\ 2 & -12 \end{bmatrix} + \begin{bmatrix} 4 & 2 \\ -5 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 6+4 & -7+2 \\ 2+(-5) & -12+(-6) \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -5 \\ -3 & -18 \end{bmatrix}$$

Example 2 **Find $A - B$ if $A = \begin{bmatrix} -2 & 8 \\ 3 & -4 \\ 10 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -3 \\ -2 & 1 \\ -6 & 8 \end{bmatrix}$.**

$$A - B = \begin{bmatrix} -2 & 8 \\ 3 & -4 \\ 10 & 7 \end{bmatrix} - \begin{bmatrix} 4 & -3 \\ -2 & 1 \\ -6 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} -2-4 & 8-(-3) \\ 3-(-2) & -4-1 \\ 10-(-6) & 7-8 \end{bmatrix} = \begin{bmatrix} -6 & 11 \\ 5 & -5 \\ 16 & -1 \end{bmatrix}$$

Exercises

Perform the indicated operations. If the matrix does not exist, write *impossible*.

1. $\begin{bmatrix} 8 & 7 \\ -10 & -6 \end{bmatrix} - \begin{bmatrix} -4 & 3 \\ 2 & -12 \end{bmatrix}$

2. $\begin{bmatrix} 6 & -5 & 9 \\ -3 & 4 & 5 \end{bmatrix} + \begin{bmatrix} -4 & 3 & 2 \\ 6 & 9 & -4 \end{bmatrix}$

3. $\begin{bmatrix} 6 \\ -3 \\ 2 \end{bmatrix} + \begin{bmatrix} -6 & 3 & -2 \end{bmatrix}$

4. $\begin{bmatrix} 5 & -2 \\ -4 & 6 \\ 7 & 9 \end{bmatrix} + \begin{bmatrix} -11 & 6 \\ 2 & -5 \\ 4 & -7 \end{bmatrix}$

5. $\begin{bmatrix} 8 & 0 & -6 \\ 4 & 5 & -11 \\ -7 & 3 & 4 \end{bmatrix} - \begin{bmatrix} -2 & 1 & 7 \\ 3 & -4 & 3 \\ -8 & 5 & 6 \end{bmatrix}$

6. $\begin{bmatrix} \frac{3}{4} & \frac{2}{5} \\ -\frac{1}{2} & \frac{4}{3} \end{bmatrix} - \begin{bmatrix} \frac{1}{2} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{2} \end{bmatrix}$

Lesson 4-2

4-2 Study Guide and Intervention *(continued)*

Operations with Matrices

Scalar Multiplication

You can multiply an $m \times n$ matrix by a scalar k.

Scalar Multiplication	$k\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} ka & kb & kc \\ kd & ke & kf \end{bmatrix}$

Example **If $A = \begin{bmatrix} 4 & 0 \\ -6 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 5 \\ 7 & 8 \end{bmatrix}$, find $3B - 2A$.**

$3B - 2A = 3\begin{bmatrix} -1 & 5 \\ 7 & 8 \end{bmatrix} - 2\begin{bmatrix} 4 & 0 \\ -6 & 3 \end{bmatrix}$ Substitution

$= \begin{bmatrix} 3(-1) & 3(5) \\ 3(7) & 3(8) \end{bmatrix} - \begin{bmatrix} 2(4) & 2(0) \\ 2(-6) & 2(3) \end{bmatrix}$ Multiply.

$= \begin{bmatrix} -3 & 15 \\ 21 & 24 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ -12 & 6 \end{bmatrix}$ Simplify.

$= \begin{bmatrix} -3 - 8 & 15 - 0 \\ 21 - (-12) & 24 - 6 \end{bmatrix}$ Subtract.

$= \begin{bmatrix} -11 & 15 \\ 33 & 18 \end{bmatrix}$ Simplify.

Exercises

Perform the indicated matrix operations. If the matrix does not exist, write *impossible*.

1. $6\begin{bmatrix} 2 & -5 & 3 \\ 0 & 7 & -1 \\ -4 & 6 & 9 \end{bmatrix}$

2. $-\frac{1}{3}\begin{bmatrix} 6 & 15 & 9 \\ 51 & -33 & 24 \\ -18 & 3 & 45 \end{bmatrix}$

3. $0.2\begin{bmatrix} 25 & -10 & -45 \\ 5 & 55 & -30 \\ 60 & 35 & -95 \end{bmatrix}$

4. $3\begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix} - 2\begin{bmatrix} -1 & 2 \\ -3 & 5 \end{bmatrix}$

5. $-2\begin{bmatrix} 3 & -1 \\ 0 & 7 \end{bmatrix} + 4\begin{bmatrix} -2 & 0 \\ 2 & 5 \end{bmatrix}$

6. $2\begin{bmatrix} 6 & -10 \\ -5 & 8 \end{bmatrix} + 5\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$

7. $4\begin{bmatrix} 1 & -2 & 5 \\ -3 & 4 & 1 \end{bmatrix} - 2\begin{bmatrix} 4 & 3 & -4 \\ 2 & -5 & -1 \end{bmatrix}$

8. $8\begin{bmatrix} 2 & 1 \\ 3 & -1 \\ -2 & 4 \end{bmatrix} + 3\begin{bmatrix} 4 & 0 \\ -2 & 3 \\ 3 & -4 \end{bmatrix}$

9. $\frac{1}{4}\left(\begin{bmatrix} 9 & 1 \\ -7 & 0 \end{bmatrix} + \begin{bmatrix} 3 & -5 \\ 1 & 7 \end{bmatrix}\right)$

4-2 Skills Practice

Operations with Matrices

Perform the indicated matrix operations. If the matrix does not exist, write *impossible*.

1. $[5 \quad -4] + [4 \quad 5]$

2. $\begin{bmatrix} 8 & 3 \\ -1 & -1 \end{bmatrix} - \begin{bmatrix} 0 & -7 \\ 6 & 2 \end{bmatrix}$

3. $[3 \quad 1 \quad 6] + \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$

4. $\begin{bmatrix} 5 & -1 & 2 \\ 1 & 8 & -6 \end{bmatrix} + \begin{bmatrix} 9 & 9 & 2 \\ 4 & 6 & 4 \end{bmatrix}$

5. $3[9 \quad 4 \quad -3]$

6. $[6 \quad -3] - 4[4 \quad 7]$

7. $-2\begin{bmatrix} -2 & 5 \\ 5 & 9 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

8. $3\begin{bmatrix} 8 \\ 0 \\ -3 \end{bmatrix} - 4\begin{bmatrix} 2 \\ 2 \\ 10 \end{bmatrix}$

9. $5\begin{bmatrix} -4 & 6 \\ 10 & 1 \\ -1 & 1 \end{bmatrix} + 2\begin{bmatrix} 6 & 5 \\ -3 & -2 \\ 1 & 0 \end{bmatrix}$

10. $3\begin{bmatrix} 3 & 1 & 3 \\ -4 & 7 & 5 \end{bmatrix} - 2\begin{bmatrix} 1 & -1 & 5 \\ 6 & 6 & -3 \end{bmatrix}$

Use $A = \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 2 \\ 1 & -2 \end{bmatrix}$, and $C = \begin{bmatrix} -3 & 4 \\ 3 & 1 \end{bmatrix}$ to find the following.

11. $A + B$

12. $B - C$

13. $B - A$

14. $A + B + C$

15. $3B$

16. $-5C$

17. $A - 4C$

18. $2B + 3A$

Lesson 4-2

NAME ______________________________ DATE______________ PERIOD _____

4-2 Practice

Operations with Matrices

Perform the indicated matrix operations. If the matrix does not exist, write *impossible*.

1. $\begin{bmatrix} 2 & -1 \\ 3 & 7 \\ 14 & -9 \end{bmatrix} + \begin{bmatrix} -6 & 9 \\ 7 & -11 \\ -8 & 17 \end{bmatrix}$

2. $\begin{bmatrix} 4 \\ -71 \\ 18 \end{bmatrix} - \begin{bmatrix} -67 \\ 45 \\ -24 \end{bmatrix}$

3. $-3\begin{bmatrix} -1 & 0 \\ 17 & -11 \end{bmatrix} + 4\begin{bmatrix} -3 & 16 \\ -21 & 12 \end{bmatrix}$

4. $7\begin{bmatrix} 2 & -1 & 8 \\ 4 & 7 & 9 \end{bmatrix} - 2\begin{bmatrix} -1 & 4 & -3 \\ 7 & 2 & -6 \end{bmatrix}$

5. $-2\begin{bmatrix} 1 \\ 2 \end{bmatrix} + 4\begin{bmatrix} 0 \\ 5 \end{bmatrix} - \begin{bmatrix} 10 \\ 18 \end{bmatrix}$

6. $\frac{3}{4}\begin{bmatrix} 8 & 12 \\ -16 & 20 \end{bmatrix} + \frac{2}{3}\begin{bmatrix} 27 & -9 \\ 54 & -18 \end{bmatrix}$

Use $A = \begin{bmatrix} 4 & -1 & 0 \\ -3 & 6 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 4 & 5 \\ 1 & 0 & -9 \end{bmatrix}$, **and** $C = \begin{bmatrix} 10 & -8 & 6 \\ -6 & -4 & 20 \end{bmatrix}$ **to find the following.**

7. $A - B$

8. $A - C$

9. $-3B$

10. $4B - A$

11. $-2B - 3C$

12. $A + 0.5C$

ECONOMICS For Exercises 13 and 14, use the table that shows loans by an economic development board to women and men starting new businesses.

	Women		Men	
	Businesses	Loan Amount ($)	Businesses	Loan Amount ($)
2003	27	$567,000	36	$864,000
2004	41	$902,000	32	$672,000
2005	35	$777,000	28	$562,000

13. Write two matrices that represent the number of new businesses and loan amounts, one for women and one for men.

14. Find the sum of the numbers of new businesses and loan amounts for both men and women over the three-year period expressed as a matrix.

15. PET NUTRITION Use the table that gives nutritional information for two types of dog food. Find the difference in the percent of protein, fat, and fiber between Mix B and Mix A expressed as a matrix.

	% Protein	% Fat	% Fiber
Mix A	22	12	5
Mix B	24	8	8

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4-2 Word Problem Practice

Operations with Matrices

1. FARES The matrix below gives general admission and planetarium fares at a science museum.

$$\begin{array}{r c} & \begin{array}{cc}\textbf{Child} & \textbf{Adult}\end{array} \\ \begin{array}{r}\textbf{General Admission} \\ \textbf{Planetarium}\end{array} & \begin{bmatrix} 5 & 10 \\ 4 & 8 \end{bmatrix} \end{array}$$

What can you do to this matrix in order to create another matrix that represents fares for 5 people?

2. NEGATION Two engineers need to negate all the entries of a matrix. One engineer tries to do this by multiplying the matrix by -1. The other engineer tries to do this by subtracting twice the matrix from itself. Which engineer, if any, will get the correct result?

3. PLANE FARES The airfares for travel between New York, Chicago, and Los Angeles are organized in the matrix on the left. The matrix on the right gives the tax surcharges for corresponding flights.

$$\begin{array}{r c} & \begin{array}{ccc}\text{NYC} & \text{Chicago} & \text{Los Angeles}\end{array} \\ \begin{array}{r}\text{NYC} \\ \text{Chicago} \\ \text{Los Angeles}\end{array} & \begin{bmatrix} 0 & 440 & 700 \\ 460 & 0 & 660 \\ 850 & 700 & 0 \end{bmatrix} \end{array} \quad \begin{array}{r c} & \begin{array}{ccc}\text{NYC} & \text{Chicago} & \text{Los Angeles}\end{array} \\ \begin{array}{r}\text{NYC} \\ \text{Chicago} \\ \text{Los Angeles}\end{array} & \begin{bmatrix} 0 & 40 & 70 \\ 46 & 0 & 60 \\ 85 & 70 & 0 \end{bmatrix} \end{array}$$

Write a matrix that represents the full cost for travel between these cities.

4. SUNFLOWERS Matrix H is a 3 by 1 matrix that contains the initial heights of three sunflowers. Matrix G is a 3 by 1 matrix that contains the numbers of inches the corresponding sunflowers grow in a week. What does matrix $H + 4G$ represent?

DINNER For Exercises 5–7, use the following information.
The menu shows prices for some dishes at a restaurant.

Il Ristorante Menu

	Regular	Half-portion
Lamb	$17.00	$9.00
Chicken	$14.00	$7.00
Steak	$22.00	$11.00

5. Make a 3 by 2 matrix to organize these data.

6. Let M be the matrix you wrote for Exercise 5. Write an expression involving M that would give prices that include an additional 20% to cover tax and tip.

7. Compute the matrix you described in Exercise 6.

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4-2 Enrichment

Sundaram's Sieve

The properties and patterns of prime numbers have fascinated many mathematicians. In 1934, a young East Indian student named Sundaram constructed the following matrix.

4	7	10	13	16	19	22	25	. . .
7	12	17	22	27	32	37	42	. . .
10	17	24	31	38	45	52	59	. . .
13	22	31	40	49	58	67	76	. . .
16	27	38	49	60	71	82	93	. . .
.	.	.	.	.	.	.	.	. . .

A surprising property of this matrix is that it can be used to determine whether or not some numbers are prime.

Complete these problems to discover this property.

1. The first row and the first column are created by using an arithmetic pattern. What is the common difference used in the pattern?

2. Find the next four numbers in the first row.

3. What are the common differences used to create the patterns in rows 2, 3, 4, and 5?

4. Write the next two rows of the matrix. Include eight numbers in each row.

5. Choose any five numbers from the matrix. For each number n, that you chose from the matrix, find $2n + 1$.

6. Write the factorization of each value of $2n + 1$ that you found in problem 5.

7. Use your results from problems 5 and 6 to complete this statement: If n occurs in the matrix, then $2n + 1$ ____________ (is/is not) a prime number.

8. Choose any five numbers that are not in the matrix. Find $2n + 1$ for each of these numbers. Show that each result is a prime number.

9. Complete this statement: If n does not occur in the matrix, then $2n + 1$ is

_____________________.

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4-3 Lesson Reading Guide

Multiplying Matrices

Get Ready for the Lesson

Read the introduction to Lesson 4-3 in your textbook.

Write a sum that shows the total points scored by the Houston Texans during the 2004 season. (The sum will include multiplications. Do not actually calculate this sum.)

Read the Lesson

1. Determine whether each indicated matrix product is defined. If so, state the dimensions of the product. If not, write *undefined*.

a. $M_{3 \times 2}$ and $N_{2 \times 3}$ MN: ______________ NM: ______________

b. $M_{1 \times 2}$ and $N_{1 \times 2}$ MN: ______________ NM: ______________

c. $M_{4 \times 1}$ and $N_{1 \times 4}$ MN: ______________ NM: ______________

d. $M_{3 \times 4}$ and $N_{4 \times 4}$ MN: ______________ NM: ______________

2. The regional sales manager for a chain of computer stores wants to compare the revenue from sales of one model of notebook computer and one model of printer for three stores in his area. The notebook computer sells for $1850 and the printer for $175. The number of computers and printers sold at the three stores during September are shown in the following table.

Store	Computers	Printers
A	128	101
B	205	166
C	97	73

Write a matrix product that the manager could use to find the total revenue for computers and printers for each of the three stores. (Do not calculate the product.)

Remember What You Learned

3. Many students find the procedure of matrix multiplication confusing at first because it is unfamiliar. Think of an easy way to use the letters R and C to remember how to multiply matrices and what the dimensions of the product will be.

4-3 Study Guide and Intervention

Multiplying Matrices

Multiply Matrices You can multiply two matrices if and only if the number of columns in the first matrix is equal to the number of rows in the second matrix.

Multiplication of Matrices	$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \cdot \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix} = \begin{bmatrix} a_1x_1 + b_1x_2 & a_1y_1 + b_1y_2 \\ a_2x_1 + b_2x_2 & a_2y_1 + b_2y_2 \end{bmatrix}$

Example **Find *AB* if $A = \begin{bmatrix} -4 & 3 \\ 2 & -2 \\ 1 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & -2 \\ -1 & 3 \end{bmatrix}$.**

$AB = \begin{bmatrix} -4 & 3 \\ 2 & -2 \\ 1 & 7 \end{bmatrix} \cdot \begin{bmatrix} 5 & -2 \\ -1 & 3 \end{bmatrix}$ Substitution

$= \begin{bmatrix} -4(5) + 3(-1) & -4(-2) + 3(3) \\ 2(5) + (-2)(-1) & 2(-2) + (-2)(3) \\ 1(5) + 7(-1) & 1(-2) + 7(3) \end{bmatrix}$ Multiply columns by rows.

$= \begin{bmatrix} -23 & 17 \\ 12 & -10 \\ -2 & 19 \end{bmatrix}$ Simplify.

Exercises

Find each product, if possible.

1. $\begin{bmatrix} 4 & 1 \\ -2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

2. $\begin{bmatrix} -1 & 0 \\ 3 & 7 \end{bmatrix} \cdot \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$

3. $\begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$

4. $\begin{bmatrix} -3 & 1 \\ 5 & -2 \end{bmatrix} \cdot \begin{bmatrix} 4 & 0 & -2 \\ -3 & 1 & 1 \end{bmatrix}$

5. $\begin{bmatrix} 3 & -2 \\ 0 & 4 \\ -5 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

6. $\begin{bmatrix} 5 & -2 \\ 2 & -3 \end{bmatrix} \cdot \begin{bmatrix} 4 & -1 \\ -2 & 5 \end{bmatrix}$

7. $\begin{bmatrix} 6 & 10 \\ -4 & 3 \\ -2 & 7 \end{bmatrix} \cdot [0 \quad 4 \quad -3]$

8. $\begin{bmatrix} 7 & -2 \\ 5 & -4 \end{bmatrix} \cdot \begin{bmatrix} 1 & -3 \\ -2 & 0 \end{bmatrix}$

9. $\begin{bmatrix} 2 & 0 & -3 \\ 1 & 4 & -2 \\ -1 & 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & -2 \\ 3 & 1 \\ -2 & 4 \end{bmatrix}$

4-3 Study Guide and Intervention *(continued)*

Multiplying Matrices

Multiplicative Properties The Commutative Property of Multiplication does *not* hold for matrices.

Properties of Matrix Multiplication	For any matrices A, B, and C for which the matrix product is defined, and any scalar c, the following properties are true.
Associative Property of Matrix Multiplication	$(AB)C = A(BC)$
Associative Property of Scalar Multiplication	$c(AB) = (cA)B = A(cB)$
Left Distributive Property	$C(A + B) = CA + CB$
Right Distributive Property	$(A + B)C = AC + BC$

Example **Use $A = \begin{bmatrix} 4 & -3 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 5 & -3 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & -2 \\ 6 & 3 \end{bmatrix}$ to find each product.**

a. $(A + B)C$

$$\begin{aligned}(A + B)C &= \left(\begin{bmatrix} 4 & -3 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 5 & -3 \end{bmatrix}\right) \cdot \begin{bmatrix} 1 & -2 \\ 6 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 6 & -3 \\ 7 & -2 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ 6 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 6(1) + (-3)(6) & 6(-2) + (-3)(3) \\ 7(1) + (-2)(6) & 7(-2) + (-2)(3) \end{bmatrix} \\ &= \begin{bmatrix} -12 & -21 \\ -5 & -20 \end{bmatrix}\end{aligned}$$

b. $AC + BC$

$$\begin{aligned}AC + BC &= \begin{bmatrix} 4 & -3 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ 6 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 5 & -3 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ 6 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 4(1) + (-3)(6) & 4(-2) + (-3)(3) \\ 2(1) + 1(6) & 2(-2) + 1(3) \end{bmatrix} + \begin{bmatrix} 2(1) + 0(6) & 2(-2) + 0(3) \\ 5(1) + (-3)(6) & 5(-2) + (-3)(3) \end{bmatrix} \\ &= \begin{bmatrix} -14 & -17 \\ 8 & -1 \end{bmatrix} + \begin{bmatrix} 2 & -4 \\ -13 & -19 \end{bmatrix} = \begin{bmatrix} -12 & -21 \\ -5 & -20 \end{bmatrix}\end{aligned}$$

Note that although the results in the example illustrate the Right Distributive Property, they do not prove it.

Exercises

Use $A = \begin{bmatrix} 3 & 2 \\ 5 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 4 \\ 2 & 1 \end{bmatrix}$, $C = \begin{bmatrix} -\frac{1}{2} & -2 \\ 1 & -3 \end{bmatrix}$, and scalar $c = -4$ to determine whether each of the following equations is true for the given matrices.

1. $c(AB) = (cA)B$

2. $AB = BA$

3. $BC = CB$

4. $(AB)C = A(BC)$

5. $C(A + B) = AC + BC$

6. $c(A + B) = cA + cB$

4-3 Skills Practice

Multiplying Matrices

Determine whether each matrix product is defined. If so, state the dimensions of the product.

1. $A_{2 \times 5} \cdot B_{5 \times 1}$

2. $M_{1 \times 3} \cdot N_{3 \times 2}$

3. $B_{3 \times 2} \cdot A_{3 \times 2}$

4. $R_{4 \times 4} \cdot S_{4 \times 1}$

5. $X_{3 \times 3} \cdot Y_{3 \times 4}$

6. $A_{6 \times 4} \cdot B_{4 \times 5}$

Find each product, if possible.

7. $\begin{bmatrix} 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

8. $\begin{bmatrix} 5 & 6 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & -5 \\ 3 & 1 \end{bmatrix}$

9. $\begin{bmatrix} 1 & 3 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

10. $\begin{bmatrix} 3 \\ -2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ -1 & 1 \end{bmatrix}$

11. $\begin{bmatrix} -3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 0 & -1 \\ 2 & 2 \end{bmatrix}$

12. $\begin{bmatrix} -1 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 2 & -3 & -2 \end{bmatrix}$

13. $\begin{bmatrix} 5 \\ 6 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 8 \end{bmatrix}$

14. $\begin{bmatrix} 2 & -2 \\ 4 & 5 \\ -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 3 \\ 3 & 0 \end{bmatrix}$

15. $\begin{bmatrix} -4 & 4 \\ -2 & 1 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & -3 \\ 0 & 2 \end{bmatrix}$

16. $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$

Use $A = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 5 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 3 & -1 \\ 1 & 0 \end{bmatrix}$, and scalar $c = 2$ to determine whether the following equations are true for the given matrices.

17. $(AC)c = A(Cc)$

18. $AB = BA$

19. $B(A + C) = AB + BC$

20. $(A - B)c = Ac - Bc$

4-3 Practice

Multiplying Matrices

Determine whether each matrix product is defined. If so, state the dimensions of the product.

1. $A_{7 \times 4} \cdot B_{4 \times 3}$

2. $A_{3 \times 5} \cdot M_{5 \times 8}$

3. $M_{2 \times 1} \cdot A_{1 \times 6}$

4. $M_{3 \times 2} \cdot A_{3 \times 2}$

5. $P_{1 \times 9} \cdot Q_{9 \times 1}$

6. $P_{9 \times 1} \cdot Q_{1 \times 9}$

Find each product, if possible.

7. $\begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 & 7 \\ 6 & 0 & -5 \end{bmatrix}$

8. $\begin{bmatrix} 2 & 4 \\ 7 & -1 \end{bmatrix} \cdot \begin{bmatrix} -3 & 0 \\ 2 & 5 \end{bmatrix}$

9. $\begin{bmatrix} -3 & 0 \\ 2 & 5 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 \\ 7 & -1 \end{bmatrix}$

10. $\begin{bmatrix} 3 & -2 & 7 \\ 6 & 0 & -5 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 & 7 \\ 6 & 0 & -5 \end{bmatrix}$

11. $\begin{bmatrix} 4 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$

12. $\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 4 & 0 & 2 \end{bmatrix}$

13. $\begin{bmatrix} -6 & 2 \\ 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

14. $\begin{bmatrix} -15 & -9 \end{bmatrix} \cdot \begin{bmatrix} 6 & 11 \\ 23 & -10 \end{bmatrix}$

Use $A = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ -2 & -1 \end{bmatrix}$, $C = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, and scalar $c = 3$ to determine whether the following equations are true for the given matrices.

15. $AC = CA$

16. $A(B + C) = BA + CA$

17. $(AB)c = c(AB)$

18. $(A + C)B = B(A + C)$

RENTALS For Exercises 19–21, use the following information.

For their one-week vacation, the Montoyas can rent a 2-bedroom condominium for \$1796, a 3-bedroom condominium for \$2165, or a 4-bedroom condominium for \$2538. The table shows the number of units in each of three complexes.

	2-Bedroom	3-Bedroom	4-Bedroom
Sun Haven	36	24	22
Surfside	29	32	42
Seabreeze	18	22	18

19. Write a matrix that represents the number of each type of unit available at each complex and a matrix that represents the weekly charge for each type of unit.

20. If all of the units in the three complexes are rented for the week at the rates given the Montoyas, express the income of each of the three complexes as a matrix.

21. What is the total income of all three complexes for the week?

4-3 Word Problem Practice

Multiplying Matrices

1. FIND THE ERROR Both A and B are 2 by 2 matrices. Maggie made the following derivation. Is this derivation valid? If not, what error did she make?

a. $(A + B)^2 = (A + B)(A + B)$

b. $= (A + B)A + (A + B)B$

c. $= AA + BA + AB + BB$

d. $= A^2 + BA + AB + B^2$

e. $= A^2 + AB + AB + B^2$

f. $= A^2 + 2AB + B^2$

2. EXAM SCORES Mr. Farey recorded the exam scores of his students in a 20 by 3 matrix. Each row listed the scores of a different student. The first exam scores were listed in the first column, and the second exam scores were listed in the second column. The final exam scores were listed in the third column. Mr. Farey needed to create a 20 by 1 matrix that contained the weighted scores of each student. The first two exams account for 25% of the weighted score, and the final exam counted 50%. To make the matrix of weighted scores, what matrix can Mr. Farey multiply his 20 by 3 matrix by on the right?

3. SPECIAL MATRICES Mandy has a 3 by 3 matrix M. She notices that for any 3 by 3 matrix X, $MX = X$. What must M be?

4. POWERS Thad just learned about matrix multiplication. He began to wonder what happens when you take powers of a matrix. He computed the first few powers of the matrix $M = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and noticed a pattern. What is M^n?

COST COMPARISONS For Exercises 5 and 6, use the following information.

Barbara and Lance need to buy pens, pencils, and erasers. They make a 2 by 3 matrix that represents the numbers of each item they would like to purchase.

$$\begin{array}{r c} & \begin{array}{ccc} \textbf{Pens} & \textbf{Pencils} & \textbf{Erasers} \end{array} \\ \begin{array}{r} \textbf{Barbara} \\ \textbf{Lance} \end{array} & \begin{bmatrix} 10 & 15 & 3 \\ 5 & 20 & 5 \end{bmatrix} \end{array}$$

They call this matrix M. Barbara and Lance find two stores that sell the items at different prices and record this information in a second matrix that they call P.

$$\begin{array}{r c} & \begin{array}{cc} \textbf{Store 1} & \textbf{Store 2} \end{array} \\ \begin{array}{r} \textbf{Pens} \\ \textbf{Pencils} \\ \textbf{Erasers} \end{array} & \begin{bmatrix} 2.20 & 1.90 \\ 0.85 & 0.95 \\ 0.60 & 0.65 \end{bmatrix} \end{array}$$

5. Compute MP.

6. What do the entries in MP mean?

4-3 Enrichment

Properties of Matrices

Computing with matrices is different from computing with real numbers. Stated below are some properties of the real number system. Are these also true for matrices? In the problems on this page, you will investigate this question.

For all real numbers a and b, $ab = 0$ if and only if $a = 0$ or $b = 0$.

Multiplication is commutative. For all real numbers a and b, $ab = ba$.

Multiplication is associative. For all real numbers a, b, and c, $a(bc) = (ab)c$.

Use the matrices *A*, *B*, and *C* for the problems. Write whether each statement is true. Assume that a 2-by-2 matrix is the 0 matrix if and only if all of its elements are zero.

$A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -3 \\ -1 & 3 \end{bmatrix}$ $C = \begin{bmatrix} 3 & 6 \\ 1 & 2 \end{bmatrix}$

1. $AB = 0$

2. $AC = 0$

3. $BC = 0$

4. $AB = BA$

5. $AC = CA$

6. $BC = CB$

7. $A(BC) = (AB)C$

8. $B(CA) = (BC)A$

9. $B(AC) = (BA)C$

10. Write a statement summarizing your findings about the properties of matrix multiplication.

Lesson 4-3

4-4 Lesson Reading Guide

Transformations with Matrices

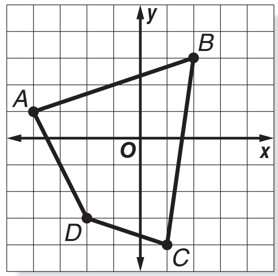
Get Ready for the Lesson

Read the introduction to Lesson 4-4 in your textbook.

Describe how you can change the orientation of a figure without changing its size or shape.

Read the Lesson

1. a. Write the vertex matrix for the quadrilateral *ABCD* shown in the graph at the right.



b. Write the vertex matrix that represents the position of the quadrilateral $A'B'C'D'$ that results when quadrilateral *ABCD* is translated 3 units to the right and 2 units down.

2. Describe the transformation that corresponds to each of the following matrices.

a. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

b. $\begin{bmatrix} 3 & 3 & 3 \\ -4 & -4 & -4 \end{bmatrix}$

c. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

d. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Remember What You Learned

3. Describe a way to remember which of the reflection matrices corresponds to reflection over the *x*-axis.

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4-4 Study Guide and Intervention

Transformations with Matrices

Translations and Dilations Matrices that represent coordinates of points on a plane are useful in describing transformations.

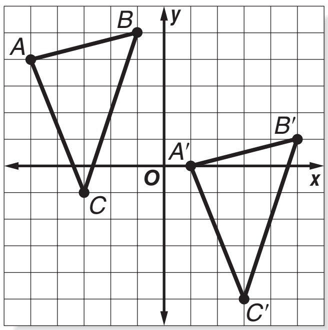
Translation	a transformation that moves a figure from one location to another on the coordinate plane

You can use matrix addition and a translation matrix to find the coordinates of the translated figure.

Dilation	a transformation in which a figure is enlarged or reduced

You can use scalar multiplication to perform dilations.

Example **Find the coordinates of the vertices of the image of $\triangle ABC$ with vertices $A(-5, 4)$, $B(-1, 5)$, and $C(-3, -1)$ if it is moved 6 units to the right and 4 units down. Then graph $\triangle ABC$ and its image $\triangle A'B'C'$.**



Write the vertex matrix for $\triangle ABC$. $\begin{bmatrix} -5 & -1 & -3 \\ 4 & 5 & -1 \end{bmatrix}$

Add the translation matrix $\begin{bmatrix} 6 & 6 & 6 \\ -4 & -4 & -4 \end{bmatrix}$ to the vertex matrix of $\triangle ABC$.

$$\begin{bmatrix} -5 & -1 & -3 \\ 4 & 5 & -1 \end{bmatrix} + \begin{bmatrix} 6 & 6 & 6 \\ -4 & -4 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 5 & 3 \\ 0 & 1 & -5 \end{bmatrix}$$

The coordinates of the vertices of $\triangle A'B'C'$ are $A'(1, 0)$, $B'(5, 1)$, and $C'(3, -5)$.

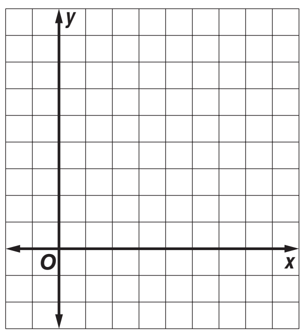
Exercises

For Exercises 1 and 2 use the following information. Quadrilateral *QUAD* with vertices $Q(-1, -3)$, $U(0, 0)$, $A(5, -1)$, and $D(2, -5)$ is translated 3 units to the left and 2 units up.

1. Write the translation matrix.

2. Find the coordinates of the vertices of $Q'U'A'D'$.

For Exercises 3–5, use the following information. The vertices of $\triangle ABC$ are $A(4, -2)$, $B(2, 8)$, and $C(8, 2)$. The triangle is dilated so that its perimeter is one-fourth the original perimeter.



3. Write the coordinates of the vertices of $\triangle ABC$ in a vertex matrix.

4. Find the coordinates of the vertices of image $\triangle A'B'C'$.

5. Graph the preimage and the image.

4-4 Study Guide and Intervention *(continued)*

Transformations with Matrices

Reflections and Rotations

Reflection Matrices	For a reflection over the:	*x*-axis	*y*-axis	line $y = x$
	multiply the vertex matrix on the left by:	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Rotation Matrices	For a counterclockwise rotation about the origin of:	90°	180°	270°
	multiply the vertex matrix on the left by:	$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Example **Find the coordinates of the vertices of the image of $\triangle ABC$ with $A(3, 5)$, $B(-2, 4)$, and $C(1, -1)$ after a reflection over the line $y = x$.**

Write the ordered pairs as a vertex matrix. Then multiply the vertex matrix by the reflection matrix for $y = x$.

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 & 1 \\ 5 & 4 & -1 \end{bmatrix} = \begin{bmatrix} 5 & 4 & -1 \\ 3 & -2 & 1 \end{bmatrix}$$

The coordinates of the vertices of $A'B'C'$ are $A'(5, 3)$, $B'(4, -2)$, and $C'(-1, 1)$.

Exercises

1. The coordinates of the vertices of quadrilateral $ABCD$ are $A(-2, 1)$, $B(-1, 3)$, $C(2, 2)$, and $D(2, -1)$. What are the coordinates of the vertices of the image $A'B'C'D'$ after a reflection over the y-axis?

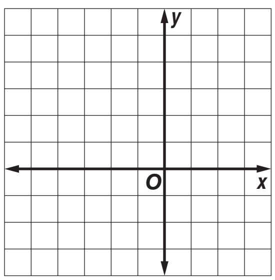
2. Triangle DEF with vertices $D(-2, 5)$, $E(1, 4)$, and $F(0, -1)$ is rotated 90° counterclockwise about the origin.

a. Write the coordinates of the triangle in a vertex matrix.

b. Write the rotation matrix for this situation.

c. Find the coordinates of the vertices of $\triangle D'E'F'$.

d. Graph $\triangle DEF$ and $\triangle D'E'F'$.

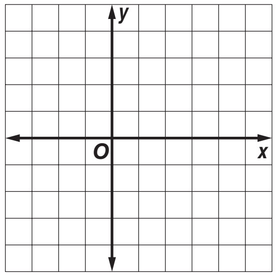


4-4 Skills Practice

Transformations with Matrices

For Exercises 1–3, use the following information.

Triangle ABC with vertices $A(2, 3)$, $B(0, 4)$, and $C(-3, -3)$ is translated 3 units right and 1 unit down.



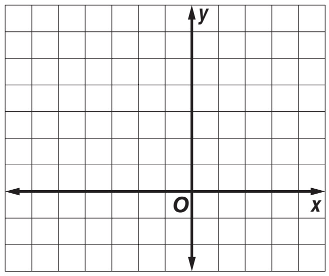
1. Write the translation matrix.

2. Find the coordinates of $\triangle A'B'C'$.

3. Graph the preimage and the image.

For Exercises 4–6, use the following information.

The vertices of $\triangle RST$ are $R(-3, 1)$, $S(2, -1)$, and $T(1, 3)$. The triangle is dilated so that its perimeter is twice the original perimeter.



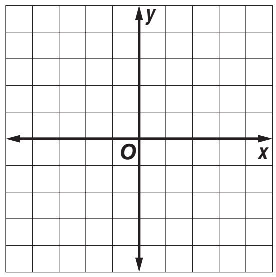
4. Write the coordinates of $\triangle RST$ in a vertex matrix.

5. Find the coordinates of the image $\triangle R'S'T'$.

6. Graph $\triangle RST$ and $\triangle R'S'T'$.

For Exercises 7–10, use the following information.

The vertices of $\triangle DEF$ are $D(4, 0)$, $E(0, -1)$, and $F(2, 3)$. The triangle is reflected over the x-axis.



7. Write the coordinates of $\triangle DEF$ in a vertex matrix.

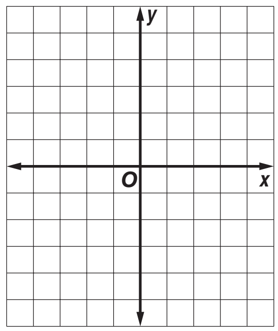
8. Write the reflection matrix for this situation.

9. Find the coordinates of $\triangle D'E'F'$.

10. Graph $\triangle DEF$ and $\triangle D'E'F'$.

For Exercises 11–14, use the following information.

Triangle XYZ with vertices $X(1, -3)$, $Y(-4, 1)$, and $Z(-2, 5)$ is rotated 180° counterclockwise about the origin.



11. Write the coordinates of the triangle in a vertex matrix.

12. Write the rotation matrix for this situation.

13. Find the coordinates of $\triangle X'Y'Z'$.

14. Graph the preimage and the image.

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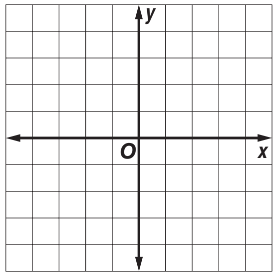
4-4 Practice

Transformations with Matrices

For Exercises 1–3, use the following information.

Quadrilateral *WXYZ* with vertices $W(-3, 2)$, $X(-2, 4)$, $Y(4, 1)$, and $Z(3, 0)$ is translated 1 unit left and 3 units down.



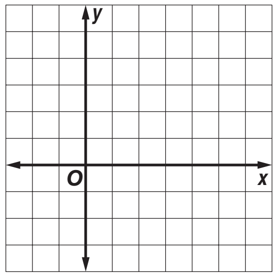
1. Write the translation matrix.

2. Find the coordinates of quadrilateral $W'X'Y'Z'$.

3. Graph the preimage and the image.

For Exercises 4–6, use the following information.

The vertices of $\triangle RST$ are $R(6, 2)$, $S(3, -3)$, and $T(-2, 5)$. The triangle is dilated so that its perimeter is one half the original perimeter.



4. Write the coordinates of $\triangle RST$ in a vertex matrix.

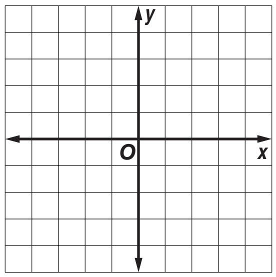
5. Find the coordinates of the image $\triangle R'S'T'$.

6. Graph $\triangle RST$ and $\triangle R'S'T'$.

For Exercises 7–10, use the following information.

The vertices of quadrilateral *ABCD* are $A(-3, 2)$, $B(0, 3)$, $C(4, -4)$, and $D(-2, -2)$. The quadrilateral is reflected over the *y*-axis.

7. Write the coordinates of *ABCD* in a vertex matrix.



8. Write the reflection matrix for this situation.

9. Find the coordinates of $A'B'C'D'$.

10. Graph *ABCD* and $A'B'C'D'$.

11. ARCHITECTURE Using architectural design software, the Bradleys plot their kitchen plans on a grid with each unit representing 1 foot. They place the corners of an island at (2, 8), (8, 11), (3, 5), and (9, 8). If the Bradleys wish to move the island 1.5 feet to the right and 2 feet down, what will the new coordinates of its corners be?

12. BUSINESS The design of a business logo calls for locating the vertices of a triangle at (1.5, 5), (4, 1), and (1, 0) on a grid. If design changes require rotating the triangle 90º counterclockwise, what will the new coordinates of the vertices be?

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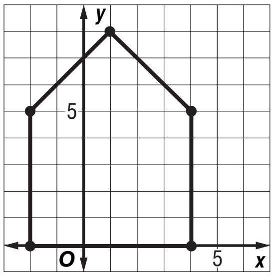
4-4 Word Problem Practice

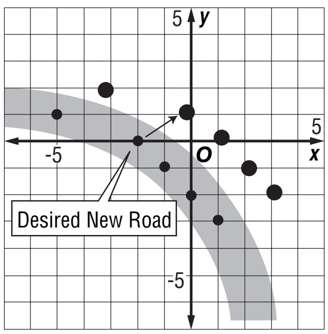
Transformations with Matrices

1. ICONS Louis needs to perform many matrix transformations to the basic house icon shown in the graph.



What is the vertex matrix for this image?

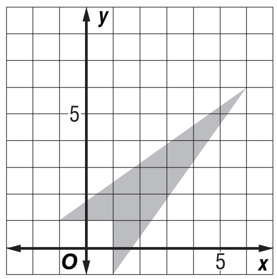
2. RELOCATION City planners are making a new road. Unfortunately, the road will pass through five ancient trees indicated by the small dots. The planners decide to move the trees to the locations indicated by the large dots. What matrix represents this translation?



3. MIRROR SYMMETRY A detective found only half of an image with mirror symmetry about the line $y = x$. The vertex matrix of the visible part is $\begin{bmatrix} 4 & 5 & -2 \\ 2 & -5 & -4 \end{bmatrix}$. What are the coordinates of the hidden vertices?

4. PHOTOGRAPHY Alejandra used a digital camera to take a picture. Because she held the camera sideways, the image on her computer screen appeared sideways. In order to transform the picture, she needed to perform a 90° clockwise rotation. What matrix represents this transformation?

ARROWS For Exercises 5–6, use the following information.
A compass arrow is pointing Northeast.



5. What is the vertex matrix for the arrow?

6. What would the vertex matrix be for the arrow if it were pointing Northwest? (Hint: Rotate 90° around the origin.)

Lesson 4-4

4-4 Enrichment

Computer Graphics Using Matrix Transformations

Computer animation creates a sensation of movement by slowly changing the position of pixels of an image on a two-dimensional video screen. By selecting the center of the screen as the origin, each pixel can be located using ordered pairs. A series of delayed matrix transformations (transformation, pause, transformation, pause) applied to the coordinates of the point (pixel) produces the desired animation effect.

Two computer programmers, Nate and Daniel, are writing a computer animation program using only the two matrices $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. Nate argues that it does not make a difference which matrix is applied first. Daniel disagrees.

1. What transformation does each matrix represent?

2. Find AB and BA. Determine whether Nate or Daniel is correct.

3. A third programmer joins Nate and Daniel. She convinces them to expand their transformation capabilities by including a reflection about the y-axis using $C = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

a. Determine a transformation using only B and C that is equivalent to the transformation A.

b. Write an expression for a 180° counterclockwise rotation in terms of the reflections A and C.

c. Is it possible to express a 270° counterclockwise rotation in terms of the reflections A and C? Explain why or why not.

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4-4 Graphing Calculator Activity

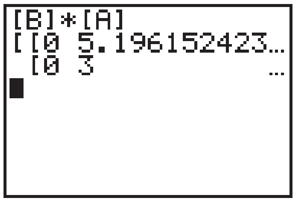
Matrices for 30°, 45°, 60° Rotations

The rotation matrix for 90° counterclockwise about the origin is $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

The general rotation matrix for any angle θ counterclockwise about the origin is $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$.

Example 1 **Find the coordinates of the image of $\triangle ABC$ with vertices $A(0, 0)$, $B(6, 0)$, and $C(3, 4)$ after a counterclockwise rotation of 30° about the origin.**

Enter the coordinates of the vertices of the in vertex matrix [A] and the rotation matrix in matrix [B]. Be sure the calculator is set in Degree mode. Keystrokes: 2nd [MATRX] ▶ ▶ ENTER 2 ENTER 3 ENTER 0 ENTER 6 ENTER 3 ENTER 0 ENTER 0 ENTER 4 ENTER 2nd [MATRX] ▶ ▶ 2 2 ENTER 2 ENTER COS 30) ENTER (–) SIN 30) ENTER SIN 30) COS 30) ENTER 2nd [QUIT] 2nd [MATRX] 2 2nd [MATRX] 1 ENTER. Hold ▶ to scroll across to see the other coordinates.

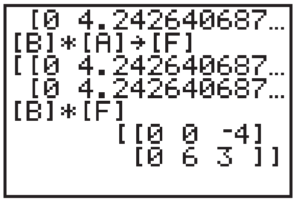


The coordinates of the image of $\triangle ABC$ are $A'(0, 0)$, $B'(5.2, 3)$, and $C'(0.6, 5.0)$.

Example 2 **Find the coordinates of the image of $\triangle ABC$ with vertices $A(0, 0)$, $B(6, 0)$, and $C(3, 4)$ after two rotations of 45° counterclockwise about the origin.**

In order to rotate the image twice, store the vertex matrix of the first image.

Keystrokes: 2nd [MATRX] ▶ ▶ 2 ENTER ENTER COS 45) ENTER (–) SIN 45) ENTER SIN 45) COS 45) ENTER 2nd [QUIT] 2nd [MATRX] 2 2nd [MATRX] 1 ENTER 2nd [ENTRY] STO▸ 2nd [MATRX] 6 ENTER 2nd [ENTRY] 2nd [ENTRY] ◀ 2nd [MATRX] 6 ENTER.



The vertices of $\triangle A'B'C'$ are $A'(0, 0)$, $B'(0, 6)$, and $C'(-4, 3)$.

Exercises

Quadrilateral $ABCD$ has vertices $A(0, 0)$, $B(4, 0)$, $C(4, 6)$, and $D(0, 6)$. Find the coordinates of the vertices of the image after each counterclockwise rotation. Round to the nearest tenth.

1. 45° **2.** 30° **3.** 60°

4. 120° **5.** 75° **6.** 225°

Lesson 4-4

4-5 Lesson Reading Guide

Determinants

Get Ready for the Lesson

Read the introduction to Lesson 4-5 in your textbook.

In this lesson, you will learn how to find the area of a triangle if you know the coordinates of its vertices using determinants. Describe a method you already know for finding the area of the Bermuda Triangle.

Read the Lesson

1. Indicate whether each of the following statements is *true* or *false*.

a. Every matrix has a determinant.

b. If both rows of a 2×2 matrix are identical, the determinant of the matrix will be 0.

c. Every element of a 3×3 matrix has a minor.

d. In order to evaluate a third-order determinant by expansion by minors it is necessary to find the minor of every element of the matrix.

e. If you evaluate a third-order determinant by expansion about the second row, the position signs you will use are $- + -$.

2. Suppose that triangle RST has vertices $R(-2, 5)$, $S(4, 1)$, and $T(0, 6)$.

a. Write a determinant that could be used in finding the area of triangle RST.

b. Explain how you would use the determinant you wrote in part **a** to find the area of the triangle.

Remember What You Learned

3. A good way to remember a complicated procedure is to break it down into steps. Write a list of steps for evaluating a third-order determinant using expansion by minors.

4-5 Study Guide and Intervention

Determinants

Determinants of 2 × 2 Matrices

Second-Order Determinant	For the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the determinant is $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$.

Example **Find the value of each determinant.**

a. $\begin{vmatrix} \mathbf{6} & \mathbf{3} \\ \mathbf{-8} & \mathbf{5} \end{vmatrix}$

$$\begin{vmatrix} 6 & 3 \\ -8 & 5 \end{vmatrix} = 6(5) - 3(-8)$$
$$= 30 - (-24) \text{ or } 54$$

b. $\begin{vmatrix} \mathbf{11} & \mathbf{-5} \\ \mathbf{9} & \mathbf{3} \end{vmatrix}$

$$\begin{vmatrix} 11 & -5 \\ 9 & 3 \end{vmatrix} = 11(-3) - (-5)(9)$$
$$= -33 - (-45) \text{ or } 78$$

Exercises

Find the value of each determinant.

1. $\begin{vmatrix} 6 & -2 \\ 5 & 7 \end{vmatrix}$

2. $\begin{vmatrix} -8 & 3 \\ -2 & 1 \end{vmatrix}$

3. $\begin{vmatrix} 3 & 9 \\ 4 & 6 \end{vmatrix}$

4. $\begin{vmatrix} 5 & 12 \\ -7 & -4 \end{vmatrix}$

5. $\begin{vmatrix} -6 & -3 \\ -4 & -1 \end{vmatrix}$

6. $\begin{vmatrix} 4 & 7 \\ 5 & 9 \end{vmatrix}$

7. $\begin{vmatrix} 14 & 8 \\ 9 & -3 \end{vmatrix}$

8. $\begin{vmatrix} 15 & 12 \\ 23 & 28 \end{vmatrix}$

9. $\begin{vmatrix} -8 & 35 \\ 5 & 20 \end{vmatrix}$

10. $\begin{vmatrix} 10 & 16 \\ 22 & 40 \end{vmatrix}$

11. $\begin{vmatrix} 24 & -8 \\ 7 & -3 \end{vmatrix}$

12. $\begin{vmatrix} 13 & 62 \\ -4 & 19 \end{vmatrix}$

13. $\begin{vmatrix} 0.2 & 8 \\ -1.5 & 15 \end{vmatrix}$

14. $\begin{vmatrix} 8.6 & 0.5 \\ 14 & 5 \end{vmatrix}$

15. $\begin{vmatrix} 20 & 110 \\ 0.1 & 1.4 \end{vmatrix}$

16. $\begin{vmatrix} 4.8 & 2.1 \\ 3.4 & 5.3 \end{vmatrix}$

17. $\begin{vmatrix} \frac{2}{3} & -\frac{1}{2} \\ \frac{1}{6} & \frac{1}{5} \end{vmatrix}$

18. $\begin{vmatrix} 6.8 & 15 \\ -0.2 & 5 \end{vmatrix}$

4-5 Study Guide and Intervention (continued)

Determinants

Determinants of 3 × 3 Matrices

Third-Order Determinants	$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$

Area of a Triangle	The area of a triangle having vertices (a, b), (c, d) and (e, f) is $\lvert A \rvert$, where $A = \frac{1}{2}\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{vmatrix}$.

Example **Evaluate** $\begin{vmatrix} 4 & 5 & -2 \\ 1 & 3 & 0 \\ 2 & -3 & 6 \end{vmatrix}$.

$$\begin{vmatrix} 4 & 5 & -2 \\ 1 & 3 & 0 \\ 2 & -3 & 6 \end{vmatrix} = 4\begin{vmatrix} 3 & 0 \\ -3 & 6 \end{vmatrix} - 5\begin{vmatrix} 1 & 0 \\ 2 & 6 \end{vmatrix} - 2\begin{vmatrix} 1 & 3 \\ 2 & -3 \end{vmatrix}$$ Third-order determinant

$= 4(18 - 0) - 5(6 - 0) - 2(-3 - 6)$ Evaluate 2 × 2 determinants.

$= 4(18) - 5(6) - 2(-9)$ Simplify.

$= 72 - 30 + 18$ Multiply.

$= 60$ Simplify.

Exercises

Evaluate each determinant.

1. $\begin{vmatrix} 3 & -2 & -2 \\ 0 & 4 & 1 \\ -1 & 5 & -3 \end{vmatrix}$

2. $\begin{vmatrix} 4 & 1 & 0 \\ -2 & 3 & 1 \\ 2 & -2 & 5 \end{vmatrix}$

3. $\begin{vmatrix} 6 & 1 & 4 \\ -2 & 3 & 0 \\ -1 & 3 & 2 \end{vmatrix}$

4. $\begin{vmatrix} 5 & -2 & 2 \\ 3 & 0 & -2 \\ 2 & 4 & -3 \end{vmatrix}$

5. $\begin{vmatrix} 6 & 1 & -4 \\ 3 & 2 & 1 \\ -2 & 2 & -1 \end{vmatrix}$

6. $\begin{vmatrix} 5 & -4 & 1 \\ 2 & 3 & -2 \\ -1 & 6 & -3 \end{vmatrix}$

7. Find the area of a triangle with vertices $X(2, -3)$, $Y(7, 4)$, and $Z(-5, 5)$.

4-5 Skills Practice

Determinants

Find the value of each determinant.

1. $\begin{vmatrix} 5 & 2 \\ 1 & 3 \end{vmatrix}$

2. $\begin{vmatrix} 10 & 9 \\ 5 & 8 \end{vmatrix}$

3. $\begin{vmatrix} 1 & 6 \\ 1 & 7 \end{vmatrix}$

4. $\begin{vmatrix} 2 & 5 \\ 3 & 1 \end{vmatrix}$

5. $\begin{vmatrix} 0 & 9 \\ 5 & 8 \end{vmatrix}$

6. $\begin{vmatrix} 3 & 12 \\ 2 & 8 \end{vmatrix}$

7. $\begin{vmatrix} -5 & 2 \\ 8 & -6 \end{vmatrix}$

8. $\begin{vmatrix} -3 & 1 \\ 8 & -7 \end{vmatrix}$

9. $\begin{vmatrix} 9 & -2 \\ -4 & 1 \end{vmatrix}$

10. $\begin{vmatrix} 1 & -5 \\ 1 & 6 \end{vmatrix}$

11. $\begin{vmatrix} 1 & -3 \\ -3 & 4 \end{vmatrix}$

12. $\begin{vmatrix} -12 & 4 \\ 1 & 4 \end{vmatrix}$

13. $\begin{vmatrix} 3 & -5 \\ 6 & -11 \end{vmatrix}$

14. $\begin{vmatrix} -1 & -3 \\ 5 & -2 \end{vmatrix}$

15. $\begin{vmatrix} -1 & -14 \\ 5 & 2 \end{vmatrix}$

16. $\begin{vmatrix} -1 & 2 \\ 0 & 4 \end{vmatrix}$

17. $\begin{vmatrix} 2 & 2 \\ -1 & 4 \end{vmatrix}$

18. $\begin{vmatrix} -1 & 6 \\ 2 & 5 \end{vmatrix}$

Evaluate each determinant using expansion by minors.

19. $\begin{vmatrix} 2 & -1 & 1 \\ 3 & 2 & -1 \\ 2 & 3 & -2 \end{vmatrix}$

20. $\begin{vmatrix} 6 & -1 & 1 \\ 5 & 2 & -1 \\ 1 & 3 & -2 \end{vmatrix}$

21. $\begin{vmatrix} 2 & 6 & 1 \\ 3 & 5 & -1 \\ 2 & 1 & -2 \end{vmatrix}$

Evaluate each determinant using diagonals.

22. $\begin{vmatrix} 2 & -1 & 6 \\ 3 & 2 & 5 \\ 2 & 3 & 1 \end{vmatrix}$

23. $\begin{vmatrix} 3 & -1 & 2 \\ 1 & 0 & 4 \\ 3 & -2 & 0 \end{vmatrix}$

24. $\begin{vmatrix} 3 & 2 & 2 \\ 1 & -1 & 4 \\ 3 & -1 & 0 \end{vmatrix}$

4-5 Practice

Determinants

Find the value of each determinant.

1. $\begin{vmatrix} 1 & 6 \\ 2 & 7 \end{vmatrix}$

2. $\begin{vmatrix} 9 & 6 \\ 3 & 2 \end{vmatrix}$

3. $\begin{vmatrix} 4 & 1 \\ -2 & -5 \end{vmatrix}$

4. $\begin{vmatrix} -14 & -3 \\ 2 & -2 \end{vmatrix}$

5. $\begin{vmatrix} 4 & -3 \\ -12 & 4 \end{vmatrix}$

6. $\begin{vmatrix} 2 & -5 \\ 5 & -11 \end{vmatrix}$

7. $\begin{vmatrix} 4 & 0 \\ -2 & 9 \end{vmatrix}$

8. $\begin{vmatrix} 3 & -4 \\ 7 & 9 \end{vmatrix}$

9. $\begin{vmatrix} -1 & -11 \\ 10 & -2 \end{vmatrix}$

10. $\begin{vmatrix} 3 & -4 \\ 3.75 & 5 \end{vmatrix}$

11. $\begin{vmatrix} 2 & -1 \\ 3 & -9.5 \end{vmatrix}$

12. $\begin{vmatrix} 0.5 & -0.7 \\ 0.4 & -0.3 \end{vmatrix}$

Evaluate each determinant using expansion by minors.

13. $\begin{vmatrix} -2 & 3 & 1 \\ 0 & 4 & -3 \\ 2 & 5 & -1 \end{vmatrix}$

14. $\begin{vmatrix} 2 & -4 & 1 \\ 3 & 0 & 9 \\ -1 & 5 & 7 \end{vmatrix}$

15. $\begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & -2 \\ 1 & 1 & -1 \end{vmatrix}$

16. $\begin{vmatrix} 0 & -4 & 0 \\ 2 & -1 & 1 \\ 3 & -2 & 5 \end{vmatrix}$

17. $\begin{vmatrix} 2 & 7 & -6 \\ 8 & 4 & 0 \\ 1 & -1 & 3 \end{vmatrix}$

18. $\begin{vmatrix} -12 & 0 & 3 \\ 7 & 5 & -1 \\ 4 & 2 & -6 \end{vmatrix}$

Evaluate each determinant using diagonals.

19. $\begin{vmatrix} -4 & 3 & -1 \\ 2 & 1 & -2 \\ 4 & 1 & -4 \end{vmatrix}$

20. $\begin{vmatrix} 2 & 2 & 3 \\ 1 & -1 & 1 \\ 3 & 1 & 1 \end{vmatrix}$

21. $\begin{vmatrix} 1 & -4 & -1 \\ 1 & -6 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

22. $\begin{vmatrix} 1 & 2 & -4 \\ 1 & 4 & -6 \\ 2 & 3 & 3 \end{vmatrix}$

23. $\begin{vmatrix} 2 & -1 & -2 \\ 4 & 0 & -2 \\ 0 & 3 & 2 \end{vmatrix}$

24. $\begin{vmatrix} 2 & 1 & 3 \\ 1 & 8 & 0 \\ 0 & 5 & -1 \end{vmatrix}$

25. GEOMETRY Find the area of a triangle whose vertices have coordinates (3, 5), (6, −5), and (−4, 10).

26. LAND MANAGEMENT A fish and wildlife management organization uses a GIS (geographic information system) to store and analyze data for the parcels of land it manages. All of the parcels are mapped on a grid in which 1 unit represents 1 acre. If the coordinates of the corners of a parcel are (−8, 10), (6, 17), and (2, −4), how many acres is the parcel?

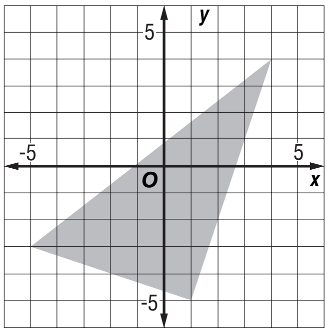
4-5 Word Problem Practice

Determinants

1. FIND THE ERROR Mark's determinant computation has sign errors. Circle the signs that must be reversed.

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 1(5)(9) - 2(6)(7) + 3(4)(8) - 3(5)(7) + 1(6)(8) - 2(4)(9)$$

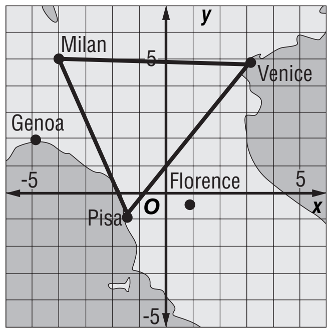
2. POOL An architect has a pool in the floor plans for a home. Set up a determinant that gives the unit area of the pool.



3. HALF-UNIT TRIANGLES For a school art project, students had to decorate a pegboard by looping strings around the pegs. Ronald wanted to make triangles with areas of one half square unit. Because Ronald had studied determinants, he knew that this was essentially the same as finding the coordinates of the vertices of a triangle (a, b), (c, d) and (e, f), so that the determinant $\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{vmatrix}$ is 1 or -1.

Give an example of such a triangle.

4. ITALY The figure shows a map of Italy overlaid on a graph. The coordinates of Milan, Venice, and Pisa are about $(-4, 5)$, $(3.25, 4.8)$, and $(-1.4, -0.8)$, respectively. Each square unit on the map represents about 400 square miles.



What is the area of the triangular region? Round your answer to the nearest square mile.

ARROWS For Exercises 5 and 6, use the following information.
Kyle is making a triangle with vertices at $(-6, 0)$, $(0, -x)$, and $(0, x)$, and $x > 0$. He plans to make the triangle using a material that costs \$2 for every square unit.

5. Write the determinant that gives the area of this triangle.

6. Evaluate the determinant you wrote for Exercise 5 and determine the value of x that results in a \$60 triangle.

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4-5 Enrichment

Matrix Transpose and Determinants

In Lesson 4-1, you learned how to represent information in matrices. A matrix contains elements of the form a_{ij} where i is the row number of the element and j is the columns number of the element.

Consider the following matrix.

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$$

In this matrix, $a_{11} = 2$, $a_{12} = -1$, $a_{21} = 3$, and $a_{22} = 4$.

The matrix transpose can be found by switching the elements around. Element a_{ij} becomes element a_{ji}. So, the matrix transpose of A, denoted by A^T is:

$$A^T = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$$

Calculate the determinant of A and A^T.

$\det(A) = 2(4) - 3(-1) = 11$

$\det(A^T) = 2(4) - (-1)(3) = 11$

1. Find each matrix transpose.

a. $B = \begin{bmatrix} -1 & 5 \\ 2 & 6 \end{bmatrix}$ **b.** $C = \begin{bmatrix} 1 & 0 \\ -3 & 4 \end{bmatrix}$ **c.** $D = \begin{bmatrix} 2 & 3 & -1 \\ -2 & -1 & 5 \\ 1 & 3 & -2 \end{bmatrix}$

2. Find the determinants of the original matrices and the transposes from Exercise 1.

3. What do you notice about the determinants? Make a conjecture about the determinant of a matrix and the determinant of its transpose.

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4-5 Spreadsheet Activity

Cramer's Rule

You have learned to solve systems of linear equations by using matrix equations and the inverse matrix. Another way to solve systems is to use *Cramer's Rule*. Study the spreadsheet below to discover Cramer's Rule.

Excel sample.xls

	A	B	C
1	6	3	-12
2	5	1	8
3			
4	=A1*B2-B1*A2		
5			
6	=C1*B2-B1*C2		
7			
8	=A1*C2-C1*A2		
9			
10	x =	=(A6/A4)	
11	y =	=(A8/A4)	

Sheet 1 | Sheet 2 | Sheet 3

To use the spreadsheet to solve a system of equations, write each equation in the form below.

$ax + by = c$

The values for the system $6x + 3y = -12$ and $5x + y = 8$ are shown. In the spreadsheet, the values of a, b, and c for the first equation are entered in cells A1, B1, and C1, respectively. The values of a, b, and c for the second equation are entered in cells A2, B2, and C2, respectively.

The values in cells B10 and B11 represent the solution for the system.

Exercises

1. Study the formula in cell A4. Write a matrix whose determinant is found using this formula.

2. Write matrices whose determinants are found using the formulas in cells A6 and A8.

3. Explain how the values of x and y are found using Cramer's rule.

Use the spreadsheet to solve each system of equations.

4. $6x + 3y = -12$
$5x + y = 8$

5. $5x - 3y = 19$
$7x + 2y = 8$

6. $8x - 3y = 11$
$6x + 9y = 15$

7. $0.3x + 1.6y = 0.44$
$0.4x + 2.5y = 0.66$

8. $3y = 4x + 28$
$5x + 7y = 8$

9. $y = -0.5x + 4$
$y = 4x - 5$

4-6 Lesson Reading Guide

Cramer's Rule

Get Ready for the Lesson

Read the introduction to Lesson 4-6 in your textbook.

A triangle is bounded by the x-axis, the line $y = \frac{1}{2}x$, and the line $y = -2x + 10$. Write three systems of equations that you could use to find the three vertices of the triangle. (Do not actually find the vertices.)

Read the Lesson

1. Suppose that you are asked to solve the following system of equations by Cramer's Rule.

$3x + 2y = 7$
$2x - 3y = 22$

Without actually evaluating any determinants, indicate which of the following ratios of determinants gives the correct value for x.

A. $\dfrac{\begin{vmatrix} 3 & 2 \\ 2 & -3 \end{vmatrix}}{\begin{vmatrix} 7 & 2 \\ 22 & -3 \end{vmatrix}}$ **B.** $\dfrac{\begin{vmatrix} 7 & 2 \\ 22 & -3 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 2 & -3 \end{vmatrix}}$ **C.** $\dfrac{\begin{vmatrix} 3 & 7 \\ 2 & 22 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 2 & -3 \end{vmatrix}}$

2. In your textbook, the statements of Cramer's Rule for two variables and three variables specify that the determinant formed from the coefficients of the variables cannot be 0. If the determinant is zero, what do you know about the system and its solutions?

Remember What You Learned

3. Some students have trouble remembering how to arrange the determinants that are used in solving a system of two linear equations by Cramer's Rule. What is a good way to remember this?

4-6 Study Guide and Intervention

Cramer's Rule

Systems of Two Linear Equations Determinants provide a way for solving systems of equations.

Cramer's Rule for Two-Variable Systems	The solution of the linear system of equations $ax + by = e$, $cx + dy = f$ is (x, y) where $x = \dfrac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$, $y = \dfrac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$, and $\begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0$.

Example **Use Cramer's Rule to solve the system of equations. $5x - 10y = 8$, $10x + 25y = -2$**

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} \qquad \text{Cramer's Rule} \qquad y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$

$$= \frac{\begin{vmatrix} 8 & -10 \\ -2 & 25 \end{vmatrix}}{\begin{vmatrix} 5 & -10 \\ 10 & 25 \end{vmatrix}} \qquad a = 5, b = -10, c = 10, d = 25, e = 8, f = -2 \qquad = \frac{\begin{vmatrix} 5 & 8 \\ 10 & -2 \end{vmatrix}}{\begin{vmatrix} 5 & -10 \\ 10 & 25 \end{vmatrix}}$$

$$= \frac{8(25) - (-2)(-10)}{5(25) - (-10)(10)} \qquad \text{Evaluate each determinant.} \qquad = \frac{5(-2) - 8(10)}{5(25) - (-10)(10)}$$

$$= \frac{180}{225} \text{ or } \frac{4}{5} \qquad \text{Simplify.} \qquad = -\frac{90}{225} \text{ or } -\frac{2}{5}$$

The solution is $\left(\frac{4}{5}, -\frac{2}{5}\right)$.

Exercises

Use Cramer's Rule to solve each system of equations.

1. $3x - 2y = 7$
$2x + 7y = 38$

2. $x - 4y = 17$
$3x - y = 29$

3. $2x - y = -2$
$4x - y = 4$

4. $2x - y = 1$
$5x + 2y = -29$

5. $4x + 2y = 1$
$5x - 4y = 24$

6. $6x - 3y = -3$
$2x + y = 21$

7. $2x + 7y = 16$
$x - 2y = 30$

8. $2x - 3y = -2$
$3x - 4y = 9$

9. $\frac{x}{3} + \frac{y}{5} = 2$
$\frac{x}{4} - \frac{y}{6} = -8$

10. $6x - 9y = -1$
$3x + 18y = 12$

11. $3x - 12y = -14$
$9x + 6y = -7$

12. $8x + 2y = \frac{3}{7}$
$5x - 4y = -\frac{27}{7}$

4-6 Study Guide and Intervention *(continued)*

Cramer's Rule

Systems of Three Linear Equations

Cramer's Rule for Three-Variable Systems	The solution of the system whose equations are $ax + by + cz = j$ $dx + ey + fz = k$ $gx + hy + iz = l$ is (x, y, z) where $x = \dfrac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}}$, $y = \dfrac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}}$, and $z = \dfrac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & l \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}}$ and $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \neq$ 0.

Example

Use Cramer's rule to solve the system of equations.

$\mathbf{6x + 4y + z = 5}$
$\mathbf{2x + 3y - 2z = -2}$
$\mathbf{8x - 2y + 2z = 10}$

Use the coefficients and constants from the equations to form the determinants. Then evaluate each determinant.

$$x = \frac{\begin{vmatrix} 5 & 4 & 1 \\ -2 & 3 & -2 \\ 10 & -2 & 2 \end{vmatrix}}{\begin{vmatrix} 6 & 4 & 1 \\ 2 & 3 & -2 \\ 8 & -2 & 2 \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} 6 & 5 & 1 \\ 2 & -2 & -2 \\ 8 & 10 & 2 \end{vmatrix}}{\begin{vmatrix} 6 & 4 & 1 \\ 2 & 3 & -2 \\ 8 & -2 & 2 \end{vmatrix}} \qquad z = \frac{\begin{vmatrix} 6 & 4 & 5 \\ 2 & 3 & -2 \\ 8 & -2 & 10 \end{vmatrix}}{\begin{vmatrix} 6 & 4 & 1 \\ 2 & 3 & -2 \\ 8 & -2 & 2 \end{vmatrix}}$$

$$= \frac{-80}{-96} \text{ or } \frac{5}{6} \qquad = \frac{32}{-96} \text{ or } -\frac{1}{3} \qquad = \frac{-128}{-96} \text{ or } \frac{4}{3}$$

The solution is $\left(\frac{5}{6}, -\frac{1}{3}, \frac{4}{3}\right)$.

Exercises

Use Cramer's rule to solve each system of equations.

1. $x - 2y + 3z = 6$
$2x - y - z = -3$
$x + y + z = 6$

2. $3x + y - 2z = -2$
$4x - 2y - 5z = 7$
$x + y + z = 1$

3. $x - 3y + z = 1$
$2x + 2y - z = -8$
$4x + 7y + 2z = 11$

4. $2x - y + 3z = -5$
$x + y - 5z = 21$
$3x - 2y - 4z = 6$

5. $3x + y - 4z = 7$
$2x - y + 5z = -24$
$10x + 3y - 2z = -2$

6. $2x - y + 4z = 9$
$3x - 2y - 5z = -13$
$x + y - 7z = 0$

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4-6 Skills Practice

Cramer's Rule

Use Cramer's Rule to solve each system of equations.

1. $2a + 3b = 6$
$2a + b = -2$

2. $3x + y = 2$
$2x - y = 3$

3. $2m + 3n = -6$
$m - 3n = 6$

4. $x - y = 2$
$2x + 3y = 9$

5. $2x + y = 4$
$7x - 2y = 3$

6. $3r - s = 7$
$5r - 2s = 8$

7. $4g + 5h = 1$
$g + 3h = 2$

8. $7x + 5y = -8$
$9x + 2y = 3$

9. $3x - 4y = 2$
$4x - 3y = 12$

10. $2x - y = 5$
$3x + y = 5$

11. $3p - 6q = 18$
$2p + 3q = 5$

12. $x - 2y = -1$
$2x + y = 3$

13. $5c + 3d = 5$
$2c + 9d = 2$

14. $5t + 2v = 2$
$2t + 3v = -8$

15. $5a - 2b = 14$
$3a + 4b = 11$

16. $65w - 8z = 83$
$9w + 4z = 0$

17. GEOMETRY The two sides of an angle are contained in the lines whose equations are $3x + 2y = 4$ and $x - 3y = 5$. Find the coordinates of the vertex of the angle.

Use Cramer's Rule to solve each system of equations.

18. $a + b + 5c = 2$
$3a + b + 2c = 3$
$4a + 2b - c = -3$

19. $x + 3y - z = 5$
$2x + 5y - z = 12$
$x - 2y - 3z = -13$

20. $3c - 5d + 2e = 4$
$2c - 3d + 4c = -3$
$4c - 2d + 3e = 0$

21. $r - 4s - t = 6$
$2r - s + 3t = 0$
$3r - 2s + t = 4$

4-6 Practice

Cramer's Rule

Use Cramer's Rule to solve each system of equations.

1. $2x + y = 0$
$3x + 2y = -2$

2. $5c + 9d = 19$
$2c - d = -20$

3. $2x + 3y = 5$
$3x - 2y = 1$

4. $20m - 3n = 28$
$2m + 3n = 16$

5. $x - 3y = 6$
$3x + y = -22$

6. $5x - 6y = -45$
$9x + 8y = 13$

7. $-2e + f = 4$
$-3e + 5f = -15$

8. $2x - y = -1$
$2x - 4y = 8$

9. $8a + 3b = 24$
$2a + b = 4$

10. $-3x + 15y = 45$
$-2x + 7y = 18$

11. $3u - 5v = 11$
$6u + 7v = -12$

12. $-6g + h = -10$
$-3g - 4h = 4$

13. $x - 3y = 8$
$x - 0.5y = 3$

14. $0.2x - 0.5y = -1$
$0.6x - 3y = -9$

15. $0.3d - 0.6g = 1.8$
$0.2d + 0.3g = 0.5$

16. GEOMETRY The two sides of an angle are contained in the lines whose equations are $x - \frac{4}{3}y = 6$ and $2x + y = 1$. Find the coordinates of the vertex of the angle.

17. GEOMETRY Two sides of a parallelogram are contained in the lines represented by the equations $0.2x - 0.5y = 1$ and $0.02x - 0.3y = -0.9$. Find the coordinates of a vertex of the parallelogram.

Use Cramer's Rule to solve each system of equations.

18. $x + 3y + 3z = 4$
$-x + 2y + z = -1$
$4x + y - 2z = -1$

19. $-5a + b - 4c = 7$
$-3a + 2b - c = 0$
$2a + 3b - c = 17$

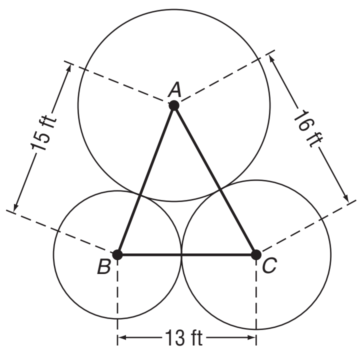
20. $2x + y - 3z = -5$
$5x + 2y - 2z = 8$
$3x - 3y + 5z = 17$

21. $2c + 3d - e = 17$
$4c + d + 5e = -9$
$c + 2d - e = 12$

22. $2j + k - 3m = -3$
$3j + 2k + 4m = 5$
$-4j - k + 2m = 4$

23. $3x - 2y + 5z = 3$
$2x + 2y - 4z = 3$
$-5x + 10y + 7z = -3$

24. LANDSCAPING A memorial garden being planted in front of a municipal library will contain three circular beds that are tangent to each other. A landscape architect has prepared a sketch of the design for the garden using CAD (computer-aided drafting) software, as shown at the right. The centers of the three circular beds are represented by points A, B, and C. The distance from A to B is 15 feet, the distance from B to C is 13 feet, and the distance from A to C is 16 feet. What is the radius of each of the circular beds?



4-6 Word Problem Practice

Cramer's Rule

1. USING CRAMER'S RULE Lucy is solving the following system of linear equations using Cramer's Rule.

$$2x + 3y = 5$$
$$x + y = 2$$

Write the three determinants she will have to compute.

2. IMPLICATIONS OF CRAMER'S RULE Cramer's Rule gives the solutions of systems of linear equations in terms of their coefficients. The formula involves addition, subtraction, multiplication, and division of those coefficients. Is it possible for an irrational number to be part of the solution of a system of linear equations whose coefficients are all rational numbers?

3. SHOPPING Sheets cost \$18.59 each and pillowcases cost \$7.24 at Carol's Linens. If Agatha buys x sheets and y pillowcases at Carol's Linens, she'll spend \$210.75. On the other hand, at Save-n-Sleep, sheets cost \$15.79 and pillowcases cost \$8.19. If Agatha buys x sheets and y pillows cases at Save-n-Sleep, she'll spend \$191.25. Use Cramer's Rule to determine how many sheets and pillow cases Agatha wants to buy.

4. BRICKS Linus owns three different types of brick that differ only in length. If he lines up 2 short, 1 medium, and 2 long bricks, the total length will be 45 inches. If he lines up 1 short, 2 medium, and 3 long bricks, the total length will be 59 inches. If he lines up 5 short, 1 medium, and 1 long brick, the total length will be 53 inches. Use Cramer's Rule to determine how long the different types of brick are.

PROMOTIONS For Exercises 5–7, use the following information.
A local zoo was trying to increase attendance by offering \$2 for every child that came. However, the zoo insisted that there be at least 1 adult for every 8 children. A school decided to take advantage of the situation by sending 1 adult for every 8 children. Let c be the number of children and let a be the number of adults. Admission for adults was d dollars. The total cost of admission for everyone was \$13.50.

5. Write a system of equations that describes the situation.

6. Is it possible that $d = 16$? Explain in terms of Cramer's Rule.

7. If adults were charged \$20.50 for admission, how many adults and children went? Use Cramer's Rule to solve.

4-6 Enrichment

Fourth-Order Determinants

To find the value of a 4 × 4 determinant, use a method called **expansion by minors**.

First write the expansion. Use the first row of the determinant.
Remember that the signs of the terms alternate.

$$\begin{vmatrix} 6 & -3 & 2 & 7 \\ 0 & 4 & 3 & 5 \\ 0 & 2 & 1 & -4 \\ 6 & 0 & -2 & 0 \end{vmatrix} = 6\begin{vmatrix} 4 & 3 & 5 \\ 2 & 1 & -4 \\ 0 & -2 & 0 \end{vmatrix} - (-3)\begin{vmatrix} 0 & 3 & 5 \\ 0 & 1 & -4 \\ 6 & -2 & 0 \end{vmatrix} + 2\begin{vmatrix} 0 & 4 & 5 \\ 0 & 2 & -4 \\ 6 & 0 & 0 \end{vmatrix} - 7\begin{vmatrix} 0 & 4 & 3 \\ 0 & 2 & 1 \\ 6 & 0 & -2 \end{vmatrix}$$

Then evaluate each 3 × 3 determinant. Use any row.

$$\begin{vmatrix} 4 & 3 & 5 \\ 2 & 1 & -4 \\ 0 & -2 & 0 \end{vmatrix} = -(-2)\begin{vmatrix} 4 & 5 \\ 2 & -4 \end{vmatrix}$$
$$= 2(-16 - 10)$$
$$= -52$$

$$\begin{vmatrix} 0 & 3 & 5 \\ 0 & 1 & -4 \\ 6 & -2 & 0 \end{vmatrix} = -3\begin{vmatrix} 0 & -4 \\ 6 & 0 \end{vmatrix} + 5\begin{vmatrix} 0 & 1 \\ 6 & -2 \end{vmatrix}$$
$$= -3(24) + 5(-6)$$
$$= -102$$

$$\begin{vmatrix} 0 & 4 & 5 \\ 0 & 2 & -4 \\ 6 & 0 & 0 \end{vmatrix} = 6\begin{vmatrix} 4 & 5 \\ 2 & -4 \end{vmatrix}$$
$$= 6(-16 - 10)$$
$$= -156$$

$$\begin{vmatrix} 0 & 4 & 3 \\ 0 & 2 & 1 \\ 6 & 0 & -2 \end{vmatrix} = -4\begin{vmatrix} 0 & 1 \\ 6 & -2 \end{vmatrix} + 3\begin{vmatrix} 0 & 2 \\ 6 & 0 \end{vmatrix}$$
$$= -4(-6) + 3(-12)$$
$$= -12$$

Finally, evaluate the original 4 × 4 determinant.

$$\begin{vmatrix} 6 & -3 & 2 & 7 \\ 0 & 4 & 3 & 5 \\ 0 & 2 & 1 & -4 \\ 6 & 0 & -2 & 0 \end{vmatrix} = 6(-52) + 3(-102) + 2(-156) - 7(-12) = -846$$

Evaluate each determinant.

1. $\begin{vmatrix} 1 & 2 & 3 & 1 \\ 4 & 3 & -1 & 0 \\ 2 & -5 & 4 & 4 \\ 1 & -2 & 0 & 2 \end{vmatrix}$

2. $\begin{vmatrix} 3 & 3 & 3 & 3 \\ 2 & 1 & 2 & 1 \\ 4 & 3 & -1 & 5 \\ 2 & 5 & 0 & 1 \end{vmatrix}$

3. $\begin{vmatrix} 1 & 4 & 3 & 0 \\ -2 & -3 & 6 & 4 \\ 5 & 1 & 1 & 2 \\ 4 & 2 & 5 & -1 \end{vmatrix}$

4-7 Lesson Reading Guide

Identity and Inverse Matrices

Get Ready for the Lesson

Read the introduction to Lesson 4-7 in your textbook.

Refer to the code table given in the introduction to this lesson. Suppose that you receive a message coded by this system as follows:

16 12 5 1 19 5   2 5   13 25   6 18 9 5 14 4.

Decode the message.

Read the Lesson

1. Indicate whether each of the following statements is *true* or *false*.

a. Every element of an identity matrix is 1.

b. There is a 3×2 identity matrix.

c. Two matrices are inverses of each other if their product is the identity matrix.

d. If M is a matrix, M^{-1} represents the reciprocal of M.

e. No 3×2 matrix has an inverse.

f. Every square matrix has an inverse.

g. If the two columns of a 2×2 matrix are identical, the matrix does not have an inverse.

2. Explain how to find the inverse of a 2×2 matrix. Do not use any special mathematical symbols in your explanation.

Remember What You Learned

3. One way to remember something is to explain it to another person. Suppose that you are studying with a classmate who is having trouble remembering how to find the inverse of a 2×2 matrix. He remembers how to move elements and change signs in the matrix, but thinks that he should multiply by the determinant of the original matrix. How can you help him remember that he must multiply by the *reciprocal* of this determinant?

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4-7 Study Guide and Intervention

Identity and Inverse Matrices

Identity and Inverse Matrices The identity matrix for matrix multiplication is a square matrix with 1s for every element of the main diagonal and zeros elsewhere.

Identity Matrix for Multiplication	If A is an $n \times n$ matrix and I is the identity matrix, then $A \cdot I = A$ and $I \cdot A = A$.

If an $n \times n$ matrix A has an inverse A^{-1}, then $A \cdot A^{-1} = A^{-1} \cdot A = I$.

Example **Determine whether $X = \begin{bmatrix} 7 & 4 \\ 10 & 6 \end{bmatrix}$ and $Y = \begin{bmatrix} 3 & -2 \\ -5 & \frac{7}{2} \end{bmatrix}$ are inverse matrices.**

Find $X \cdot Y$.

$$X \cdot Y = \begin{bmatrix} 7 & 4 \\ 10 & 6 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 \\ -5 & \frac{7}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 21 - 20 & -14 + 14 \\ 30 - 30 & -20 + 21 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Find $Y \cdot X$.

$$Y \cdot X = \begin{bmatrix} 3 & -2 \\ -5 & \frac{7}{2} \end{bmatrix} \cdot \begin{bmatrix} 7 & 4 \\ 10 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 21 - 20 & 12 - 12 \\ -35 + 35 & -20 + 21 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Since $X \cdot Y = Y \cdot X = I$, X and Y are inverse matrices.

Exercises

Determine whether each pair of matrices are inverses.

1. $\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$ and $\begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$

2. $\begin{bmatrix} 3 & 2 \\ 5 & 4 \end{bmatrix}$ and $\begin{bmatrix} 2 & -1 \\ -\frac{5}{2} & \frac{3}{2} \end{bmatrix}$

3. $\begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$ and $\begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$

4. $\begin{bmatrix} 8 & 11 \\ 3 & 4 \end{bmatrix}$ and $\begin{bmatrix} -4 & 11 \\ 3 & -8 \end{bmatrix}$

5. $\begin{bmatrix} 4 & -1 \\ 5 & 3 \end{bmatrix}$ and $\begin{bmatrix} 1 & 2 \\ 3 & 8 \end{bmatrix}$

6. $\begin{bmatrix} 5 & 2 \\ 11 & 4 \end{bmatrix}$ and $\begin{bmatrix} -2 & 1 \\ \frac{11}{2} & -\frac{5}{2} \end{bmatrix}$

7. $\begin{bmatrix} 4 & 2 \\ 6 & -2 \end{bmatrix}$ and $\begin{bmatrix} -\frac{1}{5} & -\frac{1}{10} \\ \frac{3}{10} & \frac{1}{10} \end{bmatrix}$

8. $\begin{bmatrix} 5 & 8 \\ 4 & 6 \end{bmatrix}$ and $\begin{bmatrix} -3 & 4 \\ 2 & -\frac{5}{2} \end{bmatrix}$

9. $\begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}$ and $\begin{bmatrix} \frac{7}{2} & -\frac{3}{2} \\ 1 & -2 \end{bmatrix}$

10. $\begin{bmatrix} 3 & 2 \\ 4 & -6 \end{bmatrix}$ and $\begin{bmatrix} 3 & 2 \\ -4 & -3 \end{bmatrix}$

11. $\begin{bmatrix} 7 & 2 \\ 17 & 5 \end{bmatrix}$ and $\begin{bmatrix} 5 & -2 \\ -17 & 7 \end{bmatrix}$

12. $\begin{bmatrix} 4 & 3 \\ 7 & 5 \end{bmatrix}$ and $\begin{bmatrix} -5 & 3 \\ 7 & -4 \end{bmatrix}$

4-7 Study Guide and Intervention *(continued)*

Identity and Inverse Matrices

Find Inverse Matrices

Inverse of a 2 × 2 Matrix	The inverse of a matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $ad - bc \neq 0$.

If $ad - bc = 0$, the matrix does not have an inverse.

Example **Find the inverse of $N = \begin{bmatrix} 7 & 2 \\ 2 & 1 \end{bmatrix}$.**

First find the value of the determinant.

$$\begin{vmatrix} 7 & 2 \\ 2 & 1 \end{vmatrix} = 7 - 4 = 3$$

Since the determinant does not equal 0, N^{-1} exists.

$$N^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & -2 \\ -2 & 7 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{7}{3} \end{bmatrix}$$

Check:

$$NN^{-1} = \begin{bmatrix} 7 & 2 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{7}{3} \end{bmatrix} = \begin{bmatrix} \frac{7}{3} - \frac{4}{3} & -\frac{14}{3} + \frac{14}{3} \\ \frac{2}{3} - \frac{2}{3} & -\frac{4}{3} + \frac{7}{3} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$N^{-1}N = \begin{bmatrix} \frac{1}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{7}{3} \end{bmatrix} \cdot \begin{bmatrix} 7 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{7}{3} - \frac{4}{3} & \frac{2}{3} - \frac{2}{3} \\ -\frac{14}{3} + \frac{14}{3} & -\frac{4}{3} + \frac{7}{3} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Exercises

Find the inverse of each matrix, if it exists.

1. $\begin{bmatrix} 24 & 12 \\ 8 & 4 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

3. $\begin{bmatrix} 40 & -10 \\ -20 & 30 \end{bmatrix}$

4. $\begin{bmatrix} 6 & 5 \\ 10 & 8 \end{bmatrix}$

5. $\begin{bmatrix} 3 & 6 \\ 4 & 8 \end{bmatrix}$

6. $\begin{bmatrix} 8 & 2 \\ 10 & 4 \end{bmatrix}$

Lesson 4-7

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4-7 Skills Practice

Identity and Inverse Matrices

Determine whether each pair of matrices are inverses.

1. $X = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, Y = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$

2. $P = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}, Q = \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$

3. $M = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}, N = \begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix}$

4. $A = \begin{bmatrix} -2 & 5 \\ -1 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & -5 \\ 1 & -2 \end{bmatrix}$

5. $V = \begin{bmatrix} 0 & 7 \\ -7 & 0 \end{bmatrix}, W = \begin{bmatrix} 0 & -\frac{1}{7} \\ \frac{1}{7} & 0 \end{bmatrix}$

6. $X = \begin{bmatrix} -1 & 4 \\ 1 & 2 \end{bmatrix}, Y = \begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix}$

7. $G = \begin{bmatrix} 4 & -3 \\ 1 & 2 \end{bmatrix}, H = \begin{bmatrix} \frac{2}{11} & \frac{3}{11} \\ -\frac{1}{11} & \frac{4}{11} \end{bmatrix}$

8. $D = \begin{bmatrix} -4 & -4 \\ -4 & 4 \end{bmatrix}, E = \begin{bmatrix} -0.125 & -0.125 \\ -0.125 & -0.125 \end{bmatrix}$

Find the inverse of each matrix, if it exists.

9. $\begin{bmatrix} 0 & 2 \\ 4 & 0 \end{bmatrix}$

10. $\begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$

11. $\begin{bmatrix} 9 & 3 \\ 6 & 2 \end{bmatrix}$

12. $\begin{bmatrix} -2 & -4 \\ 6 & 0 \end{bmatrix}$

13. $\begin{bmatrix} 1 & -1 \\ 3 & 3 \end{bmatrix}$

14. $\begin{bmatrix} 3 & 6 \\ -1 & -2 \end{bmatrix}$

15. $\begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix}$

16. $\begin{bmatrix} -4 & 5 \\ 1 & 2 \end{bmatrix}$

17. $\begin{bmatrix} 0 & -7 \\ -7 & 0 \end{bmatrix}$

18. $\begin{bmatrix} 10 & 8 \\ 5 & 4 \end{bmatrix}$

19. $\begin{bmatrix} 10 & 8 \\ 10 & -8 \end{bmatrix}$

20. $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

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4-7 Practice

Identity and Inverse Matrices

Determine whether each pair of matrices are inverses.

1. $M = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}, N = \begin{bmatrix} -2 & 1 \\ 3 & -2 \end{bmatrix}$

2. $X = \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}, Y = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$

3. $A = \begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}, B = \begin{bmatrix} \frac{1}{5} & -\frac{1}{10} \\ \frac{2}{5} & \frac{3}{10} \end{bmatrix}$

4. $P = \begin{bmatrix} 6 & -2 \\ -2 & 3 \end{bmatrix}, Q = \begin{bmatrix} \frac{3}{14} & \frac{1}{7} \\ \frac{1}{7} & \frac{3}{7} \end{bmatrix}$

Determine whether each statement is *true* or *false*.

5. All square matrices have multiplicative inverses.

6. All square matrices have multiplicative identities.

Find the inverse of each matrix, if it exists.

7. $\begin{bmatrix} 4 & 5 \\ -4 & -3 \end{bmatrix}$

8. $\begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$

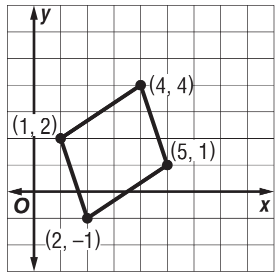
9. $\begin{bmatrix} -1 & 3 \\ 4 & -7 \end{bmatrix}$

10. $\begin{bmatrix} 2 & 5 \\ -1 & 3 \end{bmatrix}$

11. $\begin{bmatrix} 2 & -5 \\ 3 & 1 \end{bmatrix}$

12. $\begin{bmatrix} 4 & 6 \\ 6 & 9 \end{bmatrix}$

GEOMETRY For Exercises 13–16, use the figure at the right.



13. Write the vertex matrix A for the rectangle.

14. Use matrix multiplication to find BA if $B = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.5 \end{bmatrix}$.

15. Graph the vertices of the transformed triangle on the previous graph. Describe the transformation.

16. Make a conjecture about what transformation B^{-1} describes on a coordinate plane.

17. CODES Use the alphabet table below and the inverse of coding matrix $C = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ to decode this message:
19 | 14 | 11 | 13 | 11 | 22 | 55 | 65 | 57 | 60 | 2 | 1 | 52 | 47 | 33 | 51 | 56 | 55.

CODE													
A	1	**B**	2	**C**	3	**D**	4	**E**	5	**F**	6	**G**	7
H	8	**I**	9	**J**	10	**K**	11	**L**	12	**M**	13	**N**	14
O	15	**P**	16	**Q**	17	**R**	18	**S**	19	**T**	20	**U**	21
V	22	**W**	23	**X**	24	**Y**	25	**Z**	26	**–**	0		

4-7 Word Problem Practice

Identity and Inverse Matrices

1. ROTATIONS Suppose R represents a counterclockwise rotation about the origin by an angle of 45°. For what values of n is R^n equal to the inverse of R?

2. SPECIAL MATRICES Norman only likes working with matrices whose determinant is 1. If $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is such a matrix, what is its inverse?

3. CRYPTOGRAPHY A friend sends you a secret message that was coded using the coding matrix $C = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$ and the alphabet table.

CODE		
A 65	J 74	S 83
B 66	K 75	T 84
C 67	L 76	U 85
D 68	M 77	V 86
E 69	N 78	W 87
F 70	O 79	X 88
G 71	P 80	Y 89
H 72	Q 81	Z 90
I 73	R 82	–91

The message is 567 | 354 | 620 | 388. What is the decoded message?

4. SELF-INVERSES Phillip notices that any matrix with ones and negative ones on the diagonal and zeroes everywhere else has the property that it is its own inverse. Give an example of a 2 by 2 matrix that is its own inverse but has at least 1 nonzero number off the diagonal.

MATRIX OPERATIONS For Exercises 5–7, use the following information.
Garth is studying determinants and inverses of matrices in math class. His teacher suggests that there are some matrices with unique properties, and challenges the class to find such matrices and describe the properties found. Garth is curious about the matrix $G = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$.

5. What is the determinant of G?

6. Does the inverse of G exist? Explain.

7. Determine a matrix operation that could be used to transform G into its Additive Identity matrix.

4-7 Enrichment

Permutation Matrices

A permutation matrix is a square matrix in which each row and each column has one entry that is 1. All the other entries are 0. Find the inverse of a permutation matrix interchanging the rows and columns. For example, row 1 is interchanged with column 1, row 2 is interchanged with column 2.

$$P = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \qquad P^{-1} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

P is a 4×4 permutation matrix. P^{-1} is the inverse of P.

Solve each problem.

1. There is just one 2×2 permutation matrix that is not also an identity matrix. Write this matrix.

2. Find the inverse of the matrix you wrote in Exercise 1. What do you notice?

3. Show that the two matrices in Exercises 1 and 2 are inverses.

4. Write the inverse of this matrix.

$$B = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

5. Use B^{-1} from problem 4. Verify that B and B^{-1} are inverses.

6. Permutation matrices can be used to write and decipher codes. To see how this is done, use the message matrix M and matrix B from problem 4. Find matrix C so that C equals the product MB. Use the rules below.

0 times a letter = 0
1 times a letter = the same letter
0 plus a letter = the same letter

$$M = \begin{bmatrix} \text{S} & \text{H} & \text{E} \\ \text{S} & \text{A} & \text{W} \\ \text{H} & \text{I} & \text{M} \end{bmatrix}$$

7. Now find the product CB^{-1}. What do you notice?

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4-8 Lesson Reading Guide

Using Matrices to Solve Systems of Equations

Get Ready for the Lesson

Read the introduction to Lesson 4-8 in your textbook.

Write a 2×2 matrix that summarizes the information given in the introduction about the food and territory requirements for the two species.

Read the Lesson

1. a. Write a matrix equation for the following system of equations.

$3x + 5y = 10$
$2x - 4y = -7$

b. Explain how to use the matrix equation you wrote above to solve the system. Use as few mathematical symbols in your explanation as you can. Do not actually solve the system.

2. Write a system of equations that corresponds to the following matrix equation.

$$\begin{bmatrix} 3 & 2 & -4 \\ 2 & -1 & 0 \\ 0 & 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 6 \\ -4 \end{bmatrix}$$

Remember What You Learned

3. Some students have trouble remembering how to set up a matrix equation to solve a system of linear equations. What is an easy way to remember the order in which to write the three matrices that make up the equation?

4-8 Study Guide and Intervention

Using Matrices to Solve Systems of Equations

Write Matrix Equations A **matrix equation** for a system of equations consists of the product of the coefficient and variable matrices on the left and the constant matrix on the right of the equals sign.

Example **Write a matrix equation for each system of equations.**

a. $\mathbf{3x - 7y = 12}$
$\mathbf{x + 5y = -8}$

Determine the coefficient, variable, and constant matrices.

$$\begin{bmatrix} 3 & -7 \\ 1 & 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ -8 \end{bmatrix}$$

b. $\mathbf{2x - y + 3z = -7}$
$\mathbf{x + 3y - 4z = 15}$
$\mathbf{7x + 2y + z = -28}$

$$\begin{bmatrix} 2 & -1 & 3 \\ 1 & 3 & -4 \\ 7 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -7 \\ 15 \\ -28 \end{bmatrix}$$

Exercises

Write a matrix equation for each system of equations.

1. $2x + y = 8$
$5x - 3y = -12$

2. $4x - 3y = 18$
$x + 2y = 12$

3. $7x - 2y = 15$
$3x + y = -10$

4. $4x - 6y = 20$
$3x + y + 8 = 0$

5. $5x + 2y = 18$
$x = -4y + 25$

6. $3x - y = 24$
$3y = 80 - 2x$

7. $2x + y + 7z = 12$
$5x - y + 3z = 15$
$x + 2y - 6z = 25$

8. $5x - y + 7z = 32$
$x + 3y - 2z = -18$
$2x + 4y - 3z = 12$

9. $4x - 3y - z = -100$
$2x + y - 3z = -64$
$5x + 3y - 2z = 8$

10. $x - 3y + 7z = 27$
$2x + y - 5z = 48$
$4x - 2y + 3z = 72$

11. $2x + 3y - 9z = -108$
$x + 5z = 40 + 2y$
$3x + 5y = 89 + 4z$

12. $z = 45 - 3x + 2y$
$2x + 3y - z = 60$
$x = 4y - 2z + 120$

Lesson 4-8

4-8 Study Guide and Intervention *(continued)*

Using Matrices to Solve Systems of Equations

Solve Systems of Equations Use inverse matrices to solve systems of equations written as matrix equations.

Solving Matrix Equations	If $AX = B$, then $X = A^{-1}B$, where A is the coefficient matrix, X is the variable matrix, and B is the constant matrix.

Example **Solve** $\begin{bmatrix} 5 & 2 \\ 6 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$.

In the matrix equation $A = \begin{bmatrix} 5 & 2 \\ 6 & 4 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, and $B = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$.

Step 1 Find the inverse of the coefficient matrix.

$$A^{-1} = \frac{1}{20 - 12}\begin{bmatrix} 4 & -2 \\ -6 & 5 \end{bmatrix} \text{ or } \frac{1}{8}\begin{bmatrix} 4 & -2 \\ -6 & 5 \end{bmatrix}.$$

Step 2 Multiply each side of the matrix equation by the inverse matrix.

$$\frac{1}{8}\begin{bmatrix} 4 & -2 \\ -6 & 5 \end{bmatrix} \cdot \begin{bmatrix} 5 & 2 \\ 6 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 4 & -2 \\ -6 & 5 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 4 \end{bmatrix} \quad \text{Multiply each side by } A^{-1}.$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 16 \\ -16 \end{bmatrix} \quad \text{Multiply matrices.}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix} \quad \text{Simplify.}$$

The solution is $(2, -2)$.

Exercises

Solve each matrix equation or system of equations by using inverse matrices.

1. $\begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 18 \end{bmatrix}$

2. $\begin{bmatrix} -4 & -8 \\ 6 & 12 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 12 \end{bmatrix}$

3. $\begin{bmatrix} 3 & 2 \\ 5 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -7 \end{bmatrix}$

4. $\begin{bmatrix} 2 & -3 \\ 2 & 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -8 \end{bmatrix}$

5. $\begin{bmatrix} 3 & 6 \\ 5 & 9 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -15 \\ 6 \end{bmatrix}$

6. $\begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -6 \end{bmatrix}$

7. $4x - 2y = 22$
$6x + 4y = -2$

8. $2x - y = 2$
$x + 2y = 46$

9. $3x + 4y = 12$
$5x + 8y = -8$

10. $x + 3y = -5$
$2x + 7y = 8$

11. $5x + 4y = 5$
$9x - 8y = 0$

12. $3x - 2y = 5$
$x - 4y = 20$

4-8 Skills Practice

Using Matrices to Solve Systems of Equations

Write a matrix equation for each system of equations.

1. $x + y = 5$
$2x - y = 1$

2. $3a + 8b = 16$
$4a + 3b = 3$

3. $m + 3n = -3$
$4m + 3n = 6$

4. $2c + 3d = 6$
$3c - 4d = 7$

5. $r - s = 1$
$2r + 3s = 12$

6. $x + y = 5$
$3x + 2y = 10$

7. $6x - y + 2z = -4$
$-3x + 2y - z = 10$
$x + y + z = 3$

8. $a - b + c = 5$
$3a + 2b - c = 0$
$2a + 3b = 8$

Solve each matrix equation or system of equations by using inverse matrices.

9. $\begin{bmatrix} 1 & 3 \\ 4 & 3 \end{bmatrix} \cdot \begin{bmatrix} w \\ z \end{bmatrix} = \begin{bmatrix} -7 \\ -1 \end{bmatrix}$

10. $\begin{bmatrix} 4 & 3 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \end{bmatrix}$

11. $\begin{bmatrix} 5 & 8 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -1 \\ 7 \end{bmatrix}$

12. $\begin{bmatrix} 7 & -3 \\ 5 & 4 \end{bmatrix} \cdot \begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} 15 \\ 23 \end{bmatrix}$

13. $\begin{bmatrix} 3 & 12 \\ 2 & -6 \end{bmatrix} \cdot \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 25 \\ 12 \end{bmatrix}$

14. $\begin{bmatrix} 5 & 6 \\ 12 & -6 \end{bmatrix} \cdot \begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} 15 \\ 2 \end{bmatrix}$

15. $p - 3q = 6$
$2p + 3q = -6$

16. $-x - 3y = 2$
$-4x - 5y = 1$

17. $2m + 2n = -8$
$6m + 4n = -18$

18. $-3a + b = -9$
$5a - 2b = 14$

Lesson 4-8

NAME ____ DATE______________ PERIOD _____

4-8 Practice

Using Matrices to Solve Systems of Equations

Write a matrix equation for each system of equations.

1. $-3x + 2y = 9$
$5x - 3y = -13$

2. $6x - 2y = -2$
$3x + 3y = 10$

3. $2a + b = 0$
$3a + 2b = -2$

4. $r + 5s = 10$
$2r - 3s = 7$

5. $3x - 2y + 5z = 3$
$x + y - 4z = 2$
$-2x + 2y + 7z = -5$

6. $2m + n - 3p = -5$
$5m + 2n - 2p = 8$
$3m - 3n + 5p = 17$

Solve each matrix equation or system of equations by using inverse matrices.

7. $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} g \\ h \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$

8. $\begin{bmatrix} -2 & 3 \\ 1 & 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7 \\ 10 \end{bmatrix}$

9. $\begin{bmatrix} -1 & -3 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 12 \\ -11 \end{bmatrix}$

10. $\begin{bmatrix} -5 & 3 \\ 6 & 4 \end{bmatrix} \cdot \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 16 \\ 34 \end{bmatrix}$

11. $\begin{bmatrix} -4 & 2 \\ 7 & 4 \end{bmatrix} \cdot \begin{bmatrix} r \\ s \end{bmatrix} = \begin{bmatrix} 17 \\ -26 \end{bmatrix}$

12. $\begin{bmatrix} 8 & 3 \\ 12 & 6 \end{bmatrix} \cdot \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

13. $2x + 3y = 5$
$3x - 2y = 1$

14. $8d + 9f = 13$
$-6d + 5f = -45$

15. $5m + 9n = 19$
$2m - n = -20$

16. $-4j + 9k = -8$
$6j + 12k = -5$

17. AIRLINE TICKETS Last Monday at 7:30 A.M., an airline flew 89 passengers on a commuter flight from Boston to New York. Some of the passengers paid $120 for their tickets and the rest paid $230 for their tickets. The total cost of all of the tickets was $14,200. How many passengers bought $120 tickets? How many bought $230 tickets?

18. NUTRITION A single dose of a dietary supplement contains 0.2 gram of calcium and 0.2 gram of vitamin C. A single dose of a second dietary supplement contains 0.1 gram of calcium and 0.4 gram of vitamin C. If a person wants to take 0.6 gram of calcium and 1.2 grams of vitamin C, how many doses of each supplement should she take?

4-8 Word Problem Practice

Using Matrices to Solve Systems of Equations

1. **TEACHING** Paula is explaining matrices to her father. She writes down the following system of equations.

$$2x + y = 4$$
$$3x + y = 5.$$

Next, Paula shows her father the matrices that correspond to this system of equations. What are the matrices?

2. **FIND THE ERROR** Paula proceeds to solve the matrix equation

$$\begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}.$$

First, she finds the inverse.

$$\begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$$

Then she computes the answer.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 11 \\ -6 \end{bmatrix}$$

When she checked her answer, she found that it was not correct. Where did she make a mistake?

3. **AGES** Hank, Laura, and Ned are ages h, l, and n, respectively. The sum of their ages is 15 years. Laura is one year younger than the sum of Hank and Ned's ages. Ned is three times as old as Hank. Use matrices to determine the age of each sibling.

4. **ANIMALS** Quinton takes care of dogs and chickens. There are a total of 28 animals, and altogether they have 68 legs. Use matrices to determine the number of dogs and the number of chickens in Quinton's care.

SALES For Exercises 5 and 6, use the following information.
The school film society is selling only granola bars and oranges to raise money at their movie review. They sell oranges for $1 and granola bars for $1.50. The person selling snacks recorded the total cost and number of items in each sale. The manager wants to know how many of each kind of snack each person bought.

5. Suppose a person spent d dollars to buy n items. Write a system of linear equations that relate d and n to the number of oranges r and granola bars g that the person purchased.

6. One recorded sale showed that 10 items were purchased for $13.00. How many oranges and granola bars were purchased for this sale?

Lesson 4-8

4-8 Enrichment

Determining Political Popularity

Systems of equations have applications in branches of science including chemistry, ecology, and physics. They can also be used to describe situations involving social studies and politics.

Consider the two candidates for City Council, Jefferson Dailey and Robert Jackson. Support for a candidate is measured by a positive number less than 1 and opposition of a candidate by a negative number greater than -1. For example, 0.75 indicates fairly high support, while -0.75 means fairly high opposition. Simultaneous support for, or opposition to, both candidates is possible. Generally, however, if one candidate is popular and is supported while the other candidate is opposed, support of the popular candidate tends to decrease as support for the "underdog" rises. Let the change in support for Jefferson Dailey be denoted by ΔJ(delta J) and the change in support for Robert Jackson is denoted by ΔR (delta R).

This situation is described by the system of equations:

$$\begin{cases} \Delta J = -0.5J + 0.25R \\ \Delta R = -0.25J - 0.5R \end{cases}$$

For example, if $\Delta J = -0.2$ and $\Delta R = 0.2$, then current support for Jefferson Daily is decreasing at a rate of 20% while Robert Jackson's support is increasing at 20%. Substituting the given values for ΔJ and ΔR and solving the first equation for R yields:

$$R = \frac{0.5J - 0.2}{0.25}$$

Substituting this expression for R in the second equation and solving for J yields:

$$0.2 = 0.25J - 0.5\left(\frac{0.5J - 0.2}{0.25}\right) \Rightarrow J = 0.267$$

Therefore, $R = -0.267$.

If the election were held today, Jefferson Daily would win.

Solve the systems of equations for the following values of ΔJ and ΔR to determine potential winners and losers.

1. $\Delta J = -0.1$, $\Delta R = 0.2$

2. $\Delta J = 0.5$, $\Delta R = -0.1$

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4 Student Recording Sheet

Use this recording sheet with pages 220–221 of the Student Edition.

1. Ⓐ Ⓑ Ⓒ Ⓓ

2. Ⓕ Ⓖ Ⓗ Ⓙ

3. Record your answer and fill in the bubbles in the grid below. Be sure to use the correct place value.

				.			
⓪	⓪	⓪	⓪		⓪	⓪	⓪
①	①	①	①		①	①	①
②	②	②	②		②	②	②
③	③	③	③		③	③	③
④	④	④	④		④	④	④
⑤	⑤	⑤	⑤		⑤	⑤	⑤
⑥	⑥	⑥	⑥		⑥	⑥	⑥
⑦	⑦	⑦	⑦		⑦	⑦	⑦
⑧	⑧	⑧	⑧		⑧	⑧	⑧
⑨	⑨	⑨	⑨		⑨	⑨	⑨

4. Ⓐ Ⓑ Ⓒ Ⓓ

5. Ⓕ Ⓖ Ⓗ Ⓙ

6. Ⓐ Ⓑ Ⓒ Ⓓ

7. Ⓕ Ⓖ Ⓗ Ⓙ

8. Ⓐ Ⓑ Ⓒ Ⓓ

9. Ⓕ Ⓖ Ⓗ Ⓙ

10. Ⓐ Ⓑ Ⓒ Ⓓ

Pre-AP

Record your answers for Question 11 on the back of this paper.

4 Rubric for Scoring Pre-AP

(Use to score the Pre-AP question on page 231 of the Student Edition.)

General Scoring Guidelines

- If a student gives only a correct numerical answer to a problem but does not show how he or she arrived at the answer, the student will be awarded only 1 credit. All extended response questions require the student to show work.
- A fully correct answer for a multiple-part question requires correct responses for all parts of the question. For example, if a question has three parts, the correct response to one or two parts of the question that required work to be shown is *not* considered a fully correct response.
- Students who use trial and error to solve a problem must show their method. Merely showing that the answer checks or is correct is not considered a complete response for full credit.

Exercise 11 Rubric

Score	Specific Criteria
4	The difference in sales of yearbooks between grades 10 and 11 is shown by subtracting 464 (yearbooks sold to 10^{th} grade) from 546 (yearbooks sold to 11^{th} grade) for a total of 82. The difference in sales of frames between grades 10 and 11 is shown by subtracting 278 (frames sold to 10^{th} grade) from 344 (frames sold to 11^{th} grade) for a toal of 66. The total number of yearbooks sold is shown by the expression $423 + 464 + 546 + 575 = 2008$. The toal number of frames is shown by the expression $256 + 278 + 344 + 497 = 1378$. The toal sales of the yearbooks is shown by multiplying the number sold by \$48 for totals of \$20,304, \$22,272, \$26,208, and \$27,600. The total sales of frames is shown by multipling the number sold by \$18 for totals of \$4608, \$5004, \$6192, and \$8946.
3	A generally correct solution, but may contain minor flaws in reasoning or computation.
2	A partially correct interpretation and/or solution to the problem.
1	A correct solution with no evidence or explanation.
0	An incorrect solution indicating no mathematical understanding of the concept or task, or no solution given.

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4 Chapter 4 Quiz 1

SCORE ______

(Lessons 4–1 and 4–2)

1. **GRADES** On the first two algebra exams of the year, Trista's grades were 80 and 95, Javier's grades were 85 and 90, and Yolanda's grades were 75 and 90. Write a 3×2 matrix to organize this information.

2. Solve the matrix equation $\begin{bmatrix} x + 5y \\ 2x - 3y \end{bmatrix} = \begin{bmatrix} -2 \\ 9 \end{bmatrix}$.

Use matrices *A*, *B*, and *C* to find the following.

$A = \begin{bmatrix} 6 & 4 & -8 & 5 \\ 1 & -3 & 9 & 7 \\ 2 & 0 & -2 & -4 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 5 & -8 \\ 12 & -11 & 9 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 7 & -5 \\ -10 & 0 & 2 \end{bmatrix}$

3. the dimensions of matrix A

4. $B - C$

5. $3C + 2B$

1. $\begin{array}{c|cc} & \text{First Exam} & \text{Second Exam} \\ \hline \text{Trista} & 80 & 95 \\ \text{Javier} & 85 & 90 \\ \text{Yolanda} & 75 & 90 \end{array}$

2. **(3, −1)**

3. **3 × 4**

4. $\begin{bmatrix} 2 & -2 & -3 \\ 22 & -11 & 7 \end{bmatrix}$

5. $\begin{bmatrix} 9 & 31 & -31 \\ -6 & -22 & 24 \end{bmatrix}$

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4 Chapter 4 Quiz 2

SCORE ______

(Lessons 4–3 and 4–4)

1. Determine whether the matrix product $A_{5\times2} \cdot B_{2\times4}$ is defined. If so, state the dimensions of the product.

2. If possible, find the product $\begin{bmatrix} 1 & 5 & -4 \\ 6 & 0 & 8 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 \\ 3 & -3 \\ 1 & 4 \end{bmatrix}$.

3. Use $A = \begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -5 & 2 \\ 4 & 3 \end{bmatrix}$, and scalar $c = 3$ to find $c(AB)$ and $(BA)c$. Then state whether the equation $c(AB) = (BA)c$ is true for the given matrices.

4. **MULTIPLE CHOICE** Triangle $A'B'C'$ is the result of a translation of triangle ABC. A table of the translations is shown. Find the coordinates of A.
   A. $(-2, 4)$ **C.** $(-1, -1)$
   B. $(-2, -4)$ **D.** $(-1, 1)$

Triangle *ABC*	Triangle *A′B′C′*
A	*A′* (1, 1)
B (1, −4)	*B′* (4, 1)
C (1, 1)	*C′* (4, 6)

5. The vertices of $\triangle XYZ$ are $X(2, 3)$, $Y(-3, 2)$, and $Z(4, -5)$. The triangle is being reflected over the line $y = x$. Find the coordinates of $\triangle X'Y'Z'$.

1. **yes; 5 × 4**

2. $\begin{bmatrix} 13 & -32 \\ 20 & 26 \end{bmatrix}$

3. $\begin{bmatrix} -39 & -12 \\ -24 & 51 \end{bmatrix}$; $\begin{bmatrix} 9 & 48 \\ 48 & 3 \end{bmatrix}$; **not true**

4. **B**

5. $X'(3, 2)$, $Y'(2, -3)$, $Z'(-5, 4)$

Assessment

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4 Chapter 4 Quiz 3

SCORE ________

(Lessons 4–5 and 4–6)

1. Find the value of $\begin{vmatrix} -3 & 4 \\ 2 & -5 \end{vmatrix}$.

A. −22 **B.** −7 **C.** −26 **D.** 7

1. 7

2. Evaluate $\begin{vmatrix} 2 & -1 & 3 \\ 4 & 0 & 1 \\ -2 & 3 & 5 \end{vmatrix}$ using expansion by minors.

2. 52

3. GEOMETRY Find the area of a triangle whose vertices are located at (−2, 5), (−4, −3), and (3, 1). Evaluate the determinant using diagonals.

3. 24 units^2

Use Cramer's Rule to solve each system of equations.

4. $x + y = 6$
$3x + 2y = 11$

5. $4x + 2y + z = -1$
$3x - 4y + 5z = -13$
$x + 4y - 3z = 7$

4. (−1, 7)

5. (−2, 3, 1)

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4 Chapter 4 Quiz 4

SCORE ________

(Lessons 4–7 and 4–8)

1. Determine whether $A = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ 0 & -\frac{1}{2} \end{bmatrix}$ are inverses.

1. yes

2. Find the inverse of $M = \begin{bmatrix} -8 & 2 \\ 4 & -1 \end{bmatrix}$, if it exists.

2. no inverse exists

For Questions 3 and 4, write a matrix equation for each system of equations.

3. $a - 3b = 15$
$a + 4b = -13$

4. $2x - 3y + 4z = -20$
$3x + z = 2$
$x - 4y - z = -6$

3. $\begin{bmatrix} 1 & -3 \\ 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 15 \\ -13 \end{bmatrix}$

4. $\begin{bmatrix} 2 & -3 & 4 \\ 3 & 0 & 1 \\ 1 & -4 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -20 \\ 2 \\ -6 \end{bmatrix}$

5. Solve the system of equations $3x + 2y = 22$ and $x - 2y = -6$ by using inverse matrices.

5. (4, 5)

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4 Chapter 4 Mid-Chapter Test

(Lessons 4–1 through 4–4)

Part I ***Write the letter for the correct answer in the blank at the right of each question.***

1. State the dimensions of matrix F if $F = \begin{bmatrix} 0 & 1 & 0 \\ 2 & -4 & 2 \\ 4 & -8 & 4 \\ 8 & -16 & 8 \end{bmatrix}$.

A. 16×8 **B.** $2 \times 2 \times 3$ **C.** 4×3 **D.** 3×4

1. C

2. Solve the matrix equation $\begin{bmatrix} 3x - 2y \\ 4x + 5y \end{bmatrix} = \begin{bmatrix} 11 \\ 7 \end{bmatrix}$ for y.

F. -1 **G.** 7 **H.** -3 **J.** 3

2. F

3. For all matrices $X_{3\times5}$, $Y_{2\times3}$, $Z_{3\times4}$, and scalars c, which statement is always true?

A. $c(YZ) = (YZ)c$ **C.** $Y + Z = Z + Y$
B. $YX = XY$ **D.** $c(ZX) = c(XY)$

3. A

4. Triangle RST with vertices $R(-3, 4)$, $S(0, -5)$, and $T(6, 7)$ is translated 3 units left and 5 units down. Find the coordinates of R'.

F. $(-6, 1)$ **G.** $(-6, -1)$ **H.** $(0, -1)$ **J.** $(0, 9)$

4. G

5. Parallelogram $ABCD$ with $A(-2, -2)$, $B(4, -2)$, $C(5, 3)$, and $D(-1, 3)$ is rotated 270° counterclockwise about the origin. Find the coordinates of B'.

A. $(2, 4)$ **B.** $(-4, 2)$ **C.** $(2, -4)$ **D.** $(-2, -4)$

5. D

Part II

Perform the indicated matrix operations. If the matrix does not exist, write *impossible*.

6. $\begin{bmatrix} 3 & 12 & 0 & 7 \\ 9 & -15 & -8 & 6 \end{bmatrix} + \begin{bmatrix} 4 & -6 & 13 & 5 \\ -5 & 11 & 0 & -4 \end{bmatrix}$

6. $\begin{bmatrix} 7 & 6 & 13 & 12 \\ 4 & -4 & -8 & 2 \end{bmatrix}$

7. $\begin{bmatrix} 0 & 6 & 13 & -11 \\ -7 & -4 & 2 & 4 \end{bmatrix} - \begin{bmatrix} 9 & -7 & -3 & 0 \\ 6 & -5 & 1 & 19 \end{bmatrix}$

7. $\begin{bmatrix} -9 & 13 & 16 & -11 \\ -13 & 1 & 1 & -15 \end{bmatrix}$

8. $-4\begin{bmatrix} 3 & -5 & 12 \\ 9 & 11 & -7 \\ -2 & 4 & 6 \end{bmatrix}$

8. $\begin{bmatrix} -12 & 20 & -48 \\ -36 & -44 & 28 \\ 8 & -16 & -24 \end{bmatrix}$

9. $3[1 \ \ 0 \ \ -9 \ \ 6] - 4[5 \ \ 12 \ \ 0 \ \ -3]$

9. $[-17 \ \ -48 \ \ -27 \ \ 30]$

10. $\begin{bmatrix} -4 & 0 \\ 1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ 3 & -1 \end{bmatrix}$

10. $\begin{bmatrix} -4 & 8 \\ 16 & -7 \end{bmatrix}$

11. $\begin{bmatrix} -1 & 2 & 1 \\ 0 & 5 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 & 6 \\ -2 & 7 \end{bmatrix}$

11. impossible

Assessment

NAME ______________________ DATE __________ PERIOD ______

4 Chapter 4 Vocabulary Test

SCORE ______

Cramer's Rule	equal matrices	matrix equation
determinant	identity matrix	reflection
dilation	inverse	rotation
dimension	isometry	scalar multiplication
element	matrix	translation

Underline or circle the correct word or phrase to complete each sentence.

1. (*Translation*, *Cramer's Rule*) is a method for solving systems of equations that uses determinants.
2. A(n) (*determinant*, *element*) is a number associated with a square matrix.
3. Each value in a matrix is called a(n) (*equal matrix*, *element*).
4. A rectangular array of variables or constants is called a(n) (*matrix*, *element*).
5. A (*reflection*, *rotation*) is a transformation in which every point of a figure is mapped to a corresponding image across a line of symmetry.
6. (*Cramer's Rule*, *Scalar multiplication*) is an operation that allows you to multiply any matrix by a constant called a scalar.
7. A(n) (*inverse*, *rotation*) occurs when a figure is moved around a center point, usually the origin.
8. An $n \times n$ matrix is a(n) (*dilation*, *inverse*) of another $n \times n$ matrix when their product is an identity matrix.
9. A (*matrix*, *translation*) occurs when a figure is moved from one location to another without changing its size, shape, or orientation.
10. A transformation that enlarges or reduces a figure is called a (*translation*, *dilation*).

Define each term in your own words.

11. identity matrix **Sample answer: An identity matrix is a square matrix in which the elements on the main diagonal are all ones and all the other elements are zeroes.**

12. equal matrices **Sample answer: When two matrices have the same dimensions and each element of one matrix is equal to the corresponding element of the other matrix.**

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4 Chapter 4 Test, Form 1

SCORE ______

Write the letter for the correct answer in the blank at the right of each question.

1. How many elements are there in a 3×4 matrix?

A. 7 **B.** 3 **C.** 12 **D.** 4 — **1.** C

2. Solve the matrix equation $\begin{bmatrix} 3 & x \\ y & 7 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 1 & z \end{bmatrix}$ for y.

F. 1 **G.** 6 **H.** 7 **J.** -1 — **2.** F

For Questions 3–9, use the matrices to find the following.

$$P = \begin{bmatrix} 4 & 1 \\ 2 & 0 \end{bmatrix} \quad Q = \begin{bmatrix} 1 & 6 \\ 0 & 2 \end{bmatrix} \quad R = \begin{bmatrix} 0 & \frac{1}{2} \\ 1 & -2 \end{bmatrix} \quad S = \begin{bmatrix} 1 & -3 \\ 0 & \frac{1}{2} \end{bmatrix}$$

$$T = \begin{bmatrix} 6 & -4 & 9 \\ 3 & -1 & -5 \end{bmatrix} \quad U = \begin{bmatrix} 10 & -5 & -3 \\ 2 & 7 & 4 \end{bmatrix} \quad V = \begin{bmatrix} 0 & 4 \\ -2 & 6 \\ 5 & -3 \end{bmatrix}$$

3. the first row of $T + U$

A. $[-4 \ \ 1 \ \ 12]$ **B.** $[1 \ \ -8 \ \ -9]$ **C.** $[16 \ \ -9 \ \ 6]$ **D.** not possible — **3.** C

4. the first row of $U - V$

F. $[10 \ \ -3 \ \ -8]$ **G.** $[-10 \ \ 2]$ **H.** $[10 \ \ -7 \ \ 2]$ **J.** not possible — **4.** J

5. the first row of $4T$

A. $[-2 \ \ 8 \ \ -5]$ **B.** $[12 \ \ -4 \ \ -20]$ **C.** $[24 \ \ -16 \ \ 36]$ **D.** not possible — **5.** C

6. the first row of $2P + S$

F. $[9 \ \ -1]$ **G.** $[10 \ \ -4]$ **H.** $[9 \ \ 5]$ **J.** not possible — **6.** F

7. the first row of TV

A. $[12 \ \ -4 \ \ -20]$ **B.** $[-23 \ \ 21]$ **C.** $[53 \ \ -27]$ **D.** not possible — **7.** C

8. the inverse of matrix R

F. P **G.** Q **H.** S **J.** not possible — **8.** F

9. the dimensions of PV

A. 1×3 **B.** 2×3 **C.** 2×1 **D.** 3×2 — **9.** B

10. In the first game, the Arrows, A, scored 52 and the Hens, H, scored 23. In the second game, the Arrows scored 13 and the Hens scored 17. Which matrix organizes the information? — **10.** G

F. $\begin{array}{cc} & \begin{array}{cc} A & H \end{array} \\ \begin{array}{c} \text{1st} \\ \text{2nd} \end{array} & \begin{bmatrix} 52 & 13 \\ 23 & 17 \end{bmatrix} \end{array}$ **G.** $\begin{array}{cc} & \begin{array}{cc} A & H \end{array} \\ \begin{array}{c} \text{1st} \\ \text{2nd} \end{array} & \begin{bmatrix} 52 & 23 \\ 13 & 17 \end{bmatrix} \end{array}$ **H.** $\begin{array}{cc} & \begin{array}{cc} A & \text{1st} \end{array} \\ \begin{array}{c} H \\ \text{2nd} \end{array} & \begin{bmatrix} 23 & 52 \\ 17 & 13 \end{bmatrix} \end{array}$ **J.** $\begin{array}{c} \begin{array}{cc} A & H \end{array} \\ [65 \ \ 40] \end{array}$

11. Find the value of $\begin{vmatrix} 5 & 1 \\ 3 & 2 \end{vmatrix}$.

A. 13 **B.** 7 **C.** 17 **D.** 3 — **11.** B

12. Which expression is true for all matrices $X_{2\times2}$, $Y_{2\times2}$ and scalars c?

F. $c(X + Y) = (Y + X)c$ **H.** $XY = YX$

G. $c(XY) = (YX)c$ **J.** $c(XY) = (cX)(cY)$ — **12.** F

Assessment

4 Chapter 4 Test, Form 1 *(continued)*

13. Evaluate $\begin{vmatrix} 1 & 3 & 2 \\ 0 & -1 & 1 \\ 2 & 4 & 1 \end{vmatrix}$ using expansion by minors.

A. 5 **B.** -7 **C.** 7 **D.** -3 **13.** A

14. Evaluate $\begin{vmatrix} 2 & 0 & 1 \\ 3 & 1 & 2 \\ 1 & -2 & 5 \end{vmatrix}$ using diagonals.

F. -2 **G.** 7 **H.** 11 **J.** -1 **14.** H

15. Triangle RST with vertices $R(2, 5)$, $S(1, 4)$, and $T(3, 1)$ is translated 3 units right. What are the coordinates of S'?

A. (4, 4) **B.** (4, 7) **C.** (1, 7) **D.** $(-2, 4)$ **15.** A

16. The vertices of XYZ are $X(3, 1)$, $Y(0, 4)$, and $Z(0, 0)$. The triangle is being reflected over the line $y = x$. Use the reflection matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ to find X'.

F. (3, 1) **G.** (1, 3) **H.** $(-3, 1)$ **J.** $(3, -1)$ **16.** G

17. Cramer's Rule is used to solve the system of equations $2m + 3n = 11$ and $3m - 5n = 6$. Which determinant represents the numerator for m?

A. $\begin{vmatrix} 11 & 2 \\ 6 & 3 \end{vmatrix}$ **B.** $\begin{vmatrix} 2 & 3 \\ 3 & -5 \end{vmatrix}$ **C.** $\begin{vmatrix} 2 & 11 \\ 3 & 6 \end{vmatrix}$ **D.** $\begin{vmatrix} 11 & 3 \\ 6 & -5 \end{vmatrix}$ **17.** D

18. Cramer's Rule is used to solve the system of equations $2x - 3y + 4z = 12$, $3x + y + 5z = 10$ and $x - 4y - z = 8$. Which determinant represents the numerator for y?

F. $\begin{vmatrix} 2 & 12 & 4 \\ 3 & 10 & 5 \\ 1 & 8 & -1 \end{vmatrix}$ **G.** $\begin{vmatrix} 12 & -3 & 4 \\ 10 & 1 & 5 \\ 8 & -4 & -1 \end{vmatrix}$ **H.** $\begin{vmatrix} 2 & -3 & 4 \\ 3 & 1 & 5 \\ 1 & -4 & -1 \end{vmatrix}$ **J.** $\begin{vmatrix} 2 & 4 & 12 \\ 3 & 5 & 10 \\ 1 & -1 & 8 \end{vmatrix}$ **18.** F

19. Which matrix would *not* be used to write a matrix equation for the system of equations $3f - 2g = 7$ and $-2f + g = -5$?

A. $\begin{bmatrix} f \\ g \end{bmatrix}$ **B.** $\begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix}$ **C.** $\begin{bmatrix} g \\ f \end{bmatrix}$ **D.** $\begin{bmatrix} 7 \\ -5 \end{bmatrix}$ **19.** C

20. Which product would be used to solve the matrix equation $\begin{bmatrix} 4 & 6 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$ by using inverse matrices?

F. $\begin{bmatrix} 4 & 6 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 0 \end{bmatrix}$ **G.** $\frac{1}{4}\begin{bmatrix} 1 & -6 \\ 0 & 4 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 0 \end{bmatrix}$ **H.** $\frac{1}{4}\begin{bmatrix} 4 & 6 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 0 \end{bmatrix}$ **J.** $4\begin{bmatrix} 1 & -6 \\ 0 & 4 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 0 \end{bmatrix}$ **20.** G

Bonus Find the value of $\begin{vmatrix} 0 & 1 & 0 \\ a & b & c \\ c & a & b \end{vmatrix}$. **B:** $c^2 - ab$

4 Chapter 4 Test, Form 2A

Write the letter for the correct answer in the blank at the right of each question.

1. Solve the matrix equation $\begin{bmatrix} 3x \\ y \end{bmatrix} = \begin{bmatrix} 10 + 2y \\ 5 - x \end{bmatrix}$ for x.

A. 20 **B.** 25 **C.** 1 **D.** 4 **1.** D

For Questions 2–9, use the matrices to find the following.

$$P = \begin{bmatrix} 0 & 14 \\ 2 & 21 \end{bmatrix} \quad Q = \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix} \quad R = \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & -\frac{1}{4} \end{bmatrix} \quad S = \begin{bmatrix} -\frac{3}{4} & \frac{1}{2} \\ \frac{1}{14} & 0 \end{bmatrix}$$

$$T = \begin{bmatrix} -11 & 7 & 5 \\ -6 & -3 & 8 \end{bmatrix} \quad U = \begin{bmatrix} 3 & 0 & -2 \\ 4 & -9 & -5 \end{bmatrix} \quad V = \begin{bmatrix} 2 & 4 \\ -1 & 0 \\ 3 & -1 \end{bmatrix}$$

2. the dimensions of matrix T

F. 6×1 **G.** 3×2 **H.** 2×3 **J.** 1×6 **2.** H

3. the first row of $T + V$

A. $[-9 \ 6 \ 8]$ **B.** $[-9 \ -2]$ **C.** $[-7 \ 7 \ 4]$ **D.** not possible **3.** D

4. the first row of $U - T$

F. $[14 \ -7 \ -7]$ **G.** $[0]$ **H.** $[10 \ -6 \ -13]$ **J.** not possible **4.** F

5. the first row of $-4U$

A. $[-4]$ **B.** $[-16 \ 36 \ 20]$ **C.** $[-12 \ 0 \ 8]$ **D.** not possible **5.** C

6. the first row of $4Q - P$

F. $[4 \ -2]$ **G.** $[4 \ -6]$ **H.** $[4 \ 22]$ **J.** not possible **6.** G

7. the first row of UV

A. $[2 \ 21]$ **B.** $[22 \ -36 \ -24]$ **C.** $[0 \ 14]$ **D.** not possible **7.** C

8. the inverse of matrix R

F. P **G.** Q **H.** S **J.** not possible **8.** G

9. the dimensions of VT

A. 2×3 **B.** 3×2 **C.** 3×3 **D.** 2×2 **9.** C

10. On the first two science tests this year, Cory's grades were 75 and 80, David's grades were 95 and 83, and Zack's grades were 88 and 93. Which matrix organizes the information? **10.** F

F. $\begin{array}{c|c} & C \ \ D \ \ Z \\ \begin{matrix} 1\text{st} \\ 2\text{nd} \end{matrix} & \begin{bmatrix} 75 & 95 & 88 \\ 80 & 83 & 93 \end{bmatrix} \end{array}$ **G.** $\begin{array}{c|c} & C \ \ D \ \ Z \\ \begin{matrix} 1\text{st} \\ 2\text{nd} \end{matrix} & \begin{bmatrix} 75 & 80 & 88 \\ 95 & 83 & 93 \end{bmatrix} \end{array}$ **H** $\begin{array}{c|c} & C \ \ D \ \ Z \\ \begin{matrix} 1\text{st} \\ 2\text{nd} \end{matrix} & \begin{bmatrix} 80 & 83 & 93 \\ 75 & 95 & 88 \end{bmatrix} \end{array}$ **J** $\begin{array}{c} C \ \ D \ \ Z \\ [75 \ 95 \ 88] \end{array}$

11. Find the value of $\begin{vmatrix} 12 & -4 \\ 7 & -3 \end{vmatrix}$.

A. 8 **B.** -64 **C.** 6 **D.** -8 **11.** D

12. GEOMETRY Find the area of a triangle whose vertices have coordinates $(-4, 3)$, $(2, 5)$, and $(-7, -1)$.

F. 9 units^2 **G.** 18 units^2 **H.** 14 units^2 **J.** 7 units^2 **12.** F

Assessment

4 Chapter 4 Test, Form 2A *(continued)*

13. Evaluate $\begin{vmatrix} 1 & 4 & -1 \\ 0 & 3 & 5 \\ 2 & 6 & -2 \end{vmatrix}$ using diagonals.

A. 58 **B.** −2 **C.** 12 **D.** 10 **13.** D

14. For all matrices $X_{3\times3}$, $Y_{2\times3}$, and $Z_{3\times3}$ and scalars q, which statement is always true?

F. $X + 2Z = 2X + Z$ **H.** $q(XZ) = (qX)Z$

G. $q(YZ) = (ZY)q$ **J.** $(XY)Z = Z(YX)$ **14.** H

15. MAP On a map, the coordinates of the corners of a town are $A(0.5, 2)$, $B(2, 3.5)$, $C(5, 1.5)$, and $D(3, 1)$. The map is dilated so that the perimeter of the town is five times its original perimeter. Find the coordinates of C'.

A. (25, 7.5) **B.** (1, 0.3) **C.** (25, 1.5) **D.** (10, 6.5) **15.** A

16. Triangle RST with vertices $R(-10, -8)$, $S(1, 7)$, and $T(5, -10)$ is rotated 90° counterclockwise about the origin. Find the coordinates of T'.

F. (−5, 10) **G.** (−10, −5) **H.** (10, 5) **J.** (10, −5) **16.** H

17. Cramer's Rule is used to solve the system of equations $3m - 5n = 12$ and $4m + 7n = -5$. Which determinant represents the numerator for n?

A. $\begin{vmatrix} 12 & 3 \\ -5 & 4 \end{vmatrix}$ **B.** $\begin{vmatrix} 3 & -5 \\ 4 & 7 \end{vmatrix}$ **C.** $\begin{vmatrix} 3 & 12 \\ 4 & -5 \end{vmatrix}$ **D.** $\begin{vmatrix} 12 & -5 \\ -5 & 7 \end{vmatrix}$ **17.** C

18. Cramer's Rule is used to solve the system of equations $3x - y + 2z = 17$, $4x + 2y - 3z = 10$, and $2x + 5y - 9z = -6$. Which determinant represents the numerator for z?

F. $\begin{vmatrix} 3 & -1 & 2 \\ 4 & 2 & -3 \\ 2 & 5 & -9 \end{vmatrix}$ **G.** $\begin{vmatrix} 3 & -1 & 17 \\ 4 & 2 & 10 \\ 2 & 5 & -6 \end{vmatrix}$ **H.** $\begin{vmatrix} 17 & 3 & -1 \\ 10 & 4 & 2 \\ -6 & 2 & 5 \end{vmatrix}$ **J.** $\begin{vmatrix} 3 & 17 & 2 \\ 4 & 10 & -3 \\ 2 & -6 & -9 \end{vmatrix}$ **18.** G

19. Which matrix would *not* be used to write a matrix equation for the system of equations $5m - 2n = 13$ and $-m + n = -2$?

A. $\begin{bmatrix} m \\ n \end{bmatrix}$ **B.** $\begin{bmatrix} 13 \\ -2 \end{bmatrix}$ **C.** $\begin{bmatrix} 5 & 13 \\ -1 & -2 \end{bmatrix}$ **D.** $\begin{bmatrix} 5 & -2 \\ -1 & 1 \end{bmatrix}$ **19.** C

20. Which product would be used to solve the matrix equation $\begin{bmatrix} 3 & -4 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \end{bmatrix}$ by using inverse matrices?

F. $\frac{1}{11}\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 7 \end{bmatrix}$ **G.** $\frac{1}{11}\begin{bmatrix} 3 & -4 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 7 \end{bmatrix}$ **H.** $\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 7 \end{bmatrix}$ **J.** $\begin{bmatrix} 3 & -4 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 7 \end{bmatrix}$ **20.** F

Bonus Find the value of $\begin{vmatrix} d & f & d \\ 1 & 1 & 1 \\ f & d & f \end{vmatrix}$. **B:** 0

NAME ____________ DATE ________ PERIOD ______

SCORE ______

4 Chapter 4 Test, Form 2B

Write the letter for the correct answer in the blank at the right of each question.

1. Solve the matrix equation $\begin{bmatrix} 2x \\ 3y \end{bmatrix} = \begin{bmatrix} 10 + 2y \\ 13 - x \end{bmatrix}$ for x.

A. 1 **B.** -1 **C.** 7 **D.** -7 1. C

For Questions 2–9, use the matrices to find the following.

$P = \begin{bmatrix} 3 & 1 \\ -4 & 0 \end{bmatrix}$ $Q = \begin{bmatrix} 4 & 4 \\ 12 & 21 \end{bmatrix}$ $R = \begin{bmatrix} 0 & -\frac{1}{4} \\ 1 & \frac{3}{4} \end{bmatrix}$ $S = \begin{bmatrix} \frac{7}{12} & -\frac{1}{9} \\ -\frac{1}{3} & \frac{1}{9} \end{bmatrix}$

$T = \begin{bmatrix} 4 & -5 & 2 \\ 8 & -1 & 3 \end{bmatrix}$ $U = \begin{bmatrix} -9 & 6 & 4 \\ -5 & -2 & 3 \end{bmatrix}$ $V = \begin{bmatrix} 3 & 1 \\ 0 & 2 \\ -4 & 5 \end{bmatrix}$

2. the dimensions of matrix V

F. 6×1 **G.** 1×6 **H.** 2×3 **J.** 3×2 2. J

3. the first row of $T + U$

A. $[-5 \ \ 1 \ \ 6]$ **B.** $[13 \ \ -11 \ \ -2]$ **C.** $[3 \ \ -3 \ \ 6]$ **D.** not possible 3. A

4. the first row of $V - T$

F. $[-1 \ \ -7]$ **G.** $[-7 \ \ 3 \ \ 2]$ **H.** $[-1 \ \ 5 \ \ -6]$ **J.** not possible 4. J

5. the first row of $-3T$

A. $[-12 \ \ -24]$ **B.** $[-12 \ \ 15 \ \ -6]$ **C.** $[-24 \ \ 3 \ \ -9]$ **D.** not possible 5. B

6. the first row of $5P - Q$

F. $[11 \ \ 1]$ **G.** $[9 \ \ 1]$ **H.** $[19 \ \ 9]$ **J.** not possible 6. F

7. the first row of TV

A. $[20 \ \ -16 \ \ 9]$ **B.** $[20 \ \ 24]$ **C.** $[4 \ \ 4]$ **D.** not possible 7. C

8. the inverse of matrix R

F. P **G.** Q **H.** S **J.** not possible 8. F

9. the dimensions of ST

A. 1×6 **B.** 2×2 **C.** 3×2 **D.** 2×3 9. D

10. Evening shows cost adults \$7.50 and children \$5.50. Matinee shows cost adults \$5.50 and children \$2.50. Which matrix organizes the information? 10. J

F. $\begin{array}{l c} & \begin{matrix} a & c \end{matrix} \\ \text{Evening} & \multirow{2}{*}{$\begin{bmatrix} 7 & 5 \\ 5 & 2 \end{bmatrix}$} \\ \text{Matinee} & \end{array}$ **G.** $\begin{array}{l c} & \begin{matrix} a & c \end{matrix} \\ \text{Evening} & \multirow{2}{*}{$\begin{bmatrix} 7.5 & 7.5 \\ 5.5 & 2.5 \end{bmatrix}$} \\ \text{Matinee} & \end{array}$ **H.** $\begin{array}{l c} & \begin{matrix} a & c \end{matrix} \\ \text{Evening} & \multirow{2}{*}{$\begin{bmatrix} 5.5 & 2.5 \\ 7.5 & 5.5 \end{bmatrix}$} \\ \text{Matinee} & \end{array}$ **J.** $\begin{array}{l c} & \begin{matrix} a & c \end{matrix} \\ \text{Evening} & \multirow{2}{*}{$\begin{bmatrix} 7.5 & 5.5 \\ 5.5 & 2.5 \end{bmatrix}$} \\ \text{Matinee} & \end{array}$

11. Find the value of $\begin{vmatrix} 15 & 4 \\ -6 & -2 \end{vmatrix}$.

A. 6 **B.** -6 **C.** -54 **D.** 54 11. B

12. GEOMETRY Find the area of a triangle whose vertices have coordinates $(2, -5)$, $(6, 1)$, and $(-3, -4)$.

F. 34 units^2 **G.** 21 units^2 **H.** 17 units^2 **J.** 42 units^2 12. H

Assessment

4 Chapter 4 Test, Form 2B *(continued)*

13. Evaluate $\begin{vmatrix} 2 & -3 & 1 \\ 4 & 0 & -2 \\ 5 & -1 & 6 \end{vmatrix}$ using diagonals.

A. –38 **B.** 94 **C.** -42 **D.** 114 **13.** B

14. For all matrices $A_{2\times2}$, $B_{2\times4}$, and $C_{2\times2}$, and scalars d, which statement is always true?

F. $A + dC = dA + C$
H. $(CB)d = d(BC)$
G. $d(A + C) = dA + dC$
J. $(A + C)B = B(A + C)$ **14.** G

15. MAP On a town map, the coordinates of the corners of the zoo are $L(1.2, 4)$, $M(2, 0.8)$, $N(4, 1.6)$, and $P(6, 6)$. The map is dilated so that the perimeter of the zoo is four times its original size. Find the coordinates of M'.

A. (8, 0.8) **B.** (0.5, 0.1) **C.** (8, 3.2) **D.** (6, 4.8) **15.** C

16. Triangle EFG with vertices $E(-4, -5)$, $F(1, 3)$, and $G(4, -1)$ is rotated 270° counterclockwise about the origin. Find the coordinates of G'.

F. (1, 4) **G.** $(-4, 1)$ **H.** $(-1, 4)$ **J.** $(-1, -4)$ **16.** J

17. Cramer's Rule is used to solve the system of equations $5f - 9g = 10$ and $4f + 3g = -6$. Which determinant represents the numerator for f?

A. $\begin{vmatrix} 10 & -9 \\ -6 & 3 \end{vmatrix}$ **B.** $\begin{vmatrix} 5 & -9 \\ 4 & 3 \end{vmatrix}$ **C.** $\begin{vmatrix} 5 & 10 \\ 4 & -6 \end{vmatrix}$ **D.** $\begin{vmatrix} -9 & 10 \\ 3 & -6 \end{vmatrix}$ **17.** A

18. Cramer's Rule is used to solve the system of equations $4x - 5y + z = 11$, $3x - 2y + 2z = -5$, and $2x + 6y + 3z = 8$. Which determinant represents the numerator for z?

F. $\begin{vmatrix} 11 & 4 & -5 \\ -5 & 3 & -2 \\ 8 & 2 & 6 \end{vmatrix}$ **G.** $\begin{vmatrix} 4 & 11 & -5 \\ 3 & -5 & -2 \\ 2 & 8 & 6 \end{vmatrix}$ **H.** $\begin{vmatrix} 4 & -5 & 11 \\ 3 & -2 & -5 \\ 2 & 6 & 8 \end{vmatrix}$ **J.** $\begin{vmatrix} 4 & -5 & 1 \\ 3 & -2 & 2 \\ 2 & 6 & 3 \end{vmatrix}$ **18.** H

19. Which matrix would *not* be used to write a matrix equation for the system of equations $2c + 5d = -11$ and $-c + 2d = 10$?

A. $\frac{1}{9}\begin{bmatrix} 2 & -5 \\ 1 & 2 \end{bmatrix}$ **B.** $\begin{bmatrix} c \\ d \end{bmatrix}$ **C.** $\begin{bmatrix} 2 & 5 \\ -1 & 2 \end{bmatrix}$ **D.** $\begin{bmatrix} -11 \\ 10 \end{bmatrix}$ **19.** A

20. Which product would be used to solve the matrix equation $\begin{bmatrix} 7 & -3 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$ by using inverse matrices?

F. $\begin{bmatrix} 1 & 3 \\ -1 & 7 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 6 \end{bmatrix}$ **G.** $\frac{1}{10}\begin{bmatrix} 1 & 3 \\ -1 & 7 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 6 \end{bmatrix}$ **H.** $\frac{1}{10}\begin{bmatrix} 7 & -3 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 6 \end{bmatrix}$ **J.** $\begin{bmatrix} 7 & -3 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 6 \end{bmatrix}$ **20.** G

Bonus Find the value of $\begin{vmatrix} -a & b & -c \\ a & -b & c \\ a & 1 & 1 \end{vmatrix}$. **B:** 0

NAME ______________________ DATE ____________ PERIOD ________

4 Chapter 4 Test, Form 2C

SCORE ________

1. **GOLF** At a municipal golf course, the fees to play a round of golf are as shown. Write a 3×2 matrix to organize this information.

	Resident	Non-resident
Weekday morning	\$20	\$25
Weekday afternoon	\$18	\$22
Weekend	\$40	\$40

1. $\begin{array}{r c} & \begin{array}{cc}\text{Resident} & \text{Non-Resident}\end{array} \\ \begin{array}{r}\text{Weekday A.M.} \\ \text{Weekday P.M.} \\ \text{Weekend}\end{array} & \begin{bmatrix} 20 & 25 \\ 18 & 22 \\ 40 & 40 \end{bmatrix}\end{array}$

2. Solve the matrix equation $\begin{bmatrix} 2x - 3y \\ 3x + 5y \end{bmatrix} = \begin{bmatrix} 18 \\ -11 \end{bmatrix}$.

2. **(3, −4)**

For Questions 3–6, perform the indicated matrix operations. If the matrix does not exist, write *impossible*.

3. $\begin{bmatrix} 6 & 3 & 10 & 4 \\ 1 & -7 & -8 & 3 \end{bmatrix} - \begin{bmatrix} 10 & -5 & 3 & 1 \\ 9 & -4 & 0 & -2 \end{bmatrix}$

3. $\begin{bmatrix} -4 & 8 & 7 & 3 \\ -8 & -3 & -8 & 5 \end{bmatrix}$

4. $\begin{bmatrix} 1 & -4 & 9 & 0 \\ 3 & 2 & -5 & 6 \end{bmatrix} + \begin{bmatrix} 3 & 12 & -11 \\ 7 & 2 & 8 \end{bmatrix}$

4. **impossible**

5. $4[2 \ \ 1 \ \ 9 \ \ 12] - 2[-5 \ \ 8 \ \ 7 \ \ 0]$

5. $[18 \ \ -12 \ \ 22 \ \ 48]$

6. $7\begin{bmatrix} 3 \\ 0 \\ -4 \end{bmatrix} + 5\begin{bmatrix} 6 \\ 1 \\ -2 \end{bmatrix} - 3\begin{bmatrix} 10 \\ 4 \\ -1 \end{bmatrix}$

6. $\begin{bmatrix} 21 \\ -7 \\ -35 \end{bmatrix}$

7. Determine whether the matrix $X_{3\times5} \cdot Y_{5\times9}$ is defineD If so, state the dimensions of the product.

7. **yes; 3 × 9**

8. Find $\begin{bmatrix} 4 & -1 \\ 2 & -5 \end{bmatrix} \cdot \begin{bmatrix} 9 & -8 & 1 \\ 2 & -4 & -6 \end{bmatrix}$, if possible.

8. $\begin{bmatrix} 34 & -28 & 10 \\ 8 & 4 & 32 \end{bmatrix}$

9. Use $A = \begin{bmatrix} 3 & 1 \\ 0 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 5 & -3 \\ -6 & 1 \end{bmatrix}$, and $C = \begin{bmatrix} 0 & -2 \\ 7 & 3 \end{bmatrix}$ to find $(AB)C$ and $A(BC)$. Then state whether $(AB)C = A(BC)$ is true for the given matrices.

9. $\begin{bmatrix} -56 & -42 \\ -28 & -60 \end{bmatrix}$; $\begin{bmatrix} -56 & -42 \\ -28 & -60 \end{bmatrix}$; **true**

10. Triangle ABC with vertices $A(-4, -3)$, $B(2, -5)$, and $C(-1, 4)$ is translated 3 units right and 2 units down. Find the coordinates of $\triangle A'B'C'$.

10. ***A′*(−1, −5), *B′*(5, −7), *C′*(2, 2)**

11. The vertices of parallelogram $RSTU$ are $R(-4, -2)$, $S(0, -2)$, $T(6, 1)$, and $U(2, 1)$. The parallelogram is reflected over the y-axis. Find the coordinates of parallelogram $R'S'T'U'$.

11. $R'(4, -2)$, $S'(0, -2)$, $T'(-6, 1)$, $U'(-2, 1)$

12. Find the value of $\begin{vmatrix} -7 & 12 \\ 4 & -11 \end{vmatrix}$.

12. 29

13. Evaluate $\begin{vmatrix} 6 & 1 & 4 \\ -5 & 9 & -3 \\ 2 & -8 & 4 \end{vmatrix}$ using expansion by minors.

13. 174

14. **GEOMETRY** Find the area of a triangle whose vertices are located at $(-7, -1)$, $(1, 6)$, and $(5, -3)$.

14. 50 units^2

For Questions 15 and 16, use Cramer's Rule to solve each system of equations.

15. $3a + 2b = 6.5$
$2a - 1.5b = 10$

15. $(3.5, -2)$

16. $2x - 5y + 3z = 27$
$4x + 3y - 7z = -37$
$x - 2y + 5z = 30$

16. $(1, -2, 5)$

17. Determine whether $P = \begin{bmatrix} 3 & 3 \\ 6 & 2 \end{bmatrix}$ and $Q = \begin{bmatrix} -\frac{1}{6} & \frac{1}{4} \\ \frac{1}{2} & -\frac{1}{4} \end{bmatrix}$ are inverses.

17. yes

18. Find the inverse of $R = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$, if it exists.

18. $\frac{1}{12}\begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix}$

19. Solve the matrix equation $\begin{bmatrix} -2 & 4 \\ -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$ by using inverse matrices.

19. $\left(\frac{1}{2}, \frac{3}{2}\right)$

20. **TRANSPORTATION** A transportation company rents cars, trucks, and vans. The company has 366 vehicles in its fleet. It has twice as many trucks as vans, and 186 more cars than vans. Let c represent the number of cars, t represent the number of trucks, and v represent the number of vans. Write a matrix equation that describes this situation.

20. $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -2 \\ 1 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} c \\ t \\ v \end{bmatrix} = \begin{bmatrix} 366 \\ 0 \\ 186 \end{bmatrix}$

Bonus Find the value of $\begin{vmatrix} x & -y & 1 \\ -x & -y & 1 \\ 1 & 0 & 1 \end{vmatrix}$.

B: $-2xy$

4 Chapter 4 Test, Form 2D

SCORE ________

1. **TICKETS** At a local ballpark, ticket prices for home games are as shown. Write a 3×2 matrix to organize this information.

	Bleachers	Box Seats
Weekday	\$9	\$21
Weekend	\$12	\$26
Double-Header	\$27	\$27

1. $\begin{array}{r} \\ \text{Weekday} \\ \text{Weekend} \\ \text{Double-Header} \end{array} \begin{array}{c} \text{Bleachers} \quad \text{Box Seats} \\ \begin{bmatrix} 9 & 21 \\ 12 & 26 \\ 27 & 27 \end{bmatrix} \end{array}$

2. Solve the matrix equation $\begin{bmatrix} 3x - 7y \\ 4x + 5y \end{bmatrix} = \begin{bmatrix} -34 \\ 12 \end{bmatrix}$.

2. (−2, 4)

For Questions 3–6, perform the indicated matrix operations. If the matrix does not exist, write *impossible*.

3. $\begin{bmatrix} 9 & -6 & 10 & 1 \\ 7 & 4 & -8 & 0 \end{bmatrix} + \begin{bmatrix} -6 & 3 & 0 & 11 \\ 4 & -7 & 12 & 5 \end{bmatrix}$

3. $\begin{bmatrix} 3 & -3 & 10 & 12 \\ 11 & -3 & 4 & 5 \end{bmatrix}$

4. $\begin{bmatrix} 4 & -5 \\ 1 & 6 \\ 9 & -2 \end{bmatrix} - \begin{bmatrix} 12 & -6 & 9 \\ 8 & 4 & 5 \\ -7 & -3 & 9 \end{bmatrix}$

4. impossible

5. $9[-1 \quad 3 \quad 7 \quad 2] - 6[4 \quad 5 \quad -8 \quad 1]$

5. $[-33 \quad -3 \quad 111 \quad 12]$

6. $4\begin{bmatrix} 0 \\ -5 \\ 3 \end{bmatrix} + 3\begin{bmatrix} 9 \\ 2 \\ -1 \end{bmatrix} - 6\begin{bmatrix} -1 \\ 3 \\ 7 \end{bmatrix}$

6. $\begin{bmatrix} 33 \\ -32 \\ -33 \end{bmatrix}$

7. Determine whether the matrix $P_{2\times4} \cdot Q_{4\times7}$ is defined. If so, state the dimensions of the product.

7. yes; 2×7

8. Find $\begin{bmatrix} 2 & 1 \\ -3 & -6 \end{bmatrix} \cdot \begin{bmatrix} 3 & -4 & -9 \\ 8 & 7 & 10 \end{bmatrix}$, if possible.

8. $\begin{bmatrix} 14 & -1 & -8 \\ -57 & -30 & -33 \end{bmatrix}$

9. Use $A = \begin{bmatrix} 4 & 9 \\ -5 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 11 & -7 \\ 2 & 1 \end{bmatrix}$, and scalar $d = -2$ to find $d(AB)$ and $(dA)B$. Then state whether $d(AB) = (dA)B$ is true for the given matrices.

9. $\begin{bmatrix} -124 & 38 \\ 102 & -74 \end{bmatrix}$; $\begin{bmatrix} -124 & 38 \\ 102 & -74 \end{bmatrix}$; true

10. Triangle ABC with vertices $A(4, -7)$, $B(2, 3)$, and $C(-3, -1)$ is translated 2 units left and 4 units up. Find the coordinates of $\triangle A'B'C'$.

10. $A'(2, -3)$, $B'(0, 7)$, $C'(-5, 3)$

Assessment

4 Chapter 4 Test, Form 2D *(continued)*

11. The vertices of parallelogram $RSTU$ are $R(-3, -5)$, $S(4, -5)$, $T(6, 4)$, and $U(-1, 4)$. The parallelogram is reflected over the x-axis. Find the coordinates of parallelogram $R'S'T'U'$.

11. **$R'(-3, 5)$; $S'(4, 5)$; $T'(6, -4)$; $U'(-1, -4)$**

12. Find the value of $\begin{vmatrix} 10 & -7 \\ 6 & -5 \end{vmatrix}$.

12. **-8**

13. Evaluate $\begin{vmatrix} 9 & -5 & 8 \\ 1 & 2 & 1 \\ 4 & -2 & -2 \end{vmatrix}$ using expansion by minors.

13. **-128**

14. GEOMETRY Find the area of a triangle whose vertices are located at $(-8, 4)$, $(2, 6)$, and $(3, -2)$.

14. **41 units^2**

For Questions 15 and 16, use Cramer's Rule to solve each system of equations.

15. $0.5x - 2y = 7$
$3x + 10y = 9$

15. **$(8, -1.5)$**

16. $4a - 3b + 5c = 2$
$2a + 4b - 7c = 20$
$a - 3b + 8c = -13$

16. **$(3, 0, -2)$**

17. Determine whether $S = \begin{bmatrix} 4 & 4 \\ 2 & -1 \end{bmatrix}$ and $T = \begin{bmatrix} \frac{1}{12} & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{3} \end{bmatrix}$ are inverses.

17. **no**

18. Find the inverse of $A = \begin{bmatrix} 4 & -2 \\ 3 & -1 \end{bmatrix}$, if it exists.

18. **$\frac{1}{2}\begin{bmatrix} -1 & 2 \\ -3 & 4 \end{bmatrix}$**

19. Solve the matrix equation $\begin{bmatrix} 6 & -8 \\ -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7 \\ 2 \end{bmatrix}$ by using inverse matrices.

19. **$\left(-\frac{1}{2}, \frac{1}{2}\right)$**

20. GARDENING A garden shop sells flowers, trees, and shrubs. The shop sold 1085 items last month. Fifteen more shrubs were sold than trees, and 8 times as many flowers were sold as shrubs. Let f represent the number of flowers, t represent the number of trees, and s represent the number of shrubs. Write a matrix equation that describes this situation.

20. **$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ 1 & 0 & -8 \end{bmatrix} \cdot \begin{bmatrix} f \\ t \\ s \end{bmatrix} = \begin{bmatrix} 1085 \\ 15 \\ 0 \end{bmatrix}$**

Bonus Find the value of $\begin{vmatrix} a & b & -c \\ 1 & 1 & 0 \\ a & -b & c \end{vmatrix}$.

B: **$2ac$**

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4 Chapter 4 Test, Form 3

SCORE ________

1. **TICKETS** A theater group is to put on four performances of its latest production. Each ticket for a Friday or Saturday evening show will cost \$10. Each ticket for a Saturday or Sunday matinee will cost \$7. Write a 3×2 matrix to organize this information.

1. $\begin{array}{lcc} & \text{Evening} & \text{Matinee} \\ \text{Fri.} & \multicolumn{2}{c}{\multirow{3}{*}{$\begin{bmatrix} 10 & 0 \\ 10 & 7 \\ 0 & 7 \end{bmatrix}$}} \\ \text{Sat.} & & \\ \text{Sun.} & & \end{array}$

2. Solve the matrix equation $\begin{bmatrix} x^2 - 1 & 4 - y \\ 2x + y & y - 1 \end{bmatrix} = \begin{bmatrix} 3 & x + 1 \\ 1 & 4 \end{bmatrix}$.

2. **(−2, 5)**

For Questions 3–6, perform the indicated matrix operations. If the matrix does not exist, write *impossible*.

3. $\begin{bmatrix} 2.37 & 4.12 \\ 1.69 & 3.97 \\ -5.18 & 6.25 \end{bmatrix} + \begin{bmatrix} 0.95 & -7.24 \\ 3.59 & 3.41 \\ 2.68 & -5.01 \end{bmatrix}$

3. $\begin{bmatrix} 3.32 & -3.12 \\ 5.28 & 7.38 \\ -2.50 & 1.24 \end{bmatrix}$

4. $\frac{1}{6}\begin{bmatrix} 12 & 0 \\ -6 & 3 \end{bmatrix} - \frac{1}{4}\begin{bmatrix} 8 & 2 \\ 1 & -1 \end{bmatrix}$

4. $\begin{bmatrix} 0 & -\frac{1}{2} \\ -\frac{5}{4} & \frac{3}{4} \end{bmatrix}$

5. $3\begin{bmatrix} -\frac{1}{2} & -1 & 0 \\ 3 & \frac{1}{5} & 2 \end{bmatrix} + 5\begin{bmatrix} -1 & 3 & \frac{1}{4} \\ \frac{2}{5} & -1 & 0 \end{bmatrix}$

5. $\begin{bmatrix} -\frac{13}{2} & 12 & \frac{5}{4} \\ 11 & -\frac{22}{5} & 6 \end{bmatrix}$

6. $\begin{bmatrix} -1 & \frac{1}{2} \\ 0 & 4 \\ -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 4 & -2 & -\frac{1}{2} \\ 6 & 1 & 0 \end{bmatrix}$

6. $\begin{bmatrix} -1 & \frac{5}{2} & \frac{1}{2} \\ 24 & 4 & 0 \\ -2 & 5 & 1 \end{bmatrix}$

7. Use $A = \begin{bmatrix} -4 & -3 \\ 2 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -1 & -\frac{3}{2} \\ 1 & 2 \end{bmatrix}$, and $C = \begin{bmatrix} 0 & -4 \\ 2 & -1 \end{bmatrix}$ to find $C(BA)$ and $(AB)C$. Then state whether $C(BA) = (AB)C$ is true for the given matrices.

7. $\begin{bmatrix} 0 & -4 \\ 2 & -1 \end{bmatrix}$; $\begin{bmatrix} 0 & -4 \\ 2 & -1 \end{bmatrix}$; **true**

8. Quadrilateral $WXYZ$ with vertices $W(-4, 2)$, $X(-1, 4)$, $Y(6, -1)$, and $Z(-2, -3)$ is translated so that W' is at $(5, -2)$. Find the coordinates of X', Y', and Z'.

8. **$X'(8, 0)$; $Y'(15, -5)$; $Z'(7, -7)$**

9. A triangle is rotated 270° counterclockwise about the origin. The coordinates of the new vertices are $R'(-4, 1)$, $S'(1, -8)$, and $T'(6, 4)$. What were the coordinates of the triangle in its original position?

9. **$R(-1, -4)$; $S(8, 1)$; $T(-4, 6)$**

10. Find the value of $\begin{vmatrix} 6.0 & -3.1 \\ -5.9 & -4.8 \end{vmatrix}$.

10. **−47.09**

11. Solve for x if $\begin{vmatrix} -4 & 2x \\ 3 & x \end{vmatrix} = 18$.

11. $-\frac{9}{5}$

Assessment

4 Chapter 4 Test, Form 3 *(continued)*

12. Evaluate $\begin{vmatrix} 7 & 5 & 4 \\ -3 & -9 & 5 \\ 2 & 0 & -3 \end{vmatrix}$ using expansion by minors.

12. **266**

13. **GEOMETRY** Find the area of a triangle whose vertices are located at $\left(-4, \frac{1}{2}\right)$, $\left(-\frac{5}{2}, -1\right)$, and $(6, -2)$. Evaluate the determinant using diagonals.

13. $\frac{45}{8}$ **units**^{2}

For Questions 14 and 15, use Cramer's Rule to solve each system of equations.

14. $\frac{1}{2}x + \frac{1}{4}y = 1$

$3x - \frac{1}{2}y = -4$

15. $3m + 4n + 6p = 15$

$2m + 3n - 5p = -11$

$5m + 6n - p = 9$

14. $\left(-\frac{1}{2}, 5\right)$

15. **(13, −9, 2)**

16. Determine whether $M = \begin{bmatrix} 5 & -5 \\ -1 & 2 \end{bmatrix}$ and $N = \begin{bmatrix} \frac{2}{5} & 1 \\ \frac{1}{5} & 1 \end{bmatrix}$ are inverses.

16. **yes**

17. Find the inverse of $A = \begin{bmatrix} \frac{1}{5} & \frac{2}{3} \\ -\frac{2}{5} & -\frac{1}{3} \end{bmatrix}$, if it exists.

17. $5\begin{bmatrix} -\frac{1}{3} & -\frac{2}{3} \\ \frac{2}{5} & \frac{1}{5} \end{bmatrix}$

For Questions 18 and 19, solve each system of equations by using inverse matrices.

18. $9a - 4b = -5$

$6a - 2b = -3$

18. $\left(-\frac{1}{3}, \frac{1}{2}\right)$

19. $5x + 3y = -4.5$

$2x + 1.2y = -1.8$

19. **infinitely many solutions**

20. **BUDGET** A foundation spent all of its operating budget on administrative expenses, renewal grants, and low-interest loans. Administrative expenses and renewal grants accounted for 78.4% of the total budget. Renewable grants accounted for 5.1% more of the total budget than low-interest loans. Let a, r, and l each represent the percent of the budget accounted for by administrative expenses, renewal grants, and low-interest loans, respectively. Write a matrix equation that describes this situation.

20. $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} a \\ r \\ l \end{bmatrix} = \begin{bmatrix} 100 \\ 78.4 \\ 5.1 \end{bmatrix}$

Bonus Find the value of $\begin{vmatrix} a+1 & a & 1 \\ b+1 & b & -1 \\ c+1 & c & 1 \end{vmatrix}$.

B: $-2a + 2c$

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4 Chapter 4 Extended-Response Test

SCORE _____

Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solutions in more than one way or investigate beyond the requirements of the problem.

1. State any conditions or restrictions on m, n, j, and k when performing the indicated operations for matrices $A_{m \times n}$ and $B_{j \times k}$.

a. adding A and B

b. multiplying A and B

c. finding the determinant of A

d. multiplying B by a scalar c

e. finding the inverse of A

2. Choose three points in the coordinate plane so that no two points lie in the same quadrant. Consider these points the vertices of $\triangle XYZ$.

a. Write the vertex matrix V for $\triangle XYZ$, and state the dimensions of V.

For parts b through f, use $T = \begin{bmatrix} 3 & 3 & 3 \\ -4 & -4 & -4 \end{bmatrix}$, $R = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ to find each sum or product. Then state the type of transformation represented by the new matrix, describing the relationship between the image and premimage of $\triangle XYZ$.

b. $T + V$ **c.** $3V$ **d.** RV **e.** CV **f.** IV

3. Suppose you are asked to find the value of a if $\begin{vmatrix} 2 & -7 & 5 \\ 7 & a & -2 \\ 1 & 0 & -1 \end{vmatrix} = 0.$

a. If you must evaluate the determinant above using expansion by minors, which row would you use for the expansion? Explain your reasoning.

b. If you were able to choose which method, either using expansion by minors or using diagonals, to evaluate this determinant, which method would you choose? Why?

c. Find the value of a using the method of your choice.

4. Christopher purchased three CDs and two videos, and spent a total of \$85. Edward purchased two CDs and one video, spending \$50.

a. Write a system of equations that describes this situation. Explain what the variables in your system represent.

b. Use Cramer's Rule to solve your system. Interpret the meaning of your solution.

c. Write a matrix equation for your system of equations. Then solve the matrix equation by using inverse matrices.

d. Which of the two methods do you prefer. Why?

Assessment

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4 Standardized Test Practice

(Chapters 1–4)

Part 1: Multiple Choice

Instructions: Fill in the appropiate oval for the best answer.

1. If $4x^2 = 13$, then what is the value of $(2x - 3)(2x + 3)$?

A 4 **B** $22 - 6\sqrt{3}$ **C** $\sqrt{13} - 9$ **D** –22

1. Ⓐ Ⓑ Ⓒ Ⓓ

2. If the measure of the edge of a cube is 3, the measure of the surface area of the cube is _____.

F 36 **G** 54 **H** 27 **J** 9

2. Ⓕ Ⓖ Ⓗ Ⓙ

3. If the average of a and b equals the average of a, b, and c, then express c in terms of a and b.

A $a + b$ **B** $2(a + b)$ **C** $\frac{a + b}{2}$ **D** $\frac{a + b}{3}$

3. Ⓐ Ⓑ Ⓒ Ⓓ

4. There are 100 items in a garage sale. 30% of the items are sold during the first hour of the sale. If 10 items are sold during the second hour, what percent of the items have not been sold?

F 10% **G** 60% **H** 70% **J** 90%

4. Ⓕ Ⓖ Ⓗ Ⓙ

5. If it takes 6 hours for 2 people to clean a house, how many hours will it take 4 people, working at the same rate, to clean another house that is the same size?

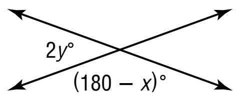
A $\frac{2}{3}$ **B** $1\frac{1}{2}$ **C** 2 **D** 3

5. Ⓐ Ⓑ Ⓒ Ⓓ

6. Which statement must be true?

F $x > y$ **H** $x < y$

G $x = y$ **J** $2y = 180 - x$



6. Ⓕ Ⓖ Ⓗ Ⓙ

7. Three less than three times a number is $\frac{5}{6}$. What is one more than twice the number?

A $2\frac{2}{3}$ **B** $3\frac{4}{9}$ **C** $3\frac{5}{9}$ **D** $2\frac{1}{2}$

7. Ⓐ Ⓑ Ⓒ Ⓓ

8. $r \triangle s$ is defined as $2r^2 + s^2 + 4rs$. What is the value of $-2 \triangle -1$?

F –17 **G** –1 **H** 1 **J** 17

8. Ⓕ Ⓖ Ⓗ Ⓙ

9. Which is closest to the value of $\frac{36.8 \times 1.5}{2.9}$?

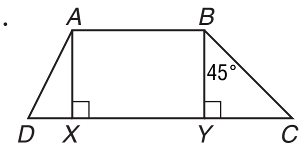
A 19 **B** 100 **C** 150 **D** 200

9. Ⓐ Ⓑ Ⓒ Ⓓ

10. In the figure, $\overline{AB} \parallel \overline{DC}$, $DX = 8$, and $AD = 17$. What is the length of BC?

F 17 **H** $8\sqrt{2}$

G $15\sqrt{2}$ **J** 15



10. Ⓕ Ⓖ Ⓗ Ⓙ

4 Standardized Test Practice *(continued)*

11. What is the value of r^3 if $\frac{1}{r} = \sqrt{0.01}$? **11.** A B C D

A 1000 **B** 100 **C** 0.01 **D** 0.0001

12. If $5^{y-4} = 25$ and $3^{2x+1} = 27$, what is the value of $\frac{x}{y}$? **12.** F G H J

F $\frac{2}{3}$ **G** 6 **H** $\frac{1}{6}$ **J** $\frac{4}{9}$

13. The Sweater Shop sells sweaters in three colors: red, \$20; blue, \$25; and white, \$28. On Tuesday, they sold 23 sweaters. They sold seven more red sweaters than blue sweaters. If they sold \$522 in sweaters, how many white sweaters did they sell? **13.** A B C D

A 4 **B** 6 **C** 9 **D** 13

14. Solve for x and y in the matrix equation $\begin{bmatrix} x \\ 5 \end{bmatrix} + \begin{bmatrix} 2y \\ -x \end{bmatrix} = \begin{bmatrix} 14 \\ -3 \end{bmatrix}$. **14.** F G H J

F $x = 3, y = -8$ **H** $x = 0, y = 7$

G $x = 8, y = 3$ **J** $x = 4, y = 5$

15. What are the dimensions of the matrix that results from the multiplication shown? **15.** A B C D

$$\begin{bmatrix} x & y & z & 1 \\ p & q & r & 1 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 4 \\ 3 \\ 2 \end{bmatrix}$$

A 2×1 **B** 4×1 **C** 4×4 **D** 5×4

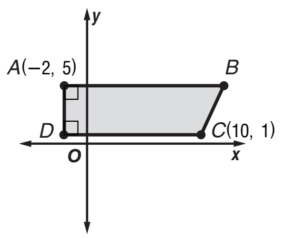
Assessment

Part 2: Grid In

Instructions: Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate circle that corresponds to that entry.

16. For what integer value of m is $2m + 14 > 42$ and $5 < \frac{m}{4} + 3 < 7$?

17. If $\overline{DC}$ is parallel to the x-axis and $BC = 5$ in the figure, how many units^2 is the area of the shaded region?



16.

		1	5	.			
0	0	0	0		0	0	0
1	1	1	1		1	1	1
2	2	2	2		2	2	2
3	3	3	3		3	3	3
4	4	4	4		4	4	4
5	5	5	5		5	5	5
6	6	6	6		6	6	6
7	7	7	7		7	7	7
8	8	8	8		8	8	8
9	9	9	9		9	9	9

17.

		5	4	.			
0	0	0	0		0	0	0
1	1	1	1		1	1	1
2	2	2	2		2	2	2
3	3	3	3		3	3	3
4	4	4	4		4	4	4
5	5	5	5		5	5	5
6	6	6	6		6	6	6
7	7	7	7		7	7	7
8	8	8	8		8	8	8
9	9	9	9		9	9	9

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4 Standardized Test Practice *(continued)*

Part 3: Short Answer

Instructions: Write your answers in the space provided.

18. The formula $S = \frac{n(n+1)}{2}$ can be used to find the sum of the first n natural numbers. Find the sum of the first 100 natural numbers.

18. **5050**

19. Evaluate $8 - |3a + b|$ if $a = 4.3$ and $b = -15$.

19. **5.9**

20. Find the slope of the line that passes through (4, –3) and (–1, –7).

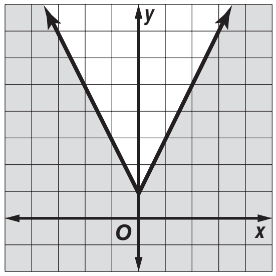
20. $\frac{4}{5}$

21. Identify the domain and range of the function $g(x) = |x| - 4$.

21. **D = all real numbers** **$\{R = y | y \geq -4\}$**

22. Graph $y \leq |2x| + 1$.

22.



23. The vertex of an angle is the point where the lines whose equations are $y = 2x$ and $y = x + 1$ meet. Find the coordinates of the vertex.

23. **(1, 2)**

24. Determine whether the ordered pair (–2, 6) is a solution of the system $y > 2x + 7$ and $y \leq 5 - 3x$.

24. **yes**

25. Solve $\begin{bmatrix} 2x - y \\ x + 4y \end{bmatrix} = \begin{bmatrix} -8 \\ 14 \end{bmatrix}$.

25. **(−2, 4)**

For Questions 26 and 27, perform the indicated matrix operations. If the matrix does not exist, write *impossible*.

26. $\begin{bmatrix} 3 & 1 \\ -2 & 17 \end{bmatrix} - \begin{bmatrix} 1 & 9 & -5 \\ -7 & 6 & 4 \end{bmatrix}$

27. $-4\begin{bmatrix} 3 & 0 & 11 \\ -9 & 2 & 6 \\ 4 & -3 & -5 \end{bmatrix}$

26. **impossible**

27. $\begin{bmatrix} -12 & 0 & -44 \\ 36 & -8 & -24 \\ -16 & 12 & 20 \end{bmatrix}$

28. Evaluate $\begin{vmatrix} -4 & 2 & -1 \\ 1 & -1 & 2 \\ -3 & 0 & 5 \end{vmatrix}$ using diagonals.

28. **1**

29a. The sum of two numbers is 37. The second number is 3 more than the first number. Write a system of equations to represent the given information.

29b. Write a matrix equation for the system of equations.

29c. What are the two numbers?

29a. **Sample answer:** $a + b = 37; b = a + 3$

29b. $\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 37 \\ 3 \end{bmatrix}$

29c. $a = 17, b = 20$

4 Unit 1 Test

(Chapters 1–4)

SCORE ______

1. Simplify $2(x + 3) - (2x - 1)$.

1. **7**

2. Name the sets of numbers to which 457 belongs.

2. **N, W, Z, Q, R**

3. The sum of a number and 17 more than twice the same number is 101. Find the number.

3. **28**

4. Evaluate $5 - |a + 5b|$ if $a = -12$ and $b = 2$.

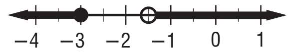
4. **3**

5. Define a variable and write an inequality. Then solve. A local summer baseball team plays 20 games each season. So far, they have won 9 games and lost 2. How many more games must they win this season to win at least 75% of all their games?

5. ***g* = the number of additional games to be won;** $\frac{g + 9}{20} \geq 0.75$**; at least six games**

6. Solve $3 + 2(1 + x) > 4$ or $2x + 14 \leq 8$. Graph the solution set on a number line.

6. $\left\{x \mid x \leq -3 \text{ or } x > -\frac{1}{2}\right\}$



7. Solve $3 + |2y - 1| \geq 1$. Graph the solution set on a number line.

7. **all real numbers**

8. If $f(x) = \frac{5x^2 - 4}{x}$, find $f(4)$.

8. **19**

9. Write an equation in slope-intercept form for the line that passes through (3, 5) and (–2, 1).

9. $y = \frac{4}{5}x + \frac{13}{5}$

10. Write an equation for the line that passes through (0, 7) and is perpendicular to the line whose equation is $y = \frac{1}{2}x - 1$.

10. $y = -2x + 7$

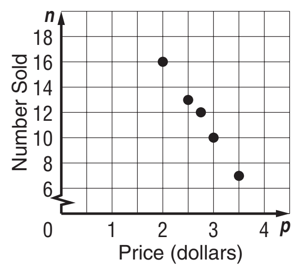
For Questions 11 and 12, use the set of data in the table.

The table shows the relationship between the price of a comic book and the number of copies sold.

Price p (in dollars)	2.00	2.50	2.75	3.00	3.50
Number sold n	16	13	12	10	7

11. Draw a scatter plot for the data.

11.



12. Use two ordered pairs to write a prediction equation. Then use your prediction equation to predict the number of comic books sold when the price is $4.50.

12. **Sample answer using (2, 16) and (3, 10):** $n = -6p + 28$; **1**

13. Evaluate $h\left(-\frac{2}{3}\right)$ if $h(x) = [\![x - 2]\!]$.

13. **−3**

14. Describe the system $6x - 2y = 10$ and $9x - 3y = 8$ as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

14. **inconsistent**

Assessment

4 Unit 1 Test *(continued)*

(Chapters 1–4)

15. Solve the system of equations at the right by using substitution. $4x - 2y = 2$, $y = -4x - 7$ — **15.** **(−1, −3)**

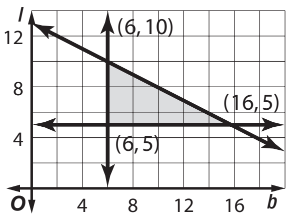
16. Solve the system of equations at the right by using elimination. $x + 3y = 5$, $3x - y = 5$ — **16.** **(2, 1)**

For Questions 17–19, use the following information.

A furniture company displays bedroom sets which require 21 square meters of space and living room sets which require 42 square meters of space. The company, which has 546 square meters of available space, wants to display at least 6 bedroom sets and at least 5 living room sets.

17. Let b represent the number of bedroom sets and ℓ represent the number of living room sets. Write a system of inequalities to represent the number of furniture sets that can be displayed. — **17.** $b \geq 6$; $l \geq 5$; $21b + 42l \leq 546$

18. Draw the graph showing the feasible region. Label the coordinates of the vertices of the feasible region. — **18.**



19. If a bedroom set sells for \$10,000 and a living room set sells for \$18,000, determine the number of bedroom sets and living room sets that must be sold to maximize the amount collected. — **19.** **16 bedroom sets 5 living room sets**

For Questions 20 and 21, use the matrices below.

$$A = \begin{bmatrix} 17 & 2 & 3 \\ 11 & 4 & -9 \end{bmatrix} \quad B = \begin{bmatrix} 10 & 6 & -7 \\ -4 & 3 & 0 \end{bmatrix} \quad C = \begin{bmatrix} -1 \\ 2 \\ -2 \end{bmatrix}$$

20. Find $A - B$. — **20.** $\begin{bmatrix} 7 & -4 & 10 \\ 15 & 1 & -9 \end{bmatrix}$

21. Find BC, if possible. — **21.** $\begin{bmatrix} 16 \\ 10 \end{bmatrix}$

22. Triangle DEF with vertices $D(2, 5)$, $E(1, –6)$, and $F(–5, 3)$ is translated 3 units right and 2 units down. Find the coordinates of $D'E'F'$. — **22.** $D'(5, 3)$; $E'(4, -8)$; $F'(-2, 1)$

23. Evaluate $\begin{vmatrix} 12 & 5 & -2 \\ -3 & 0 & 1 \\ -5 & 4 & 2 \end{vmatrix}$ using expansion by minors. — **23.** **−19**

24. Use Cramer's Rule to solve the system of equations $3x - 5y = 21$ and $4x + 2y = 2$. — **24.** **(2, −3)**

25. Solve the matrix equation $\begin{bmatrix} 4 & -5 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} 32 \\ -5 \end{bmatrix}$ using inverse matrices. — **25.** **(3, −4)**

Assessment

NAME ______________ DATE ________ PERIOD _____

4 Anticipation Guide

Matrices

STEP 1 ***Before you begin Chapter 4***

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

STEP 1 A, D, or NS	Statement	STEP 2 A or D
	1. A matrix contains constants or variables in horizontal rows and vertical columns.	A
	2. Each value in a matrix is called a term.	D
	3. If two matrices contain the same numbers but have a different number of rows or columns, then they are not equal.	A
	4. Two matrices with different dimensions can be added or subtracted by adding zeros so that both matrices have the same dimensions.	D
	5. The product of a matrix and a constant can be found by multiplying each element of the matrix by that constant.	A
	6. The associative, commutative, and distributive properties of multiplication are all true for matrices.	D
	7. A translation is a transformation in which a figure is turned around a single point.	D
	8. A vertex matrix is a matrix containing the coordinates of the vertices of a figure.	A
	9. A third-order determinant of a matrix contains three columns and any number of rows.	D
	10. Each element of an identity matrix for multiplication is 1.	D
	11. Two matrices are inverses of each other if their product is the identity matrix.	A
	12. To solve a system of equations using matrices, you must write a matrix for the coefficients, one for the variables, and one for the constants.	A

STEP 2 ***After you complete Chapter 4***

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

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4-1 Lesson Reading Guide

Introduction to Matrices

Get Ready for the Lesson

Read the introduction to Lesson 4-1 in your textbook.

a. What is the base price of a Mid-Size SUV? **$27,350**
b. What is the exterior length of a Compact SUV? **175.2 in.**

Read the Lesson

1. Give the dimensions of each matrix.

a. $\begin{bmatrix} 3 & 2 & 5 \\ -1 & 0 & 6 \end{bmatrix}$ **2×3**

b. $\begin{bmatrix} 1 & 4 & 0 & -8 & 2 \end{bmatrix}$ **1×5**

2. Identify each matrix with as many of the following descriptions that apply: *row matrix*, *column matrix*, *square matrix*, *zero matrix*.

a. $\begin{bmatrix} 6 & 5 & 4 & 3 \end{bmatrix}$ **row matrix**

b. $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ **column matrix; zero matrix**

c. $\begin{bmatrix} 0 \end{bmatrix}$ **row matrix; column matrix; square matrix; zero matrix**

3. Write a system of equations that you could use to solve the following matrix equation for x, y, and z. (Do not actually solve the system.)

$\begin{bmatrix} 3x \\ x + y \\ y - z \end{bmatrix} = \begin{bmatrix} -9 \\ 5 \\ 6 \end{bmatrix}$ **$3x = -9$, $x + y = 5$, $y - z = 6$**

Remember What You Learned

4. Some students have trouble remembering which number comes first in writing the dimensions of a matrix. Think of an easy way to remember this.
Sample answer: Read the matrix from top to bottom, then from left to right. Reading down gives the number of rows, which is written first in the dimensions of the matrix. Reading across gives the number of columns, which is written second.

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4-1 Study Guide and Intervention

Introduction to Matrices

Organize Data

Matrix	a rectangular array of variables or constants in horizontal rows and vertical columns, usually enclosed in brackets.

A matrix can be described by its **dimensions**. A matrix with m rows and n columns is an $m \times n$ matrix.

Example 1 **Owls' eggs incubate for 30 days and their fledgling period is also 30 days. Swifts' eggs incubate for 20 days and their fledgling period is 44 days. Pigeon eggs incubate for 15 days, and their fledgling period is 17 days. Eggs of the king penguin incubate for 53 days, and the fledgling time for a king penguin is 360 days. Write a 2 × 4 matrix to organize this information.** Source: *The Cambridge Factfinder*

$$\begin{array}{r} \\ \text{Incubation} \\ \text{Fledgling} \end{array} \begin{array}{c} \begin{array}{cccc} \text{Owl} & \text{Swift} & \text{Pigeon} & \text{King Penguin} \end{array} \\ \begin{bmatrix} 30 & 20 & 15 & 53 \\ 30 & 44 & 17 & 360 \end{bmatrix} \end{array}$$

Example 2 **What are the dimensions of matrix A if $A = \begin{bmatrix} 13 & 10 & -3 & 45 \\ 2 & 8 & 15 & 80 \end{bmatrix}$?**

Since matrix A has 2 rows and 4 columns, the dimensions of A are 2×4.

Exercises

State the dimensions of each matrix.

1. $\begin{bmatrix} 15 & 5 & 27 & -4 \\ 23 & 6 & 0 & 5 \\ 14 & 70 & 24 & -3 \\ 63 & 3 & 42 & 90 \end{bmatrix}$ **4 × 4**

2. $[16 \quad 12 \quad 0]$ **1 × 3**

3. $\begin{bmatrix} 71 & 44 \\ 39 & 27 \\ 45 & 16 \\ 92 & 53 \\ 78 & 65 \end{bmatrix}$ **5 × 2**

4. A travel agent provides for potential travelers the normal high temperatures for the months of January, April, July, and October for various cities. In Boston these figures are 36°, 56°, 82°, and 63°. In Dallas they are 54°, 76°, 97°, and 79°. In Los Angeles they are 68°, 72°, 84°, and 79°. In Seattle they are 46°, 58°, 74°, and 60°, and in St. Louis they are 38°, 67°, 89°, and 69°. Organize this information in a 4 × 5 matrix. Source: *The New York Times Almanac*

$$\begin{array}{r} \\ \text{January} \\ \text{April} \\ \text{July} \\ \text{October} \end{array} \begin{array}{c} \begin{array}{ccccc} \text{Boston} & \text{Dallas} & \text{Los Angeles} & \text{Seattle} & \text{St. Louis} \end{array} \\ \begin{bmatrix} 36 & 54 & 68 & 46 & 38 \\ 56 & 76 & 72 & 58 & 67 \\ 82 & 97 & 84 & 74 & 89 \\ 63 & 79 & 79 & 60 & 69 \end{bmatrix} \end{array}$$

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4-1 Study Guide and Intervention *(continued)*

Introduction to Matrices

Equations Involving Matrices

Equal Matrices	Two matrices are equal if they have the same dimensions and each element of one matrix is equal to the corresponding element of the other matrix.

You can use the definition of equal matrices to solve matrix equations.

Example **Solve $\begin{bmatrix} 4x \\ y \end{bmatrix} = \begin{bmatrix} -2y + 2 \\ x - 8 \end{bmatrix}$ for x and y.**

Since the matrices are equal, the corresponding elements are equal. When you write the sentences to show the equality, two linear equations are formed.

$4x = -2y + 2$
$y = x - 8$

This system can be solved using substitution.

$4x = -2y + 2$	First equation
$4x = -2(x - 8) + 2$	Substitute $x - 8$ for y.
$4x = -2x + 16 + 2$	Distributive Property
$6x = 18$	Add $2x$ to each side.
$x = 3$	Divide each side by 6.

To find the value of y, substitute 3 for x in either equation.

$y = x - 8$	Second equation
$y = 3 - 8$	Substitute 3 for x.
$y = -5$	Subtract.

The solution is $(3, -5)$.

Exercises

Solve each equation.

1. $[5x \ \ 4y] = [20 \ \ 20]$ **(4, 5)**

2. $\begin{bmatrix} 3x \\ y \end{bmatrix} = \begin{bmatrix} 28 + 4y \\ -3x - 2 \end{bmatrix}$ **$\left(\frac{4}{3}, -6\right)$**

3. $\begin{bmatrix} -2y \\ x \end{bmatrix} = \begin{bmatrix} 4 - 5x \\ y + 5 \end{bmatrix}$ **(−2, −7)**

4. $\begin{bmatrix} x - 2y \\ 3x - 4y \end{bmatrix} = \begin{bmatrix} -1 \\ 22 \end{bmatrix}$ **(24, 12.5)**

5. $\begin{bmatrix} 2x + 3y \\ x - 2y \end{bmatrix} = \begin{bmatrix} 3 \\ 12 \end{bmatrix}$ **(6, −3)**

6. $\begin{bmatrix} 5x + 3y \\ 2x - y \end{bmatrix} = \begin{bmatrix} -1 \\ -18 \end{bmatrix}$ **(−5, 8)**

7. $\begin{bmatrix} 8x - y & 16x \\ 12 & y - 4x \end{bmatrix} = \begin{bmatrix} 18 & 20 \\ 12 & -13 \end{bmatrix}$ **$\left(\frac{5}{4}, -8\right)$**

8. $\begin{bmatrix} 8x - 6y \\ 12x + 4y \end{bmatrix} = \begin{bmatrix} -3 \\ -11 \end{bmatrix}$ **$\left(-\frac{3}{4}, -\frac{1}{2}\right)$**

9. $\begin{bmatrix} \frac{x}{3} + \frac{y}{7} \\ \frac{x}{2} + 2y \end{bmatrix} = \begin{bmatrix} 9 \\ 51 \end{bmatrix}$ **(18, 21)**

10. $\begin{bmatrix} 3x + 1.5 \\ 2y - 2.4 \end{bmatrix} = \begin{bmatrix} 7.5 \\ 8.0 \end{bmatrix}$ **(2, 5.2)**

11. $\begin{bmatrix} 2x + 3y \\ -4x + 0.5y \end{bmatrix} = \begin{bmatrix} 17 \\ -8 \end{bmatrix}$ **(2.5, 4)**

12. $\begin{bmatrix} x - y \\ x + y \end{bmatrix} = \begin{bmatrix} 0 \\ -25 \end{bmatrix}$ **(−12.5, −12.5)**

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4-1 Skills Practice

Introduction to Matrices

State the dimensions of each matrix.

1. $\begin{bmatrix} 3 & 2 & 4 \\ -1 & 4 & 0 \end{bmatrix}$ **2 × 3**

2. $[0 \quad 15]$ **1 × 2**

3. $\begin{bmatrix} 3 & 2 \\ 1 & 8 \end{bmatrix}$ **2 × 2**

4. $\begin{bmatrix} 6 & 1 & 2 \\ -3 & 4 & 5 \\ -2 & 7 & 9 \end{bmatrix}$ **3 × 3**

5. $\begin{bmatrix} 9 & 3 & -3 & -6 \\ 3 & 4 & -4 & 5 \end{bmatrix}$ **2 × 4**

6. $\begin{bmatrix} -1 \\ -1 \\ -1 \\ -3 \end{bmatrix}$ **4 × 1**

Solve each equation.

7. $[5x \quad 3y] = [15 \quad 12]$ **(3, 4)**

8. $[3x - 2] = [7]$ **3**

9. $\begin{bmatrix} 7x \\ 14 \end{bmatrix} = \begin{bmatrix} -14 \\ 2y \end{bmatrix}$ **(−2, 7)**

10. $[2x \quad -8y \quad z] = [10 \quad 16 \quad -1]$ **(5, −2, −1)**

11. $\begin{bmatrix} 8 - x \\ 2y - 8 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$ **(4, 5)**

12. $\begin{bmatrix} 20 \\ 56 - 6y \end{bmatrix} = \begin{bmatrix} 10x \\ 32 \end{bmatrix}$ **(2, 4)**

13. $\begin{bmatrix} 5x \\ 24 \end{bmatrix} = \begin{bmatrix} -20 \\ 8y \end{bmatrix}$ **(−4, 3)**

14. $\begin{bmatrix} 3x + 2 \\ 7y - 2 \end{bmatrix} = \begin{bmatrix} 5x + 2 \\ 3y - 10 \end{bmatrix}$ **(0, −2)**

15. $\begin{bmatrix} 4x - 1 \\ 9y + 5 \end{bmatrix} = \begin{bmatrix} 3x \\ y - 3 \end{bmatrix}$ **(1, −1)**

16. $\begin{bmatrix} 3x + 1 & 18 \\ 12 & 4z \end{bmatrix} = \begin{bmatrix} 7 & 2y - 4 \\ 12 & 28 \end{bmatrix}$ **(2, 11, 7)**

17. $\begin{bmatrix} x \\ 16 \\ 3z \end{bmatrix} = \begin{bmatrix} 9 \\ 4y \\ 9 \end{bmatrix}$ **(9, 4, 3)**

18. $\begin{bmatrix} 5x \\ 4y - 3 \\ 8z \end{bmatrix} = \begin{bmatrix} 4x + 1 \\ 13 \\ 4z \end{bmatrix}$ **(1, 4, 0)**

19. $\begin{bmatrix} 2x \\ y + 2 \end{bmatrix} = \begin{bmatrix} 6y \\ x \end{bmatrix}$ **(3, 1)**

20. $\begin{bmatrix} x \\ 3y \end{bmatrix} = \begin{bmatrix} 4y \\ x - 3 \end{bmatrix}$ **(12, 3)**

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4-1 Practice

Introduction to Matrices

State the dimensions of each matrix.

1. $[-3 \quad -3 \quad 7]$ **1 × 3**

2. $\begin{bmatrix} 5 & 8 & -1 \\ -2 & 1 & 8 \end{bmatrix}$ **2 × 3**

3. $\begin{bmatrix} -2 & 2 & -2 & 3 \\ 5 & 16 & 0 & 0 \\ 4 & 7 & -1 & 4 \end{bmatrix}$ **3 × 4**

Solve each equation.

4. $[4x \quad 42] = [24 \quad 6y]$ **(6, 7)**

5. $[-2x \quad 22 \quad -3z] = [6x \quad -2y \quad 45]$ **(0, −11, −15)**

6. $\begin{bmatrix} 6x \\ 2y + 3 \end{bmatrix} = \begin{bmatrix} -36 \\ 17 \end{bmatrix}$ **(−6, 7)**

7. $\begin{bmatrix} 7x - 8 \\ 8y - 3 \end{bmatrix} = \begin{bmatrix} 20 \\ 2y + 3 \end{bmatrix}$ **(4, 1)**

8. $\begin{bmatrix} -4x - 3 \\ 6y \end{bmatrix} = \begin{bmatrix} -3x \\ -2y + 16 \end{bmatrix}$ **(−3, 2)**

9. $\begin{bmatrix} 6x - 12 \\ -3y + 6 \end{bmatrix} = \begin{bmatrix} -3x - 21 \\ 8y - 5 \end{bmatrix}$ **(−1, 1)**

10. $\begin{bmatrix} -5 & 3x + 1 \\ 2y - 1 & 3z - 2 \end{bmatrix} = \begin{bmatrix} -5 & x - 1 \\ 3y & 5z - 4 \end{bmatrix}$ **(−1, −1, 1)**

11. $\begin{bmatrix} 3x \\ y + 4 \end{bmatrix} = \begin{bmatrix} y + 8 \\ 17 \end{bmatrix}$ **(7, 13)**

12. $\begin{bmatrix} 5x + 8y \\ 3x - 11 \end{bmatrix} = \begin{bmatrix} -1 \\ y \end{bmatrix}$ **(3, −2)**

13. $\begin{bmatrix} 2x + y \\ 3x + 2y \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$ **(2, −4)**

14. TICKET PRICES The table at the right gives ticket prices for a concert. Write a 2 × 3 matrix that represents the cost of a ticket.

	Child	Student	Adult
Cost Purchased in Advance	\$6	\$12	\$18
Cost Purchased at the Door	\$8	\$15	\$22

$\begin{bmatrix} 6 & 12 & 18 \\ 8 & 15 & 22 \end{bmatrix}$

CONSTRUCTION For Exercises 15 and 16, use the following information.

During each of the last three weeks, a road-building crew has used three truckloads of gravel. The table at the right shows the amount of gravel in each load.

Week 1		Week 2		Week 3	
Load 1	40 tons	Load 1	40 tons	Load 1	32 tons
Load 2	32 tons	Load 2	40 tons	Load 2	24 tons
Load 3	24 tons	Load 3	32 tons	Load 3	24 tons

15. Write a matrix for the amount of gravel in each load.

$\begin{bmatrix} 40 & 32 & 24 \\ 40 & 40 & 32 \\ 32 & 24 & 24 \end{bmatrix}$ **or** $\begin{bmatrix} 40 & 40 & 32 \\ 32 & 40 & 24 \\ 24 & 32 & 24 \end{bmatrix}$

16. What are the dimensions of the matrix? **3 × 3**

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4-1 Word Problem Practice

Introduction to Matrices

1. HAWAII The table shows the population and area of some of the islands in Hawaii. What would be the dimensions of a matrix that represented this information?

Island	Population	Area
Hawaii	120,317	4,038
Maui	91,361	729
Oahu	836,231	594
Kauai	50,947	549
Lanai	2,426	140

Source: www.vthawaii.com

5 by 2 or 2 by 5

2. LAUNDRY Carl is looking for a Laundromat. SuperWash has 20 small washers, 10 large washers, and 20 dryers. QuickClean has 40 small washers, 5 large washers, and 50 dryers. ToughSuds has 15 small washers, 40 large washers, and 100 dryers. Write a matrix to organize this information.

Sample answer: $\begin{bmatrix} 20 & 10 & 20 \\ 40 & 5 & 50 \\ 15 & 40 & 100 \end{bmatrix}$

3. CITY DISTANCES The incomplete matrix shown gives the approximate distances between Chicago, Los Angeles, and New York City. Complete the matrix.

$$\begin{array}{r} \\ \text{NYC} \\ \text{Chicago} \\ \text{Los Angeles} \end{array} \begin{array}{c} \begin{array}{ccc} \text{NYC} & \text{Chicago} & \text{Los Angeles} \end{array} \\ \begin{bmatrix} 0 & 810 & 2790 \\ 810 & 0 & 2050 \\ 2790 & 2050 & 0 \end{bmatrix} \end{array}$$

4. INVENTORY A store manager records the number of light bulbs in stock for 3 different brands over a five-day period. The manager decides to make a matrix of this information. Each row represents a different brand, and each column represents a different day. The entry in column N represents the inventories at the beginning of day N.

$$\begin{bmatrix} 25 & 24 & 22 & 20 & 19 \\ 30 & 27 & 25 & 22 & 21 \\ 28 & 25 & 21 & 19 & 19 \end{bmatrix}$$

Assuming that the inventories were never replenished, which brand holds the record for most light bulbs sold on a given day?

the third row brand

SHOE SALES For Exercises 5 and 6, use the following information.
A shoe store manager keeps track of the amount of money made by each of three salespeople for each day of a workweek. Monday through Friday, Carla made \$40, \$70, \$35, \$50, and \$20. John made \$30, \$60, \$20, \$45, and \$30. Mary made \$35, \$90, \$30, \$40, and \$30.

5. Organize this data in a 3 by 5 matrix.
Sample answer:

$$\begin{array}{r} \\ \text{Carla} \\ \text{John} \\ \text{Mary} \end{array} \begin{array}{c} \begin{array}{ccccc} \text{M} & \text{Tu} & \text{W} & \text{Th} & \text{F} \end{array} \\ \begin{bmatrix} 40 & 70 & 35 & 50 & 20 \\ 30 & 60 & 20 & 45 & 30 \\ 35 & 90 & 30 & 40 & 30 \end{bmatrix} \end{array}$$

6. Which salesperson made the most money that week?
Mary

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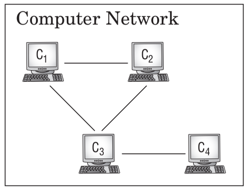
4-1 Enrichment

Matrices and Networks

Graph theory is a branch of mathematics that explores situations represented by points called vertices and the segments that may connect them, called edges. For example, graphs can be used to represent computer networks or airline routes between major cities.

An incidence matrix is a matrix used to represent the vertices, edges, and relationships among the vertices of a graph. The vertices name each row and column. For example, the incidence matrix for the computer network shown in the figure is shown below. The numbers represent how many edges connect the vertices.



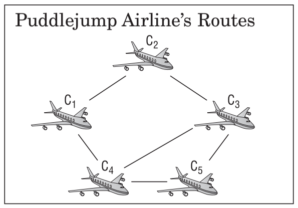
Indicates one edge from C_1 to C_2.

$$\begin{array}{r} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}$$

Indicates no edge from C_4 to C_1.

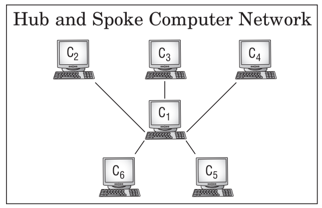
Complete the incidence matrix for each pictured network.

1. Puddlejump Airlines daily flights between cities 1–5.



	c_1	c_2	c_3	c_4	c_5
c_1	0	1	0	1	0
c_2	1	0	1	0	0
c_3	0	1	0	1	1
c_4	1	0	1	0	1
c_5	0	0	1	1	0

2. Computers in a Hub and Spoke network.



	c_1	c_2	c_3	c_4	c_5	c_6
c_1	0	1	1	1	1	1
c_2	1	0	0	0	0	0
c_3	1	0	0	0	0	0
c_4	1	0	0	0	0	0
c_5	1	0	0	0	0	0
c_6	1	0	0	0	0	0

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4-2 Lesson Reading Guide

Operations with Matrices

Get Ready for the Lesson

Read the introduction to Lesson 4-2 in your textbook.

- Write a sum that represents the total number of Calories in the patient's diet for Day 2. (Do not actually calculate the sum.) **482 + 622 + 987**
- Write the sum that represents the total fat content in the patient's diet for Day 3. (Do not actually calculate the sum.) **11 + 12 + 38**

Read the Lesson

1. For each pair of matrices, give the dimensions of the indicated sum, difference, or scalar product. If the indicated sum, difference, or scalar product does not exist, write *impossible*.

$A = \begin{bmatrix} 3 & 5 & 6 \\ -2 & 8 & 1 \end{bmatrix}$ $\quad B = \begin{bmatrix} -4 & 0 \\ 0 & -5 \end{bmatrix}$

$C = \begin{bmatrix} 5 & 10 \\ -3 & 6 \\ 4 & 12 \end{bmatrix}$ $\quad D = \begin{bmatrix} -3 & 6 & 0 \\ -8 & 4 & 0 \end{bmatrix}$

$A + D$: **2 × 3** $\quad C + D$: **impossible** $\quad 5B$: **2 × 2**

$-4C$: **3 × 2** $\quad 2D - 3A$: **2 × 3**

2. Suppose that M, N, and P are nonzero 2×4 matrices and k is a negative real number. Indicate whether each of the following statements is *true* or *false*.

a. $M + (N + P) = M + (P + N)$ **true** $\quad$ **b.** $M - N = N - M$ **false**

c. $M - (N - P) = (M - N) - P$ **false** $\quad$ **d.** $k(M - N) = kM - kN$ **true**

Remember What You Learned

3. The mathematical term *scalar* may be unfamiliar, but its meaning is related to the word *scale* as used in a *scale of miles* on a map. How can this usage of the word *scale* help you remember the meaning of *scalar*?

Sample answer: A scale of miles tells you how to multiply the distances you measure on a map by a certain number to get the actual distance between two locations. This multiplier is often called a *scale* factor. A *scalar* represents the same idea: It is a real number by which a matrix can be multiplied to change all the elements of the matrix by a uniform *scale* factor.

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4-2 Study Guide and Intervention

Operations with Matrices

Add and Subtract Matrices

Addition of Matrices	$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} + \begin{bmatrix} j & k & l \\ m & n & o \\ p & q & r \end{bmatrix} = \begin{bmatrix} a+j & b+k & c+l \\ d+m & e+n & f+o \\ g+p & h+q & i+r \end{bmatrix}$
Subtraction of Matrices	$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} - \begin{bmatrix} j & k & l \\ m & n & o \\ p & q & r \end{bmatrix} = \begin{bmatrix} a-j & b-k & c-l \\ d-m & e-n & f-o \\ g-p & h-q & i-r \end{bmatrix}$

Example 1 **Find $A + B$ if $A = \begin{bmatrix} 6 & -7 \\ 2 & -12 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 2 \\ -5 & -6 \end{bmatrix}$.**

$$A + B = \begin{bmatrix} 6 & -7 \\ 2 & -12 \end{bmatrix} + \begin{bmatrix} 4 & 2 \\ -5 & -6 \end{bmatrix}$$
$$= \begin{bmatrix} 6 + 4 & -7 + 2 \\ 2 + (-5) & -12 + (-6) \end{bmatrix}$$
$$= \begin{bmatrix} 10 & -5 \\ -3 & -18 \end{bmatrix}$$

Example 2 **Find $A - B$ if $A = \begin{bmatrix} -2 & 8 \\ 3 & -4 \\ 10 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -3 \\ -2 & 1 \\ -6 & 8 \end{bmatrix}$.**

$$A - B = \begin{bmatrix} -2 & 8 \\ 3 & -4 \\ 10 & 7 \end{bmatrix} - \begin{bmatrix} 4 & -3 \\ -2 & 1 \\ -6 & 8 \end{bmatrix}$$
$$= \begin{bmatrix} -2 - 4 & 8 - (-3) \\ 3 - (-2) & -4 - 1 \\ 10 - (-6) & 7 - 8 \end{bmatrix} = \begin{bmatrix} -6 & 11 \\ 5 & -5 \\ 16 & -1 \end{bmatrix}$$

Exercises

Perform the indicated operations. If the matrix does not exist, write *impossible*.

1. $\begin{bmatrix} 8 & 7 \\ -10 & -6 \end{bmatrix} - \begin{bmatrix} -4 & 3 \\ 2 & -12 \end{bmatrix}$ $\mathbf{\begin{bmatrix} 12 & 4 \\ -12 & 6 \end{bmatrix}}$

2. $\begin{bmatrix} 6 & -5 & 9 \\ -3 & 4 & 5 \end{bmatrix} + \begin{bmatrix} -4 & 3 & 2 \\ 6 & 9 & -4 \end{bmatrix}$ $\mathbf{\begin{bmatrix} 2 & -2 & 11 \\ 3 & 13 & 1 \end{bmatrix}}$

3. $\begin{bmatrix} 6 \\ -3 \\ 2 \end{bmatrix} + [-6 \quad 3 \quad -2]$ **impossible**

4. $\begin{bmatrix} 5 & -2 \\ -4 & 6 \\ 7 & 9 \end{bmatrix} + \begin{bmatrix} -11 & 6 \\ 2 & -5 \\ 4 & -7 \end{bmatrix}$ $\mathbf{\begin{bmatrix} -6 & 4 \\ -2 & 1 \\ 11 & 2 \end{bmatrix}}$

5. $\begin{bmatrix} 8 & 0 & -6 \\ 4 & 5 & -11 \\ -7 & 3 & 4 \end{bmatrix} - \begin{bmatrix} -2 & 1 & 7 \\ 3 & -4 & 3 \\ -8 & 5 & 6 \end{bmatrix}$

$\mathbf{\begin{bmatrix} 10 & -1 & -13 \\ 1 & 9 & -14 \\ 1 & -2 & -2 \end{bmatrix}}$

6. $\begin{bmatrix} \frac{3}{4} & \frac{2}{5} \\ -\frac{1}{2} & \frac{4}{3} \end{bmatrix} - \begin{bmatrix} \frac{1}{2} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{2} \end{bmatrix}$

$\mathbf{\begin{bmatrix} \frac{1}{4} & -\frac{4}{15} \\ -\frac{7}{6} & \frac{11}{6} \end{bmatrix}}$

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4-2 Study Guide and Intervention *(continued)*

Operations with Matrices

Scalar Multiplication You can multiply an $m \times n$ matrix by a scalar k.

Scalar Multiplication	$k\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} ka & kb & kc \\ kd & ke & kf \end{bmatrix}$

Example **If $A = \begin{bmatrix} 4 & 0 \\ -6 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 5 \\ 7 & 8 \end{bmatrix}$, find $3B - 2A$.**

$3B - 2A = 3\begin{bmatrix} -1 & 5 \\ 7 & 8 \end{bmatrix} - 2\begin{bmatrix} 4 & 0 \\ -6 & 3 \end{bmatrix}$ Substitution

$= \begin{bmatrix} 3(-1) & 3(5) \\ 3(7) & 3(8) \end{bmatrix} - \begin{bmatrix} 2(4) & 2(0) \\ 2(-6) & 2(3) \end{bmatrix}$ Multiply.

$= \begin{bmatrix} -3 & 15 \\ 21 & 24 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ -12 & 6 \end{bmatrix}$ Simplify.

$= \begin{bmatrix} -3-8 & 15-0 \\ 21-(-12) & 24-6 \end{bmatrix}$ Subtract.

$= \begin{bmatrix} -11 & 15 \\ 33 & 18 \end{bmatrix}$ Simplify.

Exercises

Perform the indicated matrix operations. If the matrix does not exist, write *impossible*.

1. $6\begin{bmatrix} 2 & -5 & 3 \\ 0 & 7 & -1 \\ -4 & 6 & 9 \end{bmatrix}$ $\begin{bmatrix} 12 & -30 & 18 \\ 0 & 42 & -6 \\ -24 & 36 & 54 \end{bmatrix}$

2. $-\frac{1}{3}\begin{bmatrix} 6 & 15 & 9 \\ 51 & -33 & 24 \\ -18 & 3 & 45 \end{bmatrix}$ $\begin{bmatrix} -2 & -5 & -3 \\ -17 & 11 & -8 \\ 6 & -1 & -15 \end{bmatrix}$

3. $0.2\begin{bmatrix} 25 & -10 & -45 \\ 5 & 55 & -30 \\ 60 & 35 & -95 \end{bmatrix}$ $\begin{bmatrix} 5 & -2 & -9 \\ 1 & 11 & -6 \\ 12 & 7 & -19 \end{bmatrix}$

4. $3\begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix} - 2\begin{bmatrix} -1 & 2 \\ -3 & 5 \end{bmatrix}$ $\begin{bmatrix} -10 & 11 \\ 12 & -1 \end{bmatrix}$

5. $-2\begin{bmatrix} 3 & -1 \\ 0 & 7 \end{bmatrix} + 4\begin{bmatrix} -2 & 0 \\ 2 & 5 \end{bmatrix}$ $\begin{bmatrix} -14 & 2 \\ 8 & 6 \end{bmatrix}$

6. $2\begin{bmatrix} 6 & -10 \\ -5 & 8 \end{bmatrix} + 5\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$ $\begin{bmatrix} 22 & -15 \\ 10 & 31 \end{bmatrix}$

7. $4\begin{bmatrix} 1 & -2 & 5 \\ -3 & 4 & 1 \end{bmatrix} - 2\begin{bmatrix} 4 & 3 & -4 \\ 2 & -5 & -1 \end{bmatrix}$ $\begin{bmatrix} -4 & -14 & 28 \\ -16 & 26 & 6 \end{bmatrix}$

8. $8\begin{bmatrix} 2 & 1 \\ 3 & -1 \\ -2 & 4 \end{bmatrix} + 3\begin{bmatrix} 4 & 0 \\ -2 & 3 \\ 3 & -4 \end{bmatrix}$ $\begin{bmatrix} 28 & 8 \\ 18 & 1 \\ -7 & 20 \end{bmatrix}$

9. $\frac{1}{4}\left(\begin{bmatrix} 9 & 1 \\ -7 & 0 \end{bmatrix} + \begin{bmatrix} 3 & -5 \\ 1 & 7 \end{bmatrix}\right)$ $\begin{bmatrix} 3 & -1 \\ -\frac{3}{2} & \frac{7}{4} \end{bmatrix}$

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4-2 Skills Practice

Operations with Matrices

Perform the indicated matrix operations. If the matrix does not exist, write *impossible*.

1. $[5 \quad -4] + [4 \quad 5]$ $[9 \quad 1]$

2. $\begin{bmatrix} 8 & 3 \\ -1 & -1 \end{bmatrix} - \begin{bmatrix} 0 & -7 \\ 6 & 2 \end{bmatrix}$ $\begin{bmatrix} 8 & 10 \\ -7 & -3 \end{bmatrix}$

3. $[3 \quad 1 \quad 6] + \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$ impossible

4. $\begin{bmatrix} 5 & -1 & 2 \\ 1 & 8 & -6 \end{bmatrix} + \begin{bmatrix} 9 & 9 & 2 \\ 4 & 6 & 4 \end{bmatrix}$ $\begin{bmatrix} 14 & 8 & 4 \\ 5 & 14 & -2 \end{bmatrix}$

5. $3[9 \quad 4 \quad -3]$ $[27 \quad 12 \quad -9]$

6. $[6 \quad -3] - 4[4 \quad 7]$ $[-10 \quad -31]$

7. $-2\begin{bmatrix} -2 & 5 \\ 5 & 9 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $\begin{bmatrix} 5 & -9 \\ -9 & -17 \end{bmatrix}$

8. $3\begin{bmatrix} 8 \\ 0 \\ -3 \end{bmatrix} - 4\begin{bmatrix} 2 \\ 2 \\ 10 \end{bmatrix}$ $\begin{bmatrix} 16 \\ -8 \\ -49 \end{bmatrix}$

9. $5\begin{bmatrix} -4 & 6 \\ 10 & 1 \\ -1 & 1 \end{bmatrix} + 2\begin{bmatrix} 6 & 5 \\ -3 & -2 \\ 1 & 0 \end{bmatrix}$ $\begin{bmatrix} -8 & 40 \\ 44 & 1 \\ -3 & 5 \end{bmatrix}$

10. $3\begin{bmatrix} 3 & 1 & 3 \\ -4 & 7 & 5 \end{bmatrix} - 2\begin{bmatrix} 1 & -1 & 5 \\ 6 & 6 & -3 \end{bmatrix}$ $\begin{bmatrix} 7 & 5 & -1 \\ -24 & 9 & 21 \end{bmatrix}$

Use $A = \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 2 \\ 1 & -2 \end{bmatrix}$, and $C = \begin{bmatrix} -3 & 4 \\ 3 & 1 \end{bmatrix}$ to find the following.

11. $A + B$ $\begin{bmatrix} 5 & 4 \\ 5 & 1 \end{bmatrix}$

12. $B - C$ $\begin{bmatrix} 5 & -2 \\ -2 & -3 \end{bmatrix}$

13. $B - A$ $\begin{bmatrix} -1 & 0 \\ -3 & -5 \end{bmatrix}$

14. $A + B + C$ $\begin{bmatrix} 2 & 8 \\ 8 & 2 \end{bmatrix}$

15. $3B$ $\begin{bmatrix} 6 & 6 \\ 3 & -6 \end{bmatrix}$

16. $-5C$ $\begin{bmatrix} 15 & -20 \\ -15 & -5 \end{bmatrix}$

17. $A - 4C$ $\begin{bmatrix} 15 & -14 \\ -8 & -1 \end{bmatrix}$

18. $2B + 3A$ $\begin{bmatrix} 13 & 10 \\ 14 & 5 \end{bmatrix}$

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4-2 Practice

Operations with Matrices

Perform the indicated matrix operations. If the matrix does not exist, write *impossible*.

1. $\begin{bmatrix} 2 & -1 \\ 3 & 7 \\ 14 & -9 \end{bmatrix} + \begin{bmatrix} -6 & 9 \\ 7 & -11 \\ -8 & 17 \end{bmatrix}$ $\begin{bmatrix} -4 & 8 \\ 10 & -4 \\ 6 & 8 \end{bmatrix}$

2. $\begin{bmatrix} 4 \\ -71 \\ 18 \end{bmatrix} - \begin{bmatrix} -67 \\ 45 \\ -24 \end{bmatrix}$ $\begin{bmatrix} 71 \\ -116 \\ 42 \end{bmatrix}$

3. $-3\begin{bmatrix} -1 & 0 \\ 17 & -11 \end{bmatrix} + 4\begin{bmatrix} -3 & 16 \\ -21 & 12 \end{bmatrix}$ $\begin{bmatrix} -9 & 64 \\ -135 & 81 \end{bmatrix}$

4. $7\begin{bmatrix} 2 & -1 & 8 \\ 4 & 7 & 9 \end{bmatrix} - 2\begin{bmatrix} -1 & 4 & -3 \\ 7 & 2 & -6 \end{bmatrix}$ $\begin{bmatrix} 16 & -15 & 62 \\ 14 & 45 & 75 \end{bmatrix}$

5. $-2\begin{bmatrix} 1 \\ 2 \end{bmatrix} + 4\begin{bmatrix} 0 \\ 5 \end{bmatrix} - \begin{bmatrix} 10 \\ 18 \end{bmatrix}$ $\begin{bmatrix} -12 \\ -2 \end{bmatrix}$

6. $\frac{3}{4}\begin{bmatrix} 8 & 12 \\ -16 & 20 \end{bmatrix} + \frac{2}{3}\begin{bmatrix} 27 & -9 \\ 54 & -18 \end{bmatrix}$ $\begin{bmatrix} 24 & 3 \\ 24 & 3 \end{bmatrix}$

Use $A = \begin{bmatrix} 4 & -1 & 0 \\ -3 & 6 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 4 & 5 \\ 1 & 0 & -9 \end{bmatrix}$, and $C = \begin{bmatrix} 10 & -8 & 6 \\ -6 & -4 & 20 \end{bmatrix}$ to find the following.

7. $A - B$ $\begin{bmatrix} 6 & -5 & -5 \\ -4 & 6 & 11 \end{bmatrix}$

8. $A - C$ $\begin{bmatrix} -6 & 7 & -6 \\ 3 & 10 & -18 \end{bmatrix}$

9. $-3B$ $\begin{bmatrix} 6 & -12 & -15 \\ -3 & 0 & 27 \end{bmatrix}$

10. $4B - A$ $\begin{bmatrix} -12 & 17 & 20 \\ 7 & -6 & -38 \end{bmatrix}$

11. $-2B - 3C$ $\begin{bmatrix} -26 & 16 & -28 \\ 16 & 12 & -42 \end{bmatrix}$

12. $A + 0.5C$ $\begin{bmatrix} 9 & -5 & 3 \\ -6 & 4 & 12 \end{bmatrix}$

ECONOMICS For Exercises 13 and 14, use the table that shows loans by an economic development board to women and men starting new businesses.

	Women		Men	
	Businesses	Loan Amount ($)	Businesses	Loan Amount ($)
2003	27	$567,000	36	$864,000
2004	41	$902,000	32	$672,000
2005	35	$777,000	28	$562,000

13. Write two matrices that represent the number of new businesses and loan amounts, one for women and one for men.
$\begin{bmatrix} 27 & 567{,}000 \\ 41 & 902{,}000 \\ 35 & 777{,}000 \end{bmatrix}$, $\begin{bmatrix} 36 & 864{,}000 \\ 32 & 672{,}000 \\ 28 & 562{,}000 \end{bmatrix}$

14. Find the sum of the numbers of new businesses and loan amounts for both men and women over the three-year period expressed as a matrix. $\begin{bmatrix} 63 & 1{,}431{,}000 \\ 73 & 1{,}574{,}000 \\ 63 & 1{,}339{,}000 \end{bmatrix}$

15. **PET NUTRITION** Use the table that gives nutritional information for two types of dog food. Find the difference in the percent of protein, fat, and fiber between Mix B and Mix A expressed as a matrix.
$\begin{bmatrix} 2 & -4 & 3 \end{bmatrix}$

	% Protein	% Fat	% Fiber
Mix A	22	12	5
Mix B	24	8	8

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4-2 Word Problem Practice

Operations with Matrices

1. **FARES** The matrix below gives general admission and planetarium fares at a science museum.

$$\begin{array}{r|cc} & \text{Child} & \text{Adult} \\ \hline \text{General Admission} & 5 & 10 \\ \text{Planetarium} & 4 & 8 \end{array}$$

What can you do to this matrix in order to create another matrix that represents fares for 5 people?
Multiply the matrix by 5.

2. **NEGATION** Two engineers need to negate all the entries of a matrix. One engineer tries to do this by multiplying the matrix by -1. The other engineer tries to do this by subtracting twice the matrix from itself. Which engineer, if any, will get the correct result?
Both engineers' methods work. For any matrix, $-M = M - 2M$.

3. **PLANE FARES** The airfares for travel between New York, Chicago, and Los Angeles are organized in the matrix on the left. The matrix on the right gives the tax surcharges for corresponding flights.

$$\begin{array}{r} \\ \text{NYC} \\ \text{Chicago} \\ \text{Los Angeles} \end{array}\begin{array}{c} \begin{array}{ccc} \text{NYC} & \text{Chicago} & \text{Los Angeles} \end{array} \\ \begin{bmatrix} 0 & 440 & 700 \\ 460 & 0 & 660 \\ 850 & 700 & 0 \end{bmatrix} \end{array} \quad \begin{array}{r} \\ \text{NYC} \\ \text{Chicago} \\ \text{Los Angeles} \end{array}\begin{array}{c} \begin{array}{ccc} \text{NYC} & \text{Chicago} & \text{Los Angeles} \end{array} \\ \begin{bmatrix} 0 & 40 & 70 \\ 46 & 0 & 60 \\ 85 & 70 & 0 \end{bmatrix} \end{array}$$

Write a matrix that represents the full cost for travel between these cities.

$$\begin{array}{r} \\ \text{NYC} \\ \text{Chicago} \\ \text{Los Angeles} \end{array}\begin{array}{c} \begin{array}{ccc} \text{NYC} & \text{Chicago} & \text{Los Angeles} \end{array} \\ \begin{bmatrix} 0 & 480 & 770 \\ 506 & 0 & 720 \\ 935 & 770 & 0 \end{bmatrix} \end{array}$$

4. **SUNFLOWERS** Matrix H is a 3 by 1 matrix that contains the initial heights of three sunflowers. Matrix G is a 3 by 1 matrix that contains the numbers of inches the corresponding sunflowers grow in a week. What does matrix $H + 4G$ represent?
the heights of the three sunflowers in four weeks time

DINNER For Exercises 5–7, use the following information.
The menu shows prices for some dishes at a restaurant.

Il Ristorante Menu

	Regular	Half-portion
Lamb	$17.00	$9.00
Chicken	$14.00	$7.00
Steak	$22.00	$11.00

5. Make a 3 by 2 matrix to organize these data.
Sample answer: $\begin{bmatrix} 17 & 9 \\ 14 & 7 \\ 22 & 11 \end{bmatrix}$

6. Let M be the matrix you wrote for Exercise 5. Write an expression involving M that would give prices that include an additional 20% to cover tax and tip.
$M + 0.2M$ or $1.2M$

7. Compute the matrix you described in Exercise 6.
Sample answer: $\begin{bmatrix} 20.40 & 10.80 \\ 16.80 & 8.40 \\ 26.40 & 13.20 \end{bmatrix}$

Lesson 4-2

Answers

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4-2 Enrichment

Sundaram's Sieve

The properties and patterns of prime numbers have fascinated many mathematicians. In 1934, a young East Indian student named Sundaram constructed the following matrix.

4	7	10	13	16	19	22	25	. . .
7	12	17	22	27	32	37	42	. . .
10	17	24	31	38	45	52	59	. . .
13	22	31	40	49	58	67	76	. . .
16	27	38	49	60	71	82	93	. . .
.	.	.	.	.	.	.	.	. . .

A surprising property of this matrix is that it can be used to determine whether or not some numbers are prime.

Complete these problems to discover this property.

1. The first row and the first column are created by using an arithmetic pattern. What is the common difference used in the pattern? **3**

2. Find the next four numbers in the first row. **28, 31, 34, 37**

3. What are the common differences used to create the patterns in rows 2, 3, 4, and 5? **5, 7, 9, 11**

4. Write the next two rows of the matrix. Include eight numbers in each row.
row 6: 19, 32, 45, 58, 71, 84, 97, 110; row 7: 22, 37, 52, 67, 82, 97, 112, 127

5. Choose any five numbers from the matrix. For each number n, that you chose from the matrix, find $2n + 1$. **Answers will vary.**

6. Write the factorization of each value of $2n + 1$ that you found in problem 5.
Answers will vary, but all numbers are composite.

7. Use your results from problems 5 and 6 to complete this statement: If n occurs in the matrix, then $2n + 1$ ___**is not**___ (is/is not) a prime number.

8. Choose any five numbers that are not in the matrix. Find $2n + 1$ for each of these numbers. Show that each result is a prime number.
Answers will vary, but all numbers are prime.

9. Complete this statement: If n does not occur in the matrix, then $2n + 1$ is ___**a prime number**___.

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4-3 Lesson Reading Guide

Multiplying Matrices

Get Ready for the Lesson

Read the introduction to Lesson 4-3 in your textbook.

Write a sum that shows the total points scored by the Houston Texans during the 2004 season. (The sum will include multiplications. Do not actually calculate this sum.)

$6 \cdot 37 + 1 \cdot 34 + 3 \cdot 17 + 2 \cdot 1 + 2 \cdot 0$

Read the Lesson

1. Determine whether each indicated matrix product is defined. If so, state the dimensions of the product. If not, write *undefined*.

a. $M_{3 \times 2}$ and $N_{2 \times 3}$ MN: **3 × 3** NM: **2 × 2**

b. $M_{1 \times 2}$ and $N_{1 \times 2}$ MN: **undefined** NM: **undefined**

c. $M_{4 \times 1}$ and $N_{1 \times 4}$ MN: **4 × 4** NM: **1 × 1**

d. $M_{3 \times 4}$ and $N_{4 \times 4}$ MN: **3 × 4** NM: **undefined**

2. The regional sales manager for a chain of computer stores wants to compare the revenue from sales of one model of notebook computer and one model of printer for three stores in his area. The notebook computer sells for \$1850 and the printer for \$175. The number of computers and printers sold at the three stores during September are shown in the following table.

Store	Computers	Printers
A	128	101
B	205	166
C	97	73

Write a matrix product that the manager could use to find the total revenue for computers and printers for each of the three stores. (Do not calculate the product.)

$$\begin{bmatrix} 128 & 101 \\ 205 & 166 \\ 97 & 73 \end{bmatrix} \cdot \begin{bmatrix} 1850 \\ 175 \end{bmatrix}$$

Remember What You Learned

3. Many students find the procedure of matrix multiplication confusing at first because it is unfamiliar. Think of an easy way to use the letters R and C to remember how to multiply matrices and what the dimensions of the product will be. **Sample answer: Just remember RC for "row, column." Multiply each *row* of the first matrix by each *column* of the second matrix. The dimensions of the product are the number of rows of the first matrix and the number of *columns* of the second matrix.**

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4-3 Study Guide and Intervention

Multiplying Matrices

Multiply Matrices You can multiply two matrices if and only if the number of columns in the first matrix is equal to the number of rows in the second matrix.

Multiplication of Matrices	$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \cdot \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix} = \begin{bmatrix} a_1x_1 + b_1x_2 & a_1y_1 + b_1y_2 \\ a_2x_1 + b_2x_2 & a_2y_1 + b_2y_2 \end{bmatrix}$

Example **Find AB if $A = \begin{bmatrix} -4 & 3 \\ 2 & -2 \\ 1 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & -2 \\ -1 & 3 \end{bmatrix}$.**

$$AB = \begin{bmatrix} -4 & 3 \\ 2 & -2 \\ 1 & 7 \end{bmatrix} \cdot \begin{bmatrix} 5 & -2 \\ -1 & 3 \end{bmatrix} \quad \text{Substitution}$$

$$= \begin{bmatrix} -4(5) + 3(-1) & -4(-2) + 3(3) \\ 2(5) + (-2)(-1) & 2(-2) + (-2)(3) \\ 1(5) + 7(-1) & 1(-2) + 7(3) \end{bmatrix} \quad \text{Multiply columns by rows.}$$

$$= \begin{bmatrix} -23 & 17 \\ 12 & -10 \\ -2 & 19 \end{bmatrix} \quad \text{Simplify.}$$

Exercises

Find each product, if possible.

1. $\begin{bmatrix} 4 & 1 \\ -2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ $\begin{bmatrix} 12 & 3 \\ -6 & 9 \end{bmatrix}$

2. $\begin{bmatrix} -1 & 0 \\ 3 & 7 \end{bmatrix} \cdot \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$ $\begin{bmatrix} -3 & -2 \\ 2 & 34 \end{bmatrix}$

3. $\begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$ $\begin{bmatrix} 7 & -7 \\ 14 & 14 \end{bmatrix}$

4. $\begin{bmatrix} -3 & 1 \\ 5 & -2 \end{bmatrix} \cdot \begin{bmatrix} 4 & 0 & -2 \\ -3 & 1 & 1 \end{bmatrix}$ $\begin{bmatrix} -15 & 1 & 7 \\ 26 & -2 & -12 \end{bmatrix}$

5. $\begin{bmatrix} 3 & -2 \\ 0 & 4 \\ -5 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ $\begin{bmatrix} -1 & 4 \\ 8 & 4 \\ -3 & -9 \end{bmatrix}$

6. $\begin{bmatrix} 5 & -2 \\ 2 & -3 \end{bmatrix} \cdot \begin{bmatrix} 4 & -1 \\ -2 & 5 \end{bmatrix}$ $\begin{bmatrix} 24 & -15 \\ 14 & -17 \end{bmatrix}$

7. $\begin{bmatrix} 6 & 10 \\ -4 & 3 \\ -2 & 7 \end{bmatrix} \cdot [0 \quad 4 \quad -3]$ not possible

8. $\begin{bmatrix} 7 & -2 \\ 5 & -4 \end{bmatrix} \cdot \begin{bmatrix} 1 & -3 \\ -2 & 0 \end{bmatrix}$ $\begin{bmatrix} 11 & -21 \\ 13 & -15 \end{bmatrix}$

9. $\begin{bmatrix} 2 & 0 & -3 \\ 1 & 4 & -2 \\ -1 & 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & -2 \\ 3 & 1 \\ -2 & 4 \end{bmatrix}$ $\begin{bmatrix} 10 & -16 \\ 18 & -6 \\ 5 & 9 \end{bmatrix}$

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4-3 Study Guide and Intervention *(continued)*

Multiplying Matrices

Multiplicative Properties The Commutative Property of Multiplication does *not* hold for matrices.

Properties of Matrix Multiplication	For any matrices A, B, and C for which the matrix product is defined, and any scalar c, the following properties are true.
Associative Property of Matrix Multiplication	$(AB)C = A(BC)$
Associative Property of Scalar Multiplication	$c(AB) = (cA)B = A(cB)$
Left Distributive Property	$C(A + B) = CA + CB$
Right Distributive Property	$(A + B)C = AC + BC$

Example **Use $A = \begin{bmatrix} 4 & -3 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 5 & -3 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & -2 \\ 6 & 3 \end{bmatrix}$ to find each product.**

a. $(A + B)C$

$$(A + B)C = \left(\begin{bmatrix} 4 & -3 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 5 & -3 \end{bmatrix}\right) \cdot \begin{bmatrix} 1 & -2 \\ 6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -3 \\ 7 & -2 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ 6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 6(1) + (-3)(6) & 6(-2) + (-3)(3) \\ 7(1) + (-2)(6) & 7(-2) + (-2)(3) \end{bmatrix}$$

$$= \begin{bmatrix} -12 & -21 \\ -5 & -20 \end{bmatrix}$$

b. $AC + BC$

$$AC + BC = \begin{bmatrix} 4 & -3 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ 6 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 5 & -3 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ 6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4(1) + (-3)(6) & 4(-2) + (-3)(3) \\ 2(1) + 1(6) & 2(-2) + 1(3) \end{bmatrix} + \begin{bmatrix} 2(1) + 0(6) & 2(-2) + 0(3) \\ 5(1) + (-3)(6) & 5(-2) + (-3)(3) \end{bmatrix}$$

$$= \begin{bmatrix} -14 & -17 \\ 8 & -1 \end{bmatrix} + \begin{bmatrix} 2 & -4 \\ -13 & -19 \end{bmatrix} = \begin{bmatrix} -12 & -21 \\ -5 & -20 \end{bmatrix}$$

Note that although the results in the example illustrate the Right Distributive Property, they do not prove it.

Exercises

Use $A = \begin{bmatrix} 3 & 2 \\ 5 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 4 \\ 2 & 1 \end{bmatrix}$, $C = \begin{bmatrix} -\frac{1}{2} & -2 \\ 1 & -3 \end{bmatrix}$, and scalar $c = -4$ to determine whether each of the following equations is true for the given matrices.

1. $c(AB) = (cA)B$ yes

2. $AB = BA$ no

3. $BC = CB$ no

4. $(AB)C = A(BC)$ yes

5. $C(A + B) = AC + BC$ no

6. $c(A + B) = cA + cB$ yes

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Lesson 4-3

Answers

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4-3 Skills Practice

Multiplying Matrices

Determine whether each matrix product is defined. If so, state the dimensions of the product.

1. $A_{2 \times 5} \cdot B_{5 \times 1}$ **2 × 1**
2. $M_{1 \times 3} \cdot N_{3 \times 2}$ **1 × 2**
3. $B_{3 \times 2} \cdot A_{3 \times 2}$ **undefined**
4. $R_{4 \times 4} \cdot S_{4 \times 1}$ **4 × 1**
5. $X_{3 \times 3} \cdot Y_{3 \times 4}$ **3 × 4**
6. $A_{6 \times 4} \cdot B_{4 \times 5}$ **6 × 5**

Find each product, if possible.

7. $\begin{bmatrix} 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 8 \end{bmatrix}$

8. $\begin{bmatrix} 5 & 6 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & -5 \\ 3 & 1 \end{bmatrix}$ $\begin{bmatrix} 28 & -19 \\ 7 & -9 \end{bmatrix}$

9. $\begin{bmatrix} 1 & 3 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ $\begin{bmatrix} -3 \\ -5 \end{bmatrix}$

10. $\begin{bmatrix} 3 \\ -2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ -1 & 1 \end{bmatrix}$ **not possible**

11. $\begin{bmatrix} -3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 0 & -1 \\ 2 & 2 \end{bmatrix}$ $\begin{bmatrix} 8 & 11 \end{bmatrix}$

12. $\begin{bmatrix} -1 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 2 & -3 & -2 \end{bmatrix}$ $\begin{bmatrix} -2 & 3 & 2 \\ 6 & -9 & -6 \end{bmatrix}$

13. $\begin{bmatrix} 5 \\ 6 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 8 \end{bmatrix}$ **not possible**

14. $\begin{bmatrix} 2 & -2 \\ 4 & 5 \\ -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 3 \\ 3 & 0 \end{bmatrix}$ $\begin{bmatrix} -6 & 6 \\ 15 & 12 \\ 3 & -9 \end{bmatrix}$

15. $\begin{bmatrix} -4 & 4 \\ -2 & 1 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & -3 \\ 0 & 2 \end{bmatrix}$ $\begin{bmatrix} -12 & 20 \\ -6 & 8 \\ 6 & 0 \end{bmatrix}$

16. $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$ $\begin{bmatrix} 4 \\ 4 \end{bmatrix}$

Use $A = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 5 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 3 & -1 \\ 1 & 0 \end{bmatrix}$, and scalar $c = 2$ to determine whether the following equations are true for the given matrices.

17. $(AC)c = A(Cc)$ **yes**
18. $AB = BA$ **no**
19. $B(A + C) = AB + BC$ **no**
20. $(A - B)c = Ac - Bc$ **yes**

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4-3 Practice

Multiplying Matrices

Determine whether each matrix product is defined. If so, state the dimensions of the product.

1. $A_{7 \times 4} \cdot B_{4 \times 3}$ **7 × 3**
2. $A_{3 \times 5} \cdot M_{5 \times 8}$ **3 × 8**
3. $M_{2 \times 1} \cdot A_{1 \times 6}$ **2 × 6**
4. $M_{3 \times 2} \cdot A_{3 \times 2}$ **undefined**
5. $P_{1 \times 9} \cdot Q_{9 \times 1}$ **1 × 1**
6. $P_{9 \times 1} \cdot Q_{1 \times 9}$ **9 × 9**

Find each product, if possible.

7. $\begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 & 7 \\ 6 & 0 & -5 \end{bmatrix}$ $\begin{bmatrix} 30 & -4 & -6 \\ 3 & -6 & 26 \end{bmatrix}$

8. $\begin{bmatrix} 2 & 4 \\ 7 & -1 \end{bmatrix} \cdot \begin{bmatrix} -3 & 0 \\ 2 & 5 \end{bmatrix}$ $\begin{bmatrix} 2 & 20 \\ -23 & -5 \end{bmatrix}$

9. $\begin{bmatrix} -3 & 0 \\ 2 & 5 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 \\ 7 & -1 \end{bmatrix}$ $\begin{bmatrix} -6 & -12 \\ 39 & 3 \end{bmatrix}$

10. $\begin{bmatrix} 3 & -2 & 7 \\ 6 & 0 & -5 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 & 7 \\ 6 & 0 & -5 \end{bmatrix}$ **not possible**

11. $\begin{bmatrix} 4 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$ $\begin{bmatrix} 2 \end{bmatrix}$

12. $\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 4 & 0 & 2 \end{bmatrix}$ $\begin{bmatrix} 4 & 0 & 2 \\ 12 & 0 & 6 \\ -4 & 0 & -2 \end{bmatrix}$

13. $\begin{bmatrix} -6 & 2 \\ 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$ $\begin{bmatrix} -30 & 10 \\ 15 & -5 \end{bmatrix}$

14. $\begin{bmatrix} -15 & -9 \end{bmatrix} \cdot \begin{bmatrix} 6 & 11 \\ 23 & -10 \end{bmatrix}$ $\begin{bmatrix} -297 & -75 \end{bmatrix}$

Use $A = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ -2 & -1 \end{bmatrix}$, $C = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, and scalar $c = 3$ to determine whether the following equations are true for the given matrices.

15. $AC = CA$ **yes**
16. $A(B + C) = BA + CA$ **no**
17. $(AB)c = c(AB)$ **yes**
18. $(A + C)B = B(A + C)$ **no**

RENTALS For Exercises 19–21, use the following information.

For their one-week vacation, the Montoyas can rent a 2-bedroom condominium for \$1796, a 3-bedroom condominium for \$2165, or a 4-bedroom condominium for \$2538. The table shows the number of units in each of three complexes.

	2-Bedroom	3-Bedroom	4-Bedroom
Sun Haven	36	24	22
Surfside	29	32	42
Seabreeze	18	22	18

19. Write a matrix that represents the number of each type of unit available at each complex and a matrix that represents the weekly charge for each type of unit. $\begin{bmatrix} 36 & 24 & 22 \\ 29 & 32 & 42 \\ 18 & 22 & 18 \end{bmatrix}$, $\begin{bmatrix} \$1796 \\ \$2165 \\ \$2538 \end{bmatrix}$

20. If all of the units in the three complexes are rented for the week at the rates given the Montoyas, express the income of each of the three complexes as a matrix. $\begin{bmatrix} 172{,}452 \\ 227{,}960 \\ 125{,}642 \end{bmatrix}$

21. What is the total income of all three complexes for the week? **\$526,054**

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4-3 Word Problem Practice

Multiplying Matrices

1. **FIND THE ERROR** Both A and B are 2 by 2 matrices. Maggie made the following derivation. Is this derivation valid? If not, what error did she make?

   a. $(A + B)^2 = (A + B)(A + B)$
   b. $= (A + B)A + (A + B)B$
   c. $= AA + BA + AB + BB$
   d. $= A^2 + BA + AB + B^2$
   e. $= A^2 + AB + AB + B^2$
   f. $= A^2 + 2AB + B^2$

   From step d to step e, Maggie commuted *A* and *B*; but for matrices, *AB* does not typically equal *BA*.

2. **EXAM SCORES** Mr. Farey recorded the exam scores of his students in a 20 by 3 matrix. Each row listed the scores of a different student. The first exam scores were listed in the first column, and the second exam scores were listed in the second column. The final exam scores were listed in the third column. Mr. Farey needed to create a 20 by 1 matrix that contained the weighted scores of each student. The first two exams account for 25% of the weighted score, and the final exam counted 50%. To make the matrix of weighted scores, what matrix can Mr. Farey multiply his 20 by 3 matrix by on the right?

   $\begin{bmatrix} 0.25 \\ 0.25 \\ 0.50 \end{bmatrix}$

3. **SPECIAL MATRICES** Mandy has a 3 by 3 matrix M. She notices that for any 3 by 3 matrix X, $MX = X$. What must M be?

   $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

4. **POWERS** Thad just learned about matrix multiplication. He began to wonder what happens when you take powers of a matrix. He computed the first few powers of the matrix $M = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and noticed a pattern. What is M^n?

   $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$

COST COMPARISONS For Exercises 5 and 6, use the following information.

Barbara and Lance need to buy pens, pencils, and erasers. They make a 2 by 3 matrix that represents the numbers of each item they would like to purchase.

$$\begin{array}{r} \\ \textbf{Barbara} \\ \textbf{Lance} \end{array} \begin{array}{c} \begin{array}{ccc} \textbf{Pens} & \textbf{Pencils} & \textbf{Erasers} \end{array} \\ \begin{bmatrix} 10 & 15 & 3 \\ 5 & 20 & 5 \end{bmatrix} \end{array}$$

They call this matrix M. Barbara and Lance find two stores that sell the items at different prices and record this information in a second matrix that they call P.

$$\begin{array}{r} \\ \textbf{Pens} \\ \textbf{Pencils} \\ \textbf{Erasers} \end{array} \begin{array}{c} \begin{array}{cc} \textbf{Store 1} & \textbf{Store 2} \end{array} \\ \begin{bmatrix} 2.20 & 1.90 \\ 0.85 & 0.95 \\ 0.60 & 0.65 \end{bmatrix} \end{array}$$

5. Compute MP.

   $\begin{bmatrix} 36.55 & 35.2 \\ 31 & 31.75 \end{bmatrix}$

6. What do the entries in MP mean?

   The first row represents the total cost of Barbara's items at stores 1 and 2, respectively, and the second row represents the total cost of Lance's items at stores 1 and 2, respectively.

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4-3 Enrichment

Properties of Matrices

Computing with matrices is different from computing with real numbers. Stated below are some properties of the real number system. Are these also true for matrices? In the problems on this page, you will investigate this question.

For all real numbers a and b, $ab = 0$ if and only if $a = 0$ or $b = 0$.

Multiplication is commutative. For all real numbers a and b, $ab = ba$.

Multiplication is associative. For all real numbers a, b, and c, $a(bc) = (ab)c$.

Use the matrices *A*, *B*, and *C* for the problems. Write whether each statement is true. Assume that a 2-by-2 matrix is the 0 matrix if and only if all of its elements are zero.

$A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -3 \\ -1 & 3 \end{bmatrix}$ $C = \begin{bmatrix} 3 & 6 \\ 1 & 2 \end{bmatrix}$

1. $AB = 0$ **no**

   $AB = \begin{bmatrix} 2 & -6 \\ -2 & 6 \end{bmatrix}$

2. $AC = 0$ **no**

   $AB = \begin{bmatrix} 10 & 20 \\ 6 & 12 \end{bmatrix}$

3. $BC = 0$ **yes**

   $AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

4. $AB = BA$ **no**

   $BA = \begin{bmatrix} 0 & -8 \\ 0 & 8 \end{bmatrix}$

   So, $AB \neq BA$.

5. $AC = CA$ **no**

   $CA = \begin{bmatrix} 15 & 21 \\ 5 & 7 \end{bmatrix}$

   So, $AC \neq CA$.

6. $BC = CB$ **no**

   $AB = \begin{bmatrix} -3 & 9 \\ -1 & 3 \end{bmatrix}$

   So, $BC \neq CB$.

7. $A(BC) = (AB)C$ **yes**

   Both products equal $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

8. $B(CA) = (BC)A$ **yes**

   Both products equal $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

9. $B(AC) = (BA)C$ **yes**

   Both products equal $\begin{bmatrix} -8 & -16 \\ 8 & 16 \end{bmatrix}$.

10. Write a statement summarizing your findings about the properties of matrix multiplication.

    Based on these examples, matrix multiplication is associative, but not commutative. Two matrices may have a product of zero even if neither of the factors equals zero.

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4-4 Lesson Reading Guide

Transformations with Matrices

Get Ready for the Lesson

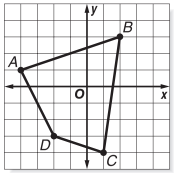
Read the introduction to Lesson 4-4 in your textbook.

Describe how you can change the orientation of a figure without changing its size or shape.

Flip (or reflect) the figure over a line.

Read the Lesson

1. a. Write the vertex matrix for the quadrilateral $ABCD$ shown in the graph at the right.



$$\begin{bmatrix} -4 & 2 & 1 & -2 \\ 1 & 3 & -4 & -3 \end{bmatrix}$$

b. Write the vertex matrix that represents the position of the quadrilateral $A'B'C'D'$ that results when quadrilateral $ABCD$ is translated 3 units to the right and 2 units down.

$$\begin{bmatrix} -1 & 5 & 4 & 1 \\ -1 & 1 & -6 & -5 \end{bmatrix}$$

2. Describe the transformation that corresponds to each of the following matrices.

a. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

counterclockwise rotation about the origin of 180°

b. $\begin{bmatrix} 3 & 3 & 3 \\ -4 & -4 & -4 \end{bmatrix}$

translation 4 units down and 3 units to the right

c. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

reflection over the y-axis

d. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

reflection over the line $y = x$

Remember What You Learned

3. Describe a way to remember which of the reflection matrices corresponds to reflection over the x-axis.

Sample answer: The only elements used in the reflection matrices are 0, 1, and −1. For such a 2 × 2 matrix M to have the property that $M \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ -y \end{bmatrix}$, the elements in the top row must be 1 and 0 (in that order), and elements in the bottom row must be 0 and −1 (in that order).

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4-4 Study Guide and Intervention

Transformations with Matrices

Translations and Dilations Matrices that represent coordinates of points on a plane are useful in describing transformations.

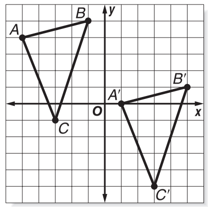
Translation	a transformation that moves a figure from one location to another on the coordinate plane

You can use matrix addition and a translation matrix to find the coordinates of the translated figure.

Dilation	a transformation in which a figure is enlarged or reduced

You can use scalar multiplication to perform dilations.

Example **Find the coordinates of the vertices of the image of $\triangle ABC$ with vertices $A(-5, 4)$, $B(-1, 5)$, and $C(-3, -1)$ if it is moved 6 units to the right and 4 units down. Then graph $\triangle ABC$ and its image $\triangle A'B'C'$.**



Write the vertex matrix for $\triangle ABC$. $\begin{bmatrix} -5 & -1 & -3 \\ 4 & 5 & -1 \end{bmatrix}$

Add the translation matrix $\begin{bmatrix} 6 & 6 & 6 \\ -4 & -4 & -4 \end{bmatrix}$ to the vertex matrix of $\triangle ABC$.

$$\begin{bmatrix} -5 & -1 & -3 \\ 4 & 5 & -1 \end{bmatrix} + \begin{bmatrix} 6 & 6 & 6 \\ -4 & -4 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 5 & 3 \\ 0 & 1 & -5 \end{bmatrix}$$

The coordinates of the vertices of $\triangle A'B'C'$ are $A'(1, 0)$, $B'(5, 1)$, and $C'(3, -5)$.

Exercises

For Exercises 1 and 2 use the following information. Quadrilateral $QUAD$ with vertices $Q(-1, -3)$, $U(0, 0)$, $A(5, -1)$, and $D(2, -5)$ is translated 3 units to the left and 2 units up.

1. Write the translation matrix. $\begin{bmatrix} -3 & -3 & -3 & -3 \\ 2 & 2 & 2 & 2 \end{bmatrix}$

2. Find the coordinates of the vertices of $Q'U'A'D'$.

$Q'(-4, -1)$, U', $(-3, 2)$, $A'(2, 1)$, $D'(-1, -3)$

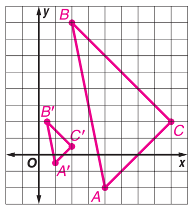
For Exercises 3–5, use the following information. The vertices of $\triangle ABC$ are $A(4, -2)$, $B(2, 8)$, and $C(8, 2)$. The triangle is dilated so that its perimeter is one-fourth the original perimeter.

3. Write the coordinates of the vertices of $\triangle ABC$ in a vertex matrix. $\begin{bmatrix} 4 & 2 & 8 \\ -2 & 8 & 2 \end{bmatrix}$

4. Find the coordinates of the vertices of image $\triangle A'B'C'$.

$A'\left(1, -\frac{1}{2}\right)$, $B'\left(\frac{1}{2}, 2\right)$, $C'\left(2, \frac{1}{2}\right)$

5. Graph the preimage and the image.



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4-4 Study Guide and Intervention *(continued)*

Transformations with Matrices

Reflections and Rotations

Reflection Matrices	For a reflection over the:	x-axis	y-axis	line $y = x$
	multiply the vertex matrix on the left by:	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Rotation Matrices	For a counterclockwise rotation about the origin of:	90°	180°	270°
	multiply the vertex matrix on the left by:	$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Example **Find the coordinates of the vertices of the image of $\triangle ABC$ with $A(3, 5)$, $B(-2, 4)$, and $C(1, -1)$ after a reflection over the line $y = x$.**

Write the ordered pairs as a vertex matrix. Then multiply the vertex matrix by the reflection matrix for $y = x$.

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 & 1 \\ 5 & 4 & -1 \end{bmatrix} = \begin{bmatrix} 5 & 4 & -1 \\ 3 & -2 & 1 \end{bmatrix}$$

The coordinates of the vertices of $A'B'C'$ are $A'(5, 3)$, $B'(4, -2)$, and $C'(-1, 1)$.

Exercises

1. The coordinates of the vertices of quadrilateral $ABCD$ are $A(-2, 1)$, $B(-1, 3)$, $C(2, 2)$, and $D(2, -1)$. What are the coordinates of the vertices of the image $A'B'C'D'$ after a reflection over the y-axis? **$A'(2, 1)$, $B'(1, 3)$, $C'(-2, 2)$, $D'(-2, -1)$**

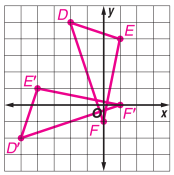
2. Triangle DEF with vertices $D(-2, 5)$, $E(1, 4)$, and $F(0, -1)$ is rotated 90° counterclockwise about the origin.

   a. Write the coordinates of the triangle in a vertex matrix.
   $\begin{bmatrix} -2 & 1 & 0 \\ 5 & 4 & -1 \end{bmatrix}$

   b. Write the rotation matrix for this situation.
   $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

   c. Find the coordinates of the vertices of $\triangle D'E'F'$.
   $D'(-5, -2)$, $E'(-4, 1)$, $F'(1, 0)$

   d. Graph $\triangle DEF$ and $\triangle D'E'F'$.



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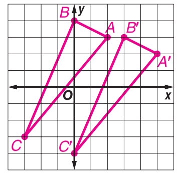
4-4 Skills Practice

Transformations with Matrices

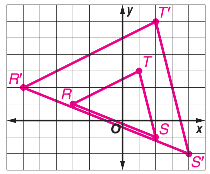
For Exercises 1–3, use the following information.
Triangle ABC with vertices $A(2, 3)$, $B(0, 4)$, and $C(-3, -3)$ is translated 3 units right and 1 unit down.

1. Write the translation matrix. $\begin{bmatrix} 3 & 3 & 3 \\ -1 & -1 & -1 \end{bmatrix}$
2. Find the coordinates of $\triangle A'B'C'$. **$A'(5, 2)$, $B'(3, 3)$, $C'(0, -4)$**
3. Graph the preimage and the image.



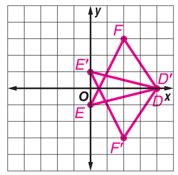
For Exercises 4–6, use the following information.
The vertices of $\triangle RST$ are $R(-3, 1)$, $S(2, -1)$, and $T(1, 3)$. The triangle is dilated so that its perimeter is twice the original perimeter.

4. Write the coordinates of $\triangle RST$ in a vertex matrix. $\begin{bmatrix} -3 & 2 & 1 \\ 1 & -1 & 3 \end{bmatrix}$
5. Find the coordinates of the image $\triangle R'S'T'$.
   $R'(-6, 2)$, $S'(4, -2)$, $T'(2, 6)$
6. Graph $\triangle RST$ and $\triangle R'S'T'$.



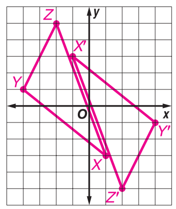
For Exercises 7–10, use the following information.
The vertices of $\triangle DEF$ are $D(4, 0)$, $E(0, -1)$, and $F(2, 3)$. The triangle is reflected over the x-axis.

7. Write the coordinates of $\triangle DEF$ in a vertex matrix. $\begin{bmatrix} 4 & 0 & 2 \\ 0 & -1 & 3 \end{bmatrix}$
8. Write the reflection matrix for this situation. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
9. Find the coordinates of $\triangle D'E'F'$. **$D'(4, 0)$, $E'(0, 1)$, $F'(2, -3)$**
10. Graph $\triangle DEF$ and $\triangle D'E'F'$.



For Exercises 11–14, use the following information.
Triangle XYZ with vertices $X(1, -3)$, $Y(-4, 1)$, and $Z(-2, 5)$ is rotated 180° counterclockwise about the origin.

11. Write the coordinates of the triangle in a vertex matrix. $\begin{bmatrix} 1 & -4 & -2 \\ -3 & 1 & 5 \end{bmatrix}$
12. Write the rotation matrix for this situation. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$
13. Find the coordinates of $\triangle X'Y'Z'$.
    $X'(-1, 3)$, $Y'(4, -1)$, $Z'(2, -5)$
14. Graph the preimage and the image.



Lesson 4-4

Answers

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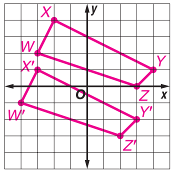
4-4 Practice

Transformations with Matrices

For Exercises 1–3, use the following information.
Quadrilateral $WXYZ$ with vertices $W(-3, 2)$, $X(-2, 4)$, $Y(4, 1)$, and $Z(3, 0)$ is translated 1 unit left and 3 units down.

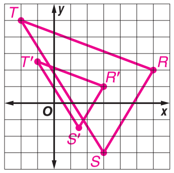


1. Write the translation matrix. $\begin{bmatrix} -1 & -1 & -1 & -1 \\ -3 & -3 & -3 & -3 \end{bmatrix}$

2. Find the coordinates of quadrilateral $W'X'Y'Z'$.
$W'(-4, -1)$, $X'(-3, 1)$, $Y'(3, -2)$, $Z'(2, -3)$

3. Graph the preimage and the image.

For Exercises 4–6, use the following information.
The vertices of $\triangle RST$ are $R(6, 2)$, $S(3, -3)$, and $T(-2, 5)$. The triangle is dilated so that its perimeter is one half the original perimeter.

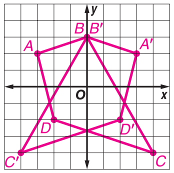


4. Write the coordinates of $\triangle RST$ in a vertex matrix. $\begin{bmatrix} 6 & 3 & -2 \\ 2 & -3 & 5 \end{bmatrix}$

5. Find the coordinates of the image $\triangle R'S'T'$.
$R'(3, 1)$, $S'(1.5, -1.5)$, $T'(-1, 2.5)$

6. Graph $\triangle RST$ and $\triangle R'S'T'$.

For Exercises 7–10, use the following information.
The vertices of quadrilateral $ABCD$ are $A(-3, 2)$, $B(0, 3)$, $C(4, -4)$, and $D(-2, -2)$. The quadrilateral is reflected over the y-axis.



7. Write the coordinates of $ABCD$ in a vertex matrix. $\begin{bmatrix} -3 & 0 & 4 & -2 \\ 2 & 3 & -4 & -2 \end{bmatrix}$

8. Write the reflection matrix for this situation. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

9. Find the coordinates of $A'B'C'D'$.
$A'(3, 2)$, $B'(0, 3)$, $C'(-4, -4)$, $D'(2, -2)$

10. Graph $ABCD$ and $A'B'C'D'$.

11. ARCHITECTURE Using architectural design software, the Bradleys plot their kitchen plans on a grid with each unit representing 1 foot. They place the corners of an island at (2, 8), (8, 11), (3, 5), and (9, 8). If the Bradleys wish to move the island 1.5 feet to the right and 2 feet down, what will the new coordinates of its corners be?
(3.5, 6), (9.5, 9), (4.5, 3), and (10.5, 6)

12. BUSINESS The design of a business logo calls for locating the vertices of a triangle at (1.5, 5), (4, 1), and (1, 0) on a grid. If design changes require rotating the triangle 90º counterclockwise, what will the new coordinates of the vertices be?
(−5, 1.5), (−1, 4), and (0, 1)

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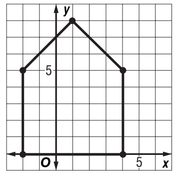
4-4 Word Problem Practice

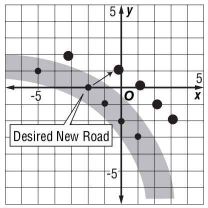
Transformations with Matrices

1. ICONS Louis needs to perform many matrix transformations to the basic house icon shown in the graph.



What is the vertex matrix for this image?
Sample answer:
$\begin{bmatrix} -2 & -2 & 1 & 4 & 4 \\ 0 & 5 & 8 & 5 & 0 \end{bmatrix}$

2. RELOCATION City planners are making a new road. Unfortunately, the road will pass through five ancient trees indicated by the small dots. The planners decide to move the trees to the locations indicated by the large dots. What matrix represents this translation?

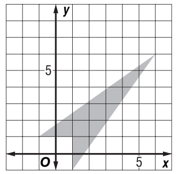


$\begin{bmatrix} 2 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$

3. MIRROR SYMMETRY A detective found only half of an image with mirror symmetry about the line $y = x$. The vertex matrix of the visible part is $\begin{bmatrix} 4 & 5 & -2 \\ 2 & -5 & -4 \end{bmatrix}$. What are the coordinates of the hidden vertices?
(2, 4), (−5, 5), (−4, −2)

4. PHOTOGRAPHY Alejandra used a digital camera to take a picture. Because she held the camera sideways, the image on her computer screen appeared sideways. In order to transform the picture, she needed to perform a 90° clockwise rotation. What matrix represents this transformation?
$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

ARROWS For Exercises 5–6, use the following information.
A compass arrow is pointing Northeast.



5. What is the vertex matrix for the arrow?
Sample answer:
$\begin{bmatrix} 6 & -1 & 1 & 1 \\ 6 & 1 & 1 & -1 \end{bmatrix}$

6. What would the vertex matrix be for the arrow if it were pointing Northwest? (Hint: Rotate 90° around the origin.)
Sample answer:
$\begin{bmatrix} -6 & -1 & -1 & 1 \\ 6 & -1 & 1 & 1 \end{bmatrix}$

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4-4 Enrichment

Computer Graphics Using Matrix Transformations

Computer animation creates a sensation of movement by slowly changing the position of pixels of an image on a two-dimensional video screen. By selecting the center of the screen as the origin, each pixel can be located using ordered pairs. A series of delayed matrix transformations (transformation, pause, transformation, pause) applied to the coordinates of the point (pixel) produces the desired animation effect.

Two computer programmers, Nate and Daniel, are writing a computer animation program using only the two matrices $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. Nate argues that it does not make a difference which matrix is applied first. Daniel disagrees.

1. What transformation does each matrix represent?
Matrix *A* represents a reflection about the *x*-axis, and matrix *B* represents a 90° counterclockwise rotation.

2. Find AB and BA. Determine whether Nate or Daniel is correct.
$AB = \begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix}$ $BA = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$**; Nate is correct because order does matter.**

3. A third programmer joins Nate and Daniel. She convinces them to expand their transformation capabilities by including a reflection about the y-axis using $C = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

a. Determine a transformation using only B and C that is equivalent to the transformation A.
CB^2

b. Write an expression for a 180° counterclockwise rotation in terms of the reflections A and C.
$B^2 = AC$ or $B^2 = CA$

c. Is it possible to express a 270° counterclockwise rotation in terms of the reflections A and C? Explain why or why not.
No; $B^3 = ABC$

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4-4 Graphing Calculator Activity

Matrices for 30°, 45°, 60° Rotations

The rotation matrix for 90° counterclockwise about the origin is $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

The general rotation matrix for any angle θ counterclockwise about the origin is $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$.

Example 1 **Find the coordinates of the image of $\triangle ABC$ with vertices $A(0, 0)$, $B(6, 0)$, and $C(3, 4)$ after a counterclockwise rotation of 30° about the origin.**

Enter the coordinates of the vertices of the in vertex matrix [A] and the rotation matrix in matrix [B]. Be sure the calculator is set in Degree mode. Keystrokes: [2nd] [MATRX] [▶] [▶] [ENTER] 2 [ENTER] 3 [ENTER] 0 [ENTER] 6 [ENTER] 3 [ENTER] 0 [ENTER] 0 [ENTER] 4 [ENTER] [2nd] [MATRX] [▶] [▶] 2 2 [ENTER] 2 [ENTER] [COS] 30 [)] [ENTER] [(–)] [SIN] 30 [)] [ENTER] [SIN] 30 [)] [COS] 30 [)] [ENTER] [2nd] [QUIT] [2nd] [MATRX] 2 [2nd] [MATRX] 1 [ENTER]. Hold [▶] to scroll across to see the other coordinates.

```
MATRIX[B] 2 ×2
[ .86603  -.5      ]
[ .5      .86603   ]

2,2=.8660254037…
```

```
[B]*[A]
[[0 5.196152423…
 [0 3          …
```

The coordinates of the image of $\triangle ABC$ are $A'(0, 0)$, $B'(5.2, 3)$, and $C'(0.6, 5.0)$.

Example 2 **Find the coordinates of the image of $\triangle ABC$ with vertices $A(0, 0)$, $B(6, 0)$, and $C(3, 4)$ after two rotations of 45° counterclockwise about the origin.**

In order to rotate the image twice, store the vertex matrix of the first image.

Keystrokes: [2nd] [MATRX] [▶] [▶] 2 [ENTER] [ENTER] [COS] 45 [)] [ENTER] [(–)] [SIN] 45 [)] [ENTER] [SIN] 45 [)] [COS] 45 [)] [ENTER] [2nd] [QUIT] [2nd] [MATRX] 2 [2nd] [MATRX] 1 [ENTER] [2nd] [ENTRY] [STO▶] [2nd] [MATRX] 6 [ENTER] [2nd] [ENTRY] [2nd] [ENTRY] [◀] [2nd] [MATRX] 6 [ENTER].

```
 [0 4.242640687…
[B]*[A]→[F]
[[0 4.242640687…
 [0 4.242640687…
[B]*[F]
     [[0 0 -4]
      [0 6 3 ]]
```

The vertices of $\triangle A'B'C'$ are $A'(0, 0)$, $B'(0, 6)$, and $C'(-4, 3)$.

Exercises

Quadrilateral *ABCD* has vertices $A(0, 0)$, $B(4, 0)$, $C(4, 6)$, and $D(0, 6)$. Find the coordinates of the vertices of the image after each counterclockwise rotation. Round to the nearest tenth.

1. 45°
$A'(0, 0)$, $B'(2.8, 2.8)$, $C'(-1.4, 7.1)$, $D'(-4.2, 4.2)$

2. 30°
$A'(0, 0)$, $B'(3.5, 2)$, $C'(0.5, 7.2)$, $D'(-3, 5.2)$

3. 60°
$A'(0, 0)$, $B'(2, 3.5)$, $C'(-3.2, 6.5)$, $D'(-5.2, 3)$

4. 120°
$A'(0, 0)$, $B'(-2, 3.5)$, $C'(-7.2, 0.5)$, $D'(-5.2, -3)$

5. 75°
$A'(0, 0)$, $B'(1, 3.9)$, $C'(-4.8, 5.4)$, $D'(-5.8, 1.6)$

6. 225°
$A'(0, 0)$, $B'(-2.8, 2.8)$, $C'(-7.1, -1.4)$, $D'(-4.2, -4.2)$

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4-5 Lesson Reading Guide

Determinants

Get Ready for the Lesson

Read the introduction to Lesson 4-5 in your textbook.

In this lesson, you will learn how to find the area of a triangle if you know the coordinates of its vertices using determinants. Describe a method you already know for finding the area of the Bermuda Triangle. **Sample answer: Use the map. Choose any side of the triangle as the base, and measure this side with a ruler. Multiply this length by the scale factor for the map. Next, draw a segment from the opposite vertex perpendicular to the base. Measure this segment, and multiply its length by the scale for the map. Finally, find the area by using the formula $A = \frac{1}{2}bh$.**

Read the Lesson

1. Indicate whether each of the following statements is *true* or *false*.

a. Every matrix has a determinant. **false**

b. If both rows of a 2 × 2 matrix are identical, the determinant of the matrix will be 0. **true**

c. Every element of a 3 × 3 matrix has a minor. **true**

d. In order to evaluate a third-order determinant by expansion by minors it is necessary to find the minor of every element of the matrix. **false**

e. If you evaluate a third-order determinant by expansion about the second row, the position signs you will use are − + −. **true**

2. Suppose that triangle RST has vertices $R(-2, 5)$, $S(4, 1)$, and $T(0, 6)$.

a. Write a determinant that could be used in finding the area of triangle RST.

$$\begin{vmatrix} -2 & 5 & 1 \\ 4 & 1 & 1 \\ 0 & 6 & 1 \end{vmatrix}$$

b. Explain how you would use the determinant you wrote in part **a** to find the area of the triangle. **Sample answer: Evaluate the determinant and multiply the result by $\frac{1}{2}$. Then take the absolute value to make sure the final answer is positive.**

Remember What You Learned

3. A good way to remember a complicated procedure is to break it down into steps. Write a list of steps for evaluating a third-order determinant using expansion by minors. **Sample answer: 1. Choose a row of the matrix. 2. Find the position signs for the row you have chosen. 3. Find the minor of each element in that row. 4. Multiply each element by its position sign and by its minor. 5. Add the results.**

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4-5 Study Guide and Intervention

Determinants

Determinants of 2 × 2 Matrices

Second-Order Determinant	For the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the determinant is $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$.

Example **Find the value of each determinant.**

a. $\begin{vmatrix} 6 & 3 \\ -8 & 5 \end{vmatrix}$

$$\begin{vmatrix} 6 & 3 \\ -8 & 5 \end{vmatrix} = 6(5) - 3(-8)$$
$$= 30 - (-24) \text{ or } 54$$

b. $\begin{vmatrix} 11 & -5 \\ 9 & 3 \end{vmatrix}$

$$\begin{vmatrix} 11 & -5 \\ 9 & 3 \end{vmatrix} = 11(-3) - (-5)(9)$$
$$= -33 - (-45) \text{ or } 78$$

Exercises

Find the value of each determinant.

1. $\begin{vmatrix} 6 & -2 \\ 5 & 7 \end{vmatrix}$ **52**

2. $\begin{vmatrix} -8 & 3 \\ -2 & 1 \end{vmatrix}$ **−2**

3. $\begin{vmatrix} 3 & 9 \\ 4 & 6 \end{vmatrix}$ **−18**

4. $\begin{vmatrix} 5 & 12 \\ -7 & -4 \end{vmatrix}$ **64**

5. $\begin{vmatrix} -6 & -3 \\ -4 & -1 \end{vmatrix}$ **−6**

6. $\begin{vmatrix} 4 & 7 \\ 5 & 9 \end{vmatrix}$ **1**

7. $\begin{vmatrix} 14 & 8 \\ 9 & -3 \end{vmatrix}$ **−114**

8. $\begin{vmatrix} 15 & 12 \\ 23 & 28 \end{vmatrix}$ **144**

9. $\begin{vmatrix} -8 & 35 \\ 5 & 20 \end{vmatrix}$ **−335**

10. $\begin{vmatrix} 10 & 16 \\ 22 & 40 \end{vmatrix}$ **48**

11. $\begin{vmatrix} 24 & -8 \\ 7 & -3 \end{vmatrix}$ **−16**

12. $\begin{vmatrix} 13 & 62 \\ -4 & 19 \end{vmatrix}$ **495**

13. $\begin{vmatrix} 0.2 & 8 \\ -1.5 & 15 \end{vmatrix}$ **15**

14. $\begin{vmatrix} 8.6 & 0.5 \\ 14 & 5 \end{vmatrix}$ **36**

15. $\begin{vmatrix} 20 & 110 \\ 0.1 & 1.4 \end{vmatrix}$ **17**

16. $\begin{vmatrix} 4.8 & 2.1 \\ 3.4 & 5.3 \end{vmatrix}$ **18.3**

17. $\begin{vmatrix} \frac{2}{3} & -\frac{1}{2} \\ \frac{1}{6} & \frac{1}{5} \end{vmatrix}$ **$\frac{13}{60}$**

18. $\begin{vmatrix} 6.8 & 15 \\ -0.2 & 5 \end{vmatrix}$ **37**

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4-5 Study Guide and Intervention (continued)

Determinants

Determinants of 3 × 3 Matrices

Third-Order Determinants	$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$
Area of a Triangle	The area of a triangle having vertices (a, b), (c, d) and (e, f) is $\lvert A \rvert$, where $A = \frac{1}{2}\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{vmatrix}$.

Example Evaluate $\begin{vmatrix} \mathbf{4} & \mathbf{5} & \mathbf{-2} \\ \mathbf{1} & \mathbf{3} & \mathbf{0} \\ \mathbf{2} & \mathbf{-3} & \mathbf{6} \end{vmatrix}$.

$\begin{vmatrix} 4 & 5 & -2 \\ 1 & 3 & 0 \\ 2 & -3 & 6 \end{vmatrix} = 4\begin{vmatrix} 3 & 0 \\ -3 & 6 \end{vmatrix} - 5\begin{vmatrix} 1 & 0 \\ 2 & 6 \end{vmatrix} - 2\begin{vmatrix} 1 & 3 \\ 2 & -3 \end{vmatrix}$ Third-order determinant

$= 4(18 - 0) - 5(6 - 0) - 2(-3 - 6)$ Evaluate 2 × 2 determinants.

$= 4(18) - 5(6) - 2(-9)$ Simplify.

$= 72 - 30 + 18$ Multiply.

$= 60$ Simplify.

Exercises

Evaluate each determinant.

1. $\begin{vmatrix} 3 & -2 & -2 \\ 0 & 4 & 1 \\ -1 & 5 & -3 \end{vmatrix}$ **−57**

2. $\begin{vmatrix} 4 & 1 & 0 \\ -2 & 3 & 1 \\ 2 & -2 & 5 \end{vmatrix}$ **80**

3. $\begin{vmatrix} 6 & 1 & 4 \\ -2 & 3 & 0 \\ -1 & 3 & 2 \end{vmatrix}$ **28**

4. $\begin{vmatrix} 5 & -2 & 2 \\ 3 & 0 & -2 \\ 2 & 4 & -3 \end{vmatrix}$ **54**

5. $\begin{vmatrix} 6 & 1 & -4 \\ 3 & 2 & 1 \\ -2 & 2 & -1 \end{vmatrix}$ **−63**

6. $\begin{vmatrix} 5 & -4 & 1 \\ 2 & 3 & -2 \\ -1 & 6 & -3 \end{vmatrix}$ **−2**

7. Find the area of a triangle with vertices $X(2, -3)$, $Y(7, 4)$, and $Z(-5, 5)$.
44.5 square units

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4-5 Skills Practice

Determinants

Find the value of each determinant.

1. $\begin{vmatrix} 5 & 2 \\ 1 & 3 \end{vmatrix}$ **13**

2. $\begin{vmatrix} 10 & 9 \\ 5 & 8 \end{vmatrix}$ **35**

3. $\begin{vmatrix} 1 & 6 \\ 1 & 7 \end{vmatrix}$ **1**

4. $\begin{vmatrix} 2 & 5 \\ 3 & 1 \end{vmatrix}$ **−13**

5. $\begin{vmatrix} 0 & 9 \\ 5 & 8 \end{vmatrix}$ **−45**

6. $\begin{vmatrix} 3 & 12 \\ 2 & 8 \end{vmatrix}$ **0**

7. $\begin{vmatrix} -5 & 2 \\ 8 & -6 \end{vmatrix}$ **14**

8. $\begin{vmatrix} -3 & 1 \\ 8 & -7 \end{vmatrix}$ **13**

9. $\begin{vmatrix} 9 & -2 \\ -4 & 1 \end{vmatrix}$ **1**

10. $\begin{vmatrix} 1 & -5 \\ 1 & 6 \end{vmatrix}$ **11**

11. $\begin{vmatrix} 1 & -3 \\ -3 & 4 \end{vmatrix}$ **−5**

12. $\begin{vmatrix} -12 & 4 \\ 1 & 4 \end{vmatrix}$ **−52**

13. $\begin{vmatrix} 3 & -5 \\ 6 & -11 \end{vmatrix}$ **−3**

14. $\begin{vmatrix} -1 & -3 \\ 5 & -2 \end{vmatrix}$ **17**

15. $\begin{vmatrix} -1 & -14 \\ 5 & 2 \end{vmatrix}$ **68**

16. $\begin{vmatrix} -1 & 2 \\ 0 & 4 \end{vmatrix}$ **−4**

17. $\begin{vmatrix} 2 & 2 \\ -1 & 4 \end{vmatrix}$ **10**

18. $\begin{vmatrix} -1 & 6 \\ 2 & 5 \end{vmatrix}$ **−17**

Evaluate each determinant using expansion by minors.

19. $\begin{vmatrix} 2 & -1 & 1 \\ 3 & 2 & -1 \\ 2 & 3 & -2 \end{vmatrix}$ **−1**

20. $\begin{vmatrix} 6 & -1 & 1 \\ 5 & 2 & -1 \\ 1 & 3 & -2 \end{vmatrix}$ **−2**

21. $\begin{vmatrix} 2 & 6 & 1 \\ 3 & 5 & -1 \\ 2 & 1 & -2 \end{vmatrix}$ **−1**

Evaluate each determinant using diagonals.

22. $\begin{vmatrix} 2 & -1 & 6 \\ 3 & 2 & 5 \\ 2 & 3 & 1 \end{vmatrix}$ **−3**

23. $\begin{vmatrix} 3 & -1 & 2 \\ 1 & 0 & 4 \\ 3 & -2 & 0 \end{vmatrix}$ **8**

24. $\begin{vmatrix} 3 & 2 & 2 \\ 1 & -1 & 4 \\ 3 & -1 & 0 \end{vmatrix}$ **40**

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4-5 Practice
Determinants

Find the value of each determinant.

1. $\begin{vmatrix} 1 & 6 \\ 2 & 7 \end{vmatrix}$ **−5**
2. $\begin{vmatrix} 9 & 6 \\ 3 & 2 \end{vmatrix}$ **0**
3. $\begin{vmatrix} 4 & 1 \\ -2 & -5 \end{vmatrix}$ **−18**
4. $\begin{vmatrix} -14 & -3 \\ 2 & -2 \end{vmatrix}$ **34**
5. $\begin{vmatrix} 4 & -3 \\ -12 & 4 \end{vmatrix}$ **−20**
6. $\begin{vmatrix} 2 & -5 \\ 5 & -11 \end{vmatrix}$ **3**
7. $\begin{vmatrix} 4 & 0 \\ -2 & 9 \end{vmatrix}$ **36**
8. $\begin{vmatrix} 3 & -4 \\ 7 & 9 \end{vmatrix}$ **55**
9. $\begin{vmatrix} -1 & -11 \\ 10 & -2 \end{vmatrix}$ **112**
10. $\begin{vmatrix} 3 & -4 \\ 3.75 & 5 \end{vmatrix}$ **30**
11. $\begin{vmatrix} 2 & -1 \\ 3 & -9.5 \end{vmatrix}$ **−16**
12. $\begin{vmatrix} 0.5 & -0.7 \\ 0.4 & -0.3 \end{vmatrix}$ **0.13**

Evaluate each determinant using expansion by minors.

13. $\begin{vmatrix} -2 & 3 & 1 \\ 0 & 4 & -3 \\ 2 & 5 & -1 \end{vmatrix}$ **−48**
14. $\begin{vmatrix} 2 & -4 & 1 \\ 3 & 0 & 9 \\ -1 & 5 & 7 \end{vmatrix}$ **45**
15. $\begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & -2 \\ 1 & 1 & -1 \end{vmatrix}$ **7**
16. $\begin{vmatrix} 0 & -4 & 0 \\ 2 & -1 & 1 \\ 3 & -2 & 5 \end{vmatrix}$ **28**
17. $\begin{vmatrix} 2 & 7 & -6 \\ 8 & 4 & 0 \\ 1 & -1 & 3 \end{vmatrix}$ **−72**
18. $\begin{vmatrix} -12 & 0 & 3 \\ 7 & 5 & -1 \\ 4 & 2 & -6 \end{vmatrix}$ **318**

Evaluate each determinant using diagonals.

19. $\begin{vmatrix} -4 & 3 & -1 \\ 2 & 1 & -2 \\ 4 & 1 & -4 \end{vmatrix}$ **10**
20. $\begin{vmatrix} 2 & 2 & 3 \\ 1 & -1 & 1 \\ 3 & 1 & 1 \end{vmatrix}$ **12**
21. $\begin{vmatrix} 1 & -4 & -1 \\ 1 & -6 & -2 \\ 2 & 3 & 1 \end{vmatrix}$ **5**
22. $\begin{vmatrix} 1 & 2 & -4 \\ 1 & 4 & -6 \\ 2 & 3 & 3 \end{vmatrix}$ **20**
23. $\begin{vmatrix} 2 & -1 & -2 \\ 4 & 0 & -2 \\ 0 & 3 & 2 \end{vmatrix}$ **−4**
24. $\begin{vmatrix} 2 & 1 & 3 \\ 1 & 8 & 0 \\ 0 & 5 & -1 \end{vmatrix}$ **0**

25. **GEOMETRY** Find the area of a triangle whose vertices have coordinates (3, 5), (6, −5), and (−4, 10). **27.5 units^2**

26. **LAND MANAGEMENT** A fish and wildlife management organization uses a GIS (geographic information system) to store and analyze data for the parcels of land it manages. All of the parcels are mapped on a grid in which 1 unit represents 1 acre. If the coordinates of the corners of a parcel are (−8, 10), (6, 17), and (2, −4), how many acres is the parcel? **133 acres**

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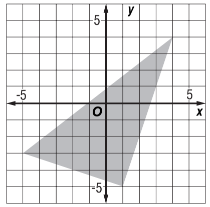
4-5 Word Problem Practice
Determinants

1. **FIND THE ERROR** Mark's determinant computation has sign errors. Circle the signs that must be reversed.

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 1(5)(9) - 2(6)(7) + 3(4)(8) - 3(5)(7) + 1(6)(8) - 2(4)(9)$$

2. **POOL** An architect has a pool in the floor plans for a home. Set up a determinant that gives the unit area of the pool.

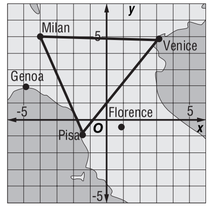


Sample answer: $\frac{1}{2}\begin{vmatrix} 4 & 4 & 1 \\ 1 & -5 & 1 \\ -5 & -3 & 1 \end{vmatrix}$

3. **HALF-UNIT TRIANGLES** For a school art project, students had to decorate a pegboard by looping strings around the pegs. Ronald wanted to make triangles with areas of one half square unit. Because Ronald had studied determinants, he knew that this was essentially the same as finding the coordinates of the vertices of a triangle (a, b), (c, d) and (e, f), so that the determinant $\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{vmatrix}$ is 1 or −1.

Give an example of such a triangle.
△*ABC* with *A*(0, 0), *B*(1, 0), and *C*(1, 1)

4. **ITALY** The figure shows a map of Italy overlaid on a graph. The coordinates of Milan, Venice, and Pisa are about (−4, 5), (3.25, 4.8), and (−1.4, −0.8), respectively. Each square unit on the map represents about 400 square miles.



What is the area of the triangular region? Round your answer to the nearest square mile.
8,306 sq. mi

ARROWS For Exercises 5 and 6, use the following information.
Kyle is making a triangle with vertices at (−6, 0), (0, $-x$), and (0, x), and $x > 0$. He plans to make the triangle using a material that costs $2 for every square unit.

5. Write the determinant that gives the area of this triangle.

Sample answer: $\frac{1}{2}\begin{vmatrix} -6 & 4 & 1 \\ 0 & -x & 1 \\ 0 & x & 1 \end{vmatrix}$

6. Evaluate the determinant you wrote for Exercise 5 and determine the value of x that results in a $60 triangle.
$6x$; $2(6x) = 60$ or $x = 5$

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4-5 Enrichment

Matrix Transpose and Determinants

In Lesson 4-1, you learned how to represent information in matrices. A matrix contains elements of the form a_{ij} where i is the row number of the element and j is the columns number of the element.

Consider the following matrix.

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$$

In this matrix, $a_{11} = 2$, $a_{12} = -1$, $a_{21} = 3$, and $a_{22} = 4$.

The matrix transpose can be found by switching the elements around. Element a_{ij} becomes element a_{ji}. So, the matrix transpose of A, denoted by A^T is:

$$A^T = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$$

Calculate the determinant of A and A^T.

$\det(A) = 2(4) - 3(-1) = 11$

$\det(A^T) = 2(4) - (-1)(3) = 11$

1. Find each matrix transpose.

a. $B = \begin{bmatrix} -1 & 5 \\ 2 & 6 \end{bmatrix}$ **b.** $C = \begin{bmatrix} 1 & 0 \\ -3 & 4 \end{bmatrix}$ **c.** $D = \begin{bmatrix} 2 & 3 & -1 \\ -2 & -1 & 5 \\ 1 & 3 & -2 \end{bmatrix}$

$B^T = \begin{bmatrix} -1 & 2 \\ 5 & 6 \end{bmatrix}$ $C^T = \begin{bmatrix} 1 & -3 \\ 0 & 4 \end{bmatrix}$ $D^T = \begin{bmatrix} 2 & -2 & 1 \\ 3 & -1 & 3 \\ -1 & 5 & -2 \end{bmatrix}$

2. Find the determinants of the original matrices and the transposes from Exercise 1.
a. $\det(B) = -16$; $\det(B^T) = -16$; **b.** $\det(C) = 7$; $\det(C^T) = 7$;
c. $\det(D) = -18$; $\det(D^T) = -18$

3. What do you notice about the determinants? Make a conjecture about the determinant of a matrix and the determinant of its transpose.
They are the same. The determinant of a matrix is the same as the determinant of its transpose.

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4-5 Spreadsheet Activity

Cramer's Rule

You have learned to solve systems of linear equations by using matrix equations and the inverse matrix. Another way to solve systems is to use *Cramer's Rule*. Study the spreadsheet below to discover Cramer's Rule.

Excel sample.xls

◇	A	B	C
1	6	3	-12
2	5	1	8
3			
4	=A1*B2-B1*A2		
5			
6	=C1*B2-B1*C2		
7			
8	=A1*C2-C1*A2		
9			
10	x =	=(A6/A4)	
11	y =	=(A8/A4)	

Sheet 1 / Sheet 2 / Sheet 3

To use the spreadsheet to solve a system of equations, write each equation in the form below.

$ax + by = c$

The values for the system $6x + 3y = -12$ and $5x + y = 8$ are shown. In the spreadsheet, the values of a, b, and c for the first equation are entered in cells A1, B1, and C1, respectively. The values of a, b, and c for the second equation are entered in cells A2, B2, and C2, respectively.

The values in cells B10 and B11 represent the solution for the system.

Exercises

1. Study the formula in cell A4. Write a matrix whose determinant is found using this formula.
$\begin{bmatrix} A1 & B1 \\ A2 & B2 \end{bmatrix}$

2. Write matrices whose determinants are found using the formulas in cells A6 and A8.
$\begin{bmatrix} C1 & B1 \\ C2 & B2 \end{bmatrix}$; $\begin{bmatrix} A1 & C1 \\ A2 & C2 \end{bmatrix}$

3. Explain how the values of x and y are found using Cramer's rule.
$$x = \frac{\begin{vmatrix} C1 & B1 \\ C2 & B2 \end{vmatrix}}{\begin{vmatrix} A1 & B1 \\ A2 & B2 \end{vmatrix}};\ y = \frac{\begin{vmatrix} A1 & C1 \\ A2 & C2 \end{vmatrix}}{\begin{vmatrix} A1 & B1 \\ A2 & B2 \end{vmatrix}}$$

Use the spreadsheet to solve each system of equations.

4. $6x + 3y = -12$
$5x + y = 8$
(4, −12)

5. $5x - 3y = 19$
$7x + 2y = 8$
(2, −3)

6. $8x - 3y = 11$
$6x + 9y = 15$
(1.6, 0.6)

7. $0.3x + 1.6y = 0.44$
$0.4x + 2.5y = 0.66$
(0.4, 0.2)

8. $3y = 4x + 28$
$5x + 7y = 8$
(−4, 4)

9. $y = -0.5x + 4$
$y = 4x - 5$
(2, 3)

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4-6 Lesson Reading Guide

Cramer's Rule

Get Ready for the Lesson

Read the introduction to Lesson 4-6 in your textbook.

A triangle is bounded by the x-axis, the line $y = \frac{1}{2}x$, and the line $y = -2x + 10$. Write three systems of equations that you could use to find the three vertices of the triangle. (Do not actually find the vertices.)

$x = 0, y = \frac{1}{2}x$; $x = 0, y = -2x + 10$; $y = \frac{1}{2}x, y = -2x + 10$

Read the Lesson

1. Suppose that you are asked to solve the following system of equations by Cramer's Rule.

$3x + 2y = 7$
$2x - 3y = 22$

Without actually evaluating any determinants, indicate which of the following ratios of determinants gives the correct value for x. **B**

A. $\dfrac{\begin{vmatrix} 3 & 2 \\ 2 & -3 \end{vmatrix}}{\begin{vmatrix} 7 & 2 \\ 22 & -3 \end{vmatrix}}$ **B.** $\dfrac{\begin{vmatrix} 7 & 2 \\ 22 & -3 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 2 & -3 \end{vmatrix}}$ **C.** $\dfrac{\begin{vmatrix} 3 & 7 \\ 2 & 22 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 2 & -3 \end{vmatrix}}$

2. In your textbook, the statements of Cramer's Rule for two variables and three variables specify that the determinant formed from the coefficients of the variables cannot be 0. If the determinant is zero, what do you know about the system and its solutions?

The system could be a dependent system and have infinitely many solutions, or it could be an inconsistent system and have no solutions.

Remember What You Learned

3. Some students have trouble remembering how to arrange the determinants that are used in solving a system of two linear equations by Cramer's Rule. What is a good way to remember this?

Sample answer: Let D be the determinant of the coefficients. Let D_x be the determinant formed by replacing the first column of D with the constants from the right-hand side of the system, and let D_y be the determinant formed by replacing the second column of D with the constants. Then $x = \frac{D_x}{D}$ and $y = \frac{D_y}{D}$.

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4-6 Study Guide and Intervention

Cramer's Rule

Systems of Two Linear Equations Determinants provide a way for solving systems of equations.

Cramer's Rule for Two-Variable Systems	The solution of the linear system of equations $ax + by = e$, $cx + dy = f$ is (x, y) where $x = \dfrac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$, $y = \dfrac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$, and $\begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0$.

Example **Use Cramer's Rule to solve the system of equations. $5x - 10y = 8$, $10x + 25y = -2$**

$x = \dfrac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$ Cramer's Rule $y = \dfrac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$

$= \dfrac{\begin{vmatrix} 8 & -10 \\ -2 & 25 \end{vmatrix}}{\begin{vmatrix} 5 & -10 \\ 10 & 25 \end{vmatrix}}$ $a = 5, b = -10, c = 10, d = 25, e = 8, f = -2$ $= \dfrac{\begin{vmatrix} 5 & 8 \\ 10 & -2 \end{vmatrix}}{\begin{vmatrix} 5 & -10 \\ 10 & 25 \end{vmatrix}}$

$= \dfrac{8(25) - (-2)(-10)}{5(25) - (-10)(10)}$ Evaluate each determinant. $= \dfrac{5(-2) - 8(10)}{5(25) - (-10)(10)}$

$= \dfrac{180}{225}$ or $\dfrac{4}{5}$ Simplify. $= -\dfrac{90}{225}$ or $-\dfrac{2}{5}$

The solution is $\left(\frac{4}{5}, -\frac{2}{5}\right)$.

Exercises

Use Cramer's Rule to solve each system of equations.

1. $3x - 2y = 7$; $2x + 7y = 38$ **(5, 4)**
2. $x - 4y = 17$; $3x - y = 29$ **(9, −2)**
3. $2x - y = -2$; $4x - y = 4$ **(3, 8)**
4. $2x - y = 1$; $5x + 2y = -29$ **(−3, −7)**
5. $4x + 2y = 1$; $5x - 4y = 24$ $\left(2, -\frac{7}{2}\right)$
6. $6x - 3y = -3$; $2x + y = 21$ **(5, 11)**
7. $2x + 7y = 16$; $x - 2y = 30$ **(22, −4)**
8. $2x - 3y = -2$; $3x - 4y = 9$ **(35, 24)**
9. $\frac{x}{3} + \frac{y}{5} = 2$; $\frac{x}{4} - \frac{y}{6} = -8$ **(−12, 30)**
10. $6x - 9y = -1$; $3x + 18y = 12$ $\left(\frac{2}{3}, \frac{5}{9}\right)$
11. $3x - 12y = -14$; $9x + 6y = -7$ $\left(-\frac{4}{3}, \frac{5}{6}\right)$
12. $8x + 2y = \frac{3}{7}$; $5x - 4y = -\frac{27}{7}$ $\left(-\frac{1}{7}, \frac{11}{14}\right)$

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4-6 Study Guide and Intervention *(continued)*

Cramer's Rule

Systems of Three Linear Equations

Cramer's Rule for Three-Variable Systems	The solution of the system whose equations are $ax + by + cz = j$, $dx + ey + fz = k$, $gx + hy + iz = l$ is (x, y, z) where $x = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}}$, $y = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}}$, and $z = \frac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & l \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}}$ and $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \neq 0$.

Example Use Cramer's rule to solve the system of equations.

$6x + 4y + z = 5$
$2x + 3y - 2z = -2$
$8x - 2y + 2z = 10$

Use the coefficients and constants from the equations to form the determinants. Then evaluate each determinant.

$$x = \frac{\begin{vmatrix} 5 & 4 & 1 \\ -2 & 3 & -2 \\ 10 & -2 & 2 \end{vmatrix}}{\begin{vmatrix} 6 & 4 & 1 \\ 2 & 3 & -2 \\ 8 & -2 & 2 \end{vmatrix}} = \frac{-80}{-96} \text{ or } \frac{5}{6} \qquad y = \frac{\begin{vmatrix} 6 & 5 & 1 \\ 2 & -2 & -2 \\ 8 & 10 & 2 \end{vmatrix}}{\begin{vmatrix} 6 & 4 & 1 \\ 2 & 3 & -2 \\ 8 & -2 & 2 \end{vmatrix}} = \frac{32}{-96} \text{ or } -\frac{1}{3} \qquad z = \frac{\begin{vmatrix} 6 & 4 & 5 \\ 2 & 3 & -2 \\ 8 & -2 & 10 \end{vmatrix}}{\begin{vmatrix} 6 & 4 & 1 \\ 2 & 3 & -2 \\ 8 & -2 & 2 \end{vmatrix}} = \frac{-128}{-96} \text{ or } \frac{4}{3}$$

The solution is $\left(\frac{5}{6}, -\frac{1}{3}, \frac{4}{3}\right)$.

Exercises

Use Cramer's rule to solve each system of equations.

1. $x - 2y + 3z = 6$
$2x - y - z = -3$
$x + y + z = 6$ **(1, 2, 3)**

2. $3x + y - 2z = -2$
$4x - 2y - 5z = 7$
$x + y + z = 1$ **(3, −5, 3)**

3. $x - 3y + z = 1$
$2x + 2y - z = -8$
$4x + 7y + 2z = 11$ **(−2, 1, 6)**

4. $2x - y + 3z = -5$
$x + y - 5z = 21$
$3x - 2y - 4z = 6$ **(4, 7, −2)**

5. $3x + y - 4z = 7$
$2x - y + 5z = -24$
$10x + 3y - 2z = -2$ **(−3, 8, −2)**

6. $2x - y + 4z = 9$
$3x - 2y - 5z = -13$
$x + y - 7z = 0$ **(5, 9, 2)**

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4-6 Skills Practice

Cramer's Rule

Lesson 4-6

Use Cramer's Rule to solve each system of equations.

1. $2a + 3b = 6$
$2a + b = -2$ **(−3, 4)**

2. $3x + y = 2$
$2x - y = 3$ **(1, −1)**

3. $2m + 3n = -6$
$m - 3n = 6$ **(0, −2)**

4. $x - y = 2$
$2x + 3y = 9$ **(3, 1)**

5. $2x + y = 4$
$7x - 2y = 3$ **(1, 2)**

6. $3r - s = 7$
$5r - 2s = 8$ **(6, 11)**

7. $4g + 5h = 1$
$g + 3h = 2$ **(−1, 1)**

8. $7x + 5y = -8$
$9x + 2y = 3$ **(1, −3)**

9. $3x - 4y = 2$
$4x - 3y = 12$ **(6, 4)**

10. $2x - y = 5$
$3x + y = 5$ **(2, −1)**

11. $3p - 6q = 18$
$2p + 3q = 5$ **(4, −1)**

12. $x - 2y = -1$
$2x + y = 3$ **(1, 1)**

13. $5c + 3d = 5$
$2c + 9d = 2$ **(1, 0)**

14. $5t + 2v = 2$
$2t + 3v = -8$ **(2, −4)**

15. $5a - 2b = 14$
$3a + 4b = 11$ **(3, 0.5)**

16. $65w - 8z = 83$
$9w + 4z = 0$ **(1, −2.25)**

17. GEOMETRY The two sides of an angle are contained in the lines whose equations are $3x + 2y = 4$ and $x - 3y = 5$. Find the coordinates of the vertex of the angle. **(2, −1)**

Use Cramer's Rule to solve each system of equations.

18. $a + b + 5c = 2$
$3a + b + 2c = 3$
$4a + 2b - c = -3$ **(2, −5, 1)**

19. $x + 3y - z = 5$
$2x + 5y - z = 12$
$x - 2y - 3z = -13$ **(3, 2, 4)**

20. $3c - 5d + 2e = 4$
$2c - 3d + 4c = -3$
$4c - 2d + 3e = 0$ **(1, −1, −2)**

21. $r - 4s - t = 6$
$2r - s + 3t = 0$
$3r - 2s + t = 4$ **(1, −1, −1)**

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4-6 Practice

Cramer's Rule

Use Cramer's Rule to solve each system of equations.

1. $2x + y = 0$
$3x + 2y = -2$ **(2, −4)**

2. $5c + 9d = 19$
$2c - d = -20$ **(−7, 6)**

3. $2x + 3y = 5$
$3x - 2y = 1$ **(1, 1)**

4. $20m - 3n = 28$
$2m + 3n = 16$ **(2, 4)**

5. $x - 3y = 6$
$3x + y = -22$ **(−6, −4)**

6. $5x - 6y = -45$
$9x + 8y = 13$ **(−3, 5)**

7. $-2e + f = 4$
$-3e + 5f = -15$ **(−5, −6)**

8. $2x - y = -1$
$2x - 4y = 8$ **(−2, −3)**

9. $8a + 3b = 24$
$2a + b = 4$ **(6, −8)**

10. $-3x + 15y = 45$
$-2x + 7y = 18$ **(5, 4)**

11. $3u - 5v = 11$
$6u + 7v = -12$ $\left(\frac{1}{3}, -2\right)$

12. $-6g + h = -10$
$-3g - 4h = 4$ $\left(\frac{4}{3}, -2\right)$

13. $x - 3y = 8$
$x - 0.5y = 3$ **(2, −2)**

14. $0.2x - 0.5y = -1$
$0.6x - 3y = -9$ **(5, 4)**

15. $0.3d - 0.6g = 1.8$
$0.2d + 0.3g = 0.5$ **(4, −1)**

16. GEOMETRY The two sides of an angle are contained in the lines whose equations are $x - \frac{4}{3}y = 6$ and $2x + y = 1$. Find the coordinates of the vertex of the angle. **(2, −3)**

17. GEOMETRY Two sides of a parallelogram are contained in the lines represented by the equations $0.2x - 0.5y = 1$ and $0.02x - 0.3y = -0.9$. Find the coordinates of a vertex of the parallelogram. **(15, 4)**

Use Cramer's Rule to solve each system of equations.

18. $x + 3y + 3z = 4$
$-x + 2y + z = -1$
$4x + y - 2z = -1$
(1, −1, 2)

19. $-5a + b - 4c = 7$
$-3a + 2b - c = 0$
$2a + 3b - c = 17$
(3, 2, −5)

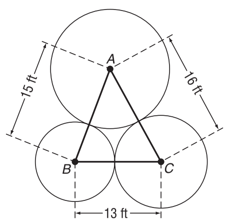
20. $2x + y - 3z = -5$
$5x + 2y - 2z = 8$
$3x - 3y + 5z = 17$
(2, 3, 4)

21. $2c + 3d - e = 17$
$4c + d + 5e = -9$
$c + 2d - e = 12$
(2, 3, −4)

22. $2j + k - 3m = -3$
$3j + 2k + 4m = 5$
$-4j - k + 2m = 4$
(−1, 2, 1)

23. $3x - 2y + 5z = 3$
$2x + 2y - 4z = 3$
$-5x + 10y + 7z = -3$
(1.2, 0.3, 0)

24. LANDSCAPING A memorial garden being planted in front of a municipal library will contain three circular beds that are tangent to each other. A landscape architect has prepared a sketch of the design for the garden using CAD (computer-aided drafting) software, as shown at the right. The centers of the three circular beds are represented by points A, B, and C. The distance from A to B is 15 feet, the distance from B to C is 13 feet, and the distance from A to C is 16 feet. What is the radius of each of the circular beds? **circle A: 9 ft, circle B: 6 ft, circle C: 7 ft**



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4-6 Word Problem Practice

Cramer's Rule

1. USING CRAMER'S RULE Lucy is solving the following system of linear equations using Cramer's Rule.

$$2x + 3y = 5$$
$$x + y = 2$$

Write the three determinants she will have to compute.

$\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 2 & 5 \\ 1 & 2 \end{vmatrix}, \begin{vmatrix} 5 & 3 \\ 2 & 1 \end{vmatrix}$

2. IMPLICATIONS OF CRAMER'S RULE Cramer's Rule gives the solutions of systems of linear equations in terms of their coefficients. The formula involves addition, subtraction, multiplication, and division of those coefficients. Is it possible for an irrational number to be part of the solution of a system of linear equations whose coefficients are all rational numbers?
No, It is impossible.

3. SHOPPING Sheets cost \$18.59 each and pillowcases cost \$7.24 at Carol's Linens. If Agatha buys x sheets and y pillowcases at Carol's Linens, she'll spend \$210.75. On the other hand, at Save-n-Sleep, sheets cost \$15.79 and pillowcases cost \$8.19. If Agatha buys x sheets and y pillows cases at Save-n-Sleep, she'll spend \$191.25. Use Cramer's Rule to determine how many sheets and pillow cases Agatha wants to buy.
9 sheets and 6 pillow cases

4. BRICKS Linus owns three different types of brick that differ only in length. If he lines up 2 short, 1 medium, and 2 long bricks, the total length will be 45 inches. If he lines up 1 short, 2 medium, and 3 long bricks, the total length will be 59 inches. If he lines up 5 short, 1 medium, and 1 long brick, the total length will be 53 inches. Use Cramer's Rule to determine how long the different types of brick are.
6.5, 9, and 11.5 in.

PROMOTIONS For Exercises 5–7, use the following information.
A local zoo was trying to increase attendance by offering \$2 for every child that came. However, the zoo insisted that there be at least 1 adult for every 8 children. A school decided to take advantage of the situation by sending 1 adult for every 8 children. Let c be the number of children and let a be the number of adults. Admission for adults was d dollars. The total cost of admission for everyone was \$13.50.

5. Write a system of equations that describes the situation.
$8a - c = 0$
$da - 2c = 13.50$

6. Is it possible that $d = 16$? Explain in terms of Cramer's Rule.
No, because Cramer's Rule would then involve division by zero.

7. If adults were charged \$20.50 for admission, how many adults and children went? Use Cramer's Rule to solve.
3 adults and 24 children

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4-6 Enrichment

Fourth-Order Determinants

To find the value of a 4 × 4 determinant, use a method called **expansion by minors**.

First write the expansion. Use the first row of the determinant.
Remember that the signs of the terms alternate.

$$\begin{vmatrix} 6 & -3 & 2 & 7 \\ 0 & 4 & 3 & 5 \\ 0 & 2 & 1 & -4 \\ 6 & 0 & -2 & 0 \end{vmatrix} = 6\begin{vmatrix} 4 & 3 & 5 \\ 2 & 1 & -4 \\ 0 & -2 & 0 \end{vmatrix} - (-3)\begin{vmatrix} 0 & 3 & 5 \\ 0 & 1 & -4 \\ 6 & -2 & 0 \end{vmatrix} + 2\begin{vmatrix} 0 & 4 & 5 \\ 0 & 2 & -4 \\ 6 & 0 & 0 \end{vmatrix} - 7\begin{vmatrix} 0 & 4 & 3 \\ 0 & 2 & 1 \\ 6 & 0 & -2 \end{vmatrix}$$

Then evaluate each 3 × 3 determinant. Use any row.

$$\begin{vmatrix} 4 & 3 & 5 \\ 2 & 1 & -4 \\ 0 & -2 & 0 \end{vmatrix} = -(-2)\begin{vmatrix} 4 & 5 \\ 2 & -4 \end{vmatrix}$$
$$= 2(-16 - 10)$$
$$= -52$$

$$\begin{vmatrix} 0 & 3 & 5 \\ 0 & 1 & -4 \\ 6 & -2 & 0 \end{vmatrix} = -3\begin{vmatrix} 0 & -4 \\ 6 & 0 \end{vmatrix} + 5\begin{vmatrix} 0 & 1 \\ 6 & -2 \end{vmatrix}$$
$$= -3(24) + 5(-6)$$
$$= -102$$

$$\begin{vmatrix} 0 & 4 & 5 \\ 0 & 2 & -4 \\ 6 & 0 & 0 \end{vmatrix} = 6\begin{vmatrix} 4 & 5 \\ 2 & -4 \end{vmatrix}$$
$$= 6(-16 - 10)$$
$$= -156$$

$$\begin{vmatrix} 0 & 4 & 3 \\ 0 & 2 & 1 \\ 6 & 0 & -2 \end{vmatrix} = -4\begin{vmatrix} 0 & 1 \\ 6 & -2 \end{vmatrix} + 3\begin{vmatrix} 0 & 2 \\ 6 & 0 \end{vmatrix}$$
$$= -4(-6) + 3(-12)$$
$$= -12$$

Finally, evaluate the original 4 × 4 determinant.

$$\begin{vmatrix} 6 & -3 & 2 & 7 \\ 0 & 4 & 3 & 5 \\ 0 & 2 & 1 & -4 \\ 6 & 0 & -2 & 0 \end{vmatrix} = 6(-52) + 3(-102) + 2(-156) - 7(-12) = -846$$

Evaluate each determinant.

1. $\begin{vmatrix} 1 & 2 & 3 & 1 \\ 4 & 3 & -1 & 0 \\ 2 & -5 & 4 & 4 \\ 1 & -2 & 0 & 2 \end{vmatrix}$

−109

2. $\begin{vmatrix} 3 & 3 & 3 & 3 \\ 2 & 1 & 2 & 1 \\ 4 & 3 & -1 & 5 \\ 2 & 5 & 0 & 1 \end{vmatrix}$

−72

3. $\begin{vmatrix} 1 & 4 & 3 & 0 \\ -2 & -3 & 6 & 4 \\ 5 & 1 & 1 & 2 \\ 4 & 2 & 5 & -1 \end{vmatrix}$

−676

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4-7 Lesson Reading Guide

Identity and Inverse Matrices

Get Ready for the Lesson

Read the introduction to Lesson 4-7 in your textbook.

Refer to the code table given in the introduction to this lesson. Suppose that you receive a message coded by this system as follows:

16 12 5 1 19 5   2 5   13 25   6 18 9 5 14 4.

Decode the message. **Please be my friend.**

Read the Lesson

1. Indicate whether each of the following statements is *true* or *false*.

a. Every element of an identity matrix is 1. **false**

b. There is a 3 × 2 identity matrix. **false**

c. Two matrices are inverses of each other if their product is the identity matrix. **true**

d. If M is a matrix, M^{-1} represents the reciprocal of M. **false**

e. No 3 × 2 matrix has an inverse. **true**

f. Every square matrix has an inverse. **false**

g. If the two columns of a 2 × 2 matrix are identical, the matrix does not have an inverse. **true**

2. Explain how to find the inverse of a 2 × 2 matrix. Do not use any special mathematical symbols in your explanation.

Sample answer: First find the determinant of the matrix. If it is zero, then the matrix has no inverse. If the determinant is not zero, form a new matrix as follows. Interchange the top left and bottom right elements. Change the signs but not the positions of the other two elements. Multiply the resulting matrix by the reciprocal of the determinant of the original matrix. The resulting matrix is the inverse of the original matrix.

Remember What You Learned

3. One way to remember something is to explain it to another person. Suppose that you are studying with a classmate who is having trouble remembering how to find the inverse of a 2 × 2 matrix. He remembers how to move elements and change signs in the matrix, but thinks that he should multiply by the determinant of the original matrix. How can you help him remember that he must multiply by the *reciprocal* of this determinant?

Sample answer: If the determinant of the matrix is 0, its reciprocal is undefined. This agrees with the fact that if the determinant of a matrix is 0, the matrix does not have an inverse.

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4-7 Study Guide and Intervention

Identity and Inverse Matrices

Identity and Inverse Matrices The identity matrix for matrix multiplication is a square matrix with 1s for every element of the main diagonal and zeros elsewhere.

Identity Matrix for Multiplication	If A is an $n \times n$ matrix and I is the identity matrix, then $A \cdot I = A$ and $I \cdot A = A$.

If an $n \times n$ matrix A has an inverse A^{-1}, then $A \cdot A^{-1} = A^{-1} \cdot A = I$.

Example **Determine whether $X = \begin{bmatrix} 7 & 4 \\ 10 & 6 \end{bmatrix}$ and $Y = \begin{bmatrix} 3 & -2 \\ -5 & \frac{7}{2} \end{bmatrix}$ are inverse matrices.**

Find $X \cdot Y$.

$$X \cdot Y = \begin{bmatrix} 7 & 4 \\ 10 & 6 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 \\ -5 & \frac{7}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 21 - 20 & -14 + 14 \\ 30 - 30 & -20 + 21 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Find $Y \cdot X$.

$$Y \cdot X = \begin{bmatrix} 3 & -2 \\ -5 & \frac{7}{2} \end{bmatrix} \cdot \begin{bmatrix} 7 & 4 \\ 10 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 21 - 20 & 12 - 12 \\ -35 + 35 & -20 + 21 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Since $X \cdot Y = Y \cdot X = I$, X and Y are inverse matrices.

Exercises

Determine whether each pair of matrices are inverses.

1. $\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$ and $\begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$ **yes**

2. $\begin{bmatrix} 3 & 2 \\ 5 & 4 \end{bmatrix}$ and $\begin{bmatrix} 2 & -1 \\ -\frac{5}{2} & \frac{3}{2} \end{bmatrix}$ **yes**

3. $\begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$ and $\begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$ **no**

4. $\begin{bmatrix} 8 & 11 \\ 3 & 4 \end{bmatrix}$ and $\begin{bmatrix} -4 & 11 \\ 3 & -8 \end{bmatrix}$ **yes**

5. $\begin{bmatrix} 4 & -1 \\ 5 & 3 \end{bmatrix}$ and $\begin{bmatrix} 1 & 2 \\ 3 & 8 \end{bmatrix}$ **no**

6. $\begin{bmatrix} 5 & 2 \\ 11 & 4 \end{bmatrix}$ and $\begin{bmatrix} -2 & 1 \\ \frac{11}{2} & -\frac{5}{2} \end{bmatrix}$ **yes**

7. $\begin{bmatrix} 4 & 2 \\ 6 & -2 \end{bmatrix}$ and $\begin{bmatrix} -\frac{1}{5} & -\frac{1}{10} \\ \frac{3}{10} & \frac{1}{10} \end{bmatrix}$ **no**

8. $\begin{bmatrix} 5 & 8 \\ 4 & 6 \end{bmatrix}$ and $\begin{bmatrix} -3 & 4 \\ 2 & -\frac{5}{2} \end{bmatrix}$ **yes**

9. $\begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}$ and $\begin{bmatrix} \frac{7}{2} & -\frac{3}{2} \\ 1 & -2 \end{bmatrix}$ **no**

10. $\begin{bmatrix} 3 & 2 \\ 4 & -6 \end{bmatrix}$ and $\begin{bmatrix} 3 & 2 \\ -4 & -3 \end{bmatrix}$ **no**

11. $\begin{bmatrix} 7 & 2 \\ 17 & 5 \end{bmatrix}$ and $\begin{bmatrix} 5 & -2 \\ -17 & 7 \end{bmatrix}$ **yes**

12. $\begin{bmatrix} 4 & 3 \\ 7 & 5 \end{bmatrix}$ and $\begin{bmatrix} -5 & 3 \\ 7 & -4 \end{bmatrix}$ **yes**

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4-7 Study Guide and Intervention *(continued)*

Identity and Inverse Matrices

Find Inverse Matrices

Inverse of a 2 × 2 Matrix	The inverse of a matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $ad - bc \neq 0$.

If $ad - bc = 0$, the matrix does not have an inverse.

Example **Find the inverse of $N = \begin{bmatrix} 7 & 2 \\ 2 & 1 \end{bmatrix}$.**

First find the value of the determinant.

$$\begin{vmatrix} 7 & 2 \\ 2 & 1 \end{vmatrix} = 7 - 4 = 3$$

Since the determinant does not equal 0, N^{-1} exists.

$$N^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & -2 \\ -2 & 7 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{7}{3} \end{bmatrix}$$

Check:

$$NN^{-1} = \begin{bmatrix} 7 & 2 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{7}{3} \end{bmatrix} = \begin{bmatrix} \frac{7}{3} - \frac{4}{3} & -\frac{14}{3} + \frac{14}{3} \\ \frac{2}{3} - \frac{2}{3} & -\frac{4}{3} + \frac{7}{3} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$N^{-1}N = \begin{bmatrix} \frac{1}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{7}{3} \end{bmatrix} \cdot \begin{bmatrix} 7 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{7}{3} - \frac{4}{3} & \frac{2}{3} - \frac{2}{3} \\ -\frac{14}{3} + \frac{14}{3} & -\frac{4}{3} + \frac{7}{3} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Exercises

Find the inverse of each matrix, if it exists.

1. $\begin{bmatrix} 24 & 12 \\ 8 & 4 \end{bmatrix}$ **no inverse exists**

2. $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

3. $\begin{bmatrix} 40 & -10 \\ -20 & 30 \end{bmatrix}$ $\frac{1}{1000}\begin{bmatrix} 30 & 10 \\ 20 & 40 \end{bmatrix}$

4. $\begin{bmatrix} 6 & 5 \\ 10 & 8 \end{bmatrix}$ $-\frac{1}{2}\begin{bmatrix} 8 & -5 \\ -10 & 6 \end{bmatrix}$

5. $\begin{bmatrix} 3 & 6 \\ 4 & 8 \end{bmatrix}$ **no inverse exists**

6. $\begin{bmatrix} 8 & 2 \\ 10 & 4 \end{bmatrix}$ $\frac{1}{12}\begin{bmatrix} 4 & -2 \\ -10 & -8 \end{bmatrix}$

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4-7 Skills Practice

Identity and Inverse Matrices

Determine whether each pair of matrices are inverses.

1. $X = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, Y = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$ **yes**

2. $P = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}, Q = \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$ **yes**

3. $M = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}, N = \begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix}$ **no**

4. $A = \begin{bmatrix} -2 & 5 \\ -1 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & -5 \\ 1 & -2 \end{bmatrix}$ **yes**

5. $V = \begin{bmatrix} 0 & 7 \\ -7 & 0 \end{bmatrix}, W = \begin{bmatrix} 0 & -\frac{1}{7} \\ \frac{1}{7} & 0 \end{bmatrix}$ **yes**

6. $X = \begin{bmatrix} -1 & 4 \\ 1 & 2 \end{bmatrix}, Y = \begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix}$ **yes**

7. $G = \begin{bmatrix} 4 & -3 \\ 1 & 2 \end{bmatrix}, H = \begin{bmatrix} \frac{2}{11} & \frac{3}{11} \\ -\frac{1}{11} & \frac{4}{11} \end{bmatrix}$ **yes**

8. $D = \begin{bmatrix} -4 & -4 \\ -4 & 4 \end{bmatrix}, E = \begin{bmatrix} -0.125 & -0.125 \\ -0.125 & -0.125 \end{bmatrix}$ **no**

Find the inverse of each matrix, if it exists.

9. $\begin{bmatrix} 0 & 2 \\ 4 & 0 \end{bmatrix}$ $-\frac{1}{8}\begin{bmatrix} 0 & -2 \\ -4 & 0 \end{bmatrix}$

10. $\begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$ $-\begin{bmatrix} 2 & -1 \\ -3 & 1 \end{bmatrix}$

11. $\begin{bmatrix} 9 & 3 \\ 6 & 2 \end{bmatrix}$ **no inverse exists**

12. $\begin{bmatrix} -2 & -4 \\ 6 & 0 \end{bmatrix}$ $\frac{1}{24}\begin{bmatrix} 0 & 4 \\ -6 & -2 \end{bmatrix}$

13. $\begin{bmatrix} 1 & -1 \\ 3 & 3 \end{bmatrix}$ $\frac{1}{6}\begin{bmatrix} 3 & 1 \\ -3 & 1 \end{bmatrix}$

14. $\begin{bmatrix} 3 & 6 \\ -1 & -2 \end{bmatrix}$ **no inverse exists**

15. $\begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix}$ $\frac{1}{2}\begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix}$

16. $\begin{bmatrix} -4 & 5 \\ 1 & 2 \end{bmatrix}$ $-\frac{1}{13}\begin{bmatrix} 2 & -5 \\ -1 & -4 \end{bmatrix}$

17. $\begin{bmatrix} 0 & -7 \\ -7 & 0 \end{bmatrix}$ $-\frac{1}{49}\begin{bmatrix} 0 & 7 \\ 7 & 0 \end{bmatrix}$

18. $\begin{bmatrix} 10 & 8 \\ 5 & 4 \end{bmatrix}$ **no inverse exists**

19. $\begin{bmatrix} 10 & 8 \\ 10 & -8 \end{bmatrix}$ $-\frac{1}{160}\begin{bmatrix} -8 & -8 \\ -10 & 10 \end{bmatrix}$

20. $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ $\frac{1}{4}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

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4-7 Practice

Identity and Inverse Matrices

Determine whether each pair of matrices are inverses.

1. $M = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}, N = \begin{bmatrix} -2 & 1 \\ 3 & -2 \end{bmatrix}$ **no**

2. $X = \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}, Y = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$ **yes**

3. $A = \begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}, B = \begin{bmatrix} \frac{1}{5} & -\frac{1}{10} \\ \frac{2}{5} & \frac{3}{10} \end{bmatrix}$ **yes**

4. $P = \begin{bmatrix} 6 & -2 \\ -2 & 3 \end{bmatrix}, Q = \begin{bmatrix} \frac{3}{14} & \frac{1}{7} \\ \frac{1}{7} & \frac{3}{7} \end{bmatrix}$ **yes**

Determine whether each statement is *true* or *false*.

5. All square matrices have multiplicative inverses. **false**

6. All square matrices have multiplicative identities. **true**

Find the inverse of each matrix, if it exists.

7. $\begin{bmatrix} 4 & 5 \\ -4 & -3 \end{bmatrix}$ $\frac{1}{8}\begin{bmatrix} -3 & -5 \\ 4 & 4 \end{bmatrix}$

8. $\begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$ $\frac{1}{10}\begin{bmatrix} 5 & 0 \\ -3 & 2 \end{bmatrix}$

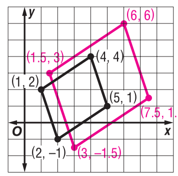
9. $\begin{bmatrix} -1 & 3 \\ 4 & -7 \end{bmatrix}$ $-\frac{1}{5}\begin{bmatrix} -7 & -3 \\ -4 & -1 \end{bmatrix}$

10. $\begin{bmatrix} 2 & 5 \\ -1 & 3 \end{bmatrix}$ $\frac{1}{11}\begin{bmatrix} 3 & -5 \\ 1 & 2 \end{bmatrix}$

11. $\begin{bmatrix} 2 & -5 \\ 3 & 1 \end{bmatrix}$ $\frac{1}{17}\begin{bmatrix} 1 & 5 \\ -3 & 2 \end{bmatrix}$

12. $\begin{bmatrix} 4 & 6 \\ 6 & 9 \end{bmatrix}$ **no inverse exists**

GEOMETRY For Exercises 13–16, use the figure at the right.



13. Write the vertex matrix A for the rectangle. $\begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 4 & 1 & -1 \end{bmatrix}$

14. Use matrix multiplication to find BA if $B = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.5 \end{bmatrix}$.
$\begin{bmatrix} 1.5 & 6 & 7.5 & 3 \\ 3 & 6 & 1.5 & -1.5 \end{bmatrix}$

15. Graph the vertices of the transformed triangle on the previous graph. Describe the transformation. **dilation by a scale factor of 1.5**

16. Make a conjecture about what transformation B^{-1} describes on a coordinate plane.
dilation by a scale factor of $\frac{2}{3}$

17. **CODES** Use the alphabet table below and the inverse of coding matrix $C = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ to decode this message:
19 | 14 | 11 | 13 | 11 | 22 | 55 | 65 | 57 | 60 | 2 | 1 | 52 | 47 | 33 | 51 | 56 | 55.

CHECK_YOUR_ANSWERS

CODE						
A 1	B 2	C 3	D 4	E 5	F 6	G 7
H 8	I 9	J 10	K 11	L 12	M 13	N 14
O 15	P 16	Q 17	R 18	S 19	T 20	U 21
V 22	W 23	X 24	Y 25	Z 26	– 0	

Lesson 4-7

Answers

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4-7 Word Problem Practice

Identity and Inverse Matrices

1. **ROTATIONS** Suppose R represents a counterclockwise rotation about the origin by an angle of 45°. For what values of n is R^n equal to the inverse of R?
n can be 1 less than any positive multiple of 8, for example 7, 15, 23, etc.

2. **SPECIAL MATRICES** Norman only likes working with matrices whose determinant is 1. If $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is such a matrix, what is its inverse?
$\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

3. **CRYPTOGRAPHY** A friend sends you a secret message that was coded using the coding matrix $C = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$ and the alphabet table.

CODE		
A 65	J 74	S 83
B 66	K 75	T 84
C 67	L 76	U 85
D 68	M 77	V 86
E 69	N 78	W 87
F 70	O 79	X 88
G 71	P 80	Y 89
H 72	Q 81	Z 90
I 73	R 82	–91

The message is 567 | 354 | 620 | 388. What is the decoded message?
HELP

4. **SELF-INVERSES** Phillip notices that any matrix with ones and negative ones on the diagonal and zeroes everywhere else has the property that it is its own inverse. Give an example of a 2 by 2 matrix that is its own inverse but has at least 1 nonzero number off the diagonal.
$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

MATRIX OPERATIONS For Exercises 5–7, use the following information.
Garth is studying determinants and inverses of matrices in math class. His teacher suggests that there are some matrices with unique properties, and challenges the class to find such matrices and describe the properties found. Garth is curious about the matrix $G = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$.

5. What is the determinant of G?
The determinant of G is 0.

6. Does the inverse of G exist? Explain.
The inverse of the matrix does not exist because $ad - bc = 0$.

7. Determine a matrix operation that could be used to transform G into its Additive Identity matrix.
G^2

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4-7 Enrichment

Permutation Matrices

A permutation matrix is a square matrix in which each row and each column has one entry that is 1. All the other entries are 0. Find the inverse of a permutation matrix interchanging the rows and columns. For example, row 1 is interchanged with column 1, row 2 is interchanged with column 2.

$$P = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \qquad P^{-1} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

P is a 4 × 4 permutation matrix. P^{-1} is the inverse of P.

Solve each problem.

1. There is just one 2 × 2 permutation matrix that is not also an identity matrix. Write this matrix.
$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

2. Find the inverse of the matrix you wrote in Exercise 1. What do you notice?
$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ **The two matrices are the same.**

3. Show that the two matrices in Exercises 1 and 2 are inverses.
$$\begin{bmatrix} 0 \cdot 1 + 1 \cdot 1 & 0 \cdot 1 + 1 \cdot 0 \\ 1 \cdot 0 + 0 \cdot 1 & 1 \cdot 1 + 0 \cdot 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

4. Write the inverse of this matrix.
$$B = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad B^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

5. Use B^{-1} from problem 4. Verify that B and B^{-1} are inverses.
$$\begin{bmatrix} 0 \cdot 0 + 0 \cdot 0 + 1 \cdot 1 & 0 \cdot 1 + 0 \cdot 0 + 1 \cdot 0 & 0 \cdot 0 + 0 \cdot 1 + 1 \cdot 0 \\ 1 \cdot 0 + 0 \cdot 0 + 0 \cdot 1 & 1 \cdot 1 + 0 \cdot 0 + 0 \cdot 0 & 1 \cdot 0 + 0 \cdot 1 + 0 \cdot 0 \\ 0 \cdot 0 + 1 \cdot 0 + 0 \cdot 1 & 0 \cdot 1 + 1 \cdot 0 + 0 \cdot 0 & 0 \cdot 0 + 1 \cdot 1 + 0 \cdot 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

6. Permutation matrices can be used to write and decipher codes. To see how this is done, use the message matrix M and matrix B from problem 4. Find matrix C so that C equals the product MB. Use the rules below.

0 times a letter = 0
1 times a letter = the same letter
0 plus a letter = the same letter

$$M = \begin{bmatrix} \text{S} & \text{H} & \text{E} \\ \text{S} & \text{A} & \text{W} \\ \text{H} & \text{I} & \text{M} \end{bmatrix} \qquad C = \begin{bmatrix} \text{H} & \text{E} & \text{S} \\ \text{A} & \text{W} & \text{S} \\ \text{I} & \text{M} & \text{H} \end{bmatrix}$$

7. Now find the product CB^{-1}. What do you notice?
$$\begin{bmatrix} \text{H} & \text{E} & \text{S} \\ \text{A} & \text{W} & \text{S} \\ \text{I} & \text{M} & \text{H} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \text{S} & \text{H} & \text{E} \\ \text{S} & \text{A} & \text{W} \\ \text{H} & \text{I} & \text{M} \end{bmatrix}$$
Multiplying M by B encodes the message. To decipher, multiply by B^{-1}.

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4-8 Lesson Reading Guide

Using Matrices to Solve Systems of Equations

Get Ready for the Lesson

Read the introduction to Lesson 4-8 in your textbook.

Write a 2×2 matrix that summarizes the information given in the introduction about the food and territory requirements for the two species.

$$\begin{bmatrix} 140 & 500 \\ 120 & 400 \end{bmatrix}$$

Read the Lesson

1. a. Write a matrix equation for the following system of equations.

$3x + 5y = 10$
$2x - 4y = -7$

$$\begin{bmatrix} 3 & 5 \\ 2 & -4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ -7 \end{bmatrix}$$

b. Explain how to use the matrix equation you wrote above to solve the system. Use as few mathematical symbols in your explanation as you can. Do not actually solve the system.

Sample answer: Find the inverse of the 2×2 matrix of coefficients. Multiply this inverse by the 2×1 matrix of constants, with the 2×2 matrix on the left. The product will be a 2×1 matrix. The number in the first row will be the value of *x*, and the number in the second row will be the value of *y*.

2. Write a system of equations that corresponds to the following matrix equation.

$$\begin{bmatrix} 3 & 2 & -4 \\ 2 & -1 & 0 \\ 0 & 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 6 \\ -4 \end{bmatrix}$$

$3x + 2y - 4z = -2$
$2x - y = 6$
$5y + 6z = -4$

Remember What You Learned

3. Some students have trouble remembering how to set up a matrix equation to solve a system of linear equations. What is an easy way to remember the order in which to write the three matrices that make up the equation?

Sample answer: Just remember "CVC" for "coefficients, variables, constants." The variable matrix is on the left side of the equals sign, just as the variables are in the system of linear equations. The constant matrix is on the right side of the equals sign, just as the constants are in the system of linear equations.

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4-8 Study Guide and Intervention

Using Matrices to Solve Systems of Equations

Write Matrix Equations A **matrix equation** for a system of equations consists of the product of the coefficient and variable matrices on the left and the constant matrix on the right of the equals sign.

Example **Write a matrix equation for each system of equations.**

a. $3x - 7y = 12$
$x + 5y = -8$

Determine the coefficient, variable, and constant matrices.

$$\begin{bmatrix} 3 & -7 \\ 1 & 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ -8 \end{bmatrix}$$

b. $2x - y + 3z = -7$
$x + 3y - 4z = 15$
$7x + 2y + z = -28$

$$\begin{bmatrix} 2 & -1 & 3 \\ 1 & 3 & -4 \\ 7 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -7 \\ 15 \\ -28 \end{bmatrix}$$

Exercises

Write a matrix equation for each system of equations.

1. $2x + y = 8$
$5x - 3y = -12$

$$\begin{bmatrix} 2 & 1 \\ 5 & -3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -12 \end{bmatrix}$$

2. $4x - 3y = 18$
$x + 2y = 12$

$$\begin{bmatrix} 4 & -3 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 18 \\ 12 \end{bmatrix}$$

3. $7x - 2y = 15$
$3x + y = -10$

$$\begin{bmatrix} 7 & -2 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ -10 \end{bmatrix}$$

4. $4x - 6y = 20$
$3x + y + 8 = 0$

$$\begin{bmatrix} 4 & -6 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 20 \\ -8 \end{bmatrix}$$

5. $5x + 2y = 18$
$x = -4y + 25$

$$\begin{bmatrix} 5 & 2 \\ 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 18 \\ 25 \end{bmatrix}$$

6. $3x - y = 24$
$3y = 80 - 2x$

$$\begin{bmatrix} 3 & -1 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 24 \\ 80 \end{bmatrix}$$

7. $2x + y + 7z = 12$
$5x - y + 3z = 15$
$x + 2y - 6z = 25$

$$\begin{bmatrix} 2 & 1 & 7 \\ 5 & -1 & 3 \\ 1 & 2 & -6 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 15 \\ 25 \end{bmatrix}$$

8. $5x - y + 7z = 32$
$x + 3y - 2z = -18$
$2x + 4y - 3z = 12$

$$\begin{bmatrix} 5 & -1 & 7 \\ 1 & 3 & -2 \\ 2 & 4 & -3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 32 \\ -18 \\ 12 \end{bmatrix}$$

9. $4x - 3y - z = -100$
$2x + y - 3z = -64$
$5x + 3y - 2z = 8$

$$\begin{bmatrix} 4 & -3 & -1 \\ 2 & 1 & -3 \\ 5 & 3 & -2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -100 \\ -64 \\ 8 \end{bmatrix}$$

10. $x - 3y + 7z = 27$
$2x + y - 5z = 48$
$4x - 2y + 3z = 72$

$$\begin{bmatrix} 1 & -3 & 7 \\ 2 & 1 & -5 \\ 4 & -2 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 27 \\ 48 \\ 72 \end{bmatrix}$$

11. $2x + 3y - 9z = -108$
$x + 5z = 40 + 2y$
$3x + 5y = 89 + 4z$

$$\begin{bmatrix} 2 & 3 & -9 \\ 1 & -2 & 5 \\ 3 & 5 & -4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -108 \\ 40 \\ 89 \end{bmatrix}$$

12. $z = 45 - 3x + 2y$
$2x + 3y - z = 60$
$x = 4y - 2z + 120$

$$\begin{bmatrix} 3 & -2 & 1 \\ 2 & 3 & -1 \\ 1 & -4 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 45 \\ 60 \\ 120 \end{bmatrix}$$

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4-8 Study Guide and Intervention *(continued)*

Using Matrices to Solve Systems of Equations

Solve Systems of Equations Use inverse matrices to solve systems of equations written as matrix equations.

Solving Matrix Equations	If $AX = B$, then $X = A^{-1}B$, where A is the coefficient matrix, X is the variable matrix, and B is the constant matrix.

Example **Solve** $\begin{bmatrix} 5 & 2 \\ 6 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$.

In the matrix equation $A = \begin{bmatrix} 5 & 2 \\ 6 & 4 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, and $B = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$.

Step 1 Find the inverse of the coefficient matrix.

$$A^{-1} = \frac{1}{20 - 12}\begin{bmatrix} 4 & -2 \\ -6 & 5 \end{bmatrix} \text{ or } \frac{1}{8}\begin{bmatrix} 4 & -2 \\ -6 & 5 \end{bmatrix}.$$

Step 2 Multiply each side of the matrix equation by the inverse matrix.

$$\frac{1}{8}\begin{bmatrix} 4 & -2 \\ -6 & 5 \end{bmatrix} \cdot \begin{bmatrix} 5 & 2 \\ 6 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 4 & -2 \\ -6 & 5 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$ Multiply each side by A^{-1}.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 16 \\ -16 \end{bmatrix}$$ Multiply matrices.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$ Simplify.

The solution is $(2, -2)$.

Exercises

Solve each matrix equation or system of equations by using inverse matrices.

1. $\begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 18 \end{bmatrix}$ **(5, −3)**

2. $\begin{bmatrix} -4 & -8 \\ 6 & 12 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 12 \end{bmatrix}$ **no solution**

3. $\begin{bmatrix} 3 & 2 \\ 5 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -7 \end{bmatrix}$ **(13, −18)**

4. $\begin{bmatrix} 2 & -3 \\ 2 & 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -8 \end{bmatrix}$ $\left(-\frac{1}{4}, -\frac{3}{2}\right)$

5. $\begin{bmatrix} 3 & 6 \\ 5 & 9 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -15 \\ 6 \end{bmatrix}$ **(57, −31)**

6. $\begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -6 \end{bmatrix}$ $\left(-\frac{9}{7}, \frac{15}{7}\right)$

7. $4x - 2y = 22$
   $6x + 4y = -2$
   (3, −5)

8. $2x - y = 2$
   $x + 2y = 46$
   (10, 18)

9. $3x + 4y = 12$
   $5x + 8y = -8$
   (32, −21)

10. $x + 3y = -5$
    $2x + 7y = 8$
    (−59, 18)

11. $5x + 4y = 5$
    $9x - 8y = 0$
    $\left(\frac{10}{19}, \frac{45}{76}\right)$

12. $3x - 2y = 5$
    $x - 4y = 20$
    (−2, −5.5)

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4-8 Skills Practice

Using Matrices to Solve Systems of Equations

Write a matrix equation for each system of equations.

1. $x + y = 5$
   $2x - y = 1$
   $\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$

2. $3a + 8b = 16$
   $4a + 3b = 3$
   $\begin{bmatrix} 3 & 8 \\ 4 & 3 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 16 \\ 3 \end{bmatrix}$

3. $m + 3n = -3$
   $4m + 3n = 6$
   $\begin{bmatrix} 1 & 3 \\ 4 & 3 \end{bmatrix} \cdot \begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} -3 \\ 6 \end{bmatrix}$

4. $2c + 3d = 6$
   $3c - 4d = 7$
   $\begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix} \cdot \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \end{bmatrix}$

5. $r - s = 1$
   $2r + 3s = 12$
   $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} r \\ s \end{bmatrix} = \begin{bmatrix} 1 \\ 12 \end{bmatrix}$

6. $x + y = 5$
   $3x + 2y = 10$
   $\begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$

7. $6x - y + 2z = -4$
   $-3x + 2y - z = 10$
   $x + y + z = 3$
   $\begin{bmatrix} 6 & -1 & 2 \\ -3 & 2 & -1 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ 10 \\ 3 \end{bmatrix}$

8. $a - b + c = 5$
   $3a + 2b - c = 0$
   $2a + 3b = 8$
   $\begin{bmatrix} 1 & -1 & 1 \\ 3 & 2 & -1 \\ 2 & 3 & 0 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 8 \end{bmatrix}$

Solve each matrix equation or system of equations by using inverse matrices.

9. $\begin{bmatrix} 1 & 3 \\ 4 & 3 \end{bmatrix} \cdot \begin{bmatrix} w \\ z \end{bmatrix} = \begin{bmatrix} -7 \\ -1 \end{bmatrix}$ **(2, −3)**

10. $\begin{bmatrix} 4 & 3 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \end{bmatrix}$ **(3, −2)**

11. $\begin{bmatrix} 5 & 8 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -1 \\ 7 \end{bmatrix}$ **(3, −2)**

12. $\begin{bmatrix} 7 & -3 \\ 5 & 4 \end{bmatrix} \cdot \begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} 15 \\ 23 \end{bmatrix}$ **(3, 2)**

13. $\begin{bmatrix} 3 & 12 \\ 2 & -6 \end{bmatrix} \cdot \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 25 \\ 12 \end{bmatrix}$ $\left(7, \frac{1}{3}\right)$

14. $\begin{bmatrix} 5 & 6 \\ 12 & -6 \end{bmatrix} \cdot \begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} 15 \\ 2 \end{bmatrix}$ $\left(1, \frac{5}{3}\right)$

15. $p - 3q = 6$
    $2p + 3q = -6$ **(0, −2)**

16. $-x - 3y = 2$
    $-4x - 5y = 1$ **(1, −1)**

17. $2m + 2n = -8$
    $6m + 4n = -18$ **(−1, −3)**

18. $-3a + b = -9$
    $5a - 2b = 14$ **(4, 3)**

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4-8 Practice

Using Matrices to Solve Systems of Equations

Write a matrix equation for each system of equations.

1. $-3x + 2y = 9$
$5x - 3y = -13$
$\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ -13 \end{bmatrix}$

2. $6x - 2y = -2$
$3x + 3y = 10$
$\begin{bmatrix} 6 & -2 \\ 3 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 10 \end{bmatrix}$

3. $2a + b = 0$
$3a + 2b = -2$
$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$

4. $r + 5s = 10$
$2r - 3s = 7$
$\begin{bmatrix} 1 & 5 \\ 2 & -3 \end{bmatrix} \cdot \begin{bmatrix} r \\ s \end{bmatrix} = \begin{bmatrix} 10 \\ 7 \end{bmatrix}$

5. $3x - 2y + 5z = 3$
$x + y - 4z = 2$
$-2x + 2y + 7z = -5$
$\begin{bmatrix} 3 & -2 & 5 \\ 1 & 1 & -4 \\ -2 & 2 & 7 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$

6. $2m + n - 3p = -5$
$5m + 2n - 2p = 8$
$3m - 3n + 5p = 17$
$\begin{bmatrix} 2 & 1 & -3 \\ 5 & 2 & -2 \\ 3 & -3 & 5 \end{bmatrix} \cdot \begin{bmatrix} m \\ n \\ p \end{bmatrix} = \begin{bmatrix} -5 \\ 8 \\ 17 \end{bmatrix}$

Solve each matrix equation or system of equations by using inverse matrices.

7. $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} g \\ h \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$ **(2, −4)**

8. $\begin{bmatrix} -2 & 3 \\ 1 & 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7 \\ 10 \end{bmatrix}$ **(5, 1)**

9. $\begin{bmatrix} -1 & -3 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 12 \\ -11 \end{bmatrix}$ **(3, −5)**

10. $\begin{bmatrix} -5 & 3 \\ 6 & 4 \end{bmatrix} \cdot \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 16 \\ 34 \end{bmatrix}$ **(1, 7)**

11. $\begin{bmatrix} -4 & 2 \\ 7 & 4 \end{bmatrix} \cdot \begin{bmatrix} r \\ s \end{bmatrix} = \begin{bmatrix} 17 \\ -26 \end{bmatrix}$ $\left(-4, \frac{1}{2}\right)$

12. $\begin{bmatrix} 8 & 3 \\ 12 & 6 \end{bmatrix} \cdot \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$ $\left(-\frac{1}{4}, \frac{1}{3}\right)$

13. $2x + 3y = 5$
$3x - 2y = 1$ **(1, 1)**

14. $8d + 9f = 13$
$-6d + 5f = -45$ **(5, −3)**

15. $5m + 9n = 19$
$2m - n = -20$ **(−7, 6)**

16. $-4j + 9k = -8$
$6j + 12k = -5$ $\left(\frac{1}{2}, -\frac{2}{3}\right)$

17. AIRLINE TICKETS Last Monday at 7:30 A.M., an airline flew 89 passengers on a commuter flight from Boston to New York. Some of the passengers paid \$120 for their tickets and the rest paid \$230 for their tickets. The total cost of all of the tickets was \$14,200. How many passengers bought \$120 tickets? How many bought \$230 tickets?
57; 32

18. NUTRITION A single dose of a dietary supplement contains 0.2 gram of calcium and 0.2 gram of vitamin C. A single dose of a second dietary supplement contains 0.1 gram of calcium and 0.4 gram of vitamin C. If a person wants to take 0.6 gram of calcium and 1.2 grams of vitamin C, how many doses of each supplement should she take?
2 doses of the first supplement and 2 doses of the second supplement

NAME ______________ DATE ________ PERIOD _____

4-8 Word Problem Practice

Using Matrices to Solve Systems of Equations

1. TEACHING Paula is explaining matrices to her father. She writes down the following system of equations.

$$2x + y = 4$$
$$3x + y = 5.$$

Next, Paula shows her father the matrices that correspond to this system of equations. What are the matrices?
$\begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

2. FIND THE ERROR Paula proceeds to solve the matrix equation

$$\begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}.$$

First, she finds the inverse.

$$\begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$$

Then she computes the answer.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 11 \\ -6 \end{bmatrix}$$

When she checked her answer, she found that it was not correct. Where did she make a mistake?
The inverse should be $\begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix}$

3. AGES Hank, Laura, and Ned are ages h, l, and n, respectively. The sum of their ages is 15 years. Laura is one year younger than the sum of Hank and Ned's ages. Ned is three times as old as Hank. Use matrices to determine the age of each sibling.
Hank is 2, Laura is 7, and Ned is 6.

4. ANIMALS Quinton takes care of dogs and chickens. There are a total of 28 animals, and altogether they have 68 legs. Use matrices to determine the number of dogs and the number of chickens in Quinton's care.
dogs, 6; chickens, 22

SALES For Exercises 5 and 6, use the following information.
The school film society is selling only granola bars and oranges to raise money at their movie review. They sell oranges for \$1 and granola bars for \$1.50. The person selling snacks recorded the total cost and number of items in each sale. The manager wants to know how many of each kind of snack each person bought.

5. Suppose a person spent d dollars to buy n items. Write a system of linear equations that relate d and n to the number of oranges r and granola bars g that the person purchased.
$h + k = n$
$h + 1.5k = d$

6. One recorded sale showed that 10 items were purchased for \$13.00. How many oranges and granola bars were purchased for this sale?
4 granola bars and 6 oranges

Lesson 4-8

Answers

NAME ______________________________ DATE ____________ PERIOD _____

4-8 Enrichment

Determining Political Popularity

Systems of equations have applications in branches of science including chemistry, ecology, and physics. They can also be used to describe situations involving social studies and politics.

Consider the two candidates for City Council, Jefferson Dailey and Robert Jackson. Support for a candidate is measured by a positive number less than 1 and opposition of a candidate by a negative number greater than -1. For example, 0.75 indicates fairly high support, while -0.75 means fairly high opposition. Simultaneous support for, or opposition to, both candidates is possible. Generally, however, if one candidate is popular and is supported while the other candidate is opposed, support of the popular candidate tends to decrease as support for the "underdog" rises. Let the change in support for Jefferson Dailey be denoted by ΔJ(delta J) and the change in support for Robert Jackson is denoted by ΔR (delta R).

This situation is described by the system of equations:

$$\begin{cases} \Delta J = -0.5J + 0.25R \\ \Delta R = -0.25J - 0.5R \end{cases}$$

For example, if $\Delta J = -0.2$ and $\Delta R = 0.2$, then current support for Jefferson Daily is decreasing at a rate of 20% while Robert Jackson's support is increasing at 20%. Substituting the given values for ΔJ and ΔR and solving the first equation for R yields:

$$R = \frac{0.5J - 0.2}{0.25}$$

Substituting this expression for R in the second equation and solving for J yields:

$$0.2 = 0.25J - 0.5\left(\frac{0.5J - 0.2}{0.25}\right) \Rightarrow J = 0.267$$

Therefore, $R = -0.267$.

If the election were held today, Jefferson Daily would win.

Solve the systems of equations for the following values of ΔJ and ΔR to determine potential winners and losers.

1. $\Delta J = -0.1$, $\Delta R = 0.2$
$J = 0$, $R = -0.4$. Jefferson Daily is "winning".

2. $\Delta J = 0.5$, $\Delta R = -0.1$
$(-1.2, -0.4)$. Neither is favored, but Robert Jackson is ahead.

Chapter 4 Assessment Answer Key

Quiz 1 (Lessons 4-1 through 4-2)
Page 65

1. $\begin{array}{l} \\ \text{Trista} \\ \text{Javier} \\ \text{Yolanda} \end{array}\begin{bmatrix} \text{First Exam} & \text{Second Exam} \\ 80 & 95 \\ 85 & 90 \\ 75 & 90 \end{bmatrix}$

2. (3, –1)

3. 3×4

4. $\begin{bmatrix} 2 & -2 & -3 \\ 22 & -11 & 7 \end{bmatrix}$

5. $\begin{bmatrix} 9 & 31 & -31 \\ -6 & -22 & 24 \end{bmatrix}$

Quiz 2 (Lessons 4-3 and 4-4)
Page 65

1. yes; 5×4

2. $\begin{bmatrix} 13 & -32 \\ 20 & 26 \end{bmatrix}$

3. $\begin{bmatrix} -39 & -12 \\ -24 & 51 \end{bmatrix}$; $\begin{bmatrix} 9 & 48 \\ 48 & 3 \end{bmatrix}$; not true

4. B

5. $X'(3, 2)$, $Y'(2, -3)$, $Z'(-5, 4)$

Quiz 3 (Lessons 4-5 and 4-6)
Page 66

1. 7

2. 52

3. 24 units^2

4. (–1, 7)

5. (–2, 3, 1)

Quiz 4 (Lessons 4-7 and 4-8)
Page 66

1. yes

2. no inverse exists

3. $\begin{bmatrix} 1 & -3 \\ 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 15 \\ -13 \end{bmatrix}$

4. $\begin{bmatrix} 2 & -3 & 4 \\ 3 & 0 & 1 \\ 1 & -4 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -20 \\ 2 \\ -6 \end{bmatrix}$

5. (4, 5)

Mid-Chapter Test
Page 67

1. C

2. F

3. A

4. G

5. D

6. $\begin{bmatrix} 7 & 6 & 13 & 12 \\ 4 & -4 & -8 & 2 \end{bmatrix}$

7. $\begin{bmatrix} -9 & 13 & 16 & -11 \\ -13 & 1 & 1 & -15 \end{bmatrix}$

8. $\begin{bmatrix} -12 & 20 & -48 \\ -36 & -44 & 28 \\ 8 & -16 & -24 \end{bmatrix}$

9. $[-17 \ -48 \ -27 \ 30]$

10. $\begin{bmatrix} -4 & 8 \\ 16 & -7 \end{bmatrix}$

11. impossible

Answers

Chapter 4 Assessment Answer Key

Vocabulary Test
Page68

1. Cramer's Rule
2. determinant
3. element
4. matrix
5. reflection
6. Scalar multiplication
7. rotation
8. inverse
9. translation
10. dilation
11. Sample answer: An identity matrix is a square matrix in which the elements on the main diagonal are all ones and all the other elements are zeroes.
12. Sample answer: When two matrices have the same dimensions and each element of one matrix is equal to the corresponding element of the other matrix.

Form 1
Page 69

1. C
2. F
3. C
4. J
5. C
6. F
7. C
8. F
9. B
10. G
11. B
12. F

Page 70

13. A
14. H
15. A
16. G
17. D
18. F
19. C
20. G

B: $c^2 - ab$

Chapter 4 Assessment Answer Key

Form 2A

Page 71

1. D
2. H
3. D
4. F
5. C
6. G
7. C
8. G
9. C
10. F
11. D
12. F

Page 72

13. D
14. H
15. A
16. H
17. C
18. G
19. C
20. F

B: 0

Form 2B

Page 73

1. C
2. J
3. A
4. J
5. B
6. F
7. C
8. F
9. D
10. J
11. B
12. H

Page 74

13. B
14. G
15. C
16. J
17. A
18. H
19. A
20. G

B: 0

Answers

Chapter 4 Assessment Answer Key

Form 2C

Page 75

1. $\begin{array}{r|cc} & \text{Resident} & \text{Non-Resident} \\ \text{Weekday A.M.} & 20 & 25 \\ \text{Weekday P.M.} & 18 & 22 \\ \text{Weekend} & 40 & 40 \end{array}$

2. (3, −4)

3. $\begin{bmatrix} -4 & 8 & 7 & 3 \\ -8 & -3 & -8 & 5 \end{bmatrix}$

4. impossible

5. [18 −12 22 48]

6. $\begin{bmatrix} 21 \\ -7 \\ -35 \end{bmatrix}$

7. yes; 3 × 9

8. $\begin{bmatrix} 34 & -28 & 10 \\ 8 & 4 & 32 \end{bmatrix}$

9. $\begin{bmatrix} -56 & -42 \\ -28 & -60 \end{bmatrix}$; $\begin{bmatrix} -56 & -42 \\ -28 & -60 \end{bmatrix}$; true

10. $A'(-1, -5)$, $B'(5, -7)$, $C'(2, 2)$

Page 76

11. $R'(4, -2)$, $S'(0, -2)$, $T'(-6, 1)$, $U'(-2, 1)$

12. 29

13. 174

14. 50 units^2

15. (3.5, −2)

16. (1, −2, 5)

17. yes

18. $\frac{1}{12}\begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix}$

19. $\left(\frac{1}{2}, \frac{3}{2}\right)$

20. $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -2 \\ 1 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} c \\ t \\ v \end{bmatrix} = \begin{bmatrix} 366 \\ 0 \\ 186 \end{bmatrix}$

B: $-2xy$

Chapter 4 Assessment Answer Key

Form 2D

Page 77

1. $\begin{array}{lc} & \begin{array}{cc}\text{Bleachers} & \text{Box Seats}\end{array} \\ \text{Weekday} & \\ \text{Weekend} & \\ \text{Double-Header} & \end{array}$ $\begin{bmatrix} 9 & 21 \\ 12 & 26 \\ 27 & 27 \end{bmatrix}$

2. (−2, 4)

3. $\begin{bmatrix} 3 & -3 & 10 & 12 \\ 11 & -3 & 4 & 5 \end{bmatrix}$

4. impossible

5. $[-33 \;\; -3 \;\; 111 \;\; 12]$

6. $\begin{bmatrix} 33 \\ -32 \\ -33 \end{bmatrix}$

7. yes; 2×7

8. $\begin{bmatrix} 14 & -1 & -8 \\ -57 & -30 & -33 \end{bmatrix}$

9. $\begin{bmatrix} -124 & 38 \\ 102 & -74 \end{bmatrix}$; $\begin{bmatrix} -124 & 38 \\ 102 & -74 \end{bmatrix}$; true

10. $A'(2, -3)$, $B'(0, 7)$, $C'(-5, 3)$

Page 78

11. $R'(-3, 5)$; $S'(4, 5)$; $T'(6, -4)$; $U'(-1, -4)$

12. −8

13. −128

14. 41 units^2

15. (8, −1.5)

16. (3, 0, −2)

17. no

18. $\frac{1}{2}\begin{bmatrix} -1 & 2 \\ -3 & 4 \end{bmatrix}$

19. $\left(-\frac{1}{2}, \frac{1}{2}\right)$

20. $\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ 1 & 0 & -8 \end{bmatrix} \cdot \begin{bmatrix} f \\ t \\ s \end{bmatrix} = \begin{bmatrix} 1085 \\ 15 \\ 0 \end{bmatrix}$

B: $2ac$

Answers

Chapter 4 Assessment Answer Key

Form 3

Page 79

1. $\begin{array}{r c} & \begin{array}{cc}\text{Evening} & \text{Matinee}\end{array} \\ \begin{array}{r}\text{Fri.}\\ \text{Sat.}\end{array} & \begin{bmatrix} 10 & 0 \\ 10 & 7 \end{bmatrix}\end{array}$

2. $(-2, 5)$

3. $\begin{bmatrix} 3.32 & -3.12 \\ 5.28 & 7.38 \\ -2.50 & 1.24 \end{bmatrix}$

4. $\begin{bmatrix} 0 & -\frac{1}{2} \\ -\frac{5}{4} & \frac{3}{4} \end{bmatrix}$

5. $\begin{bmatrix} -\frac{13}{2} & 12 & \frac{5}{4} \\ 11 & -\frac{22}{5} & 6 \end{bmatrix}$

6. $\begin{bmatrix} -1 & \frac{5}{2} & \frac{1}{2} \\ 24 & 4 & 0 \\ -2 & 5 & 1 \end{bmatrix}$

7. $\begin{bmatrix} 0 & -4 \\ 2 & -1 \end{bmatrix}$; $\begin{bmatrix} 0 & -4 \\ 2 & -1 \end{bmatrix}$; true

8. $X'(8, 0)$; $Y'(15, -5)$; $Z'(7, -7)$

9. $R(-1, -4)$; $S(8, 1)$; $T(-4, 6)$

10. -47.09

11. $-\frac{9}{5}$

Page 80

12. 266

13. $\frac{45}{8}$ units^2

14. $\left(-\frac{1}{2}, 5\right)$

15. $(13, -9, 2)$

16. yes

17. $5\begin{bmatrix} -\frac{1}{3} & -\frac{2}{3} \\ \frac{2}{5} & \frac{1}{5} \end{bmatrix}$

18. $\left(-\frac{1}{3}, \frac{1}{2}\right)$

19. infinitely many solutions

20. $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} a \\ r \\ l \end{bmatrix} = \begin{bmatrix} 100 \\ 78.4 \\ 5.1 \end{bmatrix}$

B: $-2a + 2c$

Chapter 4 Assessment Answer Key

Page 81, Extended-Response Test
Scoring Rubric

Score	General Description	Specific Criteria
4	**Superior** A correct solution that is supported by well-developed, accurate explanations	• Shows thorough understanding of the concepts of *adding, subtracting, and multiplying matrices; multiplying by a scalar; transforming geometric figures using matrices; evaluating determinants; solving systems of equations using Cramer's Rule; identifying and finding inverse matrices;* and *using inverse matrices to solve matrix equations.* • Uses appropriate strategies to solve problems. • Computations are correct. • Written explanations are exemplary. • Goes beyond requirements of some or all problems.
3	**Satisfactory** A generally correct solution, but may contain minor flaws in reasoning or computation	• Shows an understanding of the concepts of *adding, subtracting, and multiplying matrices; multiplying by a scalar; transforming geometric figures using matrices; evaluating determinants; solving systems of equations using Cramer's Rule; identifying and finding inverse matrices;* and *using inverse matrices to solve matrix equations.* • Uses appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are effective. • Satisfies all requirements of problems.
2	**Nearly Satisfactory** A partially correct interpretation and/or solution to the problem	• Shows an understanding of most of the concepts of *adding, subtracting, and multiplying matrices; multiplying by a scalar; transforming geometric figures using matrices; evaluating determinants; solving systems of equations using Cramer's Rule; identifying and finding inverse matrices;* and *using inverse matrices to solve matrix equations.* • May not use appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are satisfactory. • Satisfies the requirements of most of the problems.
1	**Nearly Unsatisfactory** A correct solution with no supporting evidence or explanation	• Final computation is correct. • No written explanations or work is shown to substantiate the final computation. • Satisfies minimal requirements of some of the problems.
0	**Unsatisfactory** An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given	• Shows little or no understanding of most of the concepts of *adding, subtracting, and multiplying matrices; multiplying by a scalar; transforming geometric figures using matrices; evaluating determinants; solving systems of equations using Cramer's Rule; identifying and finding inverse matrices;* and *using inverse matrices to solve matrix equations.* • Does not use appropriate strategies to solve problems. • Computations are incorrect. • Written explanations are unsatisfactory. • Does not satisfy requirements of problems. • No answer may be given.

Answers

Chapter 4 Assessment Answer Key

Page 81, Extended-Response Test
Sample Answers

In addition to the scoring rubric found on page 64, the following sample answers may be used as guidance in evaluating open-ended assessment items.

1. Student responses should include:

1a. A and B must have the same dimensions, so $m = j$ and $n = k$.

1b. Inner dimensions must be equal. Thus, $n = j$ for AB and $k = m$ for BA.

1c. The determinant of A exists only if A is a square matrix, so $m = n$.

1d. Matrix size for scalar multiplication is irrelevant, so there are no restrictions on j and k.

1e. Only square matrices (potentially) have inverses, so $m = n$ if A has an inverse.

2a. Student matrices must be of the form $V = \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix}$, where $P_1(x_1, y_1)$, $P_2(x_2, y_2)$, and $P_3(x_3, y_3)$ are the points, no two in the same quadrant, which the student has chosen. All students should state that V is a 2×3 matrix.

2b. Student matrices must be of the form $T + V$, which represents a translation of the original triangle 3 units right and 4 units down.

$T + V = \begin{bmatrix} x_1 + 3 & x_2 + 3 & x_3 + 3 \\ y_1 - 4 & y_2 - 4 & y_3 - 4 \end{bmatrix}$

2c. Student matrices must be of the form $3V$, which represents a dilation of the original triangle which triples the length of each side.

$3V = \begin{bmatrix} 3x_1 & 3x_2 & 3x_3 \\ 3y_1 & 3y_2 & 3y_3 \end{bmatrix}$

2d. Student matrices must be of the form RV, which represents a reflection of the original triangle over the y-axis.

$RV = \begin{bmatrix} -x_1 & -x_2 & -x_3 \\ y_1 & y_2 & y_3 \end{bmatrix}$

2e. Student matrices must be of the form CV, which represents a 90° counterclockwise rotation of the original triangle about the origin.

$CV = \begin{bmatrix} -y_1 & -y_2 & -y_3 \\ x_1 & x_2 & x_3 \end{bmatrix}$

2f. Students should indicate that I is the identity matrix, so that $IV = V$. This means that the image and the preimage of the triangle are exactly the same in all respects.

3a. Sample answer: Expansion using the bottom (3rd) row would be easiest since the elements are 1, 0 and -1.

3b. Student responses should indicate the reason for choosing one method over the other.

3c. Students should correctly apply the chosen method to arrive at the solution $a = -5$.

4a. $3c + 2v = 85$
$2c + v = 50$
(Variables may vary); c: the cost of one CD, v: the cost of one video

4b. Students should correctly apply Cramer's Rule to determine that $c = 15$ and $v = 20$, meaning that the cost of one CD is \$15 and the cost of one video is \$20.

4c. $\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} c \\ v \end{bmatrix} = \begin{bmatrix} 85 \\ 50 \end{bmatrix}$; Students should demonstrate the correct method of solving the equation using inverse matrices, and again conclude that $c = 15$ and $v = 20$.

4d. Student responses should indicate the reason for choosing one method over the other.

Chapter 4 Assessment Answer Key

Standardized Test Practice

Page 82

1. A B C D

2. F G H J

3. A B C D

4. F G H J

5. A B C D

6. F G H J

7. A B C D

8. F G H J

9. A B C D

10. F G H J

Page 83

11. A B C D

12. F G H J

13. A B C D

14. F G H J

15. A B C D

16.

		1	5	.			
0	0	0	0		0	0	0
1	1	1	1		1	1	1
2	2	2	2		2	2	2
3	3	3	3		3	3	3
4	4	4	4		4	4	4
5	5	5	5		5	5	5
6	6	6	6		6	6	6
7	7	7	7		7	7	7
8	8	8	8		8	8	8
9	9	9	9		9	9	9

17.

		5	4	.			
0	0	0	0		0	0	0
1	1	1	1		1	1	1
2	2	2	2		2	2	2
3	3	3	3		3	3	3
4	4	4	4		4	4	4
5	5	5	5		5	5	5
6	6	6	6		6	6	6
7	7	7	7		7	7	7
8	8	8	8		8	8	8
9	9	9	9		9	9	9

Answers

Chapter 4 Assessment Answer Key

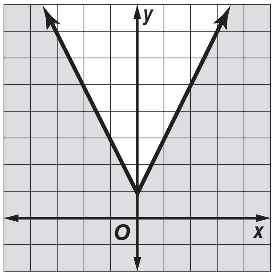
Standardized Test Practice *(continued)*
Page 84

18. 5050

19. 5.9

20. $\frac{4}{5}$

21. D = all real numbers
{R = $y | y \geq -4$}

22. 

23. (1, 2)

24. yes

25. $(-2, 4)$

26. impossible

27. $\begin{bmatrix} -12 & 0 & -44 \\ 36 & -8 & -24 \\ -16 & 12 & 20 \end{bmatrix}$

28. 1

29a. Sample answer:
$a + b = 37; b = a + 3$

29b. $\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 37 \\ 3 \end{bmatrix}$

29c. $a = 17$, $b = 20$

Chapter 4 Assessment Answer Key

Unit 1 Test

Page 85

1. 7

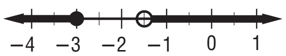
2. N, W, Z, Q, R

3. 28

4. 3

5. g = the number of additional games to be won;
$\dfrac{g + 9}{20} \geq 0.75$;
at least six games

6. $\left\{x \mid x \leq -3 \text{ or } x > -\dfrac{1}{2}\right\}$



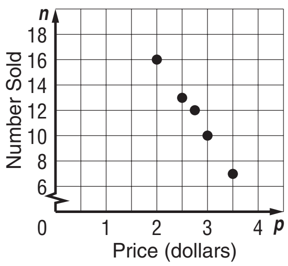
7. all real numbers

8. 19

9. $y = \frac{4}{5}x + \frac{13}{5}$

10. $y = -2x + 7$

11.



12. Sample answer using (2, 16) and (3, 10):
$n = -6p + 28$; 1

13. −3

14. inconsistent

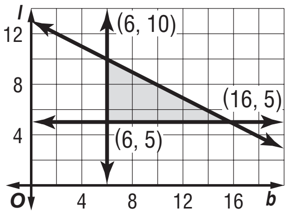
Page 86

15. (−1, −3)

16. (2, 1)

17. $b \geq 6$; $l \geq 5$;
$21b + 42l \leq 546$

18.



19. 16 bedroom sets
5 living room sets

20. $\begin{bmatrix} 7 & -4 & 10 \\ 15 & 1 & -9 \end{bmatrix}$

21. $\begin{bmatrix} 16 \\ 10 \end{bmatrix}$

22. $D'(5, 3)$; $E'(4, -8)$; $F'(-2, 1)$

23. −19

24. (2, −3)

25. (3, −4)