Glencoe Mathematics

Algebra 2

Chapter 7 Resource Masters

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Consumable Workbooks Many of the worksheets contained in the Chapter Resource Masters are available as consumable workbooks in both English and Spanish.

	ISBN10	ISBN13
Study Guide and Intervention Workbook	0-07-877355-5	978-0-07-877355-6
Skills Practice Workbook	0-07-877357-1	978-0-07-877357-0
Practice Workbook	0-07-877358-X	978-0-07-877358-7
Word Problem Practice Workbook	0-07-877360-1	978-0-07-877360-0
Spanish Versions		
Study Guide and Intervention Workbook	0-07-877356-3	978-0-07-877356-3
Practice Workbook	0-07-877359-8	978-0-07-877359-4

Answers for Workbooks The answers for Chapter 7 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

StudentWorks Plus™ This CD-ROM includes the entire Student Edition test along with the English workbooks listed above.

TeacherWorks Plus™ All of the materials found in this booklet are included for viewing, printing, and editing in this CD-ROM.

Spanish Assessment Masters (ISBN10: 0-07-0-07-877361-X, ISBN13: 978-0-07-877361-7) These masters contain a Spanish version of Chapter 7 Test Form 2A and Form 2C.

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Contents

Teacher's Guide to Using the *Chapter 7 Resource Masters*

The *Chapter 7 Resource Masters* includes the core materials needed for Chapter 7. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing on the *TeacherWorks Plus*™ CD-ROM.

Chapter Resources

Student-Built Glossary (pages 1–2) These masters are a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to recording definitions and/or examples for each term. You may suggest that student highlight or star the terms with which they are not familiar. Give to students before beginning Lesson 7-1. Encourage them to add these pages to their mathematics study notebooks. Remind them to complete the appropriate words as they study each lesson.

Anticipation Guide (pages 3–4) This master presented in both English and Spanish is a survey used before beginning the chapter to pinpoint what students may or may not know about the concepts in the chapter. Students will revisit this survey after they complete the chapter to see if their perceptions have changed.

Lesson Resources

Lesson Reading Guide Get Ready for the Lesson reiterates the questions from the beginning of the Student Edition lesson. Read the Lesson asks students to interpret the context of and relationships among terms in the lesson. Finally, Remember What You Learned asks students to summarize what they have learned using various representation techniques. Use as a study tool for note taking or as an informal reading assignment. It is also a helpful tool for ELL (English Language Learners).

Study Guide and Intervention These masters provide vocabulary, key concepts, additional worked-out examples and Check Your Progress exercises to use as a reteaching activity. It can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice This master focuses more on the computational nature of the lesson. Use as an additional practice option or as homework for second-day teaching of the lesson.

Practice This master closely follows the types of problems found in the Exercises section of the Student Edition and includes word problems. Use as an additional practice option or as homework for second-day teaching of the lesson.

Word Problem Practice This master includes additional practice in solving word problems that apply the concepts of the lesson. Use as an additional practice or as homework for second-day teaching of the lesson.

Enrichment These activities may extend the concepts of the lesson, offer an historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. They are written for use with all levels of students.

Graphing Calculator, Scientific Calculator,* or *Spreadsheet Activities
These activities present ways in which technology can be used with the concepts in some lessons of this chapter. Use as an alternative approach to some concepts or as an integral part of your lesson presentation.

Assessment Options

The assessment masters in the *Chapter 7 Resource Masters* offer a wide range of assessment tools for formative (monitoring) assessment and summative (final) assessment.

Student Recording Sheet This master corresponds with the standardized test practice at the end of the chapter.

Pre-AP Rubric This master provides information for teachers and students on how to assess performance on open-ended questions.

Quizzes Four free-response quizzes offer assessment at appropriate intervals in the chapter.

Mid-Chapter Test This 1-page test provides an option to assess the first half of the chapter. It parallels the timing of the Mid-Chapter Quiz in the Student Edition and includes both multiple-choice and free-response questions.

Vocabulary Test This test is suitable for all students. It includes a list of vocabulary words and 10 questions to assess students' knowledge of those words. This can also be used in conjunction with one of the leveled chapter tests.

Leveled Chapter Tests

- *Form 1* contains multiple-choice questions and is intended for use with below grade level students.
- *Forms 2A and 2B* contain multiple-choice questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- *Forms 2C and 2D* contain free-response questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- *Form 3* is a free-response test for use with above grade level students.

All of the above mentioned tests include a free-response Bonus question.

Extended-Response Test Performance assessment tasks are suitable for all students. Samples answers and a scoring rubric are included for evaluation.

Standardized Test Practice These three pages are cumulative in nature. It includes three parts: multiple-choice questions with bubble-in answer format, griddable questions with answer grids, and short-answer free-response questions.

Answers

- The answers for the Anticipation Guide and Lesson Resources are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment masters.

7 Student-Built Glossary

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 7. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Algebra Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
composite of functions		
conjugates		
extraneous solution		
identity function		
inverse function		
inverse relation		
like radical expressions		

(continued on the next page)

7 Student-Built Glossary *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
nth root		
one-to-one		
principal root		
radical equation		
radical inequality		
rationalizing the denominator		
square root function		
square root inequality		

7 Anticipation Guide

Radical Equations

STEP 1 ***Before you begin Chapter 7***

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

STEP 1 A, D, or NS	Statement	STEP 2 A or D
	1. Functions can be added or subtracted in the same way as polynomials.	
	2. A composition of functions, $f[g(x)]$, is found by multiplying $f(x)$ by $g(x)$.	
	3. The inverse of a function is the set of ordered pairs obtained by taking the opposite of each coordinate in the original ordered pairs.	
	4. Two functions are inverses of each other only if their compositions are the identity function.	
	5. The domain of $y = \sqrt{(x - 3)}$ would be $x \geq 3$.	
	6. The principal root of any nth root is always positive.	
	7. The radical expression $\sqrt{\frac{m}{m^3}}$ is in simplest form.	
	8. $4 + \sqrt{3}$ and $4 - \sqrt{3}$ are conjugates of each other.	
	9. $5^{\frac{2}{3}}$ is the same as $\sqrt[2]{5^3}$.	
	10. To solve an equation containing the square root of the variable, square both sides of the equation.	

STEP 2 ***After you complete Chapter 7***

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

7

Ejercicios preparatorios

Ecuaciones radicales

PASO 1 ***Antes de comenzar el Capítulo 7***

- Lee cada enunciado.
- Decide si estás de acuerdo (A) o en desacuerdo (D) con el enunciado.
- Escribe A o D en la primera columna O si no estás seguro(a) de la respuesta, escribe NS (No estoy seguro(a).

PASO 1 A, D o NS	Enunciado	PASO 2 A o D
	1. Las funciones se pueden sumar o restar de la misma manera que los polinomios.	
	2. Una composición de funciones, $f[g(x)]$, se encuentra al multiplicar $f(x)$ por $g(x)$.	
	3. El inverso de una función es el conjunto de pares ordenados que se obtienen al tomar el opuesto de cada coordenada en los pares ordenados originales.	
	4. Dos funciones son inversas entre sí sólo si sus composiciones son la función identidad.	
	5. El dominio de $y = \sqrt{(x - 3)}$ sería $x \geq 3$.	
	6. La raíz principal de cualquier enésima raíz es siempre positiva.	
	7. La expresión radical $\sqrt{\frac{m}{m^3}}$ está en forma reducida.	
	8. $4 + \sqrt{3}$ y $4 - \sqrt{3}$ son conjugados entre sí.	
	9. $5^{\frac{2}{3}}$ es lo mismo que $\sqrt[2]{5^3}$.	
	10. Para resolver una ecuación que contiene la raíz cuadrada de la variable, eleva al cuadrado ambos lados de la ecuación.	

PASO 2 ***Después de completar el Capítulo 7***

- Vuelve a leer cada enunciado y completa la última columna con una A o una D.
- ¿Cambió cualquiera de tus opiniones sobre los enunciados de la primera columna?
- En una hoja de papel aparte, escribe un ejemplo de por qué estás en desacuerdo con los enunciados que marcaste con una D.

7-1 Lesson Reading Guide

Operations on Functions

Get Ready for the Lesson

Read the introduction to Lesson 7-1 in your textbook.

Describe two ways to calculate Ms. Coffmon's profit from the sale of 50 birdhouses. (Do not actually calculate her profit.)

Read the Lesson

1. Determine whether each statement is *true* or *false*. (Remember that *true* means *always* true.)

   a. If f and g are polynomial functions, then $f + g$ is a polynomial function.

   b. If f and g are polynomial functions, then $\frac{f}{g}$ is a polynomial function.

   c. If f and g are polynomial functions, the domain of the function $f \cdot g$ is the set of all real numbers.

   d. If $f(x) = 3x + 2$ and $g(x) = x - 4$, the domain of the function $\frac{f}{g}$ is the set of all real numbers.

   e. If f and g are polynomial functions, then $(f \circ g)(x) = (g \circ f)(x)$.

   f. If f and g are polynomial functions, then $(f \cdot g)(x) = (g \cdot f)(x)$.

2. Let $f(x) = 2x - 5$ and $g(x) = x^2 + 1$.

   a. Explain in words how you would find $(f \circ g)(-3)$. (Do not actually do any calculations.)

   b. Explain in words how you would find $(g \circ f)(-3)$. (Do not actually do any calculations.)

Remember What You Learned

3. Some students have trouble remembering the correct order in which to apply the two original functions when evaluating a composite function. Write three sentences, each of which explains how to do this in a slightly different way. (Hint: Use the word *closest* in the first sentence, the words *inside* and *outside* in the second, and the words *left* and *right* in the third.)

7-1 Study Guide and Intervention

Operations on Functions

Arithmetic Operations

Operations with Functions	Sum	$(f + g)(x) = f(x) + g(x)$
	Difference	$(f - g)(x) = f(x) - g(x)$
	Product	$(f \cdot g)(x) = f(x) \cdot g(x)$
	Quotient	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$

Example **Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for $f(x) = x^2 + 3x - 4$ and $g(x) = 3x - 2$.**

$(f + g)(x) = f(x) + g(x)$ Addition of functions
$= (x^2 + 3x - 4) + (3x - 2)$ $f(x) = x^2 + 3x - 4$, $g(x) = 3x - 2$
$= x^2 + 6x - 6$ Simplify.

$(f - g)(x) = f(x) - g(x)$ Subtraction of functions
$= (x^2 + 3x - 4) - (3x - 2)$ $f(x) = x^2 + 3x - 4$, $g(x) = 3x - 2$
$= x^2 - 2$ Simplify.

$(f \cdot g)(x) = f(x) \cdot g(x)$ Multiplication of functions
$= (x^2 + 3x - 4)(3x - 2)$ $f(x) = x^2 + 3x - 4$, $g(x) = 3x - 2$
$= x^2(3x - 2) + 3x(3x - 2) - 4(3x - 2)$ Distributive Property
$= 3x^3 - 2x^2 + 9x^2 - 6x - 12x + 8$ Distributive Property
$= 3x^3 + 7x^2 - 18x + 8$ Simplify.

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ Division of functions
$= \frac{x^2 + 3x - 4}{3x - 2}, x \neq \frac{2}{3}$ $f(x) = x^2 + 3x - 4$ and $g(x) = 3x - 2$

Exercises

Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$.

1. $f(x) = 8x - 3; g(x) = 4x + 5$

2. $f(x) = x^2 + x - 6; g(x) = x - 2$

3. $f(x) = 3x^2 - x + 5; g(x) = 2x - 3$

4. $f(x) = 2x - 1; g(x) = 3x^2 + 11x - 4$

5. $f(x) = x^2 - 1; g(x) = \frac{1}{x + 1}$

7-1 Study Guide and Intervention *(continued)*

Operations on Functions

Composition of Functions

Composition of Functions	Suppose f and g are functions such that the range of g is a subset of the domain of f. Then the composite function $f \circ g$ can be described by the equation $[f \circ g](x) = f[g(x)]$.

Example 1 **For $f = \{(1, 2), (3, 3), (2, 4), (4, 1)\}$ and $g = \{(1, 3), (3, 4), (2, 2), (4, 1)\}$, find $f \circ g$ and $g \circ f$ if they exist.**

$f[g(1)] = f(3) = 3$ $\quad$ $f[g(2)] = f(2) = 4$ $\quad$ $f[g(3)] = f(4) = 1$ $\quad$ $f[g(4)] = f(1) = 2$

$f \circ g = \{(1, 3), (2, 4), (3, 1), (4, 2)\}$

$g[f(1)] = g(2) = 2$ $\quad$ $g[f(2)] = g(4) = 1$ $\quad$ $g[f(3)] = g(3) = 4$ $\quad$ $g[f(4)] = g(1) = 3$

$g \circ f = \{(1, 2), (2, 1), (3, 4), (4, 3)\}$

Example 2 **Find $[g \circ h](x)$ and $[h \circ g](x)$ for $g(x) = 3x - 4$ and $h(x) = x^2 - 1$.**

$$\begin{aligned}[g \circ h](x) &= g[h(x)]\\ &= g(x^2 - 1)\\ &= 3(x^2 - 1) - 4\\ &= 3x^2 - 7\end{aligned}$$

$$\begin{aligned}[h \circ g](x) &= h[g(x)]\\ &= h(3x - 4)\\ &= (3x - 4)^2 - 1\\ &= 9x^2 - 24x + 16 - 1\\ &= 9x^2 - 24x + 15\end{aligned}$$

Exercises

For each set of ordered pairs, find $f \circ g$ and $g \circ f$ if they exist.

1. $f = \{(-1, 2), (5, 6), (0, 9)\}$,
$g = \{(6, 0), (2, -1), (9, 5)\}$

2. $f = \{(5, -2), (9, 8), (-4, 3), (0, 4)\}$,
$g = \{(3, 7), (-2, 6), (4, -2), (8, 10)\}$

Find $[f \circ g](x)$ and $[g \circ f](x)$.

3. $f(x) = 2x + 7$; $g(x) = -5x - 1$

4. $f(x) = x^2 - 1$; $g(x) = -4x^2$

5. $f(x) = x^2 + 2x$; $g(x) = x - 9$

6. $f(x) = 5x + 4$; $g(x) = 3 - x$

Lesson 7-1

7-1 Skills Practice

Operations on Functions

Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$.

1. $f(x) = x + 5$
$g(x) = x - 4$

2. $f(x) = 3x + 1$
$g(x) = 2x - 3$

3. $f(x) = x^2$
$g(x) = 4 - x$

4. $f(x) = 3x^2$
$g(x) = \frac{5}{x}$

For each set of ordered pairs, find $f \circ g$ and $g \circ f$ if they exist.

5. $f = \{(0, 0), (4, -2)\}$
$g = \{(0, 4), (-2, 0), (5, 0)\}$

6. $f = \{(0, -3), (1, 2), (2, 2)\}$
$g = \{(-3, 1), (2, 0)\}$

7. $f = \{(-4, 3), (-1, 1), (2, 2)\}$
$g = \{(1, -4), (2, -1), (3, -1)\}$

8. $f = \{(6, 6), (-3, -3), (1, 3)\}$
$g = \{(-3, 6), (3, 6), (6, -3)\}$

Find $[g \circ h](x)$ and $[h \circ g](x)$.

9. $g(x) = 2x$
$h(x) = x + 2$

10. $g(x) = -3x$
$h(x) = 4x - 1$

11. $g(x) = x - 6$
$h(x) = x + 6$

12. $g(x) = x - 3$
$h(x) = x^2$

13. $g(x) = 5x$
$h(x) = x^2 + x - 1$

14. $g(x) = x + 2$
$h(x) = 2x^2 - 3$

If $f(x) = 3x$, $g(x) = x + 4$, and $h(x) = x^2 - 1$, find each value.

15. $f[g(1)]$

16. $g[h(0)]$

17. $g[f(-1)]$

18. $h[f(5)]$

19. $g[h(-3)]$

20. $h[f(10)]$

21. $f[h(8)]$

22. $[f \circ (h \circ g)](1)$

23. $[f \circ (g \circ h)](-2)$

7-1 Practice

Operations on Functions

Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$.

1. $f(x) = 2x + 1$
$g(x) = x - 3$

2. $f(x) = 8x^2$
$g(x) = \frac{1}{x^2}$

3. $f(x) = x^2 + 7x + 12$
$g(x) = x^2 - 9$

For each set of ordered pairs, find $f \circ g$ and $g \circ f$ if they exist.

4. $f = \{(-9, -1), (-1, 0), (3, 4)\}$
$g = \{(0, -9), (-1, 3), (4, -1)\}$

5. $f = \{(-4, 3), (0, -2), (1, -2)\}$
$g = \{(-2, 0), (3, 1)\}$

6. $f = \{(-4, -5), (0, 3), (1, 6)\}$
$g = \{(6, 1), (-5, 0), (3, -4)\}$

7. $f = \{(0, -3), (1, -3), (6, 8)\}$
$g = \{(8, 2), (-3, 0), (-3, 1)\}$

Find $[g \circ h](x)$ and $[h \circ g](x)$.

8. $g(x) = 3x$
$h(x) = x - 4$

9. $g(x) = -8x$
$h(x) = 2x + 3$

10. $g(x) = x + 6$
$h(x) = 3x^2$

11. $g(x) = x + 3$
$h(x) = 2x^2$

12. $g(x) = -2x$
$h(x) = x^2 + 3x + 2$

13. $g(x) = x - 2$
$h(x) = 3x^2 + 1$

If $f(x) = x^2$, $g(x) = 5x$, and $h(x) = x + 4$, find each value.

14. $f[g(1)]$

15. $g[h(-2)]$

16. $h[f(4)]$

17. $f[h(-9)]$

18. $h[g(-3)]$

19. $g[f(8)]$

20. $h[f(20)]$

21. $[f \circ (h \circ g)](-1)$

22. $[f \circ (g \circ h)](4)$

23. BUSINESS The function $f(x) = 1000 - 0.01x^2$ models the manufacturing cost per item when x items are produced, and $g(x) = 150 - 0.001x^2$ models the service cost per item. Write a function $C(x)$ for the total manufacturing and service cost per item.

24. MEASUREMENT The formula $f = \frac{n}{12}$ converts inches n to feet f, and $m = \frac{f}{5280}$ converts feet to miles m. Write a composition of functions that converts inches to miles.

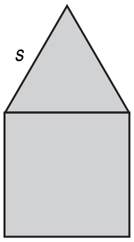
Lesson 7-1

NAME ____ DATE______________ PERIOD _____

7-1 Word Problem Practice

Operations on Functions

1. AREA Bernard wants to know the area of a figure made by joining an equilateral triangle and square along an edge. The function $f(s) = \frac{\sqrt{3}}{4}s^2$ gives the area of an equilateral triangle with side s. The function $g(s) = s^2$ gives the area of a square with side s. What function $h(s)$ gives the area of the figure as a function of its side length s?



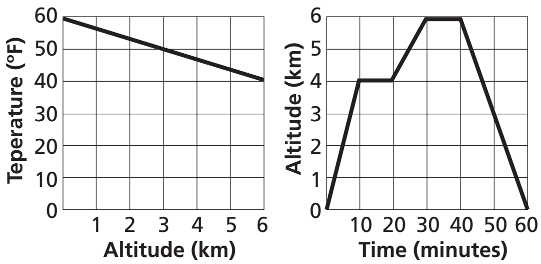
2. PRICING A computer company decides to continuously adjust the pricing of and discounts to its products in an effort to remain competitive. The function $P(t)$ gives the sale price of its Super2000 computer as a function of time. The function $D(t)$ gives the value of a special discount it offers to valued customers. How much would valued customers have to pay for one Super2000 computer?

3. LAVA A freshly ejected lava rock immediately begins to cool down. The temperature of the lava rock in degrees Fahrenheit as a function of time is given by $T(t)$. Let $C(F)$ be the function that gives degrees Celsius as a function of degrees Fahrenheit. What function gives the temperature of the lava rock in degrees Celsius as a function of time?

4. ENGINEERING A group of engineers is designing a staple gun. One team determines that the speed of impact s of the staple (in feet per second) as a function of the handle length ℓ (in inches) is given by $s(\ell) = 40 + 3\ell$. A second team determines that the number of sheets N that can be stapled as a function of the impact speed is given by $N(s) = \frac{s - 10}{3}$. What function gives N as a function of ℓ?

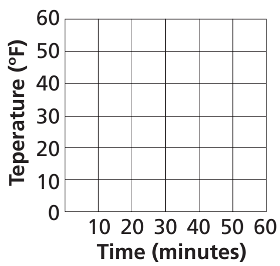
HOT AIR BALLOONS For Exercises 5 and 6, use the following information.

Hannah and Terry went on a one-hour hot air balloon ride. Let $T(A)$ be the outside air temperature as a function of altitude and let $A(t)$ be the altitude of the balloon as a function of time.



5. What function describes the air temperature Hannah and Terry felt at different times during their trip?

6. Sketch a graph of the function you wrote for Exercise 5 based on the graphs for $T(A)$ and $A(t)$ that are given.

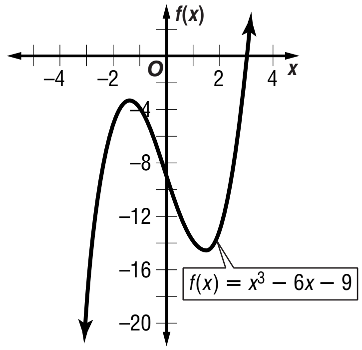


7-1 Enrichment

Relative Maximum Values

The graph of $f(x) = x^3 - 6x - 9$ shows a relative maximum value somewhere between $f(-2)$ and $f(-1)$. You can obtain a closer approximation by comparing values such as those shown in the table.

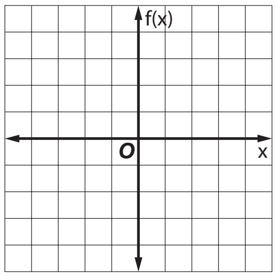
To the nearest tenth a relative maximum value for $f(x)$ is -3.3.



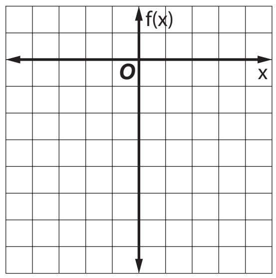
x	$f(x)$
-2	-5
-1.5	-3.375
-1.4	-3.344
-1.3	-3.397
-1	-4

Using a calculator to find points, graph each function. To the nearest tenth, find a relative maximum value of the function.

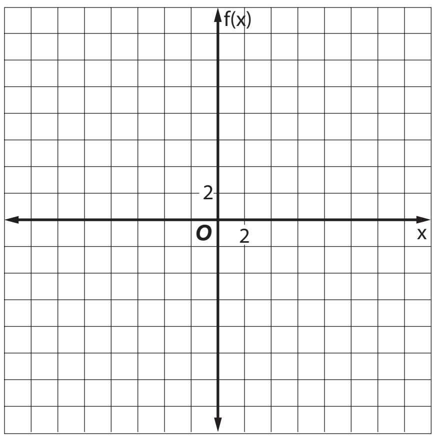
1. $f(x) = x(x^2 - 3)$



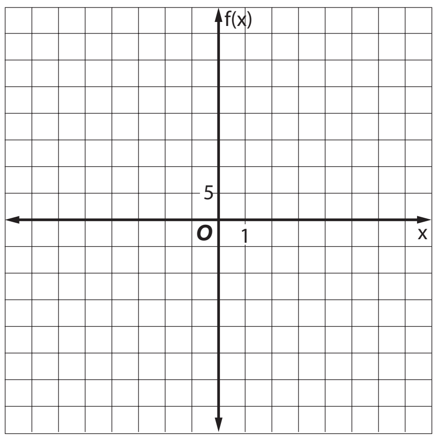
2. $f(x) = x^3 - 3x - 3$



3. $f(x) = x^3 - 9x - 2$



4. $f(x) = x^3 + 2x^2 - 12x - 24$



7-1 Spreadsheet Activity

Operations on Functions

It is possible to perform operations on functions such as addition, subtraction, multiplication and division. You can use a spreadsheet to investigate the relationships among functions.

Consider the functions $f(x) = 3x + 2$, $g(x) = x^2 - 2x$, and $h(x) = x^2 + x + 2$. Find the function values of each function for several values of x. Does it appear that $f(x) + g(x) = h(x)$?

Use Column A for the chosen values of x. Columns B, C, and E are $f(x)$, $g(x)$, and $h(x)$ respectively. Use Column D for $f(x) + g(x)$.

For every value of x, $f(x) + g(x) = h(x)$.

Functions.xls

	A	B	C	D	E
1	x	f(x)	g(x)	f(x) + g(x)	h(x)
2	–4	–10	24	14	14
3	–2.5	–5.5	11.25	5.75	5.75
4	–1	–1	3	2	2
5	0	2	0	2	2
6	1	5	–1	4	4
7	4	14	8	22	22
8	12	38	120	158	158

Sheet 1 | Sheet 2 | Sheet 3

Exercises

Study and use the spreadsheet above.

1. Find $k(x) = (3x + 2) + (x^2 - 2x)$. How does it compare to $h(x)$?

2. Change the functions in the spreadsheet to $f(x) = \frac{x}{2}$, $g(x) = 1 - x^2$, and $h(x) = 1 + \frac{x}{2} - x^2$. How are these functions related? Is it true that $f(x) + g(x) = h(x)$?

3. Make a conjecture about $(f + g)(x)$ for any functions $f(x)$ and $g(x)$.

4. Make a conjecture about $(f - g)(x)$ for any functions $f(x)$ and $g(x)$. Use the spreadsheet to test your conjecture. Does it appear to be true? Explain your answer.

Find $(f + g)(x)$, $(f - g)(x)$, for each $f(x)$ and $g(x)$. Use the spreadsheet to find function values to verify your solutions.

5. $f(x) = 6x + 8$
$g(x) = 9 + x$

6. $f(x) = x^2 + 1$
$g(x) = 3x - 4$

7. $f(x) = 10x^2$
$g(x) = 6 - x^2$

7-2 Lesson Reading Guide

Inverse Functions and Relations

Get Ready for the Lesson

Read the introduction to Lesson 7-2 in your textbook.

A function multiplies a number by 3 and then adds 5 to the result. What does the inverse function do, and in what order?

Read the Lesson

1. Complete each statement.

a. If two relations are inverses, the domain of one relation is the _________________ of the other.

b. Suppose that g is a relation and that the point $(4, -2)$ is on its graph. Then a point on the graph of g^{-1} is _________________.

c. The _________________ test can be used on the graph of a function to determine whether the function has an inverse function.

d. If you are given the graph of a function, you can find the graph of its inverse by reflecting the original graph over the line with equation _________________.

e. If f and g are inverse functions, then $(f \circ g)(x) =$ _________________ and $(g \circ f)(x) =$ _________________.

f. A function has an inverse that is also a function only if the given function is _________________.

g. Suppose that $h(x)$ is a function whose inverse is also a function. If $h(5) = 12$, then $h^{-1}(12) =$ _________________.

2. Assume that $f(x)$ is a one-to-one function defined by an algebraic equation. Write the four steps you would follow in order to find the equation for $f^{-1}(x)$.

1. ____
2. ____
3. ____
4. ____

Remember What You Learned

3. A good way to remember something new is to relate it to something you already know. How are the vertical and horizontal line tests related?

Lesson 7-2

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7-2 Study Guide and Intervention

Inverse Functions and Relations

Find Inverses

Inverse Relations	Two relations are inverse relations if and only if whenever one relation contains the element (a, b), the other relation contains the element (b, a).
Property of Inverse Functions	Suppose f and f^{-1} are inverse functions. Then $f(a) = b$ if and only if $f^{-1}(b) = a$.

Example **Find the inverse of the function $f(x) = \frac{2}{5}x - \frac{1}{5}$. Then graph the function and its inverse.**

Step 1 Replace $f(x)$ with y in the original equation.

$$f(x) = \frac{2}{5}x - \frac{1}{5} \rightarrow y = \frac{2}{5}x - \frac{1}{5}$$

Step 2 Interchange x and y.

$$x = \frac{2}{5}y - \frac{1}{5}$$

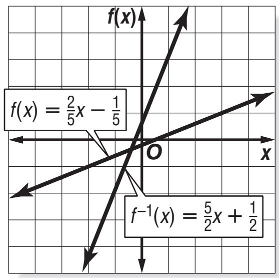
Step 3 Solve for y.

$x = \frac{2}{5}y - \frac{1}{5}$ Inverse

$5x = 2y - 1$ Multiply each side by 5.

$5x + 1 = 2y$ Add 1 to each side.

$\frac{1}{2}(5x + 1) = y$ Divide each side by 2.

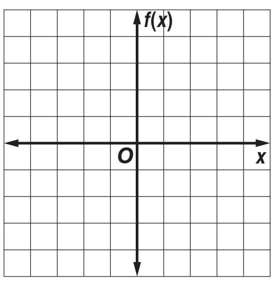


The inverse of $f(x) = \frac{2}{5}x - \frac{1}{5}$ is $f^{-1}(x) = \frac{1}{2}(5x + 1)$.

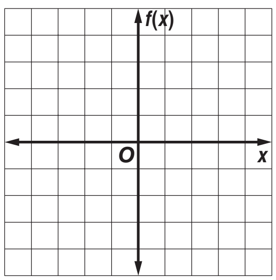
Exercises

Find the inverse of each function. Then graph the function and its inverse.

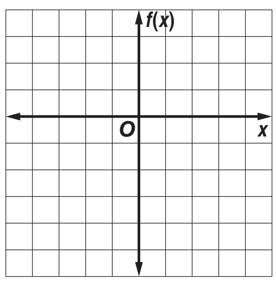
1. $f(x) = \frac{2}{3}x - 1$



2. $f(x) = 2x - 3$



3. $f(x) = \frac{1}{4}x - 2$



7-2 Study Guide and Intervention *(continued)*

Inverse Functions and Relations

Inverses of Relations and Functions

Inverse Functions	Two functions f and g are inverse functions if and only if $[f \circ g](x) = x$ and $[g \circ f](x) = x$.

Example 1 **Determine whether $f(x) = 2x - 7$ and $g(x) = \frac{1}{2}(x + 7)$ are inverse functions.**

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f\left[\frac{1}{2}(x + 7)\right] \\ &= 2\left[\frac{1}{2}(x + 7)\right] - 7 \\ &= x + 7 - 7 \\ &= x\end{aligned}$$

$$\begin{aligned}[g \circ f](x) &= g[f(x)] \\ &= g(2x - 7) \\ &= \frac{1}{2}(2x - 7 + 7) \\ &= x\end{aligned}$$

The functions are inverses since both $[f \circ g](x) = x$ and $[g \circ f](x) = x$.

Example 2 **Determine whether $f(x) = 4x + \frac{1}{3}$ and $g(x) = \frac{1}{4}x - 3$ are inverse functions.**

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f\left(\frac{1}{4}x - 3\right) \\ &= 4\left(\frac{1}{4}x - 3\right) + \frac{1}{3} \\ &= x - 12 + \frac{1}{3} \\ &= x - 11\frac{2}{3}\end{aligned}$$

Since $[f \circ g](x) \neq x$, the functions are not inverses.

Exercises

Determine whether each pair of functions are inverse functions.

1. $f(x) = 3x - 1$, $g(x) = \frac{1}{3}x + \frac{1}{3}$

2. $f(x) = \frac{1}{4}x + 5$, $g(x) = 4x - 20$

3. $f(x) = \frac{1}{2}x - 10$, $g(x) = 2x + \frac{1}{10}$

4. $f(x) = 2x + 5$, $g(x) = 5x + 2$

5. $f(x) = 8x - 12$, $g(x) = \frac{1}{8}x + 12$

6. $f(x) = -2x + 3$, $g(x) = -\frac{1}{2}x + \frac{3}{2}$

7. $f(x) = 4x - \frac{1}{2}$, $g(x) = \frac{1}{4}x + \frac{1}{8}$

8. $f(x) = 2x - \frac{3}{5}$, $g(x) = \frac{1}{10}(5x + 3)$

9. $f(x) = 4x + \frac{1}{2}$, $g(x) = \frac{1}{2}x - \frac{3}{2}$

10. $f(x) = 10 - \frac{x}{2}$, $g(x) = 20 - 2x$

11. $f(x) = 4x - \frac{4}{5}$, $g(x) = \frac{x}{4} + \frac{1}{5}$

12. $f(x) = 9 + \frac{3}{2}x$, $g(x) = \frac{2}{3}x - 6$

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7-2 Skills Practice

Inverse Functions and Relations

Find the inverse of each relation.

1. $\{(3, 1), (4, -3), (8, -3)\}$

2. $\{(-7, 1), (0, 5), (5, -1)\}$

3. $\{(-10, -2), (-7, 6), (-4, -2), (-4, 0)\}$

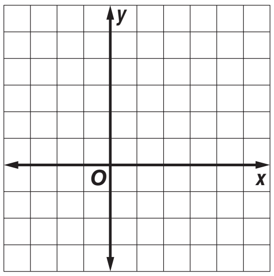
4. $\{(0, -9), (5, -3), (6, 6), (8, -3)\}$

5. $\{(-4, 12), (0, 7), (9, -1), (10, -5)\}$

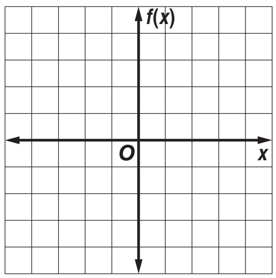
6. $\{(-4, 1), (-4, 3), (0, -8), (8, -9)\}$

Find the inverse of each function. Then graph the function and its inverse.

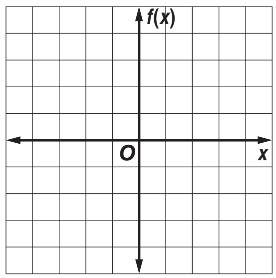
7. $y = 4$



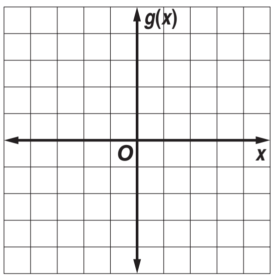
8. $f(x) = 3x$



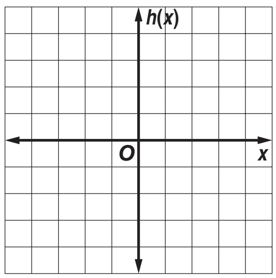
9. $f(x) = x + 2$



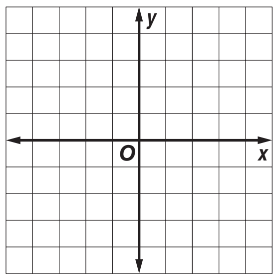
10. $g(x) = 2x - 1$



11. $h(x) = \frac{1}{4}x$



12. $y = \frac{2}{3}x + 2$



Determine whether each pair of functions are inverse functions.

13. $f(x) = x - 1$
$g(x) = 1 - x$

14. $f(x) = 2x + 3$
$g(x) = \frac{1}{2}(x - 3)$

15. $f(x) = 5x - 5$
$g(x) = \frac{1}{5}x + 1$

16. $f(x) = 2x$
$g(x) = \frac{1}{2}x$

17. $h(x) = 6x - 2$
$g(x) = \frac{1}{6}x + 3$

18. $f(x) = 8x - 10$
$g(x) = \frac{1}{8}x + \frac{5}{4}$

7-2 Practice

Inverse Functions and Relations

Find the inverse of each relation.

1. $\{(0, 3), (4, 2), (5, -6)\}$

2. $\{(-5, 1), (-5, -1), (-5, 8)\}$

3. $\{(-3, -7), (0, -1), (5, 9), (7, 13)\}$

4. $\{(8, -2), (10, 5), (12, 6), (14, 7)\}$

5. $\{(-5, -4), (1, 2), (3, 4), (7, 8)\}$

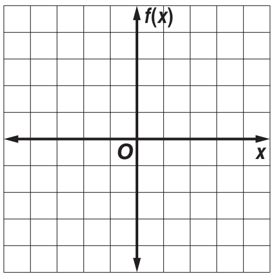
6. $\{(-3, 9), (-2, 4), (0, 0), (1, 1)\}$

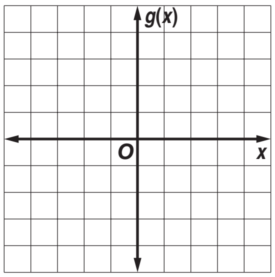
Find the inverse of each function. Then graph the function and its inverse.

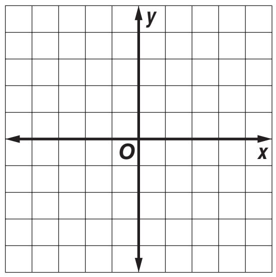
7. $f(x) = \frac{3}{4}x$

8. $g(x) = 3 + x$

9. $y = 3x - 2$







Determine whether each pair of functions are inverse functions.

10. $f(x) = x + 6$
$g(x) = x - 6$

11. $f(x) = -4x + 1$
$g(x) = \frac{1}{4}(1 - x)$

12. $g(x) = 13x - 13$
$h(x) = \frac{1}{13}x - 1$

13. $f(x) = 2x$
$g(x) = -2x$

14. $f(x) = \frac{6}{7}x$
$g(x) = \frac{7}{6}x$

15. $g(x) = 2x - 8$
$h(x) = \frac{1}{2}x + 4$

16. MEASUREMENT The points (63, 121), (71, 180), (67, 140), (65, 108), and (72, 165) give the weight in pounds as a function of height in inches for 5 students in a class. Give the points for these students that represent height as a function of weight.

REMODELING For Exercises 17 and 18, use the following information.
The Clearys are replacing the flooring in their 15 foot by 18 foot kitchen. The new flooring costs $17.99 per square yard. The formula $f(x) = 9x$ converts square yards to square feet.

17. Find the inverse $f^{-1}(x)$. What is the significance of $f^{-1}(x)$ for the Clearys?

18. What will the new flooring cost the Cleary's?

7-2 Word Problem Practice

Inverse Functions and Relations

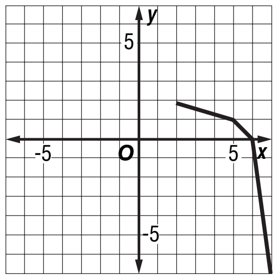
1. **VOLUME** Jason wants to make a spherical water cooler that can hold half a cubic meter of water. He knows that $V = \frac{4}{3}\pi r^3$, but he needs to know how to find r given V. Find this inverse function.

2. **EXERCISE** Alex began a new exercise routine. To gain the maximum benefit from his exercise, Alex calculated his maximum target heart rate using the function. $f(x) = 0.85(220 - x)$ where x represents his age. Find the inverse of this function.

3. **ROCKETS** The altitude of a rocket in feet as a function of time is given by $f(t) = 49t^2$, where $t \geq 0$. Find the inverse of this function and determine the times when the rocket will be 10, 100, and 1000 feet high. Round your answers to the nearest hundredth of a second.

4. **SELF-INVERTIBLE** Karen finds the incomplete graph of a function in the back of her engineering handbook. The function is graphed in the figure below.

   Karen knows that this function is its own inverse. Armed with this knowledge, extend the graph for values of x between -7 and 2.



PLANETS For Exercises 5 and 6, use the following information.

The approximate distance of a planet from the Sun is given by $d = T^{\frac{2}{3}}$ where d is distance in astronomical units and T is Earth years. An astronomical unit is the distance of the Earth from the Sun.

5. Solve for T in terms of d.

6. Pluto is about 39.44 times as far from the Sun as the Earth. About how many years does it take Pluto to orbit the Sun?

7-2 Enrichment

Reading Algebra

In mathematics, the term *group* has a special meaning. The following numbered sentences discuss the idea of group and one interesting example of a group.

01 To be a group, a set of elements and a binary operation must satisfy four conditions: the set must be closed under the operation, the operation must be associative, there must be an identity element, and every element must have an inverse.

02 The following six functions form a group under the operation of composition of functions: $f_1(x) = x$, $f_2(x) = \frac{1}{x}$, $f_3(x) = 1 - x$, $f_4(x) = \frac{(x-1)}{x}$, $f_5(x) = \frac{x}{(x-1)}$, and $f_6(x) = \frac{1}{(1-x)}$.

03 This group is an example of a noncommutative group. For example, $f_3 \circ f_2 = f_4$, but $f_2 \circ f_3 = f_6$.

04 Some experimentation with this group will show that the identity element is f_1.

05 Every element is its own inverse except for f_4 and f_6, each of which is the inverse of the other.

Use the paragraph to answer these questions.

1. Explain what it means to say that a set is *closed* under an operation. Is the set of positive integers closed under subtraction?

2. Subtraction is a noncommutative operation for the set of integers. Write an informal definition of noncommutative.

3. For the set of integers, what is the identity element for the operation of multiplication? Justify your answer.

4. Explain how the following statement relates to sentence 05:

$$(f_6 \cdot f_4)(x) = f_6[f_4(x)] = f_6\left(\frac{1}{(1-x)}\right) = \frac{1}{\frac{1-(x-1)}{x}} = x = f_1(x).$$

7-3 Lesson Reading Guide

Square Root Functions

Get Ready for the Lesson

Read the introduction to Lesson 7-3 in your textbook.

If the weight to be supported by a steel cable is doubled, should the diameter of the support cable also be doubled? If not, by what number should the diameter be multiplied?

Read the Lesson

1. Match each square root function from the list on the left with its domain and range from the list on the right.

a. $y = \sqrt{x}$	**i.** domain: $x \geq 0$; range: $y \geq 3$
b. $y = \sqrt{x + 3}$	**ii.** domain: $x \geq 0$; range: $y \leq 0$
c. $y = \sqrt{x} + 3$	**iii.** domain: $x \geq 0$; range: $y \leq -3$
d. $y = \sqrt{x - 3}$	**iv.** domain: $x \geq 0$; range: $y \geq 0$
e. $y = -\sqrt{x}$	**v.** domain: $x \geq 3$; range: $y \geq 0$
f. $y = -\sqrt{x - 3}$	**vi.** domain: $x \leq 3$; range: $y \geq 3$
g. $y = \sqrt{3 - x} + 3$	**vii.** domain: $x \geq 3$; range: $y \leq 0$
h. $y = -\sqrt{x} - 3$	**viii.** domain: $x \geq -3$; range: $y \geq 0$

2. The graph of the inequality $y \leq \sqrt{3x + 6}$ is a shaded region. Which of the following points lie inside this region?

(3, 0) (2, 4) (5, 2) (4, −2) (6, 6)

Remember What You Learned

3. A good way to remember something is to explain it to someone else. Suppose you are studying this lesson with a classmate who thinks that you cannot have square root functions because every positive real number has two square roots. How would you explain the idea of square root functions to your classmate?

7-3 Study Guide and Intervention

Square Root Functions and Inequalities

Square Root Functions A function that contains the square root of a variable expression is a **square root function**.

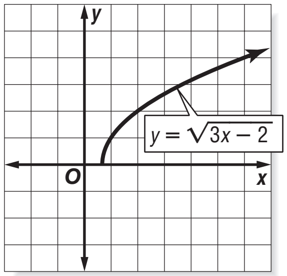
Example **Graph $y = \sqrt{3x - 2}$. State its domain and range.**

Since the radicand cannot be negative, $3x - 2 \geq 0$ or $x \geq \frac{2}{3}$.

The x-intercept is $\frac{2}{3}$. The range is $y \geq 0$.

Make a table of values and graph the function.

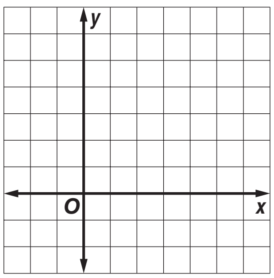
x	y
$\frac{2}{3}$	0
1	1
2	2
3	$\sqrt{7}$



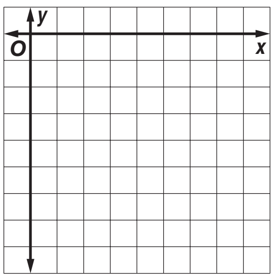
Exercises

Graph each function. State the domain and range of the function.

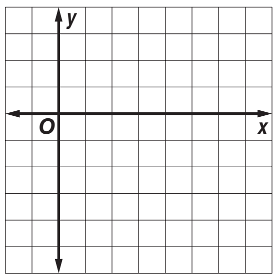
1. $y = \sqrt{2x}$



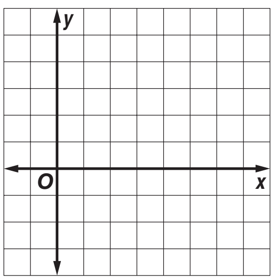
2. $y = -3\sqrt{x}$



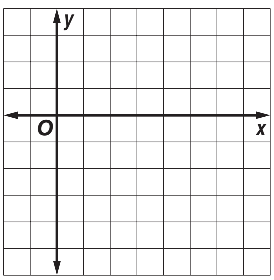
3. $y = -\sqrt{\frac{x}{2}}$



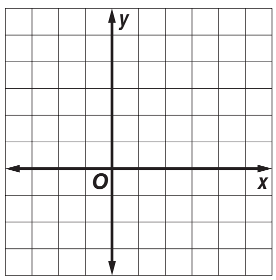
4. $y = 2\sqrt{x - 3}$



5. $y = -\sqrt{2x - 3}$



6. $y = \sqrt{2x + 5}$



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7-3 Study Guide and Intervention *(continued)*

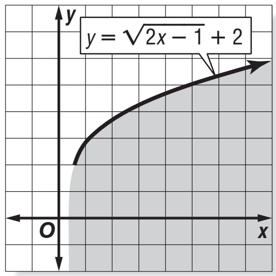
Square Root Functions and Inequalities

Square Root Inequalities A **square root inequality** is an inequality that contains the square root of a variable expression. Use what you know about graphing square root functions and quadratic inequalities to graph square root inequalities.

Example **Graph $y \leq \sqrt{2x - 1} + 2$.**

Graph the related equation $y = \sqrt{2x - 1} + 2$. Since the boundary should be included, the graph should be solid.

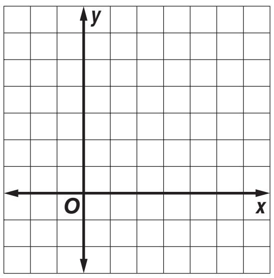
The domain includes values for $x \geq \frac{1}{2}$, so the graph is to the right of $x = \frac{1}{2}$.



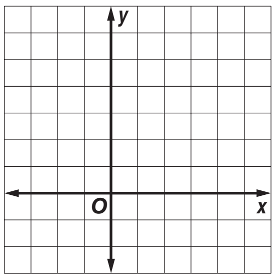
Exercises

Graph each inequality.

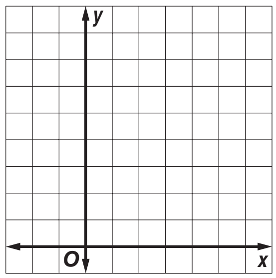
1. $y < 2\sqrt{x}$



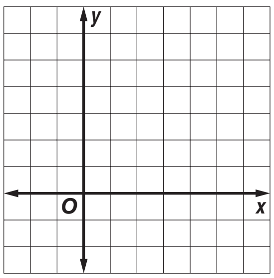
2. $y > \sqrt{x + 3}$



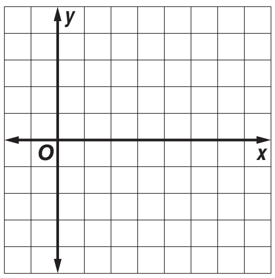
3. $y < 3\sqrt{2x - 1}$



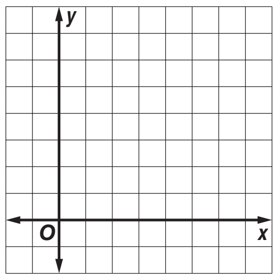
4. $y < \sqrt{3x - 4}$



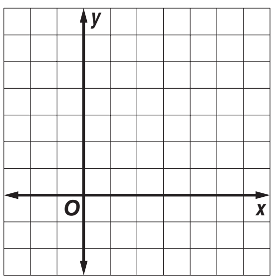
5. $y \geq \sqrt{x + 1} - 4$



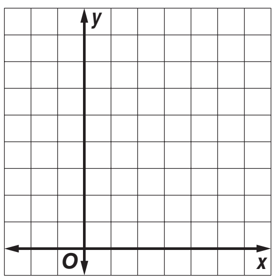
6. $y > 2\sqrt{2x - 3}$



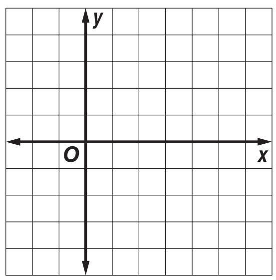
7. $y \geq \sqrt{3x + 1} - 2$



8. $y \leq \sqrt{4x - 2} + 1$



9. $y < 2\sqrt{2x - 1} - 4$



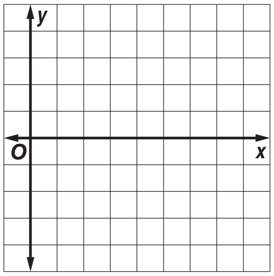
NAME ____ DATE______________ PERIOD _____

Skills Practice

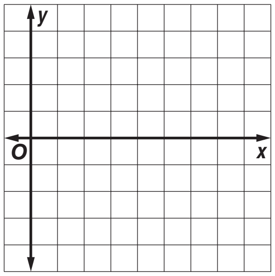
Square Root Functions and Inequalities

Graph each function. State the domain and range of each function.

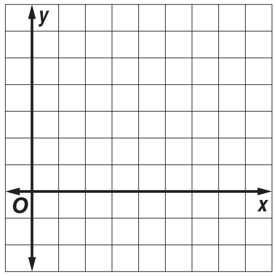
1. $y = \sqrt{2x}$



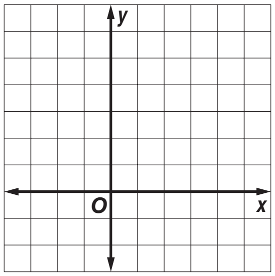
2. $y = -\sqrt{3x}$



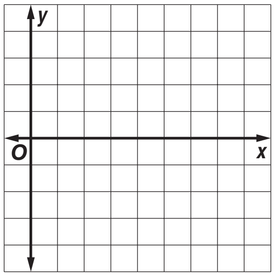
3. $y = 2\sqrt{x}$



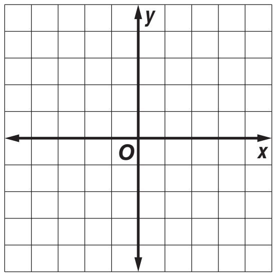
4. $y = \sqrt{x + 3}$



5. $y = -\sqrt{2x - 5}$

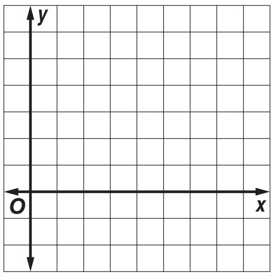


6. $y = \sqrt{x + 4} - 2$

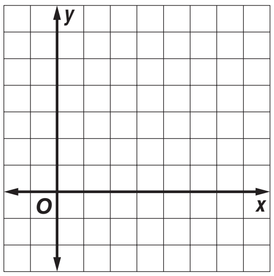


Graph each inequality.

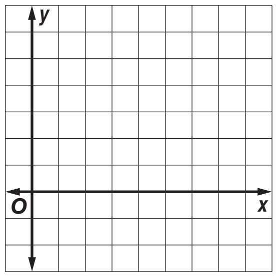
7. $y < \sqrt{4x}$



8. $y \geq \sqrt{x + 1}$



9. $y \leq \sqrt{4x - 3}$



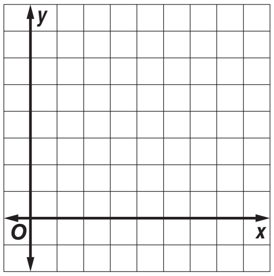
NAME ______________________________ DATE ____________ PERIOD _____

7-3 Practice

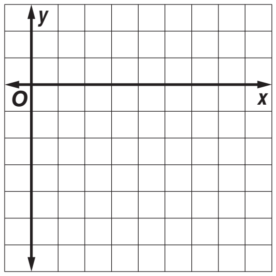
Square Root Functions and Inequalities

Graph each function. State the domain and range of each function.

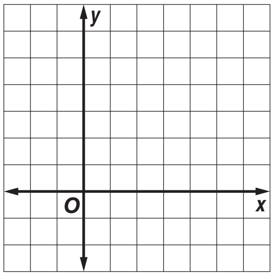
1. $y = \sqrt{5x}$



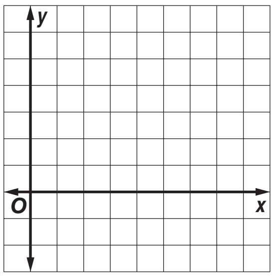
2. $y = -\sqrt{x - 1}$



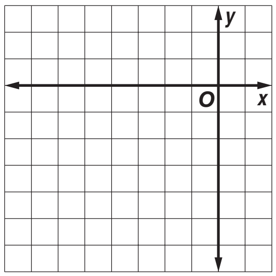
3. $y = 2\sqrt{x + 2}$



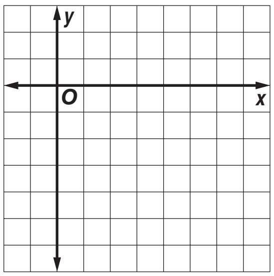
4. $y = \sqrt{3x - 4}$



5. $y = \sqrt{x + 7} - 4$

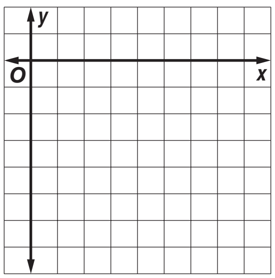


6. $y = 1 - \sqrt{2x + 3}$

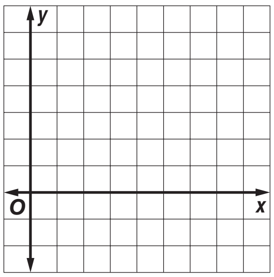


Graph each inequality.

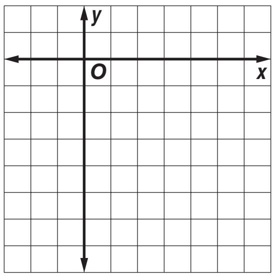
7. $y \geq -\sqrt{6x}$



8. $y \leq \sqrt{x - 5} + 3$



9. $y > -2\sqrt{3x + 2}$



10. ROLLER COASTERS The velocity of a roller coaster as it moves down a hill is $v = \sqrt{v_0^2 + 64h}$, where v_0 is the initial velocity and h is the vertical drop in feet. If $v = 70$ feet per second and $v_0 = 8$ feet per second, find h.

11. WEIGHT Use the formula $d = \sqrt{\frac{3960^2 W_E}{W_s}} - 3960$, which relates distance from Earth d in miles to weight. If an astronaut's weight on Earth W_E is 148 pounds and in space W_s is 115 pounds, how far from Earth is the astronaut?

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7-3 Word Problem Practice

Square Root Functions and Inequalities

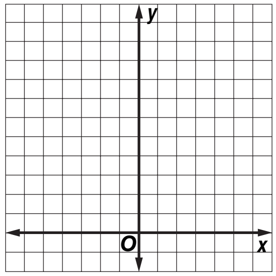
1. **SQUARES** Cathy is building a square roof for her garage. The roof will occupy 625 square feet. What are the dimensions of the roof?

2. **PENDULUMS** The period of a pendulum, the time it takes to complete one swing, is given by the formula $p = 2\pi\sqrt{\frac{L}{g}}$ where L is the length of the pendulum and g is acceleration due to gravity, 9.8 m/s^2. Find the period of a pendulum that is 0.65 meters long. Round to the nearest tenth.

3. **REFLEXES** Rachel and Ashley are testing one another's reflexes. Rachel drops a ruler from a given height so that it falls between Ashley's thumb and index finger. Ashley tries to catch the ruler before it falls through her hand. The time required to catch the ruler is given by $t = \frac{\sqrt{d}}{4}$ where d is measured *in feet*. Complete the table. Round your answers to the nearest hundredth.

Distance (in.)	Reflex Time (seconds)
3 in.	
6 in.	
9 in.	
12 in.	

4. **DISTANCE** Lance is standing at the side of a road watching a cyclist go by. The distance between Lance and the cyclist as a function of time is given by $d = \sqrt{9 + 36t^2}$. Graph this function. Find the distance between Lance and the cyclist after 3 seconds.



STARS For Exercises 5-7, use the following information.

The intensity of the light from an object varies inversely with the square of the distance. In other words, $I = \frac{k}{d^2}$.

5. Solve the equation to find d in terms of I.

6. Two stars give off the same amount of light. However, from Earth their intensities differ. Let I_1 and I_2 be their intensities and let d_1 and d_2 be their respective distances from Earth. What is the ratio of d_2 to d_1?

7. If one star appears 9 times as intense as the other, how much closer is it to Earth?

Lesson 7-3

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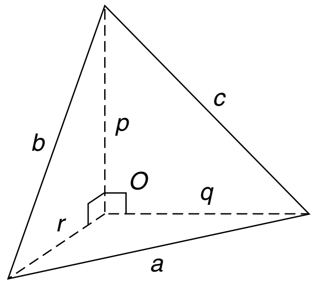
7-3 Enrichment

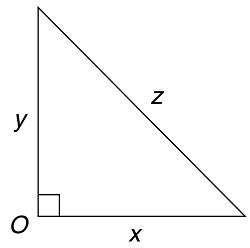
Reading Algebra

If two mathematical problems have basic structural similarities, they are said to be **analogous.** Using analogies is one way of discovering and proving new theorems.

The following numbered sentences discuss a three-dimensional analogy to the Pythagorean theorem.



01 Consider a tetrahedron with three perpendicular faces that meet at vertex O.
02 Suppose you want to know how the areas A, B, and C of the three faces that meet at vertex O are related to the area D of the face opposite vertex O.
03 It is natural to expect a formula analogous to the Pythagorean theorem $z^2 = x^2 + y^2$, which is true for a similar situation in two dimensions.
04 To explore the three-dimensional case, you might guess a formula and then try to prove it.
05 Two reasonable guesses are $D^3 = A^3 + B^3 + C^3$ and $D^2 = A^2 + B^2 + C^2$.



Refer to the numbered sentences to answer the questions.

1. Use sentence 01 and the top diagram. The prefix *tetra-* means four. Write an informal definition of tetrahedron.

2. Use sentence 02 and the top diagram. What are the lengths of the sides of each face of the tetrahedron?

3. Rewrite sentence 01 to state a two-dimensional analogue.

4. Refer to the top diagram and write expressions for the areas A, B, and C mentioned in sentence 02.

5. To explore the three-dimensional case, you might begin by expressing a, b, and c in terms of p, q, and r. Use the Pythagorean theorem to do this.

6. Which guess in sentence 05 seems more likely? Justify your answer.

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7-4 Lesson Reading Guide

*n*th *Roots*

Get Ready for the Lesson

Read the introduction to Lesson 7-4 in your textbook.

A basketball has a volume of about 382 cubic inches. Explain how you would find the radius of the basketball using a calculator. (Do not actually calculate the radius.)

Read the Lesson

1. For each radical below, identify the radicand and the index.

a. $\sqrt[3]{23}$ radicand: ____________ index: ____________

b. $\sqrt{15x^2}$ radicand: ____________ index: ____________

c. $\sqrt[5]{-343}$ radicand: ____________ index: ____________

2. Complete the following table. (Do not actually find any of the indicated roots.)

Number	Number of Positive Square Roots	Number of Negative Square Roots	Number of Positive Cube Roots	Number of Negative Cube Roots
27				
−16				

3. State whether each of the following is *true* or *false*.

a. A negative number has no real fourth roots.

b. $\pm\sqrt{121}$ represents both square roots of 121.

c. When you take the fifth root of x^5, you must take the absolute value of x to identify the principal fifth root.

Remember What You Learned

4. What is an easy way to remember that a negative number has no real square roots but has one real cube root?

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7-4 Study Guide and Intervention

*n*th Roots

Simplify Radicals

Square Root	For any real numbers a and b, if $a^2 = b$, then a is a square root of b.
***n*th Root**	For any real numbers a and b, and any positive integer n, if $a^n = b$, then a is an nth root of b.
Real *n*th Roots of *b*, $\sqrt[n]{b}$, $-\sqrt[n]{b}$	1. If n is even and $b > 0$, then b has one positive root and one negative root. 2. If n is odd and $b > 0$, then b has one positive root. 3. If n is even and $b < 0$, then b has no real roots. 4. If n is odd and $b < 0$, then b has one negative root.

Example 1 **Simplify $\sqrt{49z^8}$.**

$\sqrt{49z^8} = \sqrt{(7z^4)^2} = 7z^4$

z^4 must be positive, so there is no need to take the absolute value.

Example 2 **Simplify $-\sqrt[3]{(2a - 1)^6}$**

$-\sqrt[3]{(2a - 1)^6} = -\sqrt[3]{[(2a - 1)^2]^3} = (2a - 1)^2$

Exercises

Simplify.

1. $\sqrt{81}$

2. $\sqrt[3]{-343}$

3. $\sqrt{144p^6}$

4. $\pm\sqrt{4a^{10}}$

5. $\sqrt[5]{243p^{10}}$

6. $-\sqrt[3]{m^6n^9}$

7. $\sqrt[3]{-b^{12}}$

8. $\sqrt{16a^{10}b^8}$

9. $\sqrt{121x^6}$

10. $\sqrt{(4k)^4}$

11. $\pm\sqrt{169r^4}$

12. $-\sqrt[3]{-27p^6}$

13. $-\sqrt{625y^2z^4}$

14. $\sqrt{36q^{34}}$

15. $\sqrt{100x^2y^4z^6}$

16. $\sqrt[3]{-0.027}$

17. $-\sqrt{-0.36}$

18. $\sqrt{0.64p^{10}}$

19. $\sqrt[4]{(2x)^8}$

20. $\sqrt{(11y^2)^4}$

21. $\sqrt[3]{(5a^2b)^6}$

22. $\sqrt{(3x - 1)^2}$

23. $\sqrt[3]{(m - 5)^6}$

24. $\sqrt{36x^2 - 12x + 1}$

7-4 Study Guide and Intervention *(continued)*

*n*th *Roots*

Approximate Radicals with a Calculator

Irrational Number	a number that cannot be expressed as a terminating or a repeating decimal

Radicals such as $\sqrt{2}$ and $\sqrt{3}$ are examples of irrational numbers. Decimal approximations for irrational numbers are often used in applications. These approximations can be easily found with a calculator.

Example **Approximate $\sqrt[5]{18.2}$ with a calculator.**

$\sqrt[5]{18.2} \approx 1.787$

Exercises

Use a calculator to approximate each value to three decimal places.

1. $\sqrt{62}$

2. $\sqrt{1050}$

3. $\sqrt[3]{0.054}$

4. $-\sqrt[4]{5.45}$

5. $\sqrt{5280}$

6. $\sqrt{18,600}$

7. $\sqrt{0.095}$

8. $\sqrt[3]{-15}$

9. $\sqrt[5]{100}$

10. $\sqrt[6]{856}$

11. $\sqrt{3200}$

12. $\sqrt{0.05}$

13. $\sqrt{12,500}$

14. $\sqrt{0.60}$

15. $-\sqrt[4]{500}$

16. $\sqrt[3]{0.15}$

17. $\sqrt[6]{4200}$

18. $\sqrt{75}$

19. LAW ENFORCEMENT The formula $r = 2\sqrt{5L}$ is used by police to estimate the speed r in miles per hour of a car if the length L of the car's skid mark is measures in feet. Estimate to the nearest tenth of a mile per hour the speed of a car that leaves a skid mark 300 feet long.

20. SPACE TRAVEL The distance to the horizon d miles from a satellite orbiting h miles above Earth can be approximated by $d = \sqrt{8000h + h^2}$. What is the distance to the horizon if a satellite is orbiting 150 miles above Earth?

7-4 Skills Practice

*n*th *Roots*

Use a calculator to approximate each value to three decimal places.

1. $\sqrt{230}$

2. $\sqrt{38}$

3. $-\sqrt{152}$

4. $\sqrt{5.6}$

5. $\sqrt[3]{88}$

6. $\sqrt[3]{-222}$

7. $-\sqrt[4]{0.34}$

8. $\sqrt[5]{500}$

Simplify.

9. $\pm\sqrt{81}$

10. $\sqrt{144}$

11. $\sqrt{(-5)^2}$

12. $\sqrt{-5^2}$

13. $\sqrt{0.36}$

14. $-\sqrt{\frac{4}{9}}$

15. $\sqrt[3]{-8}$

16. $-\sqrt[3]{27}$

17. $\sqrt[3]{0.064}$

18. $\sqrt[5]{32}$

19. $\sqrt[4]{81}$

20. $\sqrt{y^2}$

21. $\sqrt[3]{125s^3}$

22. $\sqrt{64x^6}$

23. $\sqrt[3]{-27a^6}$

24. $\sqrt{m^8n^4}$

25. $-\sqrt{100p^4q^2}$

26. $\sqrt[4]{16w^4v^8}$

27. $\sqrt{(-3c)^4}$

28. $\sqrt{(a + b)^2}$

7-4 Practice

*n*th *Roots*

Use a calculator to approximate each value to three decimal places.

1. $\sqrt{7.8}$ **2.** $-\sqrt{89}$ **3.** $\sqrt[3]{25}$ **4.** $\sqrt[3]{-4}$

5. $\sqrt[4]{1.1}$ **6.** $\sqrt[5]{-0.1}$ **7.** $\sqrt[6]{5555}$ **8.** $\sqrt[4]{(0.94)^2}$

Simplify.

9. $\sqrt{0.81}$ **10.** $-\sqrt{324}$ **11.** $-\sqrt[4]{256}$ **12.** $\sqrt[6]{64}$

13. $\sqrt[3]{-64}$ **14.** $\sqrt[3]{0.512}$ **15.** $\sqrt[5]{-243}$ **16.** $-\sqrt[4]{1296}$

17. $\sqrt[5]{\frac{-1024}{243}}$ **18.** $\sqrt[5]{243x^{10}}$ **19.** $\sqrt{(14a)^2}$ **20.** $\sqrt{-(14a)^2}$

21. $\sqrt{49m^2t^8}$ **22.** $\sqrt{\frac{16m^2}{25}}$ **23.** $\sqrt[3]{-64r^6w^{15}}$ **24.** $\sqrt{(2x)^8}$

25. $-\sqrt[4]{625s^8}$ **26.** $\sqrt[3]{216p^3q^9}$ **27.** $\sqrt{676x^4y^6}$ **28.** $\sqrt[3]{-27x^9y^{12}}$

29. $-\sqrt{144m^8n^6}$ **30.** $\sqrt[5]{-32x^5y^{10}}$ **31.** $\sqrt[6]{(m+4)^6}$ **32.** $\sqrt[3]{(2x+1)^3}$

33. $-\sqrt{49a^{10}b^{16}}$ **34.** $\sqrt[4]{(x-5)^8}$ **35.** $\sqrt[3]{343d^6}$ **36.** $\sqrt{x^2+10x+25}$

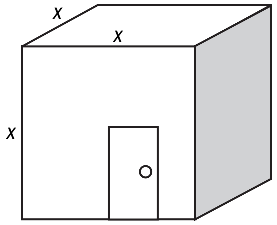
37. RADIANT TEMPERATURE Thermal sensors measure an object's *radiant* temperature, which is the amount of energy radiated by the object. The *internal* temperature of an object is called its *kinetic* temperature. The formula $T_r = T_k\sqrt[4]{e}$ relates an object's radiant temperature T_r to its kinetic temperature T_k. The variable e in the formula is a measure of how well the object radiates energy. If an object's kinetic temperature is 30°C and $e = 0.94$, what is the object's radiant temperature to the nearest tenth of a degree?

38. HERO'S FORMULA Salvatore is buying fertilizer for his triangular garden. He knows the lengths of all three sides, so he is using Hero's formula to find the area. Hero's formula states that the area of a triangle is $\sqrt{s(s-a)(s-b)(s-c)}$, where a, b, and c are the lengths of the sides of the triangle and s is half the perimeter of the triangle. If the lengths of the sides of Salvatore's garden are 15 feet, 17 feet, and 20 feet, what is the area of the garden? Round your answer to the nearest whole number.

7-4 Word Problem Practice

*n*th *Roots*

1. **CUBES** Cathy is building a cubic storage room. She wants the volume of the space to be 1728 cubic feet. What should the dimensions of the cube be?



2. **ASTRONOMY** A special form of Kepler's Third Law of Planetary Motion is given by $a = \sqrt[3]{P^2}$ where a is the average distance of an object from the Sun in AU (astronomical units) and P is the period of the orbit in years. If an object is orbiting the Sun with a period of 12 years, what is its distance from the Sun?

3. **TUNING** Two notes are an octave apart if the frequency of the higher note is twice the frequency of the lower note. Casey is experimenting with an instrument that has 6 notes tuned so that the frequency of each successive note increases by the same factor and the first and last note are an octave apart. By what factor does the frequency increase from note to note?

4. **MARKUPS** A wholesaler manufactures a part for D dollars. The wholesaler sells the part to a dealer for a P percent markup. The dealer sells the part to a retailer at an additional P percent markup. The retailer in turn sells the part to its customers marking up the price yet another P percent. What is the price that customers see? If the customer buys the part for $80 and the markup is 40%, what approximately was the original cost to make the part?

PENDULUMS For Exercises 5 and 6, use the following information.

Mr. Topalian's physics class is experimenting with pendulums. The class learned the formula $T = 2\pi\sqrt{\frac{L}{g}}$ which relates the time T that it takes for a pendulum to swing back and forth based on gravity g equal to 32 feet per second squared, and the length of the pendulum L in feet.

5. One group in the class made a 2-foot long pendulum. Use the formula to determine how long it will take for their pendulum to swing back and forth.

6. Another group decided they wanted to make a pendulum that took about 1.76 seconds to go back and forth. Approximately how long should their pendulum be?

7-4 Enrichment

Approximating Square Roots

Consider the following expansion.

$$\left(a + \frac{b}{2a}\right)^2 = a^2 + \frac{2ab}{2a} + \frac{b^2}{4a^2}$$
$$= a^2 + b + \frac{b^2}{4a^2}$$

Think what happens if a is very great in comparison to b. The term $\frac{b^2}{4a^2}$ is very small and can be disregarded in an approximation.

$$\left(a + \frac{b}{2a}\right)^2 \approx a^2 + b$$
$$a + \frac{b}{2a} \approx \sqrt{a^2 + b}$$

Suppose a number can be expressed as $a^2 + b$, $a > b$. Then an approximate value of the square root is $a + \frac{b}{2a}$. You should also see that $a - \frac{b}{2a} \approx \sqrt{a^2 - b}$.

Example **Use the formula $\sqrt{a^2 \pm b} \approx a \pm \frac{b}{2a}$ to approximate $\sqrt{101}$ and $\sqrt{622}$.**

a. $\sqrt{101} = \sqrt{100 + 1} = \sqrt{10^2 + 1}$

Let $a = 10$ and $b = 1$.

$\sqrt{101} \approx 10 + \frac{1}{2(10)}$

≈ 10.05

b. $\sqrt{622} = \sqrt{625 - 3} = \sqrt{25^2 - 3}$

Let $a = 25$ and $b = 3$.

$\sqrt{622} \approx 25 - \frac{3}{2(25)}$

≈ 24.94

Exercises

Use the formula to find an approximation for each square root to the nearest hundredth. Check your work with a calculator.

1. $\sqrt{626}$

2. $\sqrt{99}$

3. $\sqrt{402}$

4. $\sqrt{1604}$

5. $\sqrt{223}$

6. $\sqrt{80}$

7. $\sqrt{4890}$

8. $\sqrt{2505}$

9. $\sqrt{3575}$

10. $\sqrt{1{,}441{,}100}$

11. $\sqrt{290}$

12. $\sqrt{260}$

13. Show that $a - \frac{b}{2a} \approx \sqrt{a^2 - b}$ for $a > b$.

7-5 Lesson Reading Guide

Operations with Radical Expressions

Get Ready for the Lesson

Read the introduction to Lesson 7-5 in your textbook.

Describe how you could use the golden ratio to find the height of a golden triangle if you knew its width.

Read the Lesson

1. Complete the conditions that must be met for a radical expression to be in simplified form.

- The ________________ n is as ________________ as possible.
- The ________________ contains no ________________ (other than 1) that are nth powers of a(n) ________________ or polynomial.
- The radicand contains no ________________.
- No ________________ appear in the ________________.

2. a. What are conjugates of radical expressions used for?

b. How would you use a conjugate to simplify the radical expression $\frac{1+\sqrt{2}}{3-\sqrt{2}}$?

c. In order to simplify the radical expression in part b, two multiplications are necessary. The multiplication in the numerator would be done by the ________________ method, and the multiplication in the denominator would be done by finding the ________________of ________________ ________________.

Remember What You Learned

3. One way to remember something is to explain it to another person. When rationalizing the denominator in the expression $\frac{1}{\sqrt[3]{2}}$, many students think they should multiply numerator and denominator by $\frac{\sqrt[3]{2}}{\sqrt[3]{2}}$. How would you explain to a classmate why this is incorrect and what he should do instead.

7-5 Study Guide and Intervention

Operations with Radical Expressions

Simplify Radical Expressions

Product Property of Radicals	For any real numbers a and b, and any integer $n > 1$: 1. if n is even and a and b are both nonnegative, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$. 2. if n is odd, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$.

To simplify a square root, follow these steps:

1. Factor the radicand into as many squares as possible.
2. Use the Product Property to isolate the perfect squares.
3. Simplify each radical.

Quotient Property of Radicals	For any real numbers a and $b \neq 0$, and any integer $n > 1$, $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$, if all roots are defined.

To eliminate radicals from a denominator or fractions from a radicand, multiply the numerator and denominator by a quantity so that the radicand has an exact root.

Example 1 **Simplify $\sqrt[3]{-16a^5b^7}$.**

$$\sqrt[3]{-16a^5b^7} = \sqrt[3]{(-2)^3 \cdot 2 \cdot a^3 \cdot a^2 \cdot (b^2)^3 \cdot b}$$
$$= -2ab^2\sqrt[3]{2a^2b}$$

Example 2 **Simplify $\sqrt{\frac{8x^3}{45y^5}}$.**

$$\sqrt{\frac{8x^3}{45y^5}} = \frac{\sqrt{8x^3}}{\sqrt{45y^5}}$$ Quotient Property

$$= \frac{\sqrt{(2x)^2 \cdot 2x}}{\sqrt{(3y^2)^2 \cdot 5y}}$$ Factor into squares.

$$= \frac{\sqrt{(2x)^2} \cdot \sqrt{2x}}{\sqrt{(3y^2)^2} \cdot \sqrt{5y}}$$ Product Property

$$= \frac{2|x|\sqrt{2x}}{3y^2\sqrt{5y}}$$ Simplify.

$$= \frac{2|x|\sqrt{2x}}{3y^2\sqrt{5y}} \cdot \frac{\sqrt{5y}}{\sqrt{5y}}$$ Rationalize the denominator.

$$= \frac{2|x|\sqrt{10xy}}{15y^3}$$ Simplify.

Exercises

Simplify.

1. $5\sqrt{54}$

2. $\sqrt[4]{32a^9b^{20}}$

3. $\sqrt{75x^4y^7}$

4. $\sqrt{\frac{36}{125}}$

5. $\sqrt{\frac{a^6b^3}{98}}$

6. $\sqrt[3]{\frac{p^5q^3}{40}}$

7-5 Study Guide and Intervention *(continued)*

Operations with Radical Expressions

Operations with Radicals When you add expressions containing radicals, you can add only like terms or **like radical expressions**. Two radical expressions are called *like radical expressions* if both the indices and the radicands are alike.

To multiply radicals, use the Product and Quotient Properties. For products of the form $(a\sqrt{b} + c\sqrt{d}) \cdot (e\sqrt{f} + g\sqrt{h})$, use the FOIL method. To rationalize denominators, use **conjugates**. Numbers of the form $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$, where a, b, c, and d are rational numbers, are called conjugates. The product of conjugates is always a rational number.

Example 1 **Simplify $2\sqrt{50} + 4\sqrt{500} - 6\sqrt{125}$.**

$2\sqrt{50} + 4\sqrt{500} - 6\sqrt{125} = 2\sqrt{5^2 \cdot 2} + 4\sqrt{10^2 \cdot 5} - 6\sqrt{5^2 \cdot 5}$ Factor using squares.

$= 2 \cdot 5 \cdot \sqrt{2} + 4 \cdot 10 \cdot \sqrt{5} - 6 \cdot 5 \cdot \sqrt{5}$ Simplify square roots.

$= 10\sqrt{2} + 40\sqrt{5} - 30\sqrt{5}$ Multiply.

$= 10\sqrt{2} + 10\sqrt{5}$ Combine like radicals.

Example 2 **Simplify $(2\sqrt{3} - 4\sqrt{2})(\sqrt{3} + 2\sqrt{2})$.**

$(2\sqrt{3} - 4\sqrt{2})(\sqrt{3} + 2\sqrt{2})$

$= 2\sqrt{3} \cdot \sqrt{3} + 2\sqrt{3} \cdot 2\sqrt{2} - 4\sqrt{2} \cdot \sqrt{3} - 4\sqrt{2} \cdot 2\sqrt{2}$

$= 6 + 4\sqrt{6} - 4\sqrt{6} - 16$

$= -10$

Example 3 **Simplify $\dfrac{2 - \sqrt{5}}{3 + \sqrt{5}}$.**

$$\frac{2 - \sqrt{5}}{3 + \sqrt{5}} = \frac{2 - \sqrt{5}}{3 + \sqrt{5}} \cdot \frac{3 - \sqrt{5}}{3 - \sqrt{5}}$$

$$= \frac{6 - 2\sqrt{5} - 3\sqrt{5} + (\sqrt{5})^2}{3^2 - (\sqrt{5})^2}$$

$$= \frac{6 - 5\sqrt{5} + 5}{9 - 5}$$

$$= \frac{11 - 5\sqrt{5}}{4}$$

Exercises

Simplify.

1. $3\sqrt{2} + \sqrt{50} - 4\sqrt{8}$

2. $\sqrt{20} + \sqrt{125} - \sqrt{45}$

3. $\sqrt{300} - \sqrt{27} - \sqrt{75}$

4. $\sqrt[3]{81} \cdot \sqrt[3]{24}$

5. $\sqrt[3]{2}(\sqrt[3]{4} + \sqrt[3]{12})$

6. $2\sqrt{3}(\sqrt{15} + \sqrt{60})$

7. $(2 + 3\sqrt{7})(4 + \sqrt{7})$

8. $(6\sqrt{3} - 4\sqrt{2})(3\sqrt{3} + \sqrt{2})$

9. $(4\sqrt{2} - 3\sqrt{5})(2\sqrt{20 + 5})$

10. $\dfrac{5\sqrt{48} + \sqrt{75}}{5\sqrt{3}}$

11. $\dfrac{4 + \sqrt{2}}{2 - \sqrt{2}}$

12. $\dfrac{5 - 3\sqrt{3}}{1 + 2\sqrt{3}}$

7-5 Skills Practice

Operations with Radical Expressions

Simplify.

1. $\sqrt{24}$

2. $\sqrt{75}$

3. $\sqrt[3]{16}$

4. $-\sqrt[4]{48}$

5. $4\sqrt{50x^5}$

6. $\sqrt[4]{64a^4b^4}$

7. $\sqrt[3]{-\frac{1}{8}d^2f^5}$

8. $\sqrt{\frac{25}{36}s^2t}$

9. $-\sqrt{\frac{3}{7}}$

10. $\sqrt[3]{\frac{2}{9}}$

11. $\sqrt{\frac{2g^3}{5z}}$

12. $(3\sqrt{3})(5\sqrt{3})$

13. $(4\sqrt{12})(3\sqrt{20})$

14. $\sqrt{2} + \sqrt{8} + \sqrt{50}$

15. $\sqrt{12} - 2\sqrt{3} + \sqrt{108}$

16. $8\sqrt{5} - \sqrt{45} - \sqrt{80}$

17. $2\sqrt{48} - \sqrt{75} - \sqrt{12}$

18. $(2 + \sqrt{3})(6 - \sqrt{2})$

19. $(1 - \sqrt{5})(1 + \sqrt{5})$

20. $(3 - \sqrt{7})(5 + \sqrt{2})$

21. $(\sqrt{2} - \sqrt{6})^2$

22. $\frac{3}{7 - \sqrt{2}}$

23. $\frac{4}{3 + \sqrt{2}}$

24. $\frac{5}{8 - \sqrt{6}}$

7-5 Practice

Operations with Radical Expressions

Simplify.

1. $\sqrt{540}$

2. $\sqrt[3]{-432}$

3. $\sqrt[3]{128}$

4. $-\sqrt[4]{405}$

5. $\sqrt[3]{-5000}$

6. $\sqrt[5]{-1215}$

7. $\sqrt[3]{125t^6w^2}$

8. $\sqrt[4]{48v^8z^{13}}$

9. $\sqrt[3]{8g^3k^8}$

10. $\sqrt{45x^3y^8}$

11. $\sqrt{\frac{11}{9}}$

12. $\sqrt[3]{\frac{216}{24}}$

13. $\sqrt{\frac{1}{128}c^4d^7}$

14. $\sqrt{\frac{9a^5}{64b^4}}$

15. $\sqrt[4]{\frac{8}{9a^3}}$

16. $(3\sqrt{15})(-4\sqrt{45})$

17. $(2\sqrt{24})(7\sqrt{18})$

18. $\sqrt{810} + \sqrt{240} - \sqrt{250}$

19. $6\sqrt{20} + 8\sqrt{5} - 5\sqrt{45}$

20. $8\sqrt{48} - 6\sqrt{75} + 7\sqrt{80}$

21. $(3\sqrt{2} + 2\sqrt{3})^2$

22. $(3 - \sqrt{7})^2$

23. $(\sqrt{5} - \sqrt{6})(\sqrt{5} + \sqrt{2})$

24. $(\sqrt{2} + \sqrt{10})(\sqrt{2} - \sqrt{10})$

25. $(1 + \sqrt{6})(5 - \sqrt{7})$

26. $(\sqrt{3} + 4\sqrt{7})^2$

27. $(\sqrt{108} - 6\sqrt{3})^2$

28. $\frac{\sqrt{3}}{\sqrt{5} - 2}$

29. $\frac{6}{\sqrt{2} - 1}$

30. $\frac{5 + \sqrt{3}}{4 + \sqrt{3}}$

31. $\frac{3 + \sqrt{2}}{2 - \sqrt{2}}$

32. $\frac{3 + \sqrt{6}}{5 - \sqrt{24}}$

33. $\frac{3 + \sqrt{x}}{2 - \sqrt{x}}$

34. BRAKING The formula $s = 2\sqrt{5\ell}$ estimates the speed s in miles per hour of a car when it leaves skid marks ℓ feet long. Use the formula to write a simplified expression for s if $\ell = 85$. Then evaluate s to the nearest mile per hour.

35. PYTHAGOREAN THEOREM The measures of the legs of a right triangle can be represented by the expressions $6x^2y$ and $9x^2y$. Use the Pythagorean Theorem to find a simplified expression for the measure of the hypotenuse.

7-5 Word Problem Practice

Operations with Radical Expressions

1. CUBES Cathy has a rectangular box with dimensions 20 inches by 35 inches by 40 inches. She would like to replace it with a box in the shape of a cube but with the same volume. What should the length of a side of the cube be? Express your answer as a radical expression in simplest form.

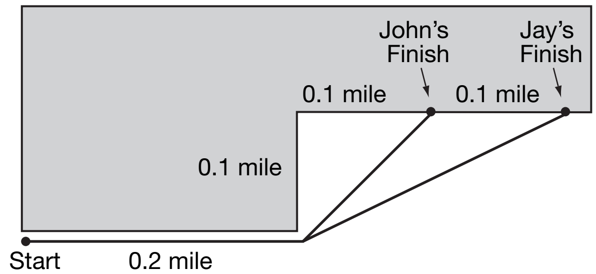
2. PHYSICS The speed of a wave traveling over a string is given by $\frac{\sqrt{t}}{\sqrt{u}}$, where t is the tension of the string and u is the density. Rewrite the expression in simplest form by rationalizing the denominator.

3. TUNING With each note higher on a piano, the frequency of the pitch increases by a factor of $\sqrt[12]{2}$. What is the ratio of the frequencies of two notes that are 6 steps apart on the piano? What is the ratio of the frequencies of two notes that are 9 steps apart on the piano? Express your answers in simplest form.

4. LIGHTS Suppose a light has a brightness intensity of I_1 when it is at a distance of d_1 and a brightness intensity of I_2 when it is at a distance of d_2. These quantities are related by the equation $\frac{d_2}{d_1} = \sqrt{\frac{I_1}{I_2}}$. Suppose $I_1 = 50$ units and $I_2 = 24$ units. What would $\frac{d_2}{d_1}$ be? Express your answer in simplest form.

RACING For Exercises 5 and 6, use the following information and express your answers in simplest form.

John is Jay's younger brother. They like to race and, after many races, they found that the fairest race was to run slightly different distances. They both start at the same place and run straight for 0.2 miles. Then they head for different finishes. In the figure, John and Jay's finishing paths are marked.



This time, they tied. Both of them finished the race in exactly 4 minutes.

5. If John and Jay continued at their average paces during the race, exactly how many minutes would it take them each to run a mile? Express your answer as a radical expression in simplest form.

6. Exactly how many times as fast did John run as Jay?

Lesson 7-5

7-5 Enrichment

Special Products with Radicals

Notice that $(\sqrt{3})(\sqrt{3}) = 3$, or $(\sqrt{3})^2 = 3$. In general, $(\sqrt{x})^2 = x$ when $x \geq 0$.

Also, notice that $(\sqrt{9})(\sqrt{4}) = \sqrt{36}$. In general, $(\sqrt{x})(\sqrt{y}) = \sqrt{xy}$ when x and y are not negative.

You can use these ideas to find the special products below.

$$(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = a - b$$
$$(\sqrt{a} + \sqrt{b})^2 = (\sqrt{a})^2 + 2\sqrt{ab} + (\sqrt{b})^2 = a + 2\sqrt{ab} + b$$
$$(\sqrt{a} - \sqrt{b})^2 = (\sqrt{a})^2 - 2\sqrt{ab} + (\sqrt{b})^2 = a - 2\sqrt{ab} + b$$

Example 1 **Find the product: $(\sqrt{2} + \sqrt{5})(\sqrt{2} - \sqrt{5})$.**

$(\sqrt{2} + \sqrt{5})(\sqrt{2} - \sqrt{5}) = (\sqrt{2})^2 - (\sqrt{5})^2 = 2 - 5 = -3$

Example 2 **Evaluate $(\sqrt{2} + \sqrt{8})^2$.**

$(\sqrt{2} + \sqrt{8})^2 = (\sqrt{2})^2 + 2\sqrt{2}\sqrt{8} + (\sqrt{8})^2$

$= 2 + 2\sqrt{16} + 8 = 2 + 2(4) + 8 = 2 + 8 + 8 = 18$

Exercises

Multiply.

1. $(\sqrt{3} - \sqrt{7})(\sqrt{3} + \sqrt{7})$

2. $(\sqrt{10} + \sqrt{2})(\sqrt{10} - \sqrt{2})$

3. $(\sqrt{2x} - \sqrt{6})(\sqrt{2x} + \sqrt{6})$

4. $(\sqrt{3} - (-7))^2$

5. $(\sqrt{1000} + \sqrt{10})^2$

6. $(\sqrt{y} + \sqrt{5})(\sqrt{y} - \sqrt{5})$

7. $(\sqrt{50} - \sqrt{x})^2$

8. $(\sqrt{x} + 20)^2$

You can extend these ideas to patterns for sums and differences of cubes. Study the pattern below. Then complete Exercises 9–12.

$(\sqrt[3]{8} - \sqrt[3]{x})(\sqrt[3]{8^2} + \sqrt[3]{8x} + \sqrt[3]{x^2}) = \sqrt[3]{8^3} - \sqrt[3]{x^3} = 8 - x$

9. $(\sqrt[3]{2} - \sqrt[3]{5})(\sqrt[3]{2^2} + \sqrt[3]{10} + \sqrt[3]{5^2})$

10. $(\sqrt[3]{y} + \sqrt[3]{w})(\sqrt[3]{y^2} - \sqrt[3]{yw} + \sqrt[3]{w^2})$

11. $(\sqrt[3]{7} + \sqrt[3]{20})(\sqrt[3]{7^2} - \sqrt[3]{140} + \sqrt[3]{20^2})$

12. $(\sqrt[3]{11} - \sqrt[3]{8})(\sqrt[3]{11^2} + \sqrt[3]{88} + \sqrt[3]{8^2})$

7-6 Lesson Reading Guide

Rational Exponents

Get Ready for the Lesson

Read the introduction to Lesson 7-6 in your textbook.

The formula in the introduction contains the exponent $\frac{2}{5}$. What do you think it might mean to raise a number to the $\frac{2}{5}$ power?

Read the Lesson

1. Complete the following definitions of rational exponents.

- For any real number b and for any positive integer n, $b^{\frac{1}{n}}$ = ________________ except when b ________________ and n is ________________.
- For any nonzero real number b, and any integers m and n, with n ________________, $b^{\frac{m}{n}}$ = ________________ = ________________, except when b ________________ and n is ________________.

2. Complete the conditions that must be met in order for an expression with rational exponents to be simplified.

- It has no ________________ exponents.
- It has no ________________ exponents in the ________________.
- It is not a ________________ fraction.
- The ________________ of any remaining ________________ is the ________________ number possible.

3. Margarita and Pierre were working together on their algebra homework. One exercise asked them to evaluate the expression $27^{\frac{4}{3}}$. Margarita thought that they should raise 27 to the fourth power first and then take the cube root of the result. Pierre thought that they should take the cube root of 27 first and then raise the result to the fourth power. Whose method is correct?

Remember What You Learned

4. Some students have trouble remembering which part of the fraction in a rational exponent gives the power and which part gives the root. How can your knowledge of integer exponents help you to keep this straight?

NAME ____ DATE_____________ PERIOD _____

7-6 Study Guide and Intervention

Rational Exponents

Rational Exponents and Radicals

Definition of $b^{\frac{1}{n}}$	For any real number b and any positive integer n, $b^{\frac{1}{n}} = \sqrt[n]{b}$, except when $b < 0$ and n is even.
Definition of $b^{\frac{m}{n}}$	For any nonzero real number b, and any integers m and n, with $n > 1$, $b^{\frac{m}{n}} = \sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m$, except when $b < 0$ and n is even.

Example 1 **Write $28^{\frac{1}{2}}$ in radical form.**

Notice that $28 > 0$.

$$\begin{aligned} 28^{\frac{1}{2}} &= \sqrt{28} \\ &= \sqrt{2^2 \cdot 7} \\ &= \sqrt{2^2} \cdot \sqrt{7} \\ &= 2\sqrt{7} \end{aligned}$$

Example 2 **Evaluate $\left(\frac{-8}{-125}\right)^{\frac{1}{3}}$.**

Notice that $-8 < 0$, $-125 < 0$, and 3 is odd.

$$\begin{aligned} \left(\frac{-8}{-125}\right)^{\frac{1}{3}} &= \frac{\sqrt[3]{-8}}{\sqrt[3]{-125}} \\ &= \frac{-2}{-5} \\ &= \frac{2}{5} \end{aligned}$$

Exercises

Write each expression in radical form.

1. $11^{\frac{1}{7}}$

2. $15^{\frac{1}{3}}$

3. $300^{\frac{3}{2}}$

Write each radical using rational exponents.

4. $\sqrt{47}$

5. $\sqrt[3]{3a^5b^2}$

6. $\sqrt[4]{162p^5}$

Evaluate each expression.

7. $-27^{\frac{2}{3}}$

8. $\dfrac{5^{-\frac{1}{2}}}{2\sqrt{5}}$

9. $(0.0004)^{\frac{1}{2}}$

10. $8^{\frac{2}{3}} \cdot 4^{\frac{3}{2}}$

11. $\dfrac{144^{-\frac{1}{2}}}{27^{-\frac{1}{3}}}$

12. $\dfrac{16^{-\frac{1}{2}}}{(0.25)^{\frac{1}{2}}}$

7-6 Study Guide and Intervention *(continued)*

Rational Exponents

Simplify Expressions All the properties of powers from Lesson 6-1 apply to rational exponents. When you simplify expressions with rational exponents, leave the exponent in rational form, and write the expression with all positive exponents. Any exponents in the denominator must be positive integers.

When you simplify radical expressions, you may use rational exponents to simplify, but your answer should be in radical form. Use the smallest index possible.

Example 1 **Simplify $y^{\frac{2}{3}} \cdot y^{\frac{3}{8}}$.**

$y^{\frac{2}{3}} \cdot y^{\frac{3}{8}} = y^{\frac{2}{3} + \frac{3}{8}} = y^{\frac{25}{24}}$

Example 2 **Simplify $\sqrt[4]{144x^6}$.**

$\sqrt[4]{144x^6} = (144x^6)^{\frac{1}{4}}$

$= (2^4 \cdot 3^2 \cdot x^6)^{\frac{1}{4}}$

$= (2^4)^{\frac{1}{4}} \cdot (3^2)^{\frac{1}{4}} \cdot (x^6)^{\frac{1}{4}}$

$= 2 \cdot 3^{\frac{1}{2}} \cdot x^{\frac{3}{2}} = 2x \cdot (3x)^{\frac{1}{2}} = 2x\sqrt{3x}$

Exercises

Simplify each expression.

1. $x^{\frac{4}{5}} \cdot x^{\frac{6}{5}}$

2. $\left(y^{\frac{2}{3}}\right)^{\frac{3}{4}}$

3. $p^{\frac{4}{5}} \cdot p^{\frac{7}{10}}$

4. $\left(m^{-\frac{6}{5}}\right)^{\frac{2}{5}}$

5. $x^{-\frac{3}{8}} \cdot x^{\frac{4}{3}}$

6. $\left(s^{-\frac{1}{6}}\right)^{-\frac{4}{3}}$

7. $\dfrac{p}{p^{\frac{1}{3}}}$

8. $\left(a^{\frac{2}{3}}\right)^{\frac{6}{5}} \cdot \left(a^{\frac{2}{5}}\right)^{3}$

9. $\dfrac{x^{-\frac{1}{2}}}{x^{-\frac{1}{3}}}$

10. $\sqrt[6]{128}$

11. $\sqrt[4]{49}$

12. $\sqrt[5]{288}$

13. $\sqrt{32} \cdot 3\sqrt{16}$

14. $\sqrt[3]{25} \cdot \sqrt{125}$

15. $\sqrt[6]{16}$

16. $\dfrac{x - \sqrt[3]{3}}{\sqrt{12}}$

17. $\sqrt{\sqrt[3]{48}}$

18. $\dfrac{a\sqrt[3]{b^4}}{\sqrt{ab^3}}$

NAME ______________________ DATE __________ PERIOD _____

7-6 Skills Practice

Rational Exponents

Write each expression in radical form.

1. $3^{\frac{1}{6}}$

2. $8^{\frac{1}{5}}$

3. $12^{\frac{2}{3}}$

4. $(s^3)^{\frac{3}{5}}$

Write each radical using rational exponents.

5. $\sqrt{51}$

6. $\sqrt[3]{37}$

7. $\sqrt[4]{15^3}$

8. $\sqrt[3]{6xy^2}$

Evaluate each expression.

9. $32^{\frac{1}{5}}$

10. $81^{\frac{1}{4}}$

11. $27^{-\frac{1}{3}}$

12. $4^{-\frac{1}{2}}$

13. $16^{\frac{3}{2}}$

14. $(-243)^{\frac{4}{5}}$

15. $27^{\frac{1}{3}} \cdot 27^{\frac{5}{3}}$

16. $\left(\frac{4}{9}\right)^{\frac{3}{2}}$

Simplify each expression.

17. $c^{\frac{12}{5}} \cdot c^{\frac{3}{5}}$

18. $m^{\frac{2}{9}} \cdot m^{\frac{16}{9}}$

19. $\left(q^{\frac{1}{2}}\right)^3$

20. $p^{-\frac{1}{5}}$

21. $x^{-\frac{6}{11}}$

22. $\dfrac{x^{\frac{2}{3}}}{x^{\frac{1}{4}}}$

23. $\dfrac{y^{-\frac{1}{2}}}{y^{\frac{1}{4}}}$

24. $\dfrac{n^{\frac{1}{3}}}{n^{\frac{1}{6}} \cdot n^{\frac{1}{2}}}$

25. $\sqrt[12]{64}$

26. $\sqrt[8]{49a^8b^2}$

7-6 Practice

Rational Exponents

Write each expression in radical form.

1. $5^{\frac{1}{3}}$ **2.** $6^{\frac{2}{5}}$ **3.** $m^{\frac{4}{7}}$ **4.** $(n^3)^{\frac{2}{5}}$

Write each radical using rational exponents.

5. $\sqrt{79}$ **6.** $\sqrt[4]{153}$ **7.** $\sqrt[3]{27m^6n^4}$ **8.** $5\sqrt{2a^{10}b}$

Evaluate each expression.

9. $81^{\frac{1}{4}}$ **10.** $1024^{-\frac{1}{5}}$ **11.** $8^{-\frac{5}{3}}$

12. $-256^{-\frac{3}{4}}$ **13.** $(-64)^{-\frac{2}{3}}$ **14.** $27^{\frac{1}{3}} \cdot 27^{\frac{4}{3}}$

15. $\left(\frac{125}{216}\right)^{\frac{2}{3}}$ **16.** $\frac{64^{\frac{2}{3}}}{343^{\frac{2}{3}}}$ **17.** $\left(25^{\frac{1}{2}}\right)\left(-64^{-\frac{1}{3}}\right)$

Simplify each expression.

18. $g^{\frac{4}{7}} \cdot g^{\frac{3}{7}}$ **19.** $s^{\frac{3}{4}} \cdot s^{\frac{13}{4}}$ **20.** $\left(u^{-\frac{1}{3}}\right)^{-\frac{4}{5}}$ **21.** $y^{-\frac{1}{2}}$

22. $b^{-\frac{3}{5}}$ **23.** $\frac{q^{\frac{3}{5}}}{q^{\frac{2}{5}}}$ **24.** $\frac{t^{\frac{2}{3}}}{5t^{\frac{1}{2}} \cdot t^{-\frac{3}{4}}}$ **25.** $\frac{2z^{\frac{1}{2}}}{z^{\frac{1}{2}} - 1}$

26. $\sqrt[10]{8^5}$ **27.** $\sqrt{12} \cdot \sqrt[5]{12^3}$ **28.** $\sqrt[4]{6} \cdot 3\sqrt[4]{6}$ **29.** $\frac{a}{\sqrt{3b}}$

30. ELECTRICITY The amount of current in amperes I that an appliance uses can be calculated using the formula $I = \left(\frac{P}{R}\right)^{\frac{1}{2}}$, where P is the power in watts and R is the resistance in ohms. How much current does an appliance use if $P = 500$ watts and $R = 10$ ohms? Round your answer to the nearest tenth.

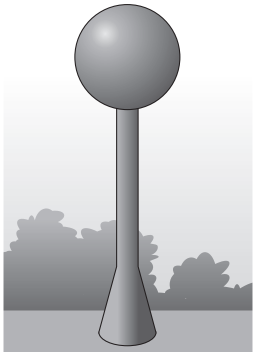
31. BUSINESS A company that produces DVDs uses the formula $C = 88n^{\frac{1}{3}} + 330$ to calculate the cost C in dollars of producing n DVDs per day. What is the company's cost to produce 150 DVDs per day? Round your answer to the nearest dollar.

7-6 Word Problem Practice

Rational Exponents

1. SQUARING THE CUBE A cube has side length s. What is the side length of the square that has an area equal to the volume of this cube? Write your answer using rational exponents.

2. WATER TOWER A large water tower stores drinking water in a big spherical tank. The mayor of the town decides that the water tower must be replaced with a larger tank. Town residents insist that the new tower be a sphere. If the new tank will hold 10 times as much water as the old tank, how many times long should the radius of the new tank be compared to the old tank? Write your answer using rational exponents.



3. BALLOONS A spherical balloon is being inflated faster and faster. The volume of the balloon as a function of time is $9\pi t^2$. What is the radius of the balloon as a function of time? Write your answer using rational exponents.

4. INTEREST Rita opened a bank account that accumulated interest at the rate of 1% compounded annually. Her money accumulated interest in that account for 8 years. She then took all of her money out of that account and placed it into another account that paid 5% interest compounded annually. After 4 years, she took all of her money out of that account. What single interest rate when compounded annually would give her the same outcome for those 12 years? Round your answer to the nearest hundredth of a percent.

CELLS For Exercises 5-7, use the following information.

The number of cells in a cell culture grows exponentially. The number of cells in the culture as a function of time is given by the expression $N\left(\frac{6}{5}\right)^t$ where t is measured in hours and N is the initial size of the culture.

5. After 3 hours, there were 1728 cells in the culture. What is N?

6. How many cells were in the culture after 20 minutes? Express your answer in simplest form.

7. How many cells were in the culture after 2.5 hours? Express your answer in simplest form.

7-6 Enrichment

Lesser-Known Geometric Formulas

Many geometric formulas involve radical expressions.

Make a drawing to illustrate each of the formulas given on this page. Then evaluate the formula for the given value of the variable. Round answers to the nearest hundredth.

1. The area of an isosceles triangle. Two sides have length a; the other side has length c. Find A when $a = 6$ and $c = 7$.
   $A = \frac{c}{4}\sqrt{4a^2 - c^2}$

2. The area of an equilateral triangle with a side of length a. Find A when $a = 8$.
   $A = \frac{a^2}{4}\sqrt{3}$

3. The area of a regular pentagon with a side of length a. Find A when $a = 4$.
   $A = \frac{a^2}{4}\sqrt{25 + 10\sqrt{5}}$

4. The area of a regular hexagon with a side of length a. Find A when $a = 9$.
   $A = \frac{3a^2}{2}\sqrt{3}$

5. The volume of a regular tetrahedron with an edge of length a. Find V when $a = 2$.
   $V = \frac{a^3}{12}\sqrt{2}$

6. The area of the curved surface of a right cone with an altitude of h and radius of base r. Find S when $r = 3$ and $h = 6$.
   $S = \pi r\sqrt{r^2 + h^2}$

7. Heron's Formula for the area of a triangle uses the semi-perimeter s, where $s = \frac{a + b + c}{2}$. The sides of the triangle have lengths a, b, and c. Find A when $a = 3$, $b = 4$, and $c = 5$.
   $A = \sqrt{s(s - a)(s - b)(s - c)}$

8. The radius of a circle inscribed in a given triangle also uses the semi-perimeter. Find r when $a = 6$, $b = 7$, and $c = 9$.
   $r = \frac{\sqrt{s(s - a)(s - b)(s - c)}}{s}$

7-6 Spreadsheet Activity

Appreciation and Depreciation

When an asset such as a house increases in value over time, it is said to *appreciate*. If the value increases by a fixed percent each year, or other period of time, the amount y of that quantity after t years is given by

$$y = a(1 + r)^t,$$

where a is the initial amount and r is the percent of increase expressed as a decimal. You can use a spreadsheet to investigate future values of an asset.

Example **Michael Blackstock is considering buying a piece of investment property for $95,000. The homes in the area are appreciating at an average rate of 4% per year. Find the expected value of the home in 1 year, 1 year and 6 months, 4 years, and 6 years and 9 months.**

Use rows 1 and 2 to enter the initial amount and the rate of increase. Then use Column A to enter the amounts of time. Enter the numbers of months as a fraction of a year since t is measured in years. Column B contains the formulas for the value of the home.

Format the cells containing the values as currency so that they are displayed as dollars and cents. The expected value of the home after each amount of time is shown in the spreadsheet.

Appreciation.xls

	A	B
1	Initial value =	$95,000.00
2	Rate =	0.04
3		
4	Years	Value
5	1	$98,800.00
6	1.5	$100,756.63
7	4	$111,136.56
8	6.75	$123,793.73

Sheet 1 | Sheet 2

Exercises

1. If Mr. Blackstock chooses another property in the neighborhood that costs $99,900, what are the expected values of that home in the same periods of time?

2. What would Mr. Blackstock's profit be on the $99,900 home if he sold it after 9 years and 3 months?

3. If an antique chair worth $165.00 increases in value an average of $3\frac{1}{2}\%$ every year, how much will it be worth next year?

4. Often assets like cars decrease in value over time. This asset is said to *depreciate*. If the value decreases by a fixed percent each year, or other period of time, the amount y of that quantity after t years is given by $y = a(1 - r)^t$, where a is the initial amount and r is the percent of decrease expressed as a decimal. Use a spreadsheet to find the value of a car purchased for $18,500 after 2 years, 2 years and 6 months, and 4 years and 3 months if the car depreciates at a rate of 12% per year.

7-7 Lesson Reading Guide

Solving Radical Equations and Inequalities

Get Ready for the lesson

Read the introduction to Lesson 7-7 in your textbook.

Explain how you would use the formula in your textbook to find the cost of producing 125,000 computer chips. (Describe the steps of the calculation in the order in which you would perform them, but do not actually do the calculation.)

Read the Lesson

1. a. What is an *extraneous solution* of a radical equation?

b. Describe two ways you can check the proposed solutions of a radical equation in order to determine whether any of them are extraneous solutions.

2. Complete the steps that should be followed in order to solve a radical inequality.

Step 1 If the ________________ of the root is ________________, identify the values of the variable for which the ________________ is ________________.

Step 2 Solve the ________________ algebraically.

Step 3 Test ________________ to check your solution.

Remember What You Learned

3. One way to remember something is to explain it to another person. Suppose that your friend Leora thinks that she does not need to check her solutions to radical equations by substitution because she knows she is very careful and seldom makes mistakes in her work. How can you explain to her that she should nevertheless check every proposed solution in the original equation?

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7-7 Study Guide and Intervention

Solving Radical Equations and Inequalities

Solve Radical Equations The following steps are used in solving equations that have variables in the radicand. Some algebraic procedures may be needed before you use these steps.

Step 1	Isolate the radical on one side of the equation.
Step 2	To eliminate the radical, raise each side of the equation to a power equal to the index of the radical.
Step 3	Solve the resulting equation.
Step 4	Check your solution in the original equation to make sure that you have not obtained any extraneous roots.

Example 1 **Solve $2\sqrt{4x + 8} - 4 = 8$.**

$2\sqrt{4x + 8} - 4 = 8$	Original equation
$2\sqrt{4x + 8} = 12$	Add 4 to each side.
$\sqrt{4x + 8} = 6$	Isolate the radical.
$4x + 8 = 36$	Square each side.
$4x = 28$	Subtract 8 from each side.
$x = 7$	Divide each side by 4.

Check

$2\sqrt{4(7) + 8} - 4 \stackrel{?}{=} 8$

$2\sqrt{36} - 4 \stackrel{?}{=} 8$

$2(6) - 4 \stackrel{?}{=} 8$

$8 = 8$

The solution $x = 7$ checks.

Example 2 **Solve $\sqrt{3x + 1} = \sqrt{5x} - 1$.**

$\sqrt{3x + 1} = \sqrt{5x} - 1$	Original equation
$3x + 1 = 5x - 2\sqrt{5x} + 1$	Square each side.
$2\sqrt{5x} = 2x$	Simplify.
$\sqrt{5x} = x$	Isolate the radical.
$5x = x^2$	Square each side.
$x^2 - 5x = 0$	Subtract $5x$ from each side.
$x(x - 5) = 0$	Factor.
$x = 0$ or $x = 5$	

Check

$\sqrt{3(0) + 1} = 1$, but $\sqrt{5(0)} - 1 = -1$, so 0 is not a solution.

$\sqrt{3(5) + 1} = 4$, and $\sqrt{5(5)} - 1 = 4$, so the solution is $x = 5$.

Exercises

Solve each equation.

1. $3 + 2x\sqrt{3} = 5$

2. $2\sqrt{3x + 4} + 1 = 15$

3. $8 + \sqrt{x + 1} = 2$

4. $\sqrt{5 - x} - 4 = 6$

5. $12 + \sqrt{2x - 1} = 4$

6. $\sqrt{12 - x} = 0$

7. $\sqrt{21} - \sqrt{5x - 4} = 0$

8. $10 - \sqrt{2x} = 5$

9. $\sqrt{x^2 + 7x} = \sqrt{7x - 9}$

10. $4\sqrt[3]{2x + 11} - 2 = 10$

11. $2\sqrt{x + 11} = \sqrt{x + 2} + \sqrt{3x - 6}$

12. $\sqrt{9x - 11} = x + 1$

7-7 Study Guide and Intervention *(continued)*

Solving Radical Equations and Inequalities

Solve Radical Inequalities A **radical inequality** is an inequality that has a variable in a radicand. Use the following steps to solve radical inequalities.

Step 1	If the index of the root is even, identify the values of the variable for which the radicand is nonnegative.
Step 2	Solve the inequality algebraically.
Step 3	Test values to check your solution.

Example **Solve $5 - \sqrt{20x + 4} \geq -3$.**

Since the radicand of a square root must be greater than or equal to zero, first solve

$20x + 4 \geq 0$.

$$20x + 4 \geq 0$$
$$20x \geq -4$$
$$x \geq -\frac{1}{5}$$

Now solve $5 - \sqrt{20x + 4} \geq -3$.

$5 - \sqrt{20x + 4} \geq -3$ Original inequality

$\sqrt{20x + 4} \leq 8$ Isolate the radical.

$20x + 4 \leq 64$ Eliminate the radical by squaring each side.

$20x \leq 60$ Subtract 4 from each side.

$x \leq 3$ Divide each side by 20.

It appears that $-\frac{1}{5} \leq x \leq 3$ is the solution. Test some values.

$x = -1$	$x = 0$	$x = 4$
$\sqrt{20(-1) + 4}$ is not a real number, so the inequality is not satisfied.	$5 - \sqrt{20(0) + 4} = 3$, so the inequality is satisfied.	$5 - \sqrt{20(4) + 4} \approx -4.2$, so the inequality is not satisfied

Therefore the solution $-\frac{1}{5} \leq x \leq 3$ checks.

Exercises

Solve each inequality.

1. $\sqrt{c - 2} + 4 \geq 7$

2. $3\sqrt{2x - 1} + 6 < 15$

3. $\sqrt{10x + 9} - 2 > 5$

4. $5\sqrt[3]{x + 2} - 8 < 2$

5. $8 - \sqrt{3x + 4} \geq 3$

6. $\sqrt{2x + 8} - 4 > 2$

7. $9 - \sqrt{6x + 3} \geq 6$

8. $\frac{20}{\sqrt{3x + 1}} \leq 4$

9. $2\sqrt{5x - 6} - 1 < 5$

10. $\sqrt{2x + 12} + 4 \geq 12$

11. $\sqrt{2d + 1} + \sqrt{d} \leq 5$

12. $4\sqrt{b + 3} - \sqrt{b - 2} \geq 10$

Lesson 7-7

7-7 Skills Practice

Solving Radical Equations and Inequalities

Solve each equation or inequality.

1. $\sqrt{x} = 5$

2. $\sqrt{x} + 3 = 7$

3. $5\sqrt{j} = 1$

4. $v^{\frac{1}{2}} + 1 = 0$

5. $18 - 3y^{\frac{1}{2}} = 25$

6. $\sqrt[3]{2w} = 4$

7. $\sqrt{b - 5} = 4$

8. $\sqrt{3n + 1} = 5$

9. $\sqrt[3]{3r - 6} = 3$

10. $2 + \sqrt{3p + 7} = 6$

11. $\sqrt{k - 4} - 1 = 5$

12. $(2d + 3)^{\frac{1}{3}} = 2$

13. $(t - 3)^{\frac{1}{3}} = 2$

14. $4 - (1 - 7u)^{\frac{1}{3}} = 0$

15. $\sqrt{3z - 2} = \sqrt{z - 4}$

16. $\sqrt{g + 1} = \sqrt{2g - 7}$

17. $\sqrt{x - 1} = 4\sqrt{x + 1}$

18. $5 + \sqrt{s - 3} \leq 6$

19. $-2 + \sqrt{3x + 3} < 7$

20. $-\sqrt{2a + 4} \geq -6$

21. $2\sqrt{4r - 3} > 10$

22. $4 - \sqrt{3x + 1} > 3$

23. $\sqrt{y + 4} - 3 \geq 3$

24. $-3\sqrt{11r + 3} \geq -15$

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7-7 Practice

Solving Radical Equations and Inequalities

Lesson 7-7

Solve each equation or inequality.

1. $\sqrt{x} = 8$

2. $4 - \sqrt{x} = 3$

3. $\sqrt{2p} + 3 = 10$

4. $4\sqrt{3h} - 2 = 0$

5. $c^{\frac{1}{2}} + 6 = 9$

6. $18 + 7h^{\frac{1}{2}} = 12$

7. $\sqrt[3]{d + 2} = 7$

8. $\sqrt[5]{w - 7} = 1$

9. $6 + \sqrt[3]{q - 4} = 9$

10. $\sqrt[4]{y - 9} + 4 = 0$

11. $\sqrt{2m - 6} - 16 = 0$

12. $\sqrt[3]{4m + 1} - 2 = 2$

13. $\sqrt{8n - 5} - 1 = 2$

14. $\sqrt{1 - 4t} - 8 = -6$

15. $\sqrt{2t - 5} - 3 = 3$

16. $(7v - 2)^{\frac{1}{4}} + 12 = 7$

17. $(3g + 1)^{\frac{1}{2}} - 6 = 4$

18. $(6u - 5)^{\frac{1}{3}} + 2 = -3$

19. $\sqrt{2d - 5} = \sqrt{d - 1}$

20. $\sqrt{4r - 6} = \sqrt{r}$

21. $\sqrt{6x - 4} = \sqrt{2x + 10}$

22. $\sqrt{2x + 5} = \sqrt{2x + 1}$

23. $3\sqrt{a} \geq 12$

24. $\sqrt{z + 5} + 4 \leq 13$

25. $8 + \sqrt{2q} \leq 5$

26. $\sqrt{2a - 3} < 5$

27. $9 - \sqrt{c + 4} \leq 6$

28. $\sqrt[3]{x - 1} < -2$

29. STATISTICS Statisticians use the formula $\sigma = \sqrt{v}$ to calculate a standard deviation σ, where v is the variance of a data set. Find the variance when the standard deviation is 15.

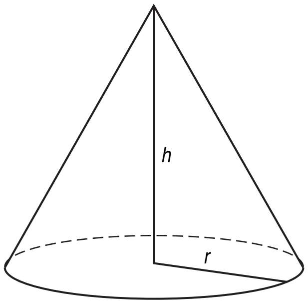
30. GRAVITATION Helena drops a ball from 25 feet above a lake. The formula $t = \frac{1}{4}\sqrt{25 - h}$ describes the time t in seconds that the ball is h feet above the water. How many feet above the water will the ball be after 1 second?

7-7 Word Problem Practice

Rational Equations and Inequalities

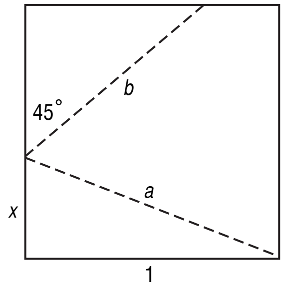
1. SIGNS A sign painter must spend $\$8n^{\frac{2}{3}} + 400$ to make n signs. How many signs can the painter make for \$1200?

2. LATERAL AREA The lateral area of a cone with base radius r and height h is given by the formula $L = \pi r\sqrt{r^2 + h^2}$. A cone has a lateral area of 65π square units and a base radius of 5 units.

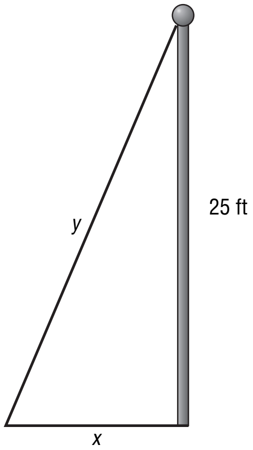


What is the height of the cone?

3. ORIGAMI Georgia wants to fold a square piece of paper into an equilateral triangle. She wants to locate the distance x up the side of the square where she can make the fold indicated by the dashed line in the figure so that $a = b$. From geometry class, she knows that $a = \sqrt{1 + x^2}$ and $b = \sqrt{2}(1 - x)$. So the equation she must solve is $\sqrt{1 + x^2} = \sqrt{2}(1 - x)$. What is x?



4. TETHERS A tether is being attached to a 25-foot pole in such a way that $x + y = 50$. By the Pythagorean Theorem, the distance $y = \sqrt{x^2 + 25^2}$. What must x be?



RANGE For Exercises 5 and 6, use the following information.

An asteroid is passing near Earth. If Earth is located at the origin of a coordinate plane, the path that the asteroid will trace out is given by $y = \frac{17}{x}$, $x > 0$. One unit corresponds to one million miles. Carl learns that he will be able to see the asteroid with his telescope when the asteroid is within $\frac{145}{12}$ million miles of Earth.

5. Write an expression that gives the distance of the asteroid from Earth as a function of x.

6. For what values of x will the asteroid be in range of Carl's telescope?

7-7 Enrichment

Truth Tables

In mathematics, the basic operations are addition, subtraction, multiplication, division, finding a root, and raising to a power. In logic, the basic operations are the following: *not* ($\sim$), *and* ($\wedge$), *or* ($\vee$), and *implies* ($\rightarrow$).

If p and q are statements, then $\sim p$ means not p; $\sim q$ means not q; $p \wedge q$ means p and q; $p \vee q$ means p or q; and $p \rightarrow q$ means p implies q. The operations are defined by truth tables. On the left below is the truth table for the statement $\sim p$. Notice that there are two possible conditions for p, true (T) or false (F). If p is true, $\sim p$ is false; if p is false, $\sim p$ is true. Also shown are the truth tables for $p \wedge q$, $p \vee q$, and $p \rightarrow q$.

p	$\sim p$
T	F
F	T

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

You can use this information to find out under what conditions a complex statement is true.

Example **Under what conditions is $\sim p \vee q$ true?**

Create the truth table for the statement. Use the information from the truth table above for $p \vee q$ to complete the last column.

p	q	$\sim p$	$\sim p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

The truth table indicates that $\sim p \vee q$ is true in all cases except where p is true and q is false.

Use truth tables to determine the conditions under which each statement is true.

1. $\sim p \vee \sim q$

2. $\sim p \rightarrow (p \rightarrow q)$

3. $(p \vee q) \vee (\sim p \wedge \sim q)$

4. $(p \rightarrow q) \vee (q \rightarrow p)$

5. $(p \rightarrow q) \wedge (q \rightarrow p)$

6. $(\sim p \wedge \sim q) \rightarrow \sim(p \vee q)$

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7 Student Recording Sheet

Use this recording sheet with pages 436–437 of the Student Edition.

Read each question. Then fill in the correct answer.

1. Ⓐ Ⓑ Ⓒ Ⓓ

2. Record your answer and fill in the bubbles in the grid below. Be sure to use the correct place value.

				.			
⓪	⓪	⓪	⓪		⓪	⓪	⓪
①	①	①	①		①	①	①
②	②	②	②		②	②	②
③	③	③	③		③	③	③
④	④	④	④		④	④	④
⑤	⑤	⑤	⑤		⑤	⑤	⑤
⑥	⑥	⑥	⑥		⑥	⑥	⑥
⑦	⑦	⑦	⑦		⑦	⑦	⑦
⑧	⑧	⑧	⑧		⑧	⑧	⑧
⑨	⑨	⑨	⑨		⑨	⑨	⑨

3. Ⓕ Ⓖ Ⓗ Ⓙ

4. Ⓐ Ⓑ Ⓒ Ⓓ

5. Ⓕ Ⓖ Ⓗ Ⓙ

6. Ⓐ Ⓑ Ⓒ Ⓓ

Pre-AP

Record your answers for Question 7 on the back of this paper.

7 Rubric for Scoring Pre-AP

(Use to score the Pre-AP question on page 437 of the Student Edition.)

General Scoring Guidelines

- If a student gives only a correct numerical answer to a problem but does not show how he or she arrived at the answer, the student will be awarded only 1 credit. All extended response questions require the student to show work.

- A fully correct answer for a multiple-part question requires correct responses for all parts of the question. For example, if a question has three parts, the correct response to one or two parts of the question that required work to be shown is *not* considered a fully correct response.

- Students who use trial and error to solve a problem must show their method. Merely showing that the answer checks or is correct is not considered a complete response for full credit.

Exercise 7 Rubric

Score	Specific Criteria
4	The period of the pendulum is found to 9.79 seconds by substituting the information into the formula $T = 2\pi\sqrt{\frac{L}{32}}$. The formula $T = 2\pi\sqrt{\frac{L}{32}}$ is correctly solved for L in terms of T, $L = 32\left(\frac{T}{2\pi}\right)^2$. The length of the pendulum is correctly determined to be 3.2 feet for a period of 2 seconds.
3	A generally correct solution, but may contain minor flaws in reasoning or computation.
2	A partially correct interpretation and/or solution to the problem.
1	A correct solution with no evidence or explanation.
0	An incorrect solution indicating no mathematical understanding of the concept or task, or no solution given.

NAME ______________ DATE ______ PERIOD ______

7 Chapter 7 Quiz 1

SCORE ______

(Lessons 7–1 and 7–2)

1. Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for $f(x) = x^2 - 3x + 2$ and $g(x) = 2x + 4$.

2. For $f(x) = \{(2, 3), (4, 4), (5, 8)\}$ and $g(x) = \{(2, 4), (3, 5), (4, 2), (8, 4)\}$, find $f \circ g$ and $g \circ f$ if they exist.

3. Find $[g \circ h](x)$ and $[h \circ g](x)$ for $g(x) = x^2 + 2x - 1$ and $h(x) = x - 4$.

4. If $f(x) = 3x - 2$ and $g(x) = x^2 + 1$, find $f[g(-3)]$ and $g[f(-3)]$.

5. Find the inverse of the relation $\{(-2, 5), (0, 4), (1, -8), (4, 7)\}$.

1. $x^2 - x + 6$; $x^2 - 5x - 2$; $2x^3 - 2x^2 - 8x + 8$; $\frac{x^2 - 3x + 2}{2x + 4}$; $x \neq -2$

2. $\{(2, 4), (3, 8), (4, 3), (8, 4)\}$; $\{(2, 5), (4, 2), (5, 4)\}$

3. $x^2 - 6x + 7$; $x^2 + 2x - 5$

4. 28; 122

5. $\{(5, -2), (4, 0), (-8, 1), (7, 4)\}$

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7 Chapter 7 Quiz 2

SCORE ______

(Lessons 7–3 and 7–4)

1. Graph $y > \sqrt{3x + 9}$. Then state the domain and range of the function.

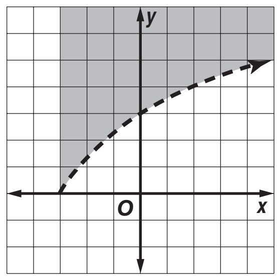
2. Graph $y = -\sqrt{2x + 5}$. Then state the domain and range of the function.

3. Simplify $\sqrt[3]{-27w^9y^6}$.

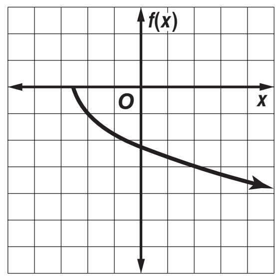
4. Use a calculator to approximate $\sqrt[3]{-56}$ to three decimal places.

5. Simplify $\sqrt{\frac{x^6}{25}}$.

1. D: $x > -3$; R: $y \geq 0$



2. D: $x \geq -\frac{5}{2}$; R: $y \leq 0$



3. $-3w^3y^2$

4. −3.826

5. $\frac{|x^3|}{5}$

Assessment

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7 Chapter 7 Quiz 3

SCORE ______

(Lessons 7–5 and 7–6)

For Questions 1–6, simplify.

1. $\sqrt{\frac{5}{2x}}$ — 1. $\frac{\sqrt{10x}}{2x}$

2. $\sqrt{18m^5n^6}$ — 2. $3m^2|n^3|\sqrt{2m}$

3. $4\sqrt{12} - \sqrt{18} + \sqrt{108} + 7\sqrt{72}$ — 3. $14\sqrt{3} + 39\sqrt{2}$

4. $(\sqrt{5} - \sqrt{7})^2$ — 4. $12 - 2\sqrt{35}$

5. $(7 - \sqrt{5})(3 + 2\sqrt{5})$ — 5. $11 + 11\sqrt{5}$

6. $\frac{2 - \sqrt{6}}{4 + \sqrt{6}}$ — 6. $\frac{7 - 3\sqrt{6}}{5}$

7. Write the expression $x^{\frac{5}{8}}$ in radical form. — 7. $\sqrt[8]{x^5}$ or $(\sqrt[8]{x})^5$

8. Write the radical $\sqrt[5]{32z^3}$ using rational exponents. — 8. $2z^{\frac{3}{5}}$

9. Evaluate $16^{-\frac{3}{2}}$. — 9 $\frac{1}{64}$

10. If t is positive, then $\frac{6t^{\frac{2}{3}} \cdot t^{\frac{4}{3}}}{t^{\frac{1}{3}}} = ?$ — 10. D

A. $6t^{\frac{7}{3}}$ B. $6t^{\frac{1}{3}}$ C. $6t^6$ D. $6t^{\frac{5}{3}}$

NAME ____________________ DATE __________ PERIOD ______

7 Chapter 7 Quiz 4

SCORE ______

(Lesson 7–7)

Solve each equation

1. $\sqrt{5y - 3} = \sqrt{7y + 9}$ — 1. no solution

2. $\sqrt[3]{2v - 7} = -2$ — 2. $-\frac{1}{2}$

3. $4(5n - 1)^{\frac{1}{3}} - 1 = 0$ — 3. $\frac{13}{64}$

For Questions 4 and 5, solve each inequality.

4. $\sqrt{2x + 5} + 1 > 4$ — 4. $x > 2$

5. Solve $5x^2 + 100 = 0$. — 5. no solution

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7 Chapter 7 Mid-Chapter Test

SCORE ______

(Lessons 7–1 through 7–4)

Part I ***Write the letter for the correct answer in the blank at the right of each question.***

1. Given $f(x) = x^2 + 3x - 5$ and $g(x) = 2x + 1$, find $(f + g)(x)$.

A. $x^2 + x - 6$ **B.** $-x^2 - 5x + 4$ **C.** $2x^2 + 4x - 5$ **D.** $x^2 + 5x - 4$ **1.** D

2. Given $f(x) = x^2 + 3x - 4$ and $g(x) = x - 1$, and $x \neq 1$, find $\left(\frac{f}{g}\right)(x)$.

F. $x + 4$ **G.** $x + 7$ **H.** $x + ($ **J.** $x + 3$ **2.** F

3. If $f(x) = \{(2, 3), (4, 8), (7, -1)\}$ and $g(x) = \{(8, 2), (-1, 4), (2, 7)\}$, find $(f \circ g)(x)$, if it exists.

A. $\{(-1, 3), (8, 8), (2, -1)\}$ **C.** $\{(2, 3), (-1, 8), (8, -1)\}$

B. $\{(8, 3), (-1, 8), (2, -1)\}$ **D.** does not exist **3.** B

4. Identify the x-intercept of the graph of $y = \sqrt{2x + 1}$.

F. $\frac{1}{2}$ **G.** 0 **H.** $-\frac{1}{2}$ **J.** 2 **4.** H

5. Identify the domain of the graph of $y > \sqrt{3x + 9}$.

A. $x \geq -3$ **B.** $x \geq 3$ **C.** $x \leq -\frac{1}{3}$ **D.** $x \geq -\frac{1}{3}$ **5.** A

6. $\sqrt[3]{216x^9}$

F. $6x^6$ **G.** $6|x^3|$ **H.** $\pm 6x^3$ **J.** $6x^3$ **6.** J

7. $\sqrt{4x^2y^2z^4}$

A. $2xyz^2$ **B.** $2|xy|z^2$ **C.** $\pm 2xyz^2$ **D.** $2x^2y^2z^4$ **7.** B

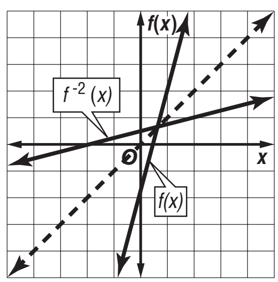
Part II

8. Find $(f - g)(x)$ and $(f \cdot g)(x)$ for $f(x) = 4x - 9$ and $g(x) = 3x^2$. **8.** $-3x^2 + 4x - 9$; $12x^3 - 27x^2$

9. Find the inverse of the function $p(x) = 4x - 8$. **9.** $p^{-1}(x) = \frac{x + 8}{4}$

10. Use a calculator to approximate $\sqrt[4]{287}$ to three decimal places. **10.** 4.116

11. Find the inverse of the function $f(x) = 4x - 2$. Then graph the function and its inverse. **11.** $f^{-1}(x) = \frac{x + 2}{4}$



12. Determine whether $g(x) = 3x - 6$ and $f(x) = \frac{1}{3}x + 2$ are inverse functions. **12.** yes

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7 Chapter 7 Vocabulary Test

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conjugates	inverse function	one-to-one	rationalizing the denominator
composition of functions	inverse relation	principal root	square root function
extraneous solution	like radical expressions	radical equation	square root inequality
identity function	*n*th root	radical inequality	

Underline or circle the correct word or phrase to complete each sentence.

1. If a function has an inverse that is also a function, then it must be a (*one-to-one, square root*) function.

2. $y = \sqrt{3x - 5}$ is a(n) (*square root, inverse*) function.

3. The process of forming a new function from two given functions by performing the two functions in succession is called (*rationalizing the denominator, composition of functions*).

4. If you square both sides of a radical equation and obtain a solution that does not satisfy the original equation, you have found a(n) (*nth root, extraneous solution*).

5. $\sqrt{3x + 5} < 0$ and $\sqrt{2x - 1} \geq 0$ are (*radical equations, radical inequalities*).

6. The expressions $7 - \sqrt{5}$ and $7 + \sqrt{5}$ are (*like radical expressions, conjugates*).

7. When no index is given as in $\sqrt{25}$, the radical sign indicates the (*principal root, nth root*).

8. Equations with radicals that have variables in the radicand are called (*like radical expressions, radical equations*).

9. A(n) (*conjugate, inverse function*) can be found by exchanging the domain and range of a function.

10. One of the steps that may be necessary to simplify a radical expression is (*composition of functions, rationalizing the denominator*).

Define each term in your own words.

11. like radical expressions **Sample answer: Two radical expressions are called like radical expressions if both the indices and the radicands are alike.**

12. inverse relation **Sample answer: The inverse relation is the set of ordered pairs obtained by reversing the coordinates of each ordered pair in a relation.**

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7 Chapter 7 Test, Form 1

SCORE ________

Write the letter for the correct answer in the blank at the right of each question.

For Questions 1 and 2, use $f(x) = x + 5$ and $g(x) = 2x$.

1. Find $(f + g)(x)$.

A. $3x + 5$ **B.** $x + 5$ **C.** $2x + 10$ **D.** $2x^2 + 5$ 1. A

2. Find $(f \cdot g)(x)$.

F. $2x^2 + 5$ **G.** $3x^2 + 10x$ **H.** $2x^2 + 10x$ **J.** $2x + 10$ 2. H

3. If $f(x) = 3x + 7$ and $g(x) = 2x - 5$, find $g[f(-3)]$.

A. -26 **B.** -9 **C.** -1 **D.** 10 3. B

4. If $f(x) = x^2$ and $g(x) = 3x - 1$ find $[g \circ f](x)$.

F. $x^2 + 3x - 1$ **H.** $9x^2 - 1$

G. $9x^2 - 6x + 1$ **J.** $3x^2 - 1$ 4. J

5. Find the inverse of $g(x) = -3x$.

A. $g^{-1}(x) = x + 1$ **C.** $g^{-1}(x) = x - 1$

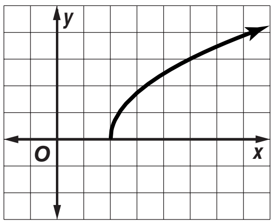
B. $g^{-1}(x) = -3x - 3$ **D.** $g^{-1}(x) = -\frac{1}{3}x$ 5. D

6. Determine which pair of functions are inverse functions.

F. $f(x) = x - 4$, $g(x) = x + 4$ **H.** $f(x) = x - 4$, $g(x) = 4x - 1$

G. $f(x) = x - 4$, $g(x) = \frac{x - 4}{4}$ **J.** $f(x) = 4x - 1$, $g(x) = 4x + 1$ 6. F

7. State the domain and range of the function graphed.



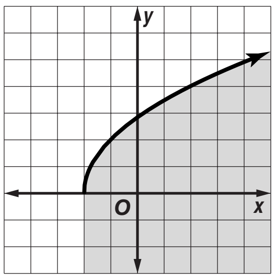
A. D: $x > 2$, R: $y > 0$

B. D: $x < 2$, R: $y > 0$

C. D: $x \geq 2$, R: $y < 0$

D. D: $x \geq 2$, R: $y \geq 0$ 7. D

8. Which inequality is graphed?



F. $y \leq \sqrt{4x + 8}$

G. $y > \sqrt{4x + 8}$

H. $y < \sqrt{4x + 8}$

J. $y \geq \sqrt{4x + 8}$ 8. F

9. Simplify $\sqrt{121}$.

A. 11 **B.** -11 **C.** ± 11 **D.** $\sqrt{11}$ 9. A

Assessment

7 Chapter 7 Test, Form 1 *(continued)*

10. Use a calculator to approximate $\sqrt{224}$ to three decimal places.
F. 15.0 **G.** 14.97 **H.** 14.966 **J.** 14.967 **10.** J

11. Simplify $(2 + \sqrt{5})(3 - \sqrt{5})$.
A. $1 + \sqrt{5}$ **B.** $1 - \sqrt{5}$ **C.** $-1 + \sqrt{5}$ **D.** $-1 - \sqrt{5}$ **11.** A

12. Simplify $\sqrt{75} + \sqrt{12}$.
F. 21 **G.** $\sqrt{87}$ **H.** $10\sqrt{3}$ **J.** $7\sqrt{3}$ **12.** J

13. Write the expression $5^{\frac{1}{7}}$ in radical form.
A. $\sqrt[7]{51}$ **B.** 35 **C.** $\sqrt[7]{5}$ **D.** $\sqrt[5]{7}$ **13.** C

14. Simplify the expression $m^{\frac{2}{5}} \cdot m^{\frac{1}{5}}$.
F. $m^{\frac{5}{3}}$ **G.** $m^{\frac{3}{5}}$ **H.** $m^{\frac{2}{25}}$ **J.** $m^{\frac{2}{5}}$ **14.** G

15. Solve $\sqrt{3x + 4} = 5$.
A. -7 **B.** 7 **C.** 21 **D.** $\frac{25}{3}$ **15.** B

16. Solve $2 + \sqrt{5x - 1} > 5$.
F. $x > 5$ **G.** $x > -2$ **H.** $x < 2$ **J.** $x > 2$ **16.** J

17. Gilda used the formula $f(x) = \frac{x}{144}$ to convert square inches to square feet. Find the inverse of $f^{-1}(x)$.
A. $f^{-1}(x) = 12x$ **B.** $f^{-1}(x) = 144x$ **C.** $f^{-1}(x) = \frac{144}{x}$ **D.** $f^{-1}(x) = (12x^2)$ **17.** B

18. About how many feet of fencing are needed to enclose a rectangular garden with a 20-foot side and a 25-foot diagonal?
F. 70 ft **G.** 80 ft **H.** 85 ft **J.** 90 ft **18.** F

19. If x is a positive number, then $\sqrt[5]{x} \div x^{\frac{1}{5}} = ?$
A. x^5 **B.** $\frac{1}{5}x$ **C.** 1 **D.** $\frac{1}{5}$ **19.** C

20. If $2^8 \cdot y = 2^5$, then $y = ?$
F. -2^{-3} **G.** -2^3 **H.** $2^{\frac{1}{3}}$ **J.** 2^{-3} **20.** J

Bonus If $g(x) = 2x + 1$, find $g[g(x)]$. **B:** $4x + 3$

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7 Chapter 7 Test, Form 2A

SCORE ______

Write the letter for the correct answer in the blank at the right of each question.

1. Find $(f \cdot g)(x)$ for $f(x) = 3x^2$ and $g(x) = 5 - x$.

A. $3x^2 - x + 5$ **C.** $3x^2 - 15x^2$

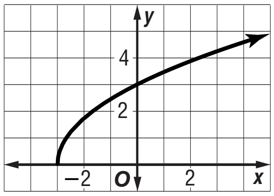
B. $75 - 30x + 3x^2$ **D.** $15x^2 - 3x^3$ **1.** D

2. If $f(x) = x^2 + 1$, and $g(x) = x - 2$, find $[f \circ g](x)$.

F. $x^2 - 4x + 5$ **H.** $x^2 - 1$

G. $x^2 - 3$ **J.** $x^3 - 2x^2 + x - 2$ **2.** F

3. State the domain and range of the function graphed at the right.



A. D: $x > -3$, R: $y > 0$

B. D: $x > -3$, R: $y < 0$

C. D: $x \geq -3$, R: $y \geq 0$

D. D: $x \geq -3$, R: $y > 0$ **3.** C

4. Find the inverse of $f(x) = 2x - 7$.

F. $f^{-1}(x) = 7x - 2$ **H.** $f^{-1}(x) = \frac{1}{2}x + 7$

G. $f^{-1}(x) = \frac{x + 7}{2}$ **J.** $f^{-1}(x) = x + \frac{7}{2}$ **4.** G

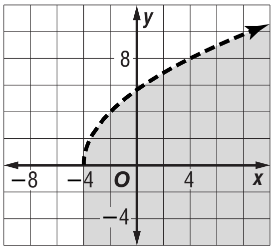
5. Determine which pair of functions are inverse functions.

A. $f(x) = 3x - 1$, $g(x) = \frac{1}{3x - 1}$ **B.** $f(x) = 2x - 5$, $g(x) = \frac{x + 5}{2}$ **C.** $f(x) = 2x + 2$, $g(x) = 2x - 2$ **D.** $f(x) = 3x - 8$, $g(x) = \frac{1}{3}x + 8$ **5.** B

6. Which inequality is graphed at the right?

F. $y \leq \sqrt{x - 4}$ **H.** $y < \sqrt{x + 4}$

G. $y \geq \sqrt{x + 4}$ **J.** $y > \sqrt{x - 4}$ **6.** H



7. Simplify $\sqrt{64n^6w^4}$.

A. $8|n^3|w^2$ **B.** $8n^3w^2$ **C.** $\pm 8n^3w^2$ **D.** $32|n^3|w^2$ **7.** A

8. Use a calculator to approximate $\sqrt[3]{257}$ to three decimal places.

F. 6.357 **G.** 4.004 **H.** 16.031 **J.** 6.358 **8.** J

9. Simplify $\sqrt[3]{625x^5}$.

A. $-25\sqrt[3]{x}$ **B.** $25x^2$ **C.** $5x\sqrt[3]{5x^2}$ **D.** $-5x\sqrt[3]{5x}$ **9.** C

10. Simplify $\sqrt{5} + \sqrt{20} - \sqrt{27} + \sqrt{147}$.

F. $5\sqrt{3} + 6$ **G.** $3\sqrt{5} + 4\sqrt{3}$ **H.** $3\sqrt{5} + 10\sqrt{3}$ **J.** $2\sqrt{5} - 3\sqrt{3}$ **10.** G

7 Chapter 7 Test, Form 2A *(continued)*

11. Simplify $\frac{6}{4 + \sqrt{2}}$.

A. $\frac{12 - 6\sqrt{2}}{7}$ **B.** $\frac{4 - \sqrt{2}}{2}$ **C.** $\frac{4 - \sqrt{2}}{3}$ **D.** $\frac{12 - 3\sqrt{2}}{7}$ **11.** D

12. Write the radical $\sqrt[6]{y^4}$ using rational exponents.

F. $y^{\frac{1}{6}}$ **G.** $y^{\frac{3}{2}}$ **H.** $y^{\frac{2}{3}}$ **J.** y^{24} **12.** H

13. Simplify the expression $\frac{m^{\frac{2}{3}}}{m^{\frac{1}{5}}}$.

A. $m^{\frac{7}{15}}$ **B.** $m^{-\frac{1}{2}}$ **C.** $m^{\frac{15}{7}}$ **D.** $m^{\frac{3}{8}}$ **13.** A

14. A correct step in the solution of the equation $(2m + 1)^{\frac{1}{4}} - 2 = 1$ is _____.

F. $(2m + 1) - 16 = 1$ **H.** $2m + 1 = 81$

G. $(2m + 1)^{\frac{1}{4}} = 1$ **J.** $2m + 1 = 3^{\frac{1}{4}}$ **14.** H

15. Solve $\sqrt{2x + 4} + 1 \geq 5$.

A. $x \geq 0$ **B.** $x \leq -2$ **C.** $-2 \leq x \leq 6$ **D.** $x \geq 6$ **15.** D

16. When inflation causes the price of an item to increase, the new cost C and the original cost c are related by the formula $C = c(1 + r)^n$, where r is the rate of inflation per year as a decimal and n is the number of years. What would be the price of a $2,000 item after six months of 5% inflation?

F. $2449.49 **G.** $2680.19 **H.** $22,781.25 **J.** $2049.39 **16.** J

17. The velocity V of an object can be defined as $v = \sqrt{\frac{2K}{m}}$, where m is the mass of an object and K is the kinetic energy. Find the velocity of an object with a mass of 11 grams and a kinetic energy of 550.

A. 100 m/s **B.** 50 m/s **C.** 15 m/s **D.** 10 m/s **17.** D

18. Find the area of a circle whose radius is $2x^{\frac{1}{5}}z^2$ feet? Use 3.14 for π.

F. $12.56x^{\frac{2}{25}}z^4$ ft^2 **G.** $6.28x^{\frac{1}{25}}z^4$ ft^2 **H.** $6.28x^{\frac{2}{5}}z^4$ ft^2 **J.** $12.56x^{\frac{2}{5}}z^4$ ft^2 **18.** J

19. If x is a positive number, then $\sqrt[5]{x^3} \div x^{\frac{3}{5}} = ?$

A. 1 **B.** $\frac{1}{3}x$ **C.** x^1 **D.** $\frac{1}{5}$ **19.** A

20. When an object is dropped from the top of a 50-foot tall building, the object will be h feet above the ground after t seconds, where $\sqrt{50 - h} = t$. How far above the ground will the object be after 3 seconds?

F. 47 ft **G.** 41 ft **H.** $\sqrt{41}$ ft **J.** 22 ft **20.** G

Bonus If $f(x) = 3x + 4$, solve $f[f(x)] = f(x)$ for x. **B:** −2

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7 Chapter 7 Test, Form 2B

SCORE ________

Write the letter for the correct answer in the blank at the right of each question.

1. Find $(f - g)(x)$ for $f(x) = x^2 + 8x$ and $g(x) = 3x + 5$.

A. $-x^2 - 5x + 5$ **B.** $x^2 + 5x + 5$ **C.** $x^2 + 5x - 5$ **D.** $x^2 + 11x + 5$ 1. C

2. If $f(x) = x^2 - 3$, and $g(x) = 2x - 1$, find $[g \circ f](x)$.

F. $2x^3 - x^2 - 6x + 3$ **H.** $4x^2 - 4x - 2$

G. $x^2 + 2x - 4$ **J.** $2x^2 - 7$ 2. J

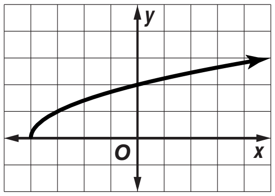
3. State the domain and range of the function graphed at the right.

A. D: $x > -4$, R: $y > 0$

B. D: $x \geq -4$, R: $y \geq 0$

C. D: $x \geq -4$, R: $y \leq 0$

D. D: $x > -4$, R: $y < 0$



3. B

4. Find the inverse of $f(x) = 3 + 5x$.

F. $f^{-1}(x) = 5 + 3x$ **H.** $f^{-1}(x) = \frac{x - 3}{5}$

G. $f^{-1}(x) = \frac{3 + 5x}{5}$ **J.** $f^{-1}(x) = -3 + \frac{1}{5}x$ 4. H

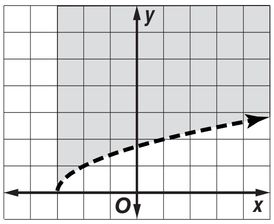
5. Determine which pair of functions are *not* inverse functions.

A. $g(x) = 2x + 9$, $h(x) = \frac{1}{2}x - 9$ **B.** $g(x) = x - 1$, $h(x) = x + 1$ **C.** $g(x) = 3x - 6$, $h(x) = \frac{1}{3}x + 2$ **D.** $g(x) = 3x + 4$, $h(x) = \frac{x - 4}{3}$ 5. A

6. Which inequality is graphed at the right?

F. $y \geq \sqrt{x - 3}$ **H.** $y < \sqrt{x + 3}$

G. $y \leq \sqrt{x - 3}$ **J.** $y > \sqrt{x + 3}$



6. J

7. Simplify $\sqrt{25p^4q^2}$.

A. $5|p^2|q$ **B.** $5p^2q$ **C.** $\pm 5p^2q$ **D.** $5p^2|q|$ 7. D

8. Use a calculator to approximate $\sqrt[4]{160}$ to three decimal places.

F. 3.556 **G.** 12.649 **H.** 3.557 **J.** 5.429 8. H

9. Simplify $\sqrt[3]{256t^4}$.

A. $4t\sqrt[3]{4t}$ **B.** $16t\sqrt[3]{t}$ **C.** $\pm 4t\sqrt[3]{4t}$ **D.** $4t\sqrt{4t}$ 9. A

10. Simplify $\sqrt{32} - \sqrt{18} + \sqrt{54} + \sqrt{150}$.

F. $7\sqrt{2} - 2\sqrt{6}$ **G.** $7\sqrt{2} + 8\sqrt{6}$ **H.** $3\sqrt{2} + 3\sqrt{6}$ **J.** $\sqrt{2} + 8\sqrt{6}$ 10. J

Assessment

7 Chapter 7 Test, Form 2B *(continued)*

11. Simplify $\frac{5}{2 - \sqrt{3}}$.

A. $10 + 5\sqrt{3}$ **B.** $10 - 5\sqrt{3}$ **C.** $-10 - 5\sqrt{3}$ **D.** $-10 + 5\sqrt{3}$ **11.** A

12. Write the radical $\sqrt[5]{m^3}$ using rational exponents.

F. m^2 **G.** $m^{\frac{5}{3}}$ **H.** $m^{\frac{3}{5}}$ **J.** m^{15} **12.** H

13. Simplify the expression $\frac{t^{\frac{3}{4}}}{t^{\frac{1}{5}}}$.

A. t^{-2} **B.** $t^{\frac{11}{20}}$ **C.** $t^{\frac{19}{20}}$ **D.** $t^{\frac{3}{20}}$ **13.** B

14. A correct step in the solution of the equation $(5z - 1)^{\frac{1}{3}} - 3 = 1$ is _____.

F. $5z - 1 = 4^{\frac{1}{3}}$ **H.** $(5z - 1) - 27 = 1$

G. $(5z - 1) - 9 = 3$ **J.** $5z - 1 = 64$ **14.** J

15. Solve $\sqrt{3x + 6} - 1 \geq 5$.

A. $x \geq 0$ **B.** $-2 \leq x \leq 10$ **C.** $x \geq 10$ **D.** $x \geq -2$ **15.** C

16. When inflation causes the price of an item to increase, the new cost C and the original cost c are related by the formula $C = c(1 + r)^n$, where r is the rate of inflation per year as a decimal and n is the number of years. What would be the price of a \$3,000 item after six months of 6% inflation?

F. \$4255.56 **G.** \$3794.73 **H.** \$3088.69 **J.** \$50,331.65 **16.** H

17. The velocity V of an object can be defined as $v = \sqrt{\frac{2K}{m}}$, where m is the mass of an object and K is the kinetic energy. Find the velocity of an object with a mass of 20 grams and a kinetic energy of 360.

A. 6 m/s **B.** 4.24 m/s **C.** 0.34 m/s **D.** 36 m/s **17.** A

18. Find the area of a circle whose radius is $3x^{\frac{3}{4}}z^5$ inches? Use 3.14 for π.

F. $28.26x^{\frac{3}{2}}z^{10}$ in.^{2} **G.** $28.26x^{\frac{9}{4}}z^{10}$ in.^{2} **H.** $9.42x^{\frac{3}{2}}z^{10}$ in.^{2} **J.** $28.26x^{\frac{9}{4}}z^{25}$ in.^{2} **18.** F

19. If x is a positive number, then $\sqrt[2]{x^5} \div x^{-\frac{5}{2}} = ?$

A. 1 **B.** $\frac{2}{5}x$ **C.** x^5 **D.** $\frac{1}{5}$ **19.** C

20. When an object is dropped from the top of a 50-foot tall building, the object will be h feet above the ground after t seconds, where $\sqrt{50 - h} = t$. How far above the ground will the object be after 7 seconds?

F. 43 ft **G.** 6.56 ft **H.** $\sqrt{43}$ ft **J.** 1 ft **20.** J

Bonus If $g(x) = 4x - 9$, solve $g[g(x)] = g(x)$ for x. **B:** 3

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7 Chapter 7 Test, Form 2C

SCORE ______

1. Find $(f \cdot g)(x)$ for $f(x) = x^2 + 4$ and $g(x) = 7 - x$. — 1. $-x^3 + 7x^2 - 4x + 28$

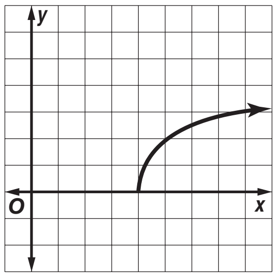
2. If $f(x) = x - 5$ and $g(x) = x^2 + 3$, find $f[g(-2)]$. — 2. 2

3. If $f(x) = 2x + 5$ and $g(x) = x^2 - 3$, find $[f \circ g](x)$. — 3. $2x^2 - 1$

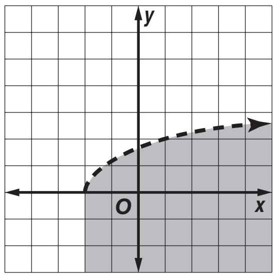
4. Find the inverse of $f(x) = 5x + 10$. — 4. $f^{-1}(x) = \frac{1}{5}x - 2$

5. Determine whether $f(x) = 5x - 3$ and $g(x) = \frac{x + 3}{5}$ are inverse functions. — 5. yes

6. Graph $y = \sqrt{2x - 8}$. Then state the domain and range of the function. — 6. D: $x \geq 4$ R: $y \geq 0$



7. Graph $y < \sqrt{x + 2}$. — 7.



For Questions 8–11, simplify.

8. $\sqrt{\frac{4}{49}}$ — 8. $\frac{2}{7}$

9. $\sqrt{49x^6y^4}$ — 9. $7|x^3|y^2$

10. $\sqrt[3]{24a^6b^5}$ — 10. $2a^2b\sqrt[3]{3b^2}$

11. $5\sqrt{72} + \sqrt{75} - \sqrt{288}$ — 11. $18\sqrt{2} + 5\sqrt{3}$

Assessment

7 Chapter 7 Test, Form 2C *(continued)*

12. **TREES** The diameter of a tree d (in inches) is related to its basal area BA (in square feet) by the formula $d = \sqrt{\frac{576(BA)}{\pi}}$. If the basal area of a tree is 12.4 square feet, what is the diameter of the tree? Use a calculator to approximate your answer to three decimal places.

12. **47.693 in.**

13. Write the radical $\sqrt[5]{32m^3}$ using rational exponents.

13. $2m^{\frac{3}{5}}$

14. Simplify the expression $\frac{x}{x^{\frac{1}{2}} \cdot x^{\frac{1}{3}}}$.

14. $x^{\frac{1}{6}}$ **or** $\sqrt[6]{x}$

15. Solve $\sqrt[3]{3m + 1} = 4$.

15. **21**

16. Solve $4 - \sqrt{5y - 10} \leq -1$.

16. $y \geq 7$

17. The velocity of v in feet per second of a roller coaster at the bottom of a hill is related to the vertical drop h in feet and the velocity v_0 in feet per second of the coaster at the top of the hill by the formula $v_0 = \sqrt{v^2 - 64h}$. What velocity must a coaster have at the top of a 150-foot hill to achieve a velocity of 100 feet per second?

17. **20 ft/s**

18. What is $\sqrt{34}$ divided by $\sqrt{51}$?

18. $\frac{\sqrt{6}}{3}$

19. A triangle has a base of $6r^{\frac{1}{2}}s^{\frac{3}{4}}$ units and a height of $8r^{\frac{1}{2}}s^{\frac{3}{4}}$ units. Find the area of the triangle.

19. $24rs^{\frac{3}{2}}$ **units²**

20. The radius r of a sphere with volume V is given by $r = \left(\frac{3V}{4\pi}\right)^{\frac{1}{3}}$. Find the radius of a ball that holds 66 cubic centimeters of air.

20. **2.5 cm**

Bonus If $g(x) = 5x - 8$, solve $g[g(x)] = g(x)$ for x.

B: **2**

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SCORE ______

7 Chapter 7 Test, Form 2D

1. Find $(f \cdot g)(x)$ for $f(x) = x^2 - 4$ and $g(x) = 6 - x$.

1. $-x^3 + 6x^2 + 4x - 24$

2. If $f(x) = 2x - 7$ and $g(x) = x^2 - 5$, find $g[f(5)]$.

2. **4**

3. If $f(x) = 3 - x$ and $g(x) = x^2 - 4$, find $[g \circ f](x)$.

3. $x^2 - 6x + 5$

4. Find the inverse of $g(x) = -2x + 4$.

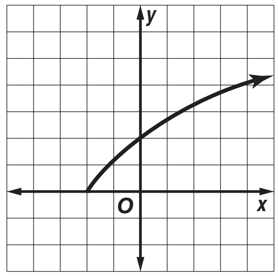
4. $g^{-1}(x) = -\frac{1}{2}x + 2$

5. Determine whether $f(x) = 4x - 8$ and $g(x) = \frac{1}{4}x + 2$ are inverse functions.

5. **yes**

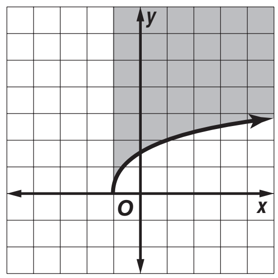
6. Graph $y = \sqrt{3x + 6}$. Then state the domain and range of the function.

6. D: $x \geq -2$
R: $y \geq 0$



7. Graph $y \geq \sqrt{2x + 2}$.

7.



For Questions 8–11, simplify.

8. $\sqrt{\frac{9}{25}}$

8. $\frac{3}{5}$

9. $\sqrt[4]{16x^4y^8}$

9. $2|x|y^2$

10. $\sqrt[3]{-64a^6b^7}$

10. $-4a^2b^2\sqrt[3]{b}$

11. $2\sqrt{50} + \sqrt{45} - \sqrt{18}$

11. $7\sqrt{2} + 3\sqrt{5}$

Assessment

7 Chapter 7 Test, Form 2D *(continued)*

12. **TREES** The diameter of a tree d (in inches) is related to its basal area BA (in square feet) by the formula $d = \sqrt{\frac{576(BA)}{\pi}}$. If the basal area of a tree is 8.9 square feet, what is the diameter of the tree? Use a calculator to approximate your answer to three decimal places.

12. **40.406 in.**

13. Write the radical $\sqrt[3]{-125x^2}$ using rational exponents.

13. $-5x^{\frac{2}{3}}$

14. Simplify the expression $\dfrac{x^{\frac{8}{5}}}{x \cdot x^{\frac{1}{2}}}$.

14. $x^{\frac{1}{10}}$ or $\sqrt[10]{x}$

15. Solve $\sqrt[4]{10s + 1} = 3$.

15. **8**

16. Solve $2 + \sqrt{3t + 6} > 5$.

16. $t > 1$

17. The velocity v in feet per second of a roller coaster at the bottom of a hill is related to the vertical drop h in feet and the velocity v_0 in feet per second of the coaster at the top of the hill by the formula $v_0 = \sqrt{v^2 - 64h}$. What velocity must a coaster have at the top of a 50-foot hill to achieve a velocity of 57 feet per second?

17. **7 ft/s**

18. What is $\sqrt{57}$ divided by $\sqrt{95}$?

18. $\dfrac{\sqrt{15}}{5}$

19. A triangle has a base of $4r^{\frac{2}{3}}s^{\frac{1}{5}}$ units and a height of $5r^{\frac{1}{3}}s^{\frac{3}{5}}$ units. Find the area of the triangle.

19. $10rs^{\frac{4}{5}}$ **units**^{2}

20. The radius r of a sphere with volume V is given by $r = \left(\frac{3V}{4\pi}\right)^{\frac{1}{3}}$. Find the radius of a ball that holds 150 cubic centimeters of air.

20. **3.30 cm**

Bonus If $g(x) = 3x + 8$, solve $g[g(x)] = g(x)$ for x.

B: **−4**

NAME ______________________ DATE ____________ PERIOD ______

7 Chapter 7 Test, Form 3

SCORE ______

1. Find $(f \cdot g)(x)$ for $f(x) = x^2 - 4$ and $g(x) = \frac{x}{x - 2}$. **1.** $x^2 + 2x$

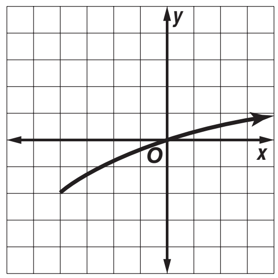
2. If $g(x) = 3x$ and $h(x) = x^3 - x^2 + x - 1$, find $[h \circ g](x)$. **2.** $27x^3 - 9x^2 + 3x - 1$

3. If $f(x) = 5x$, $g(x) = 2x - 1$, and $h(x) = x^2 - 1$, find $[h \circ (g \circ f)](-3)$. **3.** 960

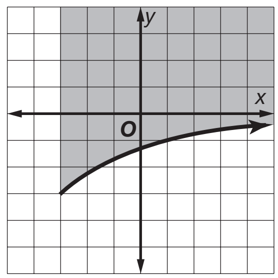
4. Find the inverse of $h(x) = \frac{2x + 6}{5}$. **4.** $h^{-1}(x) = \frac{5x - 6}{2}$

5. Determine whether $f(x) = \frac{1}{2}x - \frac{7}{3}$ and $g(x) = 2x + \frac{14}{3}$ are inverse functions. **5.** yes

6. Graph $y = \sqrt{x + 4} - 2$. Then state the domain and range of the function. **6.** D: $x \geq -4$ R: $y \geq -2$



7. Graph $y \geq \sqrt{x + 3} - 3$. **7.**



For Questions 8–11, simplify.

8. $\sqrt{4x^2 - 20x + 25}$ **8.** $|2x - 5|$

69. $\sqrt[3]{-27x^6y^3}$ **9.** $-3x^2y$

10. $\sqrt[3]{x^5y^7}$ **10.** $xy^2\sqrt[3]{x^2y}$

11. $2\sqrt{15} + \sqrt{60} - 3\sqrt{45}$ **11.** $4\sqrt{15} - 9\sqrt{5}$

7 Chapter 7 Test, Form 3 *(continued)*

12. GEOMETRY The volume V of a sphere and the length of its radius r are the related by the formula $r = \sqrt[3]{\frac{3V}{4\pi}}$. Use the formula to find radius of a sphere with volume 800 cubic meters. Approximate your answer to three decimal places.

12. **5.760 m**

13. Write the expression $\sqrt[4]{16x^9y^4}$ using rational exponents.

13. $\mathbf{2x^{\frac{9}{4}}y}$

14. Simplify the expression $\dfrac{3^{\frac{1}{2}} - 1}{2 + 3^{\frac{1}{2}}}$.

14. $\mathbf{-5 + 3\sqrt{3}}$

15. Solve $\sqrt{x + 11} - 10 = 14$.

15. **565**

16. Solve $\sqrt{x + 2} < 5 - \sqrt{2x + 5}$.

16. $\mathbf{-2 \le x < 2}$

17. Simplify $\frac{x - 9}{\sqrt{x} + 3}$.

17. $\mathbf{\sqrt{x} - 3}$

18. The formula for finding the area of an isosceles triangle for which two sides have length a; the other side has length c is $A = \frac{c}{4}\sqrt{4a^2 - c^2}$. Find A when $a = 8$ and $c = 9$.

18. $\mathbf{\frac{45\sqrt{7}}{4}}$ **units²**

19. The formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ can be used to find the distance between two points on a coordinate grid. Find the distance from A to B for $A(8, 5)$ and $B(-2, 4)$, to the nearest whole unit.

19. **10 units**

20. The force due to gravity decreases with the square of the distance from the center of the Earth. As an object moves farther from Earth, its weight decreases. The radius of Earth is approximately 3960 miles. The formula relating weight and distance is $r = \sqrt{\frac{3960^2\, W_E}{W_S}} - 3960$, where W_E represents the weight of a body on Earth, W_S represents its weight a certain distance from the center of Earth, and r represents the distance above Earth's surface. An astronaut weighs 155 pounds on Earth. To the nearest pound, what is his weight in space if he is 100 miles above the surface of the Earth?

20. **147 pounds**

Bonus Simplify $(5 + 2\sqrt{3})(2 - 4\sqrt{3})(4 + 2\sqrt{3})(3 - 2\sqrt{3})$.

B: $\mathbf{-28\sqrt{3} + 96}$

7 Chapter 7 Extended-Response Test

SCORE _____

Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. Jorge works for the A-Glide Sled Company. This company estimates its monthly profit for the sale of x sleds, in hundreds of dollars, is given by the expression $\sqrt{3x + 19}$. Tia works for a competing sled manufacturer, SnowFun. Tia's company estimates that its monthly profit for the sale of x sleds, in hundreds of dollars, is given by the expression $3 + \sqrt{2x}$. Mark has been offered a job at both companies and decides he will work for the company that has the greatest monthly profit. Before he makes his decision, however, he asks Jorge and Tia the average number of sleds sold each month by each of their companies.

a. Why is the number of sleds sold important to Mark?

b. Assume both companies make the same number of sleds in a certain month. Determine the number of sleds that would make Mark want to work for SnowFun, and give the profit, to the nearest dollar, earned by each company during that month.

c. After talking to Jorge and Tia, Mark decided to work for A-Glide. Assume that both companies average the same number of sleds sold per month. Write and solve an inequality to determine the possible responses Mark might have heard from Jorge and Tia. What does this tell you about the number of sleds sold each month?

2. a. Write a first-degree function $g(x)$ and a second-degree function $h(x)$.

Find $g(2x + 3)$, $h(3a)$, $(g + h)(x)$, $(g - h)(x)$, $(g \cdot h)(x)$, $\left(\frac{h}{g}\right)(x)$, $(h \circ g)(x)$, $g[h(x)]$, $[h \circ (g \circ g)](2)$, and $g^{-1}(x)$.

b. Explain, then show, how to prove that $g(x)$ and $g^{-1}(x)$ are, in fact, inverse functions. Then explain the relationship between the graphs of these two functions.

3. a. Replace a and b in the square root function $y = \sqrt{ax + b}$, with positive integers.

b. Graph your square root function.

c. State the domain, range, and x- and y-intercepts.

d. Rewrite your square root function as a square root inequality.

e. How does the graph of your inequality differ from the graph of your square root function?

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7 Standardized Test Practice

(Chapters 1–7)

Part 1: Multiple Choice

Instructions: Fill in the appropriate circle for the best answer.

1. If $r^2 + 1 = -2r$, then $\left(r + \frac{1}{2}\right)^2 = \underline{\ \ ?\ \ }$.

A $-\frac{1}{4}$ **C** 1

B $\frac{1}{4}$ **D** cannot be determined

1. (A) **(B)** (C) (D)

2. How many fourths is $26\frac{2}{3}\%$?

F $\frac{1}{4}$ **G** 4 **H** $\frac{16}{15}$ **J** $\frac{4}{15}$

2. (F) (G) **(H)** (J)

3. Find p in terms of m if $\frac{m}{p} = q$, $q = p$, $p \neq 0$, and $m \geq 0$.

A $\pm\sqrt{m}$ **B** $\pm\sqrt{mq}$ **C** m **D** $\pm\sqrt{p}$

3. **(A)** (B) (C) (D)

4. Find the average of $\frac{a}{3}, \frac{a}{6}, and\ \frac{a}{9}$.

F $\frac{11a}{54}$ **G** $\frac{11a}{18}$ **H** $\frac{a}{54}$ **J** $\frac{11a}{6}$

4. **(F)** (G) (H) (J)

5. What is the ones digit in 3^{50}?

A 1 **B** 3 **C** 7 **D** 9

5. (A) (B) (C) **(D)**

6. What is the value of $a^2 - b^2$ if $a + b = 6$ and $a - b = -3$?

F -18 **G** 3 **H** 9 **J** 18

6. **(F)** (G) (H) (J)

7. If $\sqrt{n}$ is an irrational number, which of the following must be irrational?

A. $\sqrt{n^2}$ **B.** $2\sqrt{n}$ **C.** $\sqrt{\frac{n}{2}}$ **D.** $\sqrt{2n}$

7. (A) **(B)** (C) (D)

8. Evaluate $4m^3 - 3m^2 + 2m - 2$ if $m = -1$.

F 1 **G** -11 **H** -1 **J** 7

8. (F) **(G)** (H) (J)

9. If the slope of the line through $A(-7, 4)$ and $B(5, y)$ is $-\frac{1}{4}$, what is the value of y?

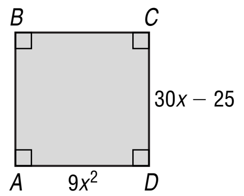
A -1 **B** 7 **C** $\frac{9}{2}$ **D** 1

9. (A) (B) (C) **(D)**

10. Find the area of square $ABCD$.

F $\frac{5}{3}$ units^2 **H** 25 units^2

G 625 units^2 **J** $\frac{25}{9}$ units^2



10. (F) **(G)** (H) (J)

7 Standardized Test Practice *(continued)*

11. Of the 24 socks in a drawer, there are three times as many black socks as brown socks. Some of the black socks are plain and some are patterned. There are five times as many plain socks as there are patterned socks. What is the probability that, without looking, you select a plain black sock from the drawer?

A $\frac{1}{4}$ **B** $\frac{5}{8}$ **C** $\frac{5}{6}$ **D** $\frac{3}{4}$

11. A **(B)** C D

12. Let d◆ and d^* be defined for any positive integer d as follows: d◆ is the number obtained by dividing d by its first digit and d^* is the sum of the digits of d. What is the value of $\frac{354◆}{354^*}$?

F $7\frac{3}{8}$ **G** $29\frac{1}{2}$ **H** $9\frac{5}{6}$ **J** $\frac{6}{59}$

12. F G **(H)** J

13. Simplify $\sqrt{49x^2y^4}$.

A $7|x|y^2$ **B** $24.5|x|y^2$ **C** $\pm 7xy^2$ **D** $|xy|$

13. **(A)** B C D

14. Write the radical $\sqrt[4]{25z^6}$ using rational exponents.

F $2.5z^{\frac{2}{3}}$ **G** $5^{\frac{1}{2}}z^{\frac{3}{2}}$ **H** $5^{\frac{1}{2}}z^{\frac{2}{3}}$ **J** $5^{\frac{1}{4}}z^{\frac{3}{2}}$

14. F **(G)** H J

15. Solve the inequality $-x^2 + 25 < 0$.

A $\{x \mid x < -5 \text{ or } x > 5\}$ **C** $\{x \mid -5 < x < 5\}$

B $\{x \mid x = -5 \text{ or } x = 5\}$ **D** $\varnothing$

15. **(A)** B C D

16. Write the expression $2n^{\frac{2}{3}} - 3n^{\frac{1}{3}} + 5$ in quadratic form, if possible.

F $\left(2n^{\frac{1}{3}}\right)^2 - 3\left(n^{\frac{1}{3}}\right) + 5$ **H** $2\left(n^{\frac{1}{3}}\right)^2 - 3\left(n^{\frac{1}{3}}\right) + 5$

G $2(n^2)^{\frac{1}{3}} - 3(n)^{\frac{1}{3}} + 5$ **J** not possible

16. F G **(H)** J

Part 2: Griddable

Instructions: Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate circle that corresponds to that entry.

17. Point X lies between points P and Q on a number line. If $XQ = 15$ and $PQ = 24$, then $PX =$ ___?___.

18. If $\frac{1}{8} = \frac{n}{0.4}$, what is the value of n?

17.

			9	.			
0	0	0	0		0	0	0
1	1	1	1		1	1	1
2	2	2	2		2	2	2
3	3	3	3		3	3	3
4	4	4	4		4	4	4
5	5	5	5		5	5	5
6	6	6	6		6	6	6
7	7	7	7		7	7	7
8	8	8	8		8	8	8
9	9	9	●		9	9	9

18.

				.	0	5	
0	0	0	0		●	0	0
1	1	1	1		1	1	1
2	2	2	2		2	2	2
3	3	3	3		3	3	3
4	4	4	4		4	4	4
5	5	5	5		5	●	5
6	6	6	6		6	6	6
7	7	7	7		7	7	7
8	8	8	8		8	8	8
9	9	9	9		9	9	9

Assessment

7 Standardized Test Practice *(continued)*

Part 3: Short Answer

Instructions: Write your answers in the space provided.

19. Define a variable and write an inequality. Then solve. Marlea received an inheritance of \$10,000. She plans to invest some in a stock that pays 7% interest annually. She will deposit the remainder in a savings account that pays 5% interest annually. What is the least amount that Marlea can invest in stock if she wants to earn at least \$550 on her investments for the year?

19. **s = amount invested in stock: $0.07s + 0.05(10{,}000 - s) \geq 550$; at least \$2500**

20. Describe the system of equations as *consistent* and *independent*, *consistent* and *dependent*, or *inconsistent*.
$6x + 2y = 4$
$9x + 3y = 6$

20. **consistent and dependent**

21. Triangle ABC with vertices at $A(-1, -3)$, $B(2, 3)$, and $C(-4, 1)$ is translated 5 units right and 3 units down. Find the coordinates of A', B', and C'.

21. **$A'(4, -6)$, $B'(7, 0)$, $C'(1, -2)$**

22. Use Cramer's Rule to solve the system of equations.
$2x + y = -1$
$-3x - y = 4$

22. **$(-3, 5)$**

23. Factor $4n^2 + 20n + 25$ completely. If the polynomial is not factorable, write *prime*. (Lesson 5-4)

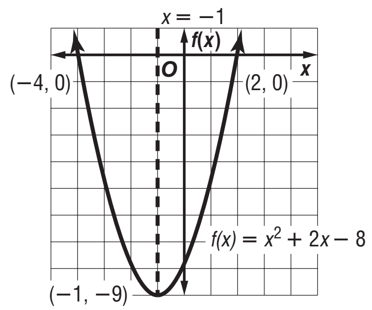
23. **$(2n + 5)^2$**

24. Solve $\sqrt{2x + 10} - 1 > 5$.

24. **$x > 13$**

25. Graph the quadratic function $f(x) = x^2 + 2x - 8$, labeling the y-intercept, vertex, and axis of symmetry.

25.



26. Write a quadratic equation with 3 and −2 as its roots. Write the equation in the form $ax^2 + bx + c = 0$, where a, b, and c are integers.

26. **$x^2 - x - 6 = 0$**

27. Find $p(-3)$ if $p(x) = x^4 - 8x^3 + 5x - 4$.

27. **278**

28. The measures of the legs of a right triangle can be represented by the expressions $3x^4y$ and $2x^4y$.

a. What is the area of a triangle?

b. What is the measure of the hypotenuse?

c. What is the perimeter of the triangle?

28a. **$3x^8y^2$**

28b. **$x^4|y|\sqrt{13}$**

28c. **$5x^4y + x^4|y|\sqrt{13}$**

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SCORE ________

7 Unit 2 Test

(Chapters 5–7)

For Questions 1–7, simplify. Assume that no denominator equals 0.

1. $(7x^2 + 3x - 9) - (-x^2 + 8x - 3)$

2. $5x^3(7x)^2$

3. $(2x - 3)^2$

4. $\dfrac{8y^3 + 27}{2xy - 10y + 3x - 15}$

5. $\sqrt{16x^2y^4}$

6. $\sqrt{12} - \sqrt{18} + 3\sqrt{50} + \sqrt{75}$

7. $\dfrac{2 + i}{1 - 3i}$

8. Use synthetic division to find $(2x^3 - 5x^2 + 7x - 1) \div (x - 1)$.

9. Write the expression $m^{\frac{7}{9}}$ in radical form.

10. Solve $\sqrt{3x + 6} + 4 \le 7$.

11. Graph $f(x) = -x^2 + 4x - 3$, labeling the y-intercept, vertex, and axis of symmetry.

12. The shape of a supporting arch can be modeled by $h(x) = -0.03x^2 + 3x$, where $h(x)$ represents the height of the arch and x represents the horizontal distance from one end of the base of the arch in meters. Find the maximum height of the arch.

13. Solve $2x^2 = 3x + 2$ by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

14. Solve $x^2 - 2x = 24$ by factoring.

15. Write a quadratic equation with $-\frac{3}{4}$ and 4 as its roots. Write the equation in the form $ax^2 + bx + c = 0$, where a, b, and c are integers.

16. Find the exact solutions to $6x^2 + x + 4 = 0$ by using the Quadratic Formula.

17. Find the value of the discriminant for $9x^2 + 1 = 6x$. Then describe the number and type of roots for the equation.

Answers

1. $8x^2 - 5x - 6$

2. $245x^5$

3. $4x^2 - 12x + 9$

4. $\dfrac{4y^2 - 6y + 9}{x - 5}$

5. $4|x|y^2$

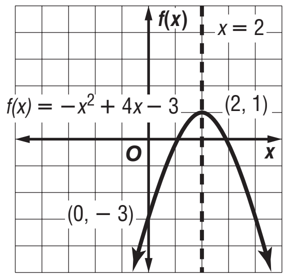
6. $7\sqrt{3} + 12\sqrt{2}$

7. $-\frac{1}{10} + \frac{7}{10}i$

8. $2x^2 - 3x + 4 + \dfrac{3}{x - 1}$

9. $\sqrt[9]{m^7}$

10. $-2 \le x \le 1$

11. 

12. 75 m

13. between −1 and 0; 2

14. {−4, 6}

15. $4x^2 - 13x - 12 = 0$

16. $\dfrac{-1 \pm i\sqrt{95}}{12}$

17. 0; 1 real root

Assessment

7 Unit 2 Test *(continued)*

(Chapters 5–7)

18. Identify the vertex, axis of symmetry, and direction of opening for $y = 2(x + 3)^2 - 5$.

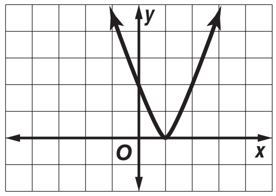
18. **$(-3, -5)$; $x = -3$; up**

19. Write $y = -4x^2 + 8x - 1$ in vertex form.

19. **$y = -4(x - 1)^2 + 3$**

20. Graph $y > x^2 - 2x + 1$.

20.

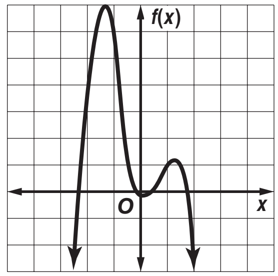


21. Find $p(-3)$ if $p(x) = x^5 + 3x^2$.

21. **-216**

22. Graph $f(x) = -(x)^4 + 4x^2 - 2x$ by making a table of values. Then estimate the x-coordinates at which the relative maxima and relative minima occur.

22.



Sample answer: rel. max at $x = -2$ and $x = 1$, rel. min. at $x = 0$

23. Solve $x^4 + 200 = 102x^2$.

23. **$10, -10; \sqrt{2}, -\sqrt{2}$**

24. Use synthetic substitution to find $f(-3)$ for $f(x) = 2x^3 - 6x^2 - 5x + 7$.

24. **-86**

25. One factor of $f(x) = x^3 + x^2 - 22x - 40$ is $x + 4$. Find the other factors.

25. **$x + 2$; $x - 5$**

26. State the number of positive real zeros, negative real zeros, and imaginary zeros for $g(x) = 9x^3 - 7x^2 + 10x - 4$.

26. **3 or 1; 0; 2 or 0**

27. List all of the possible rational zeros of $f(x) = 3x^5 - 7x^3 + 2x - 15$.

27. **$\pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{3}, \pm \frac{5}{3}$**

28. If $f(x) = 3x$ and $g(x) = 4x - 3$, find $f[g(5)]$ and $g[f(5)]$.

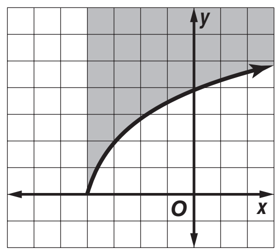
28. **51; 57**

29. Find the inverse of $f(x) = 7x - 2$.

29. **$f^{-1}(x) = \frac{x + 2}{7}$**

30. Graph $y \geq \sqrt{3x + 12}$.

30.



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7 Anticipation Guide

Radical Equations

STEP 1 ***Before you begin Chapter 7***

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

STEP 1 A, D, or NS	Statement	STEP 2 A or D
	1. Functions can be added or subtracted in the same way as polynomials.	**A**
	2. A composition of functions, $f[g(x)]$, is found by multiplying $f(x)$ by $g(x)$.	**D**
	3. The inverse of a function is the set of ordered pairs obtained by taking the opposite of each coordinate in the original ordered pairs.	**D**
	4. Two functions are inverses of each other only if their compositions are the identity function.	**A**
	5. The domain of $y = \sqrt{(x-3)}$ would be $x \geq 3$.	**A**
	6. The principal root of any nth root is always positive.	**D**
	7. The radical expression $\sqrt{\frac{m}{m^3}}$ is in simplest form.	**D**
	8. $4 + \sqrt{3}$ and $4 - \sqrt{3}$ are conjugates of each other.	**A**
	9. $5^{\frac{2}{3}}$ is the same as $\sqrt[2]{5^3}$.	**D**
	10. To solve an equation containing the square root of the variable, square both sides of the equation.	**A**

STEP 2 ***After you complete Chapter 7***

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

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7-1 Lesson Reading Guide

Operations on Functions

Get Ready for the Lesson

Read the introduction to Lesson 7-1 in your textbook.

Describe two ways to calculate Ms. Coffmon's profit from the sale of 50 birdhouses. (Do not actually calculate her profit.) **Sample answer:**
1. Find the revenue by substituting 50 for *x* in the expression 125*x*. Next, find the cost by substituting 50 for *x* in the expression 65*x* + 5400. Finally, subtract the cost from the revenue to find the profit.
2. Form the profit function $p(x) = r(x) - c(x) = 125x - (65x + 5400) = 60x - 5400$. Substitute 50 for *x* in the expression $60x - 5400$.

Read the Lesson

1. Determine whether each statement is *true* or *false*. (Remember that *true* means *always* true.)

a. If f and g are polynomial functions, then $f + g$ is a polynomial function. **true**

b. If f and g are polynomial functions, then $\frac{f}{g}$ is a polynomial function. **false**

c. If f and g are polynomial functions, the domain of the function $f \cdot g$ is the set of all real numbers. **true**

d. If $f(x) = 3x + 2$ and $g(x) = x - 4$, the domain of the function $\frac{f}{g}$ is the set of all real numbers. **false**

e. If f and g are polynomial functions, then $(f \circ g)(x) = (g \circ f)(x)$. **false**

f. If f and g are polynomial functions, then $(f \cdot g)(x) = (g \cdot f)(x)$. **true**

2. Let $f(x) = 2x - 5$ and $g(x) = x^2 + 1$.

a. Explain in words how you would find $(f \circ g)(-3)$. (Do not actually do any calculations.) **Sample answer: Square −3 and add 1. Take the number you get, multiply it by 2, and subtract 5.**

b. Explain in words how you would find $(g \circ f)(-3)$. (Do not actually do any calculations.) **Sample answer: Multiply −3 by 2 and subtract 5. Take the number you get, square it, and add 1.**

Remember What You Learned

3. Some students have trouble remembering the correct order in which to apply the two original functions when evaluating a composite function. Write three sentences, each of which explains how to do this in a slightly different way. (Hint: Use the word *closest* in the first sentence, the words *inside* and *outside* in the second, and the words *left* and *right* in the third.) **Sample answer: 1. The function that is written closest to the variable is applied first. 2. Work from the inside to the outside. 3. Work from right to left.**

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7-1 Study Guide and Intervention

Operations on Functions

Arithmetic Operations

Operations with Functions	
	Sum $(f + g)(x) = f(x) + g(x)$
	Difference $(f - g)(x) = f(x) - g(x)$
	Product $(f \cdot g)(x) = f(x) \cdot g(x)$
	Quotient $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$

Example **Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for $f(x) = x^2 + 3x - 4$ and $g(x) = 3x - 2$.**

$(f + g)(x) = f(x) + g(x)$ — Addition of functions
$= (x^2 + 3x - 4) + (3x - 2)$ — $f(x) = x^2 + 3x - 4$, $g(x) = 3x - 2$
$= x^2 + 6x - 6$ — Simplify.

$(f - g)(x) = f(x) - g(x)$ — Subtraction of functions
$= (x^2 + 3x - 4) - (3x - 2)$ — $f(x) = x^2 + 3x - 4$, $g(x) = 3x - 2$
$= x^2 - 2$ — Simplify.

$(f \cdot g)(x) = f(x) \cdot g(x)$ — Multiplication of functions
$= (x^2 + 3x - 4)(3x - 2)$ — $f(x) = x^2 + 3x - 4$, $g(x) = 3x - 2$
$= x^2(3x - 2) + 3x(3x - 2) - 4(3x - 2)$ — Distributive Property
$= 3x^3 - 2x^2 + 9x^2 - 6x - 12x + 8$ — Distributive Property
$= 3x^3 + 7x^2 - 18x + 8$ — Simplify.

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ — Division of functions
$= \frac{x^2 + 3x - 4}{3x - 2}, x \neq \frac{2}{3}$ — $f(x) = x^2 + 3x - 4$ and $g(x) = 3x - 2$

Exercises

Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$.

1. $f(x) = 8x - 3$; $g(x) = 4x + 5$
$12x + 2$; $4x - 8$; $32x^2 + 28x - 15$; $\frac{8x - 3}{4x + 5}, x \neq -\frac{5}{4}$

2. $f(x) = x^2 + x - 6$; $g(x) = x - 2$
$x^2 + 2x - 8$; $x^2 - 4$; $x^3 - x^2 - 8x + 12$; $x + 3, x \neq 2$

3. $f(x) = 3x^2 - x + 5$; $g(x) = 2x - 3$
$3x^2 + x + 2$; $3x^2 - 3x + 8$; $6x^3 - 11x^2 + 13x - 15$; $\frac{3x^2 - x + 5}{2x - 3}, x \neq \frac{3}{2}$

4. $f(x) = 2x - 1$; $g(x) = 3x^2 + 11x - 4$
$3x^2 + 13x - 5$; $-3x^2 - 9x + 3$; $6x^3 + 19x^2 - 19x + 4$; $\frac{2x - 1}{(3x - 1)(x + 4)}, x \neq \frac{1}{3}, -4$

5. $f(x) = x^2 - 1$; $g(x) = \frac{1}{x + 1}$
$x^2 - 1 + \frac{1}{x + 1}, x \neq -1$; $x^2 - 1 - \frac{1}{x + 1}, x \neq -1$; $x - 1, x \neq -1$; $x^3 + x^2 - x - 1, x \neq -1$

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7-1 Study Guide and Intervention *(continued)*

Operations on Functions

Composition of Functions

Composition of Functions	Suppose f and g are functions such that the range of g is a subset of the domain of f. Then the composite function $f \circ g$ can be described by the equation $[f \circ g](x) = f[g(x)]$.

Example 1 **For $f = \{(1, 2), (3, 3), (2, 4), (4, 1)\}$ and $g = \{(1, 3), (3, 4), (2, 2), (4, 1)\}$, find $f \circ g$ and $g \circ f$ if they exist.**

$f[g(1)] = f(3) = 3$ $f[g(2)] = f(2) = 4$ $f[g(3)] = f(4) = 1$ $f[g(4)] = f(1) = 2$
$f \circ g = \{(1, 3), (2, 4), (3, 1), (4, 2)\}$
$g[f(1)] = g(2) = 2$ $g[f(2)] = g(4) = 1$ $g[f(3)] = g(3) = 4$ $g[f(4)] = g(1) = 3$
$g \circ f = \{(1, 2), (2, 1), (3, 4), (4, 3)\}$

Example 2 **Find $[g \circ h](x)$ and $[h \circ g](x)$ for $g(x) = 3x - 4$ and $h(x) = x^2 - 1$.**

$[g \circ h](x) = g[h(x)]$
$= g(x^2 - 1)$
$= 3(x^2 - 1) - 4$
$= 3x^2 - 7$

$[h \circ g](x) = h[g(x)]$
$= h(3x - 4)$
$= (3x - 4)^2 - 1$
$= 9x^2 - 24x + 16 - 1$
$= 9x^2 - 24x + 15$

Exercises

For each set of ordered pairs, find $f \circ g$ and $g \circ f$ if they exist.

1. $f = \{(-1, 2), (5, 6), (0, 9)\}$,
$g = \{(6, 0), (2, -1), (9, 5)\}$
$f \circ g = \{(2, 2), (6, 9), (9, 6)\}$;
$g \circ f = \{(-1, -1), (0, 5), (5, 0)\}$

2. $f = \{(5, -2), (9, 8), (-4, 3), (0, 4)\}$,
$g = \{(3, 7), (-2, 6), (4, -2), (8, 10)\}$
$f \circ g$ does not exist;
$g \circ f = \{(-4, 7), (0, -2), (5, 6), (9, 10)\}$

Find $[f \circ g](x)$ and $[g \circ f](x)$.

3. $f(x) = 2x + 7$; $g(x) = -5x - 1$
$[f \circ g](x) = -10x + 5$,
$[g \circ f](x) = -10x - 36$

4. $f(x) = x^2 - 1$; $g(x) = -4x^2$
$[f \circ g](x) = 16x^4 - 1$,
$[g \circ f](x) = -4x^4 + 8x^2 - 4$

5. $f(x) = x^2 + 2x$; $g(x) = x - 9$
$[f \circ g](x) = x^2 - 16x + 63$,
$[g \circ f](x) = x^2 + 2x - 9$

6. $f(x) = 5x + 4$; $g(x) = 3 - x$
$[f \circ g](x) = 19 - 5x$,
$[g \circ f](x) = -1 - 5x$

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7-1 Skills Practice

Operations on Functions

Find $(f+g)(x)$, $(f-g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$.

1. $f(x) = x + 5$, $g(x) = x - 4$ — $2x + 1$; 9; $x^2 + x - 20$; $\frac{x+5}{x-4}$, $x \neq 4$

2. $f(x) = 3x + 1$, $g(x) = 2x - 3$ — $5x - 2$; $x + 4$; $6x^2 - 7x - 3$; $\frac{3x+1}{2x-3}$, $x \neq \frac{3}{2}$

3. $f(x) = x^2$, $g(x) = 4 - x$ — $x^2 - x + 4$; $x^2 + x - 4$; $4x^2 - x^3$; $\frac{x^2}{4-x}$, $x \neq 4$

4. $f(x) = 3x^2$, $g(x) = \frac{5}{x}$ — $\frac{3x^3+5}{x}$, $x \neq 0$; $\frac{3x^3-5}{x}$, $x \neq 0$; $15x$, $x \neq 0$; $\frac{3x^3}{5}$, $x \neq 0$

For each set of ordered pairs, find $f \circ g$ and $g \circ f$ if they exist.

5. $f = \{(0, 0), (4, -2)\}$
$g = \{(0, 4), (-2, 0), (5, 0)\}$
$\{(0, -2), (-2, 0), (5, 0)\}$;
$\{(0, 4), (4, 0)\}$

6. $f = \{(0, -3), (1, 2), (2, 2)\}$
$g = \{(-3, 1), (2, 0)\}$
$\{(-3, 2), (2, -3)\}$;
$\{(0, 1), (1, 0), (2, 0)\}$

7. $f = \{(-4, 3), (-1, 1), (2, 2)\}$
$g = \{(1, -4), (2, -1), (3, -1)\}$
$\{(1, 3), (2, 1), (3, 1)\}$;
$\{(-4, -1), (-1, -4), (2, -1)\}$

8. $f = \{(6, 6), (-3, -3), (1, 3)\}$
$g = \{(-3, 6), (3, 6), (6, -3)\}$
$\{(-3, 6), (3, 6), (6, -3)\}$;
$\{(6, -3), (-3, 6), (1, 6)\}$

Find $[g \circ h](x)$ and $[h \circ g](x)$.

9. $g(x) = 2x$, $h(x) = x + 2$ — $2x + 4$; $2x + 2$

10. $g(x) = -3x$, $h(x) = 4x - 1$ — $-12x + 3$; $-12x - 1$

11. $g(x) = x - 6$, $h(x) = x + 6$ — x; x

12. $g(x) = x - 3$, $h(x) = x^2$ — $x^2 - 3$; $x^2 - 6x + 9$

13. $g(x) = 5x$, $h(x) = x^2 + x - 1$ — $5x^2 + 5x - 5$; $25x^2 + 5x - 1$

14. $g(x) = x + 2$, $h(x) = 2x^2 - 3$ — $2x^2 - 1$; $2x^2 + 8x + 5$

If $f(x) = 3x$, $g(x) = x + 4$, and $h(x) = x^2 - 1$, find each value.

15. $f[g(1)]$ **15**
16. $g[h(0)]$ **3**
17. $g[f(-1)]$ **1**
18. $h[f(5)]$ **224**
19. $g[h(-3)]$ **12**
20. $h[f(10)]$ **899**
21. $f[h(8)]$ **189**
22. $[f \circ (h \circ g)](1)$ **72**
23. $[f \circ (g \circ h)](-2)$ **21**

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7-1 Practice

Operations on Functions

Find $(f+g)(x)$, $(f-g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$.

1. $f(x) = 2x + 1$, $g(x) = x - 3$
$3x - 2$; $x + 4$;
$2x^2 - 5x - 3$;
$\frac{2x+1}{x-3}$, $x \neq 3$

2. $f(x) = 8x^2$, $g(x) = \frac{1}{x^2}$
$\frac{8x^4+1}{x^2}$, $x \neq 0$;
$\frac{8x^4-1}{x^2}$, $x \neq 0$;
8, $x \neq 0$; $8x^4$, $x \neq 0$

3. $f(x) = x^2 + 7x + 12$, $g(x) = x^2 - 9$
$2x^2 + 7x + 3$; $7x + 21$;
$x^4 + 7x^3 + 3x^2 - 63x - 108$;
$\frac{x+4}{x-3}$, $x \neq \pm 3$

For each set of ordered pairs, find $f \circ g$ and $g \circ f$ if they exist.

4. $f = \{(-9, -1), (-1, 0), (3, 4)\}$
$g = \{(0, -9), (-1, 3), (4, -1)\}$
$\{(0, -1), (-1, 4), (4, 0)\}$;
$\{(-9, 3), (-1, -9), (3, -1)\}$

5. $f = \{(-4, 3), (0, -2), (1, -2)\}$
$g = \{(-2, 0), (3, 1)\}$
$\{(-2, -2), (3, -2)\}$;
$\{(-4, 1), (0, 0), (1, 0)\}$

6. $f = \{(-4, -5), (0, 3), (1, 6)\}$
$g = \{(6, 1), (-5, 0), (3, -4)\}$
$\{(6, 6), (-5, 3), (3, -5)\}$;
$\{(-4, 0), (0, -4), (1, 1)\}$

7. $f = \{(0, -3), (1, -3), (6, 8)\}$
$g = \{(8, 2), (-3, 0), (-3, 1)\}$
does not exist;
$\{(0, 0), (1, 0), (6, 2)\}$

Find $[g \circ h](x)$ and $[h \circ g](x)$.

8. $g(x) = 3x$, $h(x) = x - 4$
$3x - 12$; $3x - 4$

9. $g(x) = -8x$, $h(x) = 2x + 3$
$-16x - 24$; $-16x + 3$

10. $g(x) = x + 6$, $h(x) = 3x^2$ — $3x^2 + 6$;
$3x^2 + 36x + 108$

11. $g(x) = x + 3$, $h(x) = 2x^2$
$2x^2 + 3$;
$2x^2 + 12x + 18$

12. $g(x) = -2x$, $h(x) = x^2 + 3x + 2$
$-2x^2 - 6x - 4$;
$4x^2 - 6x + 2$

13. $g(x) = x - 2$, $h(x) = 3x^2 + 1$
$3x^2 - 1$;
$3x^2 - 12x + 13$

If $f(x) = x^2$, $g(x) = 5x$, and $h(x) = x + 4$, find each value.

14. $f[g(1)]$ **25**
15. $g[h(-2)]$ **10**
16. $h[f(4)]$ **20**
17. $f[h(-9)]$ **25**
18. $h[g(-3)]$ **−11**
19. $g[f(8)]$ **320**
20. $h[f(20)]$ **404**
21. $[f \circ (h \circ g)](-1)$ **1**
22. $[f \circ (g \circ h)](4)$ **1600**

23. BUSINESS The function $f(x) = 1000 - 0.01x^2$ models the manufacturing cost per item when x items are produced, and $g(x) = 150 - 0.001x^2$ models the service cost per item. Write a function $C(x)$ for the total manufacturing and service cost per item.
$C(x) = 1150 - 0.011x^2$

24. MEASUREMENT The formula $f = \frac{n}{12}$ converts inches n to feet f, and $m = \frac{f}{5280}$ converts feet to miles m. Write a composition of functions that converts inches to miles.
$[m \circ f]n = \frac{n}{63{,}360}$

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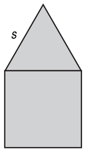
7-1 Word Problem Practice

Operations on Functions

1. AREA Bernard wants to know the area of a figure made by joining an equilateral triangle and square along an edge. The function $f(s) = \frac{\sqrt{3}}{4}s^2$ gives the area of an equilateral triangle with side s. The function $g(s) = s^2$ gives the area of a square with side s. What function $h(s)$ gives the area of the figure as a function of its side length s?



$h(s) = (f + g)(s) = (\frac{\sqrt{3}}{4} + 1)s^2$

2. PRICING A computer company decides to continuously adjust the pricing of and discounts to its products in an effort to remain competitive. The function $P(t)$ gives the sale price of its Super2000 computer as a function of time. The function $D(t)$ gives the value of a special discount it offers to valued customers. How much would valued customers have to pay for one Super2000 computer?

$(P - D)(t)$

3. LAVA A freshly ejected lava rock immediately begins to cool down. The temperature of the lava rock in degrees Fahrenheit as a function of time is given by $T(t)$. Let $C(F)$ be the function that gives degrees Celsius as a function of degrees Fahrenheit. What function gives the temperature of the lava rock in degrees Celsius as a function of time?

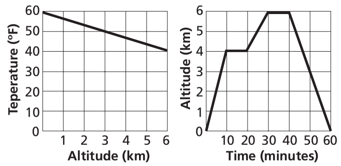
$C(T(t))$

4. ENGINEERING A group of engineers is designing a staple gun. One team determines that the speed of impact s of the staple (in feet per second) as a function of the handle length ℓ (in inches) is given by $s(\ell) = 40 + 3\ell$. A second team determines that the number of sheets N that can be stapled as a function of the impact speed is given by $N(s) = \frac{s - 10}{3}$. What function gives N as a function of ℓ?

$N(s(\ell)) = 10 + \ell$

HOT AIR BALLOONS For Exercises 5 and 6, use the following information.

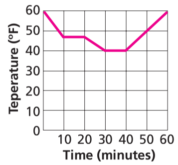
Hannah and Terry went on a one-hour hot air balloon ride. Let $T(A)$ be the outside air temperature as a function of altitude and let $A(t)$ be the altitude of the balloon as a function of time.



5. What function describes the air temperature Hannah and Terry felt at different times during their trip?

$T(A(t))$

6. Sketch a graph of the function you wrote for Exercise 5 based on the graphs for $T(A)$ and $A(t)$ that are given.



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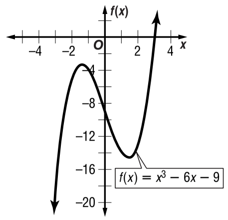
7-1 Enrichment

Relative Maximum Values

The graph of $f(x) = x^3 - 6x - 9$ shows a relative maximum value somewhere between $f(-2)$ and $f(-1)$. You can obtain a closer approximation by comparing values such as those shown in the table.

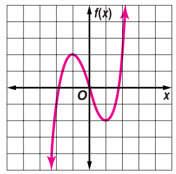
To the nearest tenth a relative maximum value for $f(x)$ is -3.3.



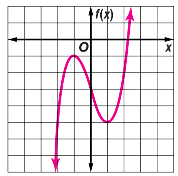
x	$f(x)$
−2	−5
−1.5	−3.375
−1.4	−3.344
−1.3	−3.397
−1	−4

Using a calculator to find points, graph each function. To the nearest tenth, find a relative maximum value of the function.

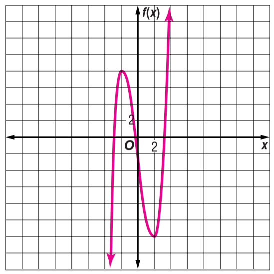
1. $f(x) = x(x^2 - 3)$ rel. max. of 2.0



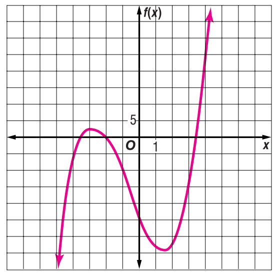
2. $f(x) = x^3 - 3x - 3$ rel. max. of −1.0



3. $f(x) = x^3 - 9x - 2$ rel. max. of 8.4



4. $f(x) = x^3 + 2x^2 - 12x - 24$ rel. max. of 3.3



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7-1 Spreadsheet Activity

Operations on Functions

It is possible to perform operations on functions such as addition, subtraction, multiplication and division. You can use a spreadsheet to investigate the relationships among functions.

Consider the functions $f(x) = 3x + 2$, $g(x) = x^2 - 2x$, and $h(x) = x^2 + x + 2$. Find the function values of each function for several values of x. Does it appear that $f(x) + g(x) = h(x)$?

Use Column A for the chosen values of x. Columns B, C, and E are $f(x)$, $g(x)$, and $h(x)$ respectively. Use Column D for $f(x) + g(x)$.

For every value of x, $f(x) + g(x) = h(x)$.

Functions.xls

◇	A	B	C	D	E
1	x	f(x)	g(x)	f(x) + g(x)	h(x)
2	–4	–10	24	14	14
3	–2.5	–5.5	11.25	5.75	5.75
4	–1	–1	3	2	2
5	0	2	0	2	2
6	1	5	–1	4	4
7	4	14	8	22	22
8	12	38	120	158	158

Sheet 1 / Sheet 2 / Sheet 3

Exercises

Study and use the spreadsheet above.

1. Find $k(x) = (3x + 2) + (x^2 - 2x)$. How does it compare to $h(x)$?
$k(x) = x^2 + x + 2 = h(x)$

2. Change the functions in the spreadsheet to $f(x) = \frac{x}{2}$, $g(x) = 1 - x^2$, and $h(x) = 1 + \frac{x}{2} - x^2$. How are these functions related? Is it true that $f(x) + g(x) = h(x)$? $(f + g)(x) = h(x)$; yes

3. Make a conjecture about $(f + g)(x)$ for any functions $f(x)$ and $g(x)$.
$(f + g)(x) = f(x) + g(x)$

4. Make a conjecture about $(f - g)(x)$ for any functions $f(x)$ and $g(x)$. Use the spreadsheet to test your conjecture. Does it appear to be true? Explain your answer. $(f - g)(x) = f(x) - g(x)$; See students' work.

Find $(f + g)(x)$, $(f - g)(x)$, for each $f(x)$ and $g(x)$. Use the spreadsheet to find function values to verify your solutions. 5-7. See students' spreadsheets.

5. $f(x) = 6x + 8$
$g(x) = 9 + x$
$7x + 17$; $5x - 1$

6. $f(x) = x^2 + 1$
$g(x) = 3x - 4$
$x^2 + 3x - 3$; $x^2 - 3x + 5$

7. $f(x) = 10x^2$
$g(x) = 6 - x^2$
$9x^2 + 6$; $11x^2 - 6$

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7-2 Lesson Reading Guide

Inverse Functions and Relations

Get Ready for the Lesson

Read the introduction to Lesson 7-2 in your textbook.

A function multiplies a number by 3 and then adds 5 to the result. What does the inverse function do, and in what order? Sample answer: It first subtracts 5 from the number and then divides the result by 3.

Read the Lesson

1. Complete each statement.

a. If two relations are inverses, the domain of one relation is the ___range___ of the other.

b. Suppose that g is a relation and that the point $(4, -2)$ is on its graph. Then a point on the graph of g^{-1} is ___(−2, 4)___.

c. The ___horizontal line___ test can be used on the graph of a function to determine whether the function has an inverse function.

d. If you are given the graph of a function, you can find the graph of its inverse by reflecting the original graph over the line with equation ___$y = x$___.

e. If f and g are inverse functions, then $(f \circ g)(x) =$ ___x___ and $(g \circ f)(x) =$ ___x___.

f. A function has an inverse that is also a function only if the given function is ___one-to-one___.

g. Suppose that $h(x)$ is a function whose inverse is also a function. If $h(5) = 12$, then $h^{-1}(12) =$ ___5___.

2. Assume that $f(x)$ is a one-to-one function defined by an algebraic equation. Write the four steps you would follow in order to find the equation for $f^{-1}(x)$.

1. Replace $f(x)$ with y in the original equation.
2. Interchange x and y.
3. Solve for y.
4. Replace y with $f^{-1}(x)$.

Remember What You Learned

3. A good way to remember something new is to relate it to something you already know. How are the vertical and horizontal line tests related? Sample answer: The vertical line test determines whether a relation is a function because the ordered pairs in a function can have no repeated x-values. The horizontal line test determines whether a function is one-to-one because a one-to-one function cannot have any repeated y-values.

Lesson 7-2

Answers

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7-2 Study Guide and Intervention
Inverse Functions and Relations

Find Inverses

Inverse Relations	Two relations are inverse relations if and only if whenever one relation contains the element (a, b), the other relation contains the element (b, a).
Property of Inverse Functions	Suppose f and f^{-1} are inverse functions. Then $f(a) = b$ if and only if $f^{-1}(b) = a$.

Example **Find the inverse of the function $f(x) = \frac{2}{5}x - \frac{1}{5}$. Then graph the function and its inverse.**

Step 1 Replace $f(x)$ with y in the original equation.

$f(x) = \frac{2}{5}x - \frac{1}{5} \rightarrow y = \frac{2}{5}x - \frac{1}{5}$

Step 2 Interchange x and y.

$x = \frac{2}{5}y - \frac{1}{5}$

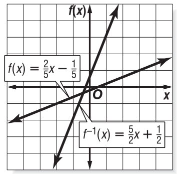
Step 3 Solve for y.

$x = \frac{2}{5}y - \frac{1}{5}$ Inverse

$5x = 2y - 1$ Multiply each side by 5.

$5x + 1 = 2y$ Add 1 to each side.

$\frac{1}{2}(5x + 1) = y$ Divide each side by 2.

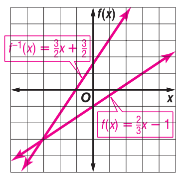


The inverse of $f(x) = \frac{2}{5}x - \frac{1}{5}$ is $f^{-1}(x) = \frac{1}{2}(5x + 1)$.

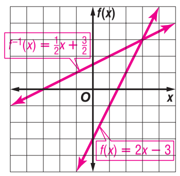
Exercises

Find the inverse of each function. Then graph the function and its inverse.

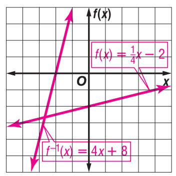
1. $f(x) = \frac{2}{3}x - 1$ **$f^{-1}(x) = \frac{3}{2}x + \frac{3}{2}$**



2. $f(x) = 2x - 3$ **$f^{-1}(x) = \frac{1}{2}x + \frac{3}{2}$**



3. $f(x) = \frac{1}{4}x - 2$ **$f^{-1}(x) = 4x + 8$**



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7-2 Study Guide and Intervention *(continued)*
Inverse Functions and Relations

Inverses of Relations and Functions

Inverse Functions	Two functions f and g are inverse functions if and only if $[f \circ g](x) = x$ and $[g \circ f](x) = x$.

Example 1 **Determine whether $f(x) = 2x - 7$ and $g(x) = \frac{1}{2}(x + 7)$ are inverse functions.**

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f\left[\frac{1}{2}(x + 7)\right] \\ &= 2\left[\frac{1}{2}(x + 7)\right] - 7 \\ &= x + 7 - 7 \\ &= x\end{aligned}$$

$$\begin{aligned}[g \circ f](x) &= g[f(x)] \\ &= g(2x - 7) \\ &= \frac{1}{2}(2x - 7 + 7) \\ &= x\end{aligned}$$

The functions are inverses since both $[f \circ g](x) = x$ and $[g \circ f](x) = x$.

Example 2 **Determine whether $f(x) = 4x + \frac{1}{3}$ and $g(x) = \frac{1}{4}x - 3$ are inverse functions.**

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f\left(\frac{1}{4}x - 3\right) \\ &= 4\left(\frac{1}{4}x - 3\right) + \frac{1}{3} \\ &= x - 12 + \frac{1}{3} \\ &= x - 11\frac{2}{3}\end{aligned}$$

Since $[f \circ g](x) \neq x$, the functions are not inverses.

Exercises

Determine whether each pair of functions are inverse functions.

1. $f(x) = 3x - 1$; $g(x) = \frac{1}{3}x + \frac{1}{3}$ **yes**

2. $f(x) = \frac{1}{4}x + 5$; $g(x) = 4x - 20$ **yes**

3. $f(x) = \frac{1}{2}x - 10$; $g(x) = 2x + \frac{1}{10}$ **no**

4. $f(x) = 2x + 5$; $g(x) = 5x + 2$ **no**

5. $f(x) = 8x - 12$; $g(x) = \frac{1}{8}x + 12$ **no**

6. $f(x) = -2x + 3$; $g(x) = -\frac{1}{2}x + \frac{3}{2}$ **yes**

7. $f(x) = 4x - \frac{1}{2}$; $g(x) = \frac{1}{4}x + \frac{1}{8}$ **yes**

8. $f(x) = 2x - \frac{3}{5}$; $g(x) = \frac{1}{10}(5x + 3)$ **yes**

9. $f(x) = 4x + \frac{1}{2}$; $g(x) = \frac{1}{2}x - \frac{3}{2}$ **no**

10. $f(x) = 10 - \frac{x}{2}$; $g(x) = 20 - 2x$ **yes**

11. $f(x) = 4x - \frac{4}{5}$; $g(x) = \frac{x}{4} + \frac{1}{5}$ **yes**

12. $f(x) = 9 + \frac{3}{2}x$; $g(x) = \frac{2}{3}x - 6$ **yes**

Lesson 7-2

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7-2 Skills Practice

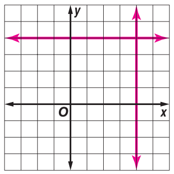
Inverse Functions and Relations

Find the inverse of each relation.

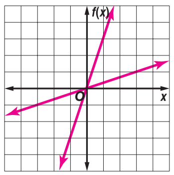
1. {(3, 1), (4, −3), (8, −3)}
   {(1, 3), (−3, 4), (−3, 8)}
2. {(−7, 1), (0, 5), (5, −1)}
   {(1, −7), (5, 0), (−1, 5)}
3. {(−10, −2), (−7, 6), (−4, −2), (−4, 0)}
   {(−2, −10), (6, −7), (−2, −4), (0, −4)}
4. {(0, −9), (5, −3), (6, 6), (8, −3)}
   {(−9, 0), (−3, 5), (6, 6), (−3, 8)}
5. {(−4, 12), (0, 7), (9, −1), (10, −5)}
   {(12, −4), (7, 0), (−1, 9), (−5, 10)}
6. {(−4, 1), (−4, 3), (0, −8), (8, −9)}
   {(1, −4), (3, −4), (−8, 0), (−9, 8)}

Find the inverse of each function. Then graph the function and its inverse.

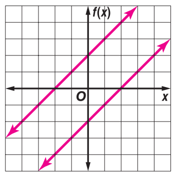
7. $y = 4$
   $x = 4$



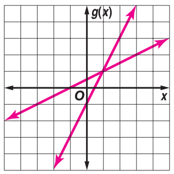
8. $f(x) = 3x$
   $f^{-1}(x) = \frac{1}{3}x$



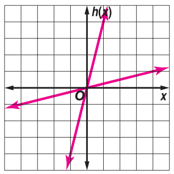
9. $f(x) = x + 2$
   $f^{-1}(x) = x - 2$



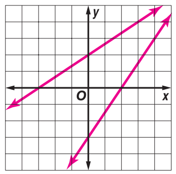
10. $g(x) = 2x - 1$
    $g^{-1}(x) = \frac{x + 1}{2}$



11. $h(x) = \frac{1}{4}x$
    $h^{-1}(x) = 4x$



12. $y = \frac{2}{3}x + 2$
    $y = \frac{3}{2}x - 3$



Determine whether each pair of functions are inverse functions.

13. $f(x) = x - 1$, $g(x) = 1 - x$ **no**
14. $f(x) = 2x + 3$, $g(x) = \frac{1}{2}(x - 3)$ **yes**
15. $f(x) = 5x - 5$, $g(x) = \frac{1}{5}x + 1$ **yes**
16. $f(x) = 2x$, $g(x) = \frac{1}{2}x$ **yes**
17. $h(x) = 6x - 2$, $g(x) = \frac{1}{6}x + 3$ **no**
18. $f(x) = 8x - 10$, $g(x) = \frac{1}{8}x + \frac{5}{4}$ **yes**

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7-2 Practice

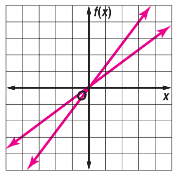
Inverse Functions and Relations

Find the inverse of each relation.

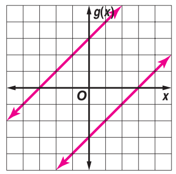
1. {(0, 3), (4, 2), (5, −6)}
   {(3, 0), (2, 4), (−6, 5)}
2. {(−5, 1), (−5, −1), (−5, 8)}
   {(1, −5), (−1, −5), (8, −5)}
3. {(−3, −7), (0, −1), (5, 9), (7, 13)}
   {(−7, −3), (−1, 0), (9, 5), (13, 7)}
4. {(8, −2), (10, 5), (12, 6), (14, 7)}
   {(−2, 8), (5, 10), (6, 12), (7, 14)}
5. {(−5, −4), (1, 2), (3, 4), (7, 8)}
   {(−4, −5), (2, 1), (4, 3), (8, 7)}
6. {(−3, 9), (−2, 4), (0, 0), (1, 1)}
   {(9, −3), (4, −2), (0, 0), (1, 1)}

Find the inverse of each function. Then graph the function and its inverse.

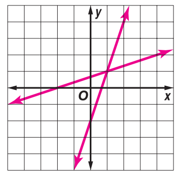
7. $f(x) = \frac{3}{4}x$
   $f^{-1}(x) = \frac{4}{3}x$



8. $g(x) = 3 + x$
   $g^{-1}(x) = x - 3$



9. $y = 3x - 2$
   $y = \frac{x + 2}{3}$



Determine whether each pair of functions are inverse functions.

10. $f(x) = x + 6$, $g(x) = x - 6$ **yes**
11. $f(x) = -4x + 1$, $g(x) = \frac{1}{4}(1 - x)$ **yes**
12. $g(x) = 13x - 13$, $h(x) = \frac{1}{13}x - 1$ **no**
13. $f(x) = 2x$, $g(x) = -2x$ **no**
14. $f(x) = \frac{6}{7}x$, $g(x) = \frac{7}{6}x$ **yes**
15. $g(x) = 2x - 8$, $h(x) = \frac{1}{2}x + 4$ **yes**

16. **MEASUREMENT** The points (63, 121), (71, 180), (67, 140), (65, 108), and (72, 165) give the weight in pounds as a function of height in inches for 5 students in a class. Give the points for these students that represent height as a function of weight.
    (121, 63), (180, 71), (140, 67), (108, 65), (165, 72)

REMODELING For Exercises 17 and 18, use the following information.

The Clearys are replacing the flooring in their 15 foot by 18 foot kitchen. The new flooring costs \$17.99 per square yard. The formula $f(x) = 9x$ converts square yards to square feet.

17. Find the inverse $f^{-1}(x)$. What is the significance of $f^{-1}(x)$ for the Clearys? **$f^{-1}(x) = \frac{x}{9}$; It will allow them to convert the square footage of their kitchen floor to square yards, so they can then calculate the cost of the new flooring.**

18. What will the new flooring cost the Cleary's? **\$539.70**

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7-2 Word Problem Practice

Inverse Functions and Relations

1. **VOLUME** Jason wants to make a spherical water cooler that can hold half a cubic meter of water. He knows that $V = \frac{4}{3}\pi r^3$, but he needs to know how to find r given V. Find this inverse function.

$r = \sqrt[3]{\frac{3V}{4\pi}}$

2. **EXERCISE** Alex began a new exercise routine. To gain the maximum benefit from his exercise, Alex calculated his maximum target heart rate using the function. $f(x) = 0.85(220 - x)$ where x represents his age. Find the inverse of this function.

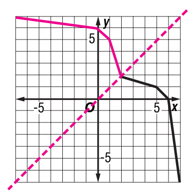
$f^{-1}(x) = 220 - \frac{x}{0.85}$

3. **ROCKETS** The altitude of a rocket in feet as a function of time is given by $f(t) = 49t^2$, where $t \geq 0$. Find the inverse of this function and determine the times when the rocket will be 10, 100, and 1000 feet high. Round your answers to the nearest hundredth of a second.

$f^{-1}(t) = \frac{\sqrt{t}}{7}$; 10 ft at 0.45 s, 100 ft at 1.43 s, 1000 ft at 4.52 s

4. **SELF-INVERTIBLE** Karen finds the incomplete graph of a function in the back of her engineering handbook. The function is graphed in the figure below.

Karen knows that this function is its own inverse. Armed with this knowledge, extend the graph for values of x between −7 and 2.



PLANETS For Exercises 5 and 6, use the following information.

The approximate distance of a planet from the Sun is given by $d = T^{\frac{2}{3}}$ where d is distance in astronomical units and T is Earth years. An astronomical unit is the distance of the Earth from the Sun.

5. Solve for T in terms of d.

$T = d^{1.5}$

6. Pluto is about 39.44 times as far from the Sun as the Earth. About how many years does it take Pluto to orbit the Sun?

248 yr

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7-2 Enrichment

Reading Algebra

In mathematics, the term *group* has a special meaning. The following numbered sentences discuss the idea of group and one interesting example of a group.

01 To be a group, a set of elements and a binary operation must satisfy four conditions: the set must be closed under the operation, the operation must be associative, there must be an identity element, and every element must have an inverse.

02 The following six functions form a group under the operation of composition of functions: $f_1(x) = x$, $f_2(x) = \frac{1}{x}$, $f_3(x) = 1 - x$, $f_4(x) = \frac{(x-1)}{x}$, $f_5(x) = \frac{x}{(x-1)}$, and $f_6(x) = \frac{1}{(1-x)}$.

03 This group is an example of a noncommutative group. For example, $f_3 \circ f_2 = f_4$, but $f_2 \circ f_3 = f_6$.

04 Some experimentation with this group will show that the identity element is f_1.

05 Every element is its own inverse except for f_4 and f_6, each of which is the inverse of the other.

Use the paragraph to answer these questions.

1. Explain what it means to say that a set is *closed* under an operation. Is the set of positive integers closed under subtraction? **Performing the operation on any two elements of the set results in an element of the same set. No, 3 and 4 are positive integers but 3 − 4 is not.**

2. Subtraction is a noncommutative operation for the set of integers. Write an informal definition of noncommutative. **The order in which the elements are used with the operation can affect the result.**

3. For the set of integers, what is the identity element for the operation of multiplication? Justify your answer.

1, because, for every integer *a*, *a* · 1 = *a* and 1 · *a* = *a*.

4. Explain how the following statement relates to sentence 05:

$(f_6 \cdot f_4)(x) = f_6[f_4(x)] = f_6\left(\frac{1}{(1-x)}\right) = \frac{1}{1 - \frac{(x-1)}{x}} = x = f_1(x)$.

It shows that f_4 is the inverse of f_6.

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7-3 Lesson Reading Guide

Square Root Functions

Get Ready for the Lesson

Read the introduction to Lesson 7-3 in your textbook.

If the weight to be supported by a steel cable is doubled, should the diameter of the support cable also be doubled? If not, by what number should the diameter be multiplied?

no; $\sqrt{2}$

Read the Lesson

1. Match each square root function from the list on the left with its domain and range from the list on the right.

a. $y = \sqrt{x}$ **iv**	**i.** domain: $x \geq 0$; range: $y \geq 3$
b. $y = \sqrt{x + 3}$ **viii**	**ii.** domain: $x \geq 0$; range: $y \leq 0$
c. $y = \sqrt{x} + 3$ **i**	**iii.** domain: $x \geq 0$; range: $y \leq -3$
d. $y = \sqrt{x - 3}$ **v**	**iv.** domain: $x \geq 0$; range: $y \geq 0$
e. $y = -\sqrt{x}$ **ii**	**v.** domain: $x \geq 3$; range: $y \geq 0$
f. $y = -\sqrt{x - 3}$ **vii**	**vi.** domain: $x \leq 3$; range: $y \geq 3$
g. $y = \sqrt{3 - x} + 3$ **vi**	**vii.** domain: $x \geq 3$; range: $y \leq 0$
h. $y = -\sqrt{x} - 3$ **iii**	**viii.** domain: $x \geq -3$; range: $y \geq 0$

2. The graph of the inequality $y \leq \sqrt{3x + 6}$ is a shaded region. Which of the following points lie inside this region?

(3, 0) (2, 4) (5, 2) (4, −2) (6, 6)

(3, 0), (5, 2), (4, −2)

Remember What You Learned

3. A good way to remember something is to explain it to someone else. Suppose you are studying this lesson with a classmate who thinks that you cannot have square root functions because every positive real number has two square roots. How would you explain the idea of square root functions to your classmate?

Sample answer: To form a square root function, choose *either* the positive or negative square root. For example, $y = \sqrt{x}$ and $y = -\sqrt{x}$ are two separate functions.

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7-3 Study Guide and Intervention

Square Root Functions and Inequalities

Square Root Functions A function that contains the square root of a variable expression is a **square root function**.

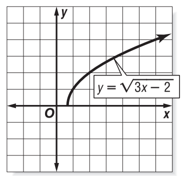
Example **Graph $y = \sqrt{3x - 2}$. State its domain and range.**

Since the radicand cannot be negative, $3x - 2 \geq 0$ or $x \geq \frac{2}{3}$.

The x-intercept is $\frac{2}{3}$. The range is $y \geq 0$.

Make a table of values and graph the function.

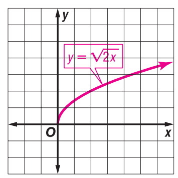
x	y
$\frac{2}{3}$	0
1	1
2	2
3	$\sqrt{7}$



Exercises

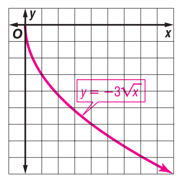
Graph each function. State the domain and range of the function.

1. $y = \sqrt{2x}$



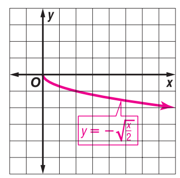
D: $x \geq 0$; R: $y \geq 0$

2. $y = -3\sqrt{x}$



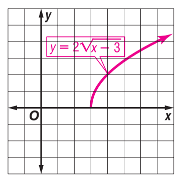
D: $x \geq 0$; R: $y \leq 0$

3. $y = -\sqrt{\frac{x}{2}}$



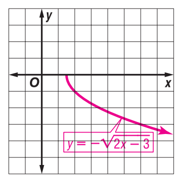
D: $x \geq 0$; R: $y \leq 0$

4. $y = 2\sqrt{x - 3}$



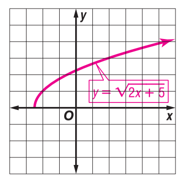
D: $x \geq 3$; R: $y \geq 0$

5. $y = -\sqrt{2x - 3}$



D: $x \geq \frac{3}{2}$; R: $y \leq 0$

6. $y = \sqrt{2x + 5}$



D: $x \geq -\frac{5}{2}$; R: $y \geq 0$

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7-3 Study Guide and Intervention *(continued)*

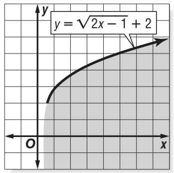
Square Root Functions and Inequalities

Square Root Inequalities A **square root inequality** is an inequality that contains the square root of a variable expression. Use what you know about graphing square root functions and quadratic inequalities to graph square root inequalities.

Example **Graph $y \leq \sqrt{2x - 1} + 2$.**

Graph the related equation $y = \sqrt{2x - 1} + 2$. Since the boundary should be included, the graph should be solid.

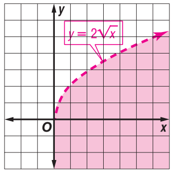
The domain includes values for $x \geq \frac{1}{2}$, so the graph is to the right of $x = \frac{1}{2}$.



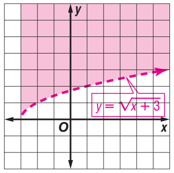
Exercises

Graph each inequality.

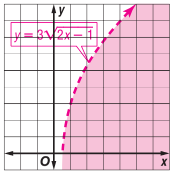
1. $y < 2\sqrt{x}$



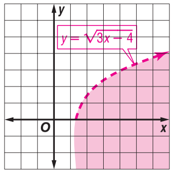
2. $y > \sqrt{x + 3}$



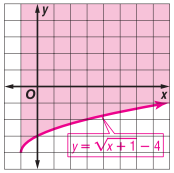
3. $y < 3\sqrt{2x - 1}$



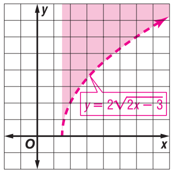
4. $y < \sqrt{3x - 4}$



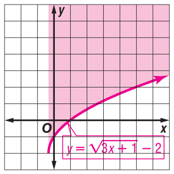
5. $y \geq \sqrt{x + 1} - 4$



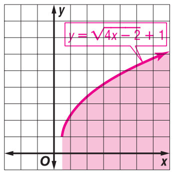
6. $y > 2\sqrt{2x - 3}$



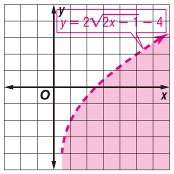
7. $y \geq \sqrt{3x + 1} - 2$



8. $y \leq \sqrt{4x - 2} + 1$



9. $y < 2\sqrt{2x - 1} - 4$



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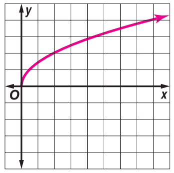
7-3 Skills Practice

Square Root Functions and Inequalities

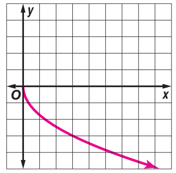
Graph each function. State the domain and range of each function.

1. $y = \sqrt{2x}$



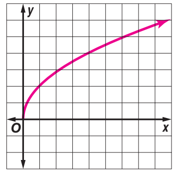
D: $x \geq 0$, R: $y \geq 0$

2. $y = -\sqrt{3x}$



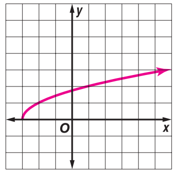
D: $x \geq 0$, R: $y \leq 0$

3. $y = 2\sqrt{x}$



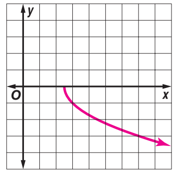
D: $x \geq 0$, R: $y \geq 0$

4. $y = \sqrt{x + 3}$



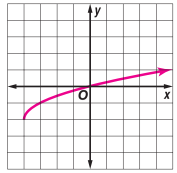
D: $x \geq -3$, R: $y \geq 0$

5. $y = -\sqrt{2x - 5}$



D: $x \geq 2.5$, R: $y \leq 0$

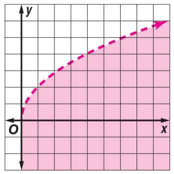
6. $y = \sqrt{x + 4} - 2$



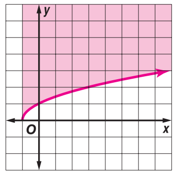
D: $x \geq -4$, R: $y \geq -2$

Graph each inequality.

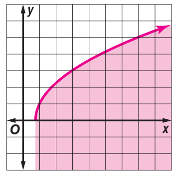
7. $y < \sqrt{4x}$



8. $y \geq \sqrt{x + 1}$



9. $y \leq \sqrt{4x} - 3$



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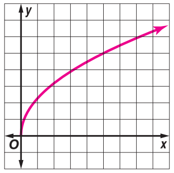
7-3 Practice

Square Root Functions and Inequalities

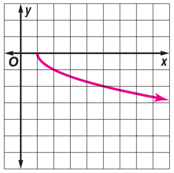
Graph each function. State the domain and range of each function.

1. $y = \sqrt{5x}$



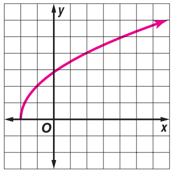
D: $x \geq 0$, R: $y \geq 0$

2. $y = -\sqrt{x - 1}$



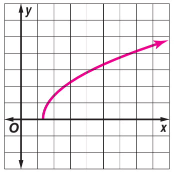
D: $x \geq 1$, R: $y \leq 0$

3. $y = 2\sqrt{x + 2}$



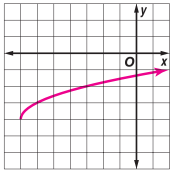
D: $x \geq -2$, R: $y \geq 0$

4. $y = \sqrt{3x - 4}$



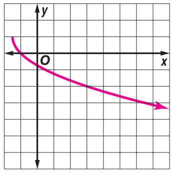
D: $x \geq \frac{4}{3}$, R: $y \geq 0$

5. $y = \sqrt{x + 7} - 4$



D: $x \geq -7$, R: $y \geq -4$

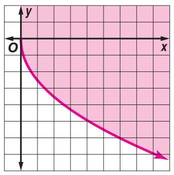
6. $y = 1 - \sqrt{2x + 3}$



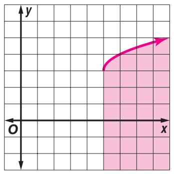
D: $x \geq -\frac{3}{2}$, R: $y \leq 1$

Graph each inequality.

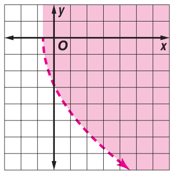
7. $y \geq -\sqrt{6x}$



8. $y \leq \sqrt{x - 5} + 3$



9. $y > -2\sqrt{3x + 2}$



10. ROLLER COASTERS The velocity of a roller coaster as it moves down a hill is $v = \sqrt{v_0^2 + 64h}$, where v_0 is the initial velocity and h is the vertical drop in feet. If $v = 70$ feet per second and $v_0 = 8$ feet per second, find h. **about 75.6 ft**

11. WEIGHT Use the formula $d = \sqrt{\frac{3960^2 W_E}{W_s}} - 3960$, which relates distance from Earth d in miles to weight. If an astronaut's weight on Earth W_E is 148 pounds and in space W_s is 115 pounds, how far from Earth is the astronaut? **about 532 mi**

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7-3 Word Problem Practice

Square Root Functions and Inequalities

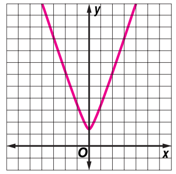
1. SQUARES Cathy is building a square roof for her garage. The roof will occupy 625 square feet. What are the dimensions of the roof?
25 ft by 25 ft

2. PENDULUMS The period of a pendulum, the time it takes to complete one swing, is given by the formula $p = 2\pi\sqrt{\frac{L}{g}}$ where L is the length of the pendulum and g is acceleration due to gravity, 9.8 m/s^2. Find the period of a pendulum that is 0.65 meters long. Round to the nearest tenth.
1.6 seconds

3. REFLEXES Rachel and Ashley are testing one another's reflexes. Rachel drops a ruler from a given height so that it falls between Ashley's thumb and index finger. Ashley tries to catch the ruler before it falls through her hand. The time required to catch the ruler is given by $t = \frac{\sqrt{d}}{4}$ where d is measured *in feet*. Complete the table. Round your answers to the nearest hundredth.

Distance (in.)	Reflex Time (seconds)
3 in.	**0.13**
6 in.	**0.18**
9 in.	**0.22**
12 in.	**0.25**

4. DISTANCE Lance is standing at the side of a road watching a cyclist go by. The distance between Lance and the cyclist as a function of time is given by $d = \sqrt{9 + 36t^2}$. Graph this function. Find the distance between Lance and the cyclist after 3 seconds.



18.25 m

STARS For Exercises 5-7, use the following information.

The intensity of the light from an object varies inversely with the square of the distance. In other words, $I = \frac{k}{d^2}$.

5. Solve the equation to find d in terms of I.
$d = \sqrt{\frac{kI}{I}}$

6. Two stars give off the same amount of light. However, from Earth their intensities differ. Let I_1 and I_2 be their intensities and let d_1 and d_2 be their respective distances from Earth. What is the ratio of d_2 to d_1?
$\frac{d_2}{d_1} = \sqrt{\frac{I_1 I_2}{I_2}}$

7. If one star appears 9 times as intense as the other, how much closer is it to Earth?
3 times as close

Lesson 7-3

Answers

Answers (Lessons 7-3 and 7-4)

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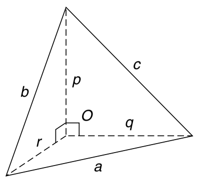
7-3 Enrichment

Reading Algebra

If two mathematical problems have basic structural similarities, they are said to be **analogous.** Using analogies is one way of discovering and proving new theorems.

The following numbered sentences discuss a three-dimensional analogy to the Pythagorean theorem.



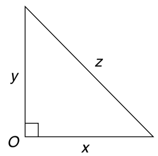
01 Consider a tetrahedron with three perpendicular faces that meet at vertex *O*.

02 Suppose you want to know how the areas *A, B,* and *C* of the three faces that meet at vertex *O* are related to the area *D* of the face opposite vertex *O*.

03 It is natural to expect a formula analogous to the Pythagorean theorem $z^2 = x^2 + y^2$, which is true for a similar situation in two dimensions.

04 To explore the three-dimensional case, you might guess a formula and then try to prove it.

05 Two reasonable guesses are $D^3 = A^3 + B^3 + C^3$ and $D^2 = A^2 + B^2 + C^2$.



Refer to the numbered sentences to answer the questions.

1. Use sentence 01 and the top diagram. The prefix *tetra-* means four. Write an informal definition of tetrahedron.
   a three-dimensional figure with four faces

2. Use sentence 02 and the top diagram. What are the lengths of the sides of each face of the tetrahedron?
   a, b,* and *c*; *a, q,* and *r*; *b, p,* and *r*; *c, p,* and *q

3. Rewrite sentence 01 to state a two-dimensional analogue.
   Consider a triangle with two perpendicular sides that meet at vertex *C*.

4. Refer to the top diagram and write expressions for the areas *A, B,* and *C* mentioned in sentence 02.
   Possible answer: $A = \frac{1}{2}pr$, $B = \frac{1}{2}pq$, $C = \frac{1}{2}rq$

5. To explore the three-dimensional case, you might begin by expressing *a, b,* and *c* in terms of *p, q,* and *r*. Use the Pythagorean theorem to do this.
   $a^2 = q^2 + r^2$, $b^2 = r^2 + p^2$, $c^2 = p^2 + q^2$

6. Which guess in sentence 05 seems more likely? Justify your answer.
   See students' explanations.

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7-4 Lesson Reading Guide

*n*th Roots

Get Ready for the Lesson

Read the introduction to Lesson 7-4 in your textbook.

A basketball has a volume of about 382 cubic inches. Explain how you would find the radius of the basketball using a calculator. (Do not actually calculate the radius.)
Sample answer: Using a calculator, find the product of 3 times the volume. Divide this number by 4π. Then find the positive cube root result. Round the answer to the nearest tenth.

Read the Lesson

1. For each radical below, identify the radicand and the index.
   a. $\sqrt[3]{23}$ radicand: **23** index: **3**
   b. $\sqrt{15x^2}$ radicand: $15x^2$ index: **2**
   c. $\sqrt[5]{-343}$ radicand: **−343** index: **5**

2. Complete the following table. (Do not actually find any of the indicated roots.)

Number	Number of Positive Square Roots	Number of Negative Square Roots	Number of Positive Cube Roots	Number of Negative Cube Roots
27	1	1	1	0
−16	0	0	0	1

3. State whether each of the following is *true* or *false*.
   a. A negative number has no real fourth roots. **true**
   b. $\pm\sqrt{121}$ represents both square roots of 121. **true**
   c. When you take the fifth root of x^5, you must take the absolute value of *x* to identify the principal fifth root. **false**

Remember What You Learned

4. What is an easy way to remember that a negative number has no real square roots but has one real cube root? **Sample answer: The square of a positive or negative number is positive, so there is no real number whose square is negative. However, the cube of a negative number is negative, so a negative number has one real cube root, which is a negative number.**

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7-4 Study Guide and Intervention

*n*th Roots

Simplify Radicals

Square Root	For any real numbers *a* and *b*, if $a^2 = b$, then *a* is a square root of *b*.
***n*th Root**	For any real numbers *a* and *b*, and any positive integer *n*, if $a^n = b$, then *a* is an *n*th root of *b*.
Real *n*th Roots of *b*, $\sqrt[n]{b}$, $-\sqrt[n]{b}$	1. If *n* is even and $b > 0$, then *b* has one positive root and one negative root. 2. If *n* is odd and $b > 0$, then *b* has one positive root. 3. If *n* is even and $b < 0$, then *b* has no real roots. 4. If *n* is odd and $b < 0$, then *b* has one negative root.

Example 1 **Simplify $\sqrt{49z^8}$.**

$\sqrt{49z^8} = \sqrt{(7z^4)^2} = 7z^4$

z^4 must be positive, so there is no need to take the absolute value.

Example 2 **Simplify $-\sqrt[3]{(2a-1)^6}$**

$-\sqrt[3]{(2a-1)^6} = -\sqrt[3]{[(2a-1)^2]^3} = (2a-1)^2$

Exercises

Simplify.

1. $\sqrt{81}$ **9**

2. $\sqrt[3]{-343}$ **−7**

3. $\sqrt{144p^6}$ **$12|p^3|$**

4. $\pm\sqrt{4a^{10}}$ **$\pm 2a^5$**

5. $\sqrt[5]{243p^{10}}$ **$3p^2$**

6. $-\sqrt[3]{m^6n^9}$ **$-m^2n^3$**

7. $\sqrt[3]{-b^{12}}$ **$-b^4$**

8. $\sqrt{16a^{10}b^8}$ **$4|a^5|b^4$**

9. $\sqrt{121x^6}$ **$11|x^3|$**

10. $\sqrt{(4k)^4}$ **$16k^2$**

11. $\pm\sqrt{169r^4}$ **$\pm 13r^2$**

12. $-\sqrt[3]{-27p^6}$ **$3p^2$**

13. $-\sqrt{625y^2z^4}$ **$-25|y|z^2$**

14. $\sqrt{36q^{34}}$ **$6|q^{17}|$**

15. $\sqrt{100x^2y^4z^6}$ **$10|x|y^2|z^3|$**

16. $\sqrt[3]{-0.027}$ **−0.3**

17. $-\sqrt{-0.36}$ **not a real number**

18. $\sqrt{0.64p^{10}}$ **$0.8|p^5|$**

19. $\sqrt[4]{(2x)^8}$ **$4x^2$**

20. $\sqrt{(11y^2)^4}$ **$121y^4$**

21. $\sqrt[3]{(5a^2b)^6}$ **$25a^4b^2$**

22. $\sqrt{(3x-1)^2}$ **$|3x-1|$**

23. $\sqrt[3]{(m-5)^6}$ **$(m-5)^2$**

24. $\sqrt{36x^2-12x+1}$ **$|6x-1|$**

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7-4 Study Guide and Intervention (continued)

*n*th Roots

Approximate Radicals with a Calculator

Irrational Number	a number that cannot be expressed as a terminating or a repeating decimal

Radicals such as $\sqrt{2}$ and $\sqrt{3}$ are examples of irrational numbers. Decimal approximations for irrational numbers are often used in applications. These approximations can be easily found with a calculator.

Example **Approximate $\sqrt[5]{18.2}$ with a calculator.**

$\sqrt[5]{18.2} \approx 1.787$

Exercises

Use a calculator to approximate each value to three decimal places.

1. $\sqrt{62}$ **7.874**

2. $\sqrt{1050}$ **32.404**

3. $\sqrt[3]{0.054}$ **0.378**

4. $-\sqrt[4]{5.45}$ **−1.528**

5. $\sqrt{5280}$ **72.664**

6. $\sqrt{18{,}600}$ **136.382**

7. $\sqrt{0.095}$ **0.308**

8. $\sqrt[3]{-15}$ **−2.466**

9. $\sqrt[5]{100}$ **2.512**

10. $\sqrt[6]{856}$ **3.081**

11. $\sqrt{3200}$ **56.569**

12. $\sqrt{0.05}$ **0.224**

13. $\sqrt{12{,}500}$ **111.803**

14. $\sqrt{0.60}$ **0.775**

15. $-\sqrt[4]{500}$ **−4.729**

16. $\sqrt[3]{0.15}$ **0.531**

17. $\sqrt[6]{4200}$ **4.017**

18. $\sqrt{75}$ **8.660**

19. LAW ENFORCEMENT The formula $r = 2\sqrt{5L}$ is used by police to estimate the speed r in miles per hour of a car if the length L of the car's skid mark is measures in feet. Estimate to the nearest tenth of a mile per hour the speed of a car that leaves a skid mark 300 feet long. **77.5 mi/h**

20. SPACE TRAVEL The distance to the horizon d miles from a satellite orbiting h miles above Earth can be approximated by $d = \sqrt{8000h + h^2}$. What is the distance to the horizon if a satellite is orbiting 150 miles above Earth? **about 1100 mi**

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7-4 Skills Practice

nth Roots

Use a calculator to approximate each value to three decimal places.

1. $\sqrt{230}$ **15.166**
2. $\sqrt{38}$ **6.164**
3. $-\sqrt{152}$ **−12.329**
4. $\sqrt{5.6}$ **2.366**
5. $\sqrt[3]{88}$ **4.448**
6. $\sqrt[3]{-222}$ **−6.055**
7. $-\sqrt[4]{0.34}$ **−0.764**
8. $\sqrt[5]{500}$ **3.466**

Simplify.

9. $\pm\sqrt{81}$ **±9**
10. $\sqrt{144}$ **12**
11. $\sqrt{(-5)^2}$ **5**
12. $\sqrt{-5^2}$ **not a real number**
13. $\sqrt{0.36}$ **0.6**
14. $-\sqrt{\frac{4}{9}}$ **$-\frac{2}{3}$**
15. $\sqrt[3]{-8}$ **−2**
16. $-\sqrt[3]{27}$ **−3**
17. $\sqrt[3]{0.064}$ **0.4**
18. $\sqrt[5]{32}$ **2**
19. $\sqrt[4]{81}$ **3**
20. $\sqrt{y^2}$ **$|y|$**
21. $\sqrt[3]{125s^3}$ **$5s$**
22. $\sqrt{64x^6}$ **$8|x^3|$**
23. $\sqrt[3]{-27a^6}$ **$-3a^2$**
24. $\sqrt{m^8n^4}$ **m^4n^2**
25. $-\sqrt{100p^4q^2}$ **$-10p^2|q|$**
26. $\sqrt[4]{16w^4v^8}$ **$2|w|v^2$**
27. $\sqrt{(-3c)^4}$ **$9c^2$**
28. $\sqrt{(a+b)^2}$ **$|a+b|$**

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7-4 Practice

nth Roots

Use a calculator to approximate each value to three decimal places.

1. $\sqrt{7.8}$ **2.793**
2. $-\sqrt{89}$ **−9.434**
3. $\sqrt[3]{25}$ **2.924**
4. $\sqrt[3]{-4}$ **−1.587**
5. $\sqrt[4]{1.1}$ **1.024**
6. $\sqrt[5]{-0.1}$ **−0.631**
7. $\sqrt[6]{5555}$ **4.208**
8. $\sqrt[4]{(0.94)^2}$ **0.970**

Simplify.

9. $\sqrt{0.81}$ **0.9**
10. $-\sqrt{324}$ **−18**
11. $-\sqrt[4]{256}$ **−4**
12. $\sqrt[6]{64}$ **2**
13. $\sqrt[3]{-64}$ **−4**
14. $\sqrt[3]{0.512}$ **0.8**
15. $\sqrt[5]{-243}$ **−3**
16. $-\sqrt[4]{1296}$ **−6**
17. $\sqrt[5]{\frac{-1024}{243}}$ **$-\frac{4}{3}$**
18. $\sqrt[5]{243x^{10}}$ **$3x^2$**
19. $\sqrt{(14a)^2}$ **$14|a|$**
20. $\sqrt{-(14a)^2}$ **not a real number**
21. $\sqrt{49m^2t^8}$ **$7|m|t^4$**
22. $\sqrt{\frac{16m^2}{25}}$ **$\frac{4|m|}{5}$**
23. $\sqrt[3]{-64r^6w^{15}}$ **$-4r^2w^5$**
24. $\sqrt{(2x)^8}$ **$16x^4$**
25. $-\sqrt[4]{625s^8}$ **$-5s^2$**
26. $\sqrt[3]{216p^3q^9}$ **$6pq^3$**
27. $\sqrt{676x^4y^6}$ **$26x^2|y^3|$**
28. $\sqrt[3]{-27x^9y^{12}}$ **$-3x^3y^4$**
29. $-\sqrt{144m^8n^6}$ **$-12m^4|n^3|$**
30. $\sqrt[5]{-32x^5y^{10}}$ **$-2xy^2$**
31. $\sqrt[6]{(m+4)^6}$ **$|m+4|$**
32. $\sqrt[3]{(2x+1)^3}$ **$2x+1$**
33. $-\sqrt{49a^{10}b^{16}}$ **$-7|a^5|b^8$**
34. $\sqrt[4]{(x-5)^8}$ **$(x-5)^2$**
35. $\sqrt[3]{343d^6}$ **$7d^2$**
36. $\sqrt{x^2+10x+25}$ **$|x+5|$**

37. **RADIANT TEMPERATURE** Thermal sensors measure an object's *radiant* temperature, which is the amount of energy radiated by the object. The *internal* temperature of an object is called its *kinetic* temperature. The formula $T_r = T_k\sqrt[4]{e}$ relates an object's radiant temperature T_r to its kinetic temperature T_k. The variable e in the formula is a measure of how well the object radiates energy. If an object's kinetic temperature is 30°C and $e = 0.94$, what is the object's radiant temperature to the nearest tenth of a degree? **29.5°C**

38. **HERO'S FORMULA** Salvatore is buying fertilizer for his triangular garden. He knows the lengths of all three sides, so he is using Hero's formula to find the area. Hero's formula states that the area of a triangle is $\sqrt{s(s-a)(s-b)(s-c)}$, where a, b, and c are the lengths of the sides of the triangle and s is half the perimeter of the triangle. If the lengths of the sides of Salvatore's garden are 15 feet, 17 feet, and 20 feet, what is the area of the garden? Round your answer to the nearest whole number. **124 ft²**

Lesson 7-4

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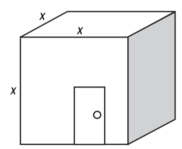
7-4 Word Problem Practice

*n*th Roots

1. **CUBES** Cathy is building a cubic storage room. She wants the volume of the space to be 1728 cubic feet. What should the dimensions of the cube be?



12 ft by 12 ft by 12 ft

2. **ASTRONOMY** A special form of Kepler's Third Law of Planetary Motion is given by $a = \sqrt[3]{P^2}$ where a is the average distance of an object from the Sun in AU (astronomical units) and P is the period of the orbit in years. If an object is orbiting the Sun with a period of 12 years, what is its distance from the Sun?
5.24 AU

3. **TUNING** Two notes are an octave apart if the frequency of the higher note is twice the frequency of the lower note. Casey is experimenting with an instrument that has 6 notes tuned so that the frequency of each successive note increases by the same factor and the first and last note are an octave apart. By what factor does the frequency increase from note to note?
$\sqrt[5]{2}$ or approximately 1.15

4. **MARKUPS** A wholesaler manufactures a part for D dollars. The wholesaler sells the part to a dealer for a P percent markup. The dealer sells the part to a retailer at an additional P percent markup. The retailer in turn sells the part to its customers marking up the price yet another P percent. What is the price that customers see? If the customer buys the part for \$80 and the markup is 40%, what approximately was the original cost to make the part?
$D(1 + P)^3$; \$29.15

PENDULUMS For Exercises 5 and 6, use the following information.

Mr. Topalian's physics class is experimenting with pendulums. The class learned the formula $T = 2\pi\sqrt{\frac{L}{g}}$ which relates the time T that it takes for a pendulum to swing back and forth based on gravity g equal to 32 feet per second squared, and the length of the pendulum L in feet.

5. One group in the class made a 2-foot long pendulum. Use the formula to determine how long it will take for their pendulum to swing back and forth.
1.57 seconds

6. Another group decided they wanted to make a pendulum that took about 1.76 seconds to go back and forth. Approximately how long should their pendulum be?
2.5 feet

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7-4 Enrichment

Approximating Square Roots

Consider the following expansion.

$$\left(a + \frac{b}{2a}\right)^2 = a^2 + \frac{2ab}{2a} + \frac{b^2}{4a^2}$$
$$= a^2 + b + \frac{b^2}{4a^2}$$

Think what happens if a is very great in comparison to b. The term $\frac{b^2}{4a^2}$ is very small and can be disregarded in an approximation.

$$\left(a + \frac{b}{2a}\right)^2 \approx a^2 + b$$
$$a + \frac{b}{2a} \approx \sqrt{a^2 + b}$$

Suppose a number can be expressed as $a^2 + b$, $a > b$. Then an approximate value of the square root is $a + \frac{b}{2a}$. You should also see that $a - \frac{b}{2a} \approx \sqrt{a^2 - b}$.

Example **Use the formula $\sqrt{a^2 \pm b} \approx a \pm \frac{b}{2a}$ to approximate $\sqrt{101}$ and $\sqrt{622}$.**

a. $\sqrt{101} = \sqrt{100 + 1} = \sqrt{10^2 + 1}$

Let $a = 10$ and $b = 1$.

$\sqrt{101} \approx 10 + \frac{1}{2(10)}$

≈ 10.05

b. $\sqrt{622} = \sqrt{625 - 3} = \sqrt{25^2 - 3}$

Let $a = 25$ and $b = 3$.

$\sqrt{622} \approx 25 - \frac{3}{2(25)}$

≈ 24.94

Exercises

Use the formula to find an approximation for each square root to the nearest hundredth. Check your work with a calculator.

1. $\sqrt{626}$ **25.02**
2. $\sqrt{99}$ **9.95**
3. $\sqrt{402}$ **20.05**
4. $\sqrt{1604}$ **40.05**
5. $\sqrt{223}$ **14.93**
6. $\sqrt{80}$ **8.94**
7. $\sqrt{4890}$ **69.93**
8. $\sqrt{2505}$ **50.05**
9. $\sqrt{3575}$ **59.79**
10. $\sqrt{1{,}441{,}100}$ **1200.46**
11. $\sqrt{290}$ **17.03**
12. $\sqrt{260}$ **16.12**
13. Show that $a - \frac{b}{2a} \approx \sqrt{a^2 - b}$ for $a > b$. **$\left(a - \frac{b}{2a}\right)^2 = a^2 - b + \frac{b^2}{4a^2}$; disregard $\frac{b^2}{4a^2}$; $\left(a - \frac{b}{2a}\right)^2 \approx a^2 - b$; $a - \frac{b}{2a} \approx \sqrt{a^2 - b}$**

Lesson 7-4

Answers

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7-5 Lesson Reading Guide

Operations with Radical Expressions

Get Ready for the Lesson

Read the introduction to Lesson 7-5 in your textbook.

Describe how you could use the golden ratio to find the height of a golden triangle if you knew its width. **Sample answer: Use a calculator to multiply the width by 2 and divide the result by the quantity of $\sqrt{5} - 1$. Round this answer to the nearest tenth.**

Read the Lesson

1. Complete the conditions that must be met for a radical expression to be in simplified form.
   - The **index** n is as **small** as possible.
   - The **radicand** contains no **factors** (other than 1) that are nth powers of a(n) **integer** or polynomial.
   - The radicand contains no **fractions**.
   - No **radicals** appear in the **denominator**.

2. a. What are conjugates of radical expressions used for? **to rationalize binomial denominators**

   b. How would you use a conjugate to simplify the radical expression $\frac{1 + \sqrt{2}}{3 - \sqrt{2}}$? **Multiply numerator and denominator by $3 + \sqrt{2}$.**

   c. In order to simplify the radical expression in part b, two multiplications are necessary. The multiplication in the numerator would be done by the **FOIL** method, and the multiplication in the denominator would be done by finding the **difference** of **two** **squares**.

Remember What You Learned

3. One way to remember something is to explain it to another person. When rationalizing the denominator in the expression $\frac{1}{\sqrt[3]{2}}$, many students think they should multiply numerator and denominator by $\frac{\sqrt[3]{2}}{\sqrt[3]{2}}$. How would you explain to a classmate why this is incorrect and what he should do instead. **Sample answer: Because you are working with cube roots, not square roots, you need to make the radicand in the denominator a perfect cube, not a perfect square. Multiply numerator and denominator by $\frac{\sqrt[3]{4}}{\sqrt[3]{4}}$ to make the denominator $\sqrt[3]{8}$, which equals 2.**

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7-5 Study Guide and Intervention

Operations with Radical Expressions

Simplify Radical Expressions

Product Property of Radicals	For any real numbers a and b, and any integer $n > 1$: 1. if n is even and a and b are both nonnegative, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$. 2. if n is odd, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$.

To simplify a square root, follow these steps:
1. Factor the radicand into as many squares as possible.
2. Use the Product Property to isolate the perfect squares.
3. Simplify each radical.

Quotient Property of Radicals	For any real numbers a and $b \neq 0$, and any integer $n > 1$, $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$, if all roots are defined.

To eliminate radicals from a denominator or fractions from a radicand, multiply the numerator and denominator by a quantity so that the radicand has an exact root.

Example 1 Simplify $\sqrt[3]{-16a^5b^7}$.

$$\sqrt[3]{-16a^5b^7} = \sqrt[3]{(-2)^3 \cdot 2 \cdot a^3 \cdot a^2 \cdot (b^2)^3 \cdot b}$$
$$= -2ab^2\sqrt[3]{2a^2b}$$

Example 2 Simplify $\sqrt{\frac{8x^3}{45y^5}}$.

$$\sqrt{\frac{8x^3}{45y^5}} = \frac{\sqrt{8x^3}}{\sqrt{45y^5}} \quad \text{Quotient Property}$$
$$= \frac{\sqrt{(2x)^2 \cdot 2x}}{\sqrt{(3y^2)^2 \cdot 5y}} \quad \text{Factor into squares.}$$
$$= \frac{\sqrt{(2x)^2} \cdot \sqrt{2x}}{\sqrt{(3y^2)^2} \cdot \sqrt{5y}} \quad \text{Product Property}$$
$$= \frac{2|x|\sqrt{2x}}{3y^2\sqrt{5y}} \quad \text{Simplify.}$$
$$= \frac{2|x|\sqrt{2x}}{3y^2\sqrt{5y}} \cdot \frac{\sqrt{5y}}{\sqrt{5y}} \quad \text{Rationalize the denominator.}$$
$$= \frac{2|x|\sqrt{10xy}}{15y^3} \quad \text{Simplify.}$$

Exercises

Simplify.

1. $5\sqrt{54}$ **$15\sqrt{6}$**
2. $\sqrt[4]{32a^9b^{20}}$ **$2a^2|b^5|\sqrt[4]{2a}$**
3. $\sqrt{75x^4y^7}$ **$5x^2y^3\sqrt{3y}$**
4. $\sqrt{\frac{36}{125}}$ **$\frac{6\sqrt{5}}{25}$**
5. $\sqrt{\frac{a^6b^3}{98}}$ **$\frac{|a^3|b\sqrt{2b}}{14}$**
6. $\sqrt[3]{\frac{p^5q^3}{40}}$ **$\frac{pq\sqrt[3]{25p^2}}{10}$**

Lesson 7-5

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7-5 Study Guide and Intervention *(continued)*

Operations with Radical Expressions

Operations with Radicals When you add expressions containing radicals, you can add only like terms or **like radical expressions**. Two radical expressions are called *like radical expressions* if both the indices and the radicands are alike.

To multiply radicals, use the Product and Quotient Properties. For products of the form $(a\sqrt{b} + c\sqrt{d}) \cdot (e\sqrt{f} + g\sqrt{h})$, use the FOIL method. To rationalize denominators, use **conjugates**. Numbers of the form $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$, where a, b, c, and d are rational numbers, are called conjugates. The product of conjugates is always a rational number.

Example 1 **Simplify $2\sqrt{50} + 4\sqrt{500} - 6\sqrt{125}$.**

$2\sqrt{50} + 4\sqrt{500} - 6\sqrt{125} = 2\sqrt{5^2 \cdot 2} + 4\sqrt{10^2 \cdot 5} - 6\sqrt{5^2 \cdot 5}$ Factor using squares.

$= 2 \cdot 5 \cdot \sqrt{2} + 4 \cdot 10 \cdot \sqrt{5} - 6 \cdot 5 \cdot \sqrt{5}$ Simplify square roots.

$= 10\sqrt{2} + 40\sqrt{5} - 30\sqrt{5}$ Multiply.

$= 10\sqrt{2} + 10\sqrt{5}$ Combine like radicals.

Example 2 **Simplify $(2\sqrt{3} - 4\sqrt{2})(\sqrt{3} + 2\sqrt{2})$.**

$(2\sqrt{3} - 4\sqrt{2})(\sqrt{3} + 2\sqrt{2})$

$= 2\sqrt{3} \cdot \sqrt{3} + 2\sqrt{3} \cdot 2\sqrt{2} - 4\sqrt{2} \cdot \sqrt{3} - 4\sqrt{2} \cdot 2\sqrt{2}$

$= 6 + 4\sqrt{6} - 4\sqrt{6} - 16$

$= -10$

Example 3 **Simplify $\dfrac{2 - \sqrt{5}}{3 + \sqrt{5}}$.**

$\dfrac{2 - \sqrt{5}}{3 + \sqrt{5}} = \dfrac{2 - \sqrt{5}}{3 + \sqrt{5}} \cdot \dfrac{3 - \sqrt{5}}{3 - \sqrt{5}}$

$= \dfrac{6 - 2\sqrt{5} - 3\sqrt{5} + (\sqrt{5})^2}{3^2 - (\sqrt{5})^2}$

$= \dfrac{6 - 5\sqrt{5} + 5}{9 - 5}$

$= \dfrac{11 - 5\sqrt{5}}{4}$

Exercises

Simplify.

1. $3\sqrt{2} + \sqrt{50} - 4\sqrt{8}$ **0**

2. $\sqrt{20} + \sqrt{125} - \sqrt{45}$ **$4\sqrt{5}$**

3. $\sqrt{300} - \sqrt{27} - \sqrt{75}$ **$2\sqrt{3}$**

4. $\sqrt[3]{81} \cdot \sqrt[3]{24}$ **$6\sqrt[3]{9}$**

5. $\sqrt[3]{2}(\sqrt[3]{4} + \sqrt[3]{12})$ **$2 + 2\sqrt[3]{3}$**

6. $2\sqrt{3}(\sqrt{15} + \sqrt{60})$ **$18\sqrt{5}$**

7. $(2 + 3\sqrt{7})(4 + \sqrt{7})$ **$29 + 14\sqrt{7}$**

8. $(6\sqrt{3} - 4\sqrt{2})(3\sqrt{3} + \sqrt{2})$ **$46 - 6\sqrt{6}$**

9. $(4\sqrt{2} - 3\sqrt{5})(2\sqrt{20} + 5)$ **$40\sqrt{2} - 30\sqrt{5}$**

10. $\dfrac{5\sqrt{48} + \sqrt{75}}{5\sqrt{3}}$ **5**

11. $\dfrac{4 + \sqrt{2}}{2 - \sqrt{2}}$ **$5 + 3\sqrt{2}$**

12. $\dfrac{5 - 3\sqrt{3}}{1 + 2\sqrt{3}}$ **$\dfrac{13\sqrt{3} - 23}{11}$**

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7-5 Skills Practice

Operations with Radical Expressions

Simplify.

1. $\sqrt{24}$ **$2\sqrt{6}$**

2. $\sqrt{75}$ **$5\sqrt{3}$**

3. $\sqrt[3]{16}$ **$2\sqrt[3]{2}$**

4. $-\sqrt[4]{48}$ **$-2\sqrt[4]{3}$**

5. $4\sqrt{50x^5}$ **$20x^2\sqrt{2x}$**

6. $\sqrt[4]{64a^4b^4}$ **$2|ab|\sqrt{2}$**

7. $\sqrt[3]{-\frac{1}{8}d^2f^5}$ **$-\frac{1}{2}f\sqrt[3]{d^2f^2}$**

8. $\sqrt{\frac{25}{36}s^2t}$ **$\frac{5}{6}|s|\sqrt{t}$**

9. $-\sqrt{\frac{3}{7}}$ **$-\dfrac{\sqrt{21}}{7}$**

10. $\sqrt[3]{\frac{2}{9}}$ **$\dfrac{\sqrt[3]{6}}{3}$**

11. $\sqrt{\frac{2g^3}{5z}}$ **$\dfrac{g\sqrt{10gz}}{5z}$**

12. $(3\sqrt{3})(5\sqrt{3})$ **45**

13. $(4\sqrt{12})(3\sqrt{20})$ **$48\sqrt{15}$**

14. $\sqrt{2} + \sqrt{8} + \sqrt{50}$ **$8\sqrt{2}$**

15. $\sqrt{12} - 2\sqrt{3} + \sqrt{108}$ **$6\sqrt{3}$**

16. $8\sqrt{5} - \sqrt{45} - \sqrt{80}$ **$\sqrt{5}$**

17. $2\sqrt{48} - \sqrt{75} - \sqrt{12}$ **$\sqrt{3}$**

18. $(2 + \sqrt{3})(6 - \sqrt{2})$ **$12 - 2\sqrt{2} + 6\sqrt{3} - \sqrt{6}$**

19. $(1 - \sqrt{5})(1 + \sqrt{5})$ **-4**

20. $(3 - \sqrt{7})(5 + \sqrt{2})$ **$15 + 3\sqrt{2} - 5\sqrt{7} - \sqrt{14}$**

21. $(\sqrt{2} - \sqrt{6})^2$ **$8 - 4\sqrt{3}$**

22. $\dfrac{3}{7 - \sqrt{2}}$ **$\dfrac{21 + 3\sqrt{2}}{47}$**

23. $\dfrac{4}{3 + \sqrt{2}}$ **$\dfrac{12 - 4\sqrt{2}}{7}$**

24. $\dfrac{5}{8 - \sqrt{6}}$ **$\dfrac{40 + 5\sqrt{6}}{58}$**

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7-5 Practice

Operations with Radical Expressions

Simplify.

1. $\sqrt{540}$ **$6\sqrt{15}$**

2. $\sqrt[3]{-432}$ **$-6\sqrt[3]{2}$**

3. $\sqrt[3]{128}$ **$4\sqrt[3]{2}$**

4. $-\sqrt[4]{405}$ **$-3\sqrt[4]{5}$**

5. $\sqrt[3]{-5000}$ **$-10\sqrt[3]{5}$**

6. $\sqrt[5]{-1215}$ **$-3\sqrt[5]{5}$**

7. $\sqrt[3]{125t^6w^2}$ **$5t^2\sqrt[3]{w^2}$**

8. $\sqrt[4]{48v^8z^{13}}$ **$2v^2z^3\sqrt[4]{3z}$**

9. $\sqrt[3]{8g^3k^8}$ **$2gk^2\sqrt[3]{k^2}$**

10. $\sqrt{45x^3y^8}$ **$3xy^4\sqrt{5x}$**

11. $\sqrt{\frac{11}{9}}$ **$\frac{\sqrt{11}}{3}$**

12. $\sqrt[3]{\frac{216}{24}}$ **$\sqrt[3]{9}$**

13. $\sqrt{\frac{1}{128}c^4d^7}$ **$\frac{1}{16}c^2d^3\sqrt{2d}$**

14. $\sqrt{\frac{9a^5}{64b^4}}$ **$\frac{3a^2\sqrt{a}}{8b^2}$**

15. $\sqrt[4]{\frac{8}{9a^3}}$ **$\frac{\sqrt[4]{72a}}{3a}$**

16. $(3\sqrt{15})(-4\sqrt{45})$ **$-180\sqrt{3}$**

17. $(2\sqrt{24})(7\sqrt{18})$ **$168\sqrt{3}$**

18. $\sqrt{810} + \sqrt{240} - \sqrt{250}$ **$4\sqrt{10} + 4\sqrt{15}$**

19. $6\sqrt{20} + 8\sqrt{5} - 5\sqrt{45}$ **$5\sqrt{5}$**

20. $8\sqrt{48} - 6\sqrt{75} + 7\sqrt{80}$ **$2\sqrt{3} + 28\sqrt{5}$**

21. $(3\sqrt{2} + 2\sqrt{3})^2$ **$30 + 12\sqrt{6}$**

22. $(3 - \sqrt{7})^2$ **$16 - 6\sqrt{7}$**

23. $(\sqrt{5} - \sqrt{6})(\sqrt{5} + \sqrt{2})$ **$5 + \sqrt{10} - \sqrt{30} - 2\sqrt{3}$**

24. $(\sqrt{2} + \sqrt{10})(\sqrt{2} - \sqrt{10})$ **-8**

25. $(1 + \sqrt{6})(5 - \sqrt{7})$ **$5 - \sqrt{7} + 5\sqrt{6} - \sqrt{42}$**

26. $(\sqrt{3} + 4\sqrt{7})^2$ **$115 + 8\sqrt{21}$**

27. $(\sqrt{108} - 6\sqrt{3})^2$ **0**

28. $\frac{\sqrt{3}}{\sqrt{5} - 2}$ **$\sqrt{15} + 2\sqrt{3}$**

29. $\frac{6}{\sqrt{2} - 1}$ **$6\sqrt{2} + 6$**

30. $\frac{5 + \sqrt{3}}{4 + \sqrt{3}}$ **$\frac{17 - \sqrt{3}}{13}$**

31. $\frac{3 + \sqrt{2}}{2 - \sqrt{2}}$ **$\frac{8 + 5\sqrt{2}}{2}$**

32. $\frac{3 + \sqrt{6}}{5 - \sqrt{24}}$ **$27 + 11\sqrt{6}$**

33. $\frac{3 + \sqrt{x}}{2 - \sqrt{x}}$ **$\frac{6 + 5\sqrt{x} + x}{4 - x}$**

34. **BRAKING** The formula $s = 2\sqrt{5\ell}$ estimates the speed s in miles per hour of a car when it leaves skid marks ℓ feet long. Use the formula to write a simplified expression for s if $\ell = 85$. Then evaluate s to the nearest mile per hour. **$10\sqrt{17}$; 41 mi/h**

35. **PYTHAGOREAN THEOREM** The measures of the legs of a right triangle can be represented by the expressions $6x^2y$ and $9x^2y$. Use the Pythagorean Theorem to find a simplified expression for the measure of the hypotenuse. **$3x^2|y|\sqrt{13}$**

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7-5 Word Problem Practice

Operations with Radical Expressions

1. **CUBES** Cathy has a rectangular box with dimensions 20 inches by 35 inches by 40 inches. She would like to replace it with a box in the shape of a cube but with the same volume. What should the length of a side of the cube be? Express your answer as a radical expression in simplest form.
$10\sqrt[3]{28}$ in.

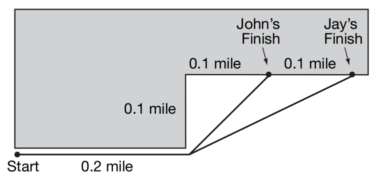
2. **PHYSICS** The speed of a wave traveling over a string is given by $\frac{\sqrt{t}}{\sqrt{u}}$, where t is the tension of the string and u is the density. Rewrite the expression in simplest form by rationalizing the denominator.
$\frac{\sqrt{tu}}{u}$

3. **TUNING** With each note higher on a piano, the frequency of the pitch increases by a factor of $\sqrt[12]{2}$. What is the ratio of the frequencies of two notes that are 6 steps apart on the piano? What is the ratio of the frequencies of two notes that are 9 steps apart on the piano? Express your answers in simplest form.
$\sqrt{2}$ and $\sqrt[4]{8}$

4. **LIGHTS** Suppose a light has a brightness intensity of I_1 when it is at a distance of d_1 and a brightness intensity of I_2 when it is at a distance of d_2. These quantities are related by the equation $\frac{d_2}{d_1} = \sqrt{\frac{I_1}{I_2}}$. Suppose $I_1 = 50$ units and $I_2 = 24$ units. What would $\frac{d_2}{d_1}$ be? Express your answer in simplest form.
$\frac{5\sqrt{3}}{6}$

RACING For Exercises 5 and 6, use the following information and express your answers in simplest form.

John is Jay's younger brother. They like to race and, after many races, they found that the fairest race was to run slightly different distances. They both start at the same place and run straight for 0.2 miles. Then they head for different finishes. In the figure, John and Jay's finishing paths are marked.



This time, they tied. Both of them finished the race in exactly 4 minutes.

5. If John and Jay continued at their average paces during the race, exactly how many minutes would it take them each to run a mile? Express your answer as a radical expression in simplest form.
John: $\frac{0.8 - 4\sqrt{0.02}}{0.02}$
or $40 - 20\sqrt{2}$ min;
Jay: $\frac{0.8 - 4\sqrt{0.05}}{-0.01}$
or $40\sqrt{5} - 80$ min

6. Exactly how many times as fast did John run as Jay?
$2 - \sqrt{2} + \sqrt{5} + \frac{\sqrt{10}}{2}$

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7-5 Enrichment

Special Products with Radicals

Notice that $(\sqrt{3})(\sqrt{3}) = 3$, or $(\sqrt{3})^2 = 3$. In general, $(\sqrt{x})^2 = x$ when $x \geq 0$.

Also, notice that $(\sqrt{9})(\sqrt{4}) = \sqrt{36}$. In general, $(\sqrt{x})(\sqrt{y}) = \sqrt{xy}$ when x and y are not negative.

You can use these ideas to find the special products below.

$(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = a - b$
$(\sqrt{a} + \sqrt{b})^2 = (\sqrt{a})^2 + 2\sqrt{ab} + (\sqrt{b})^2 = a + 2\sqrt{ab} + b$
$(\sqrt{a} - \sqrt{b})^2 = (\sqrt{a})^2 - 2\sqrt{ab} + (\sqrt{b})^2 = a - 2\sqrt{ab} + b$

Example 1 **Find the product: $(\sqrt{2} + \sqrt{5})(\sqrt{2} - \sqrt{5})$.**

$(\sqrt{2} + \sqrt{5})(\sqrt{2} - \sqrt{5}) = (\sqrt{2})^2 - (\sqrt{5})^2 = 2 - 5 = -3$

Example 2 **Evaluate $(\sqrt{2} + \sqrt{8})^2$.**

$(\sqrt{2} + \sqrt{8})^2 = (\sqrt{2})^2 + 2\sqrt{2}\sqrt{8} + (\sqrt{8})^2$
$= 2 + 2\sqrt{16} + 8 = 2 + 2(4) + 8 = 2 + 8 + 8 = 18$

Exercises

Multiply.

1. $(\sqrt{3} - \sqrt{7})(\sqrt{3} + \sqrt{7})$ **-4**
2. $(\sqrt{10} + \sqrt{2})(\sqrt{10} - \sqrt{2})$ **8**
3. $(\sqrt{2x} - \sqrt{6})(\sqrt{2x} + \sqrt{6})$ **$2x - 6$**
4. $(\sqrt{3} - (-7))^2$ **$52 + 14\sqrt{3}$**
5. $(\sqrt{1000} + \sqrt{10})^2$ **1210**
6. $(\sqrt{y} + \sqrt{5})(\sqrt{y} - \sqrt{5})$ **$y - 5$**
7. $(\sqrt{50} - \sqrt{x})^2$ **$50 - 10\sqrt{2x} + x$**
8. $(\sqrt{x} + 20)^2$ **$x + 40\sqrt{x} + 400$**

You can extend these ideas to patterns for sums and differences of cubes. Study the pattern below. Then complete Exercises 9–12.

$(\sqrt[3]{8} - \sqrt[3]{x})(\sqrt[3]{8^2} + \sqrt[3]{8x} + \sqrt[3]{x^2}) = \sqrt[3]{8^3} - \sqrt[3]{x^3} = 8 - x$

9. $(\sqrt[3]{2} - \sqrt[3]{5})(\sqrt[3]{2^2} + \sqrt[3]{10} + \sqrt[3]{5^2})$ **-3**
10. $(\sqrt[3]{y} + \sqrt[3]{w})(\sqrt[3]{y^2} - \sqrt[3]{yw} + \sqrt[3]{w^2})$ **$y + w$**
11. $(\sqrt[3]{7} + \sqrt[3]{20})(\sqrt[3]{7^2} - \sqrt[3]{140} + \sqrt[3]{20^2})$ **27**
12. $(\sqrt[3]{11} - \sqrt[3]{8})(\sqrt[3]{11^2} + \sqrt[3]{88} + \sqrt[3]{8^2})$ **3**

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7-6 Lesson Reading Guide

Rational Exponents

Lesson 7-6

Get Ready for the Lesson

Read the introduction to Lesson 7-6 in your textbook.

The formula in the introduction contains the exponent $\frac{2}{5}$. What do you think it might mean to raise a number to the $\frac{2}{5}$ power?

Sample answer: Take the fifth root of the number and square it.

Read the Lesson

1. Complete the following definitions of rational exponents.
   - For any real number b and for any positive integer n, $b^{\frac{1}{n}} =$ **$\sqrt[n]{b}$** except when b **< 0** and n is **even**.
   - For any nonzero real number b, and any integers m and n, with n **> 1**, $b^{\frac{m}{n}} =$ **$\sqrt[n]{b^m}$** $=$ **$(\sqrt[n]{b})^m$**, except when b **< 0** and n is **even**.

2. Complete the conditions that must be met in order for an expression with rational exponents to be simplified.
   - It has no **negative** exponents.
   - It has no **fractional** exponents in the **denominator**.
   - It is not a **complex** fraction.
   - The **index** of any remaining **radical** is the **least** number possible.

3. Margarita and Pierre were working together on their algebra homework. One exercise asked them to evaluate the expression $27^{\frac{4}{3}}$. Margarita thought that they should raise 27 to the fourth power first and then take the cube root of the result. Pierre thought that they should take the cube root of 27 first and then raise the result to the fourth power. Whose method is correct? **Both methods are correct.**

Remember What You Learned

4. Some students have trouble remembering which part of the fraction in a rational exponent gives the power and which part gives the root. How can your knowledge of integer exponents help you to keep this straight? **Sample answer: An integer exponent can be written as a rational exponent. For example, $2^3 = 2^{\frac{3}{1}}$. You know that this means that 2 is raised to the third power, so the numerator must give the power, and, therefore, the denominator must give the root.**

Answers

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7-6 Study Guide and Intervention

Rational Exponents

Rational Exponents and Radicals

Definition of $b^{\frac{1}{n}}$	For any real number *b* and any positive integer *n*, $b^{\frac{1}{n}} = \sqrt[n]{b}$, except when $b < 0$ and *n* is even.
Definition of $b^{\frac{m}{n}}$	For any nonzero real number *b*, and any integers *m* and *n*, with $n > 1$, $b^{\frac{m}{n}} = \sqrt[n]{b^m} = (\sqrt[n]{b})^m$, except when $b < 0$ and *n* is even.

Example 1 **Write $28^{\frac{1}{2}}$ in radical form.**

Notice that $28 > 0$.

$$28^{\frac{1}{2}} = \sqrt{28} = \sqrt{2^2 \cdot 7} = \sqrt{2^2} \cdot \sqrt{7} = 2\sqrt{7}$$

Example 2 **Evaluate $\left(\frac{-8}{-125}\right)^{\frac{1}{3}}$.**

Notice that $-8 < 0$, $-125 < 0$, and 3 is odd.

$$\left(\frac{-8}{-125}\right)^{\frac{1}{3}} = \frac{\sqrt[3]{-8}}{\sqrt[3]{-125}} = \frac{-2}{-5}$$

Exercises

$= \frac{2}{5}$

Write each expression in radical form.

1. $11^{\frac{1}{7}}$ **$\sqrt[7]{11}$**
2. $15^{\frac{1}{3}}$ **$\sqrt[3]{15}$**
3. $300^{\frac{3}{2}}$ **$3000\sqrt{3}$**

Write each radical using rational exponents.

4. $\sqrt{47}$ **$47^{\frac{1}{2}}$**
5. $\sqrt[3]{3a^5b^2}$ **$3^{\frac{1}{3}}a^{\frac{5}{3}}b^{\frac{2}{3}}$**
6. $\sqrt[4]{162p^5}$ **$3 \cdot 2^{\frac{1}{4}} \cdot p^{\frac{5}{4}}$**

Evaluate each expression.

7. $-27^{\frac{2}{3}}$ **9**
8. $\frac{5^{-\frac{1}{2}}}{2\sqrt{5}}$ **$\frac{1}{10}$**
9. $(0.0004)^{\frac{1}{2}}$ **0.02**

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7-6 Study Guide and Intervention *(continued)*

Rational Exponents

Lesson 7-6

Simplify Expressions All the properties of powers from Lesson 6-1 apply to rational exponents. When you simplify expressions with rational exponents, leave the exponent in rational form, and write the expression with all positive exponents. Any exponents in the denominator must be positive integers.

When you simplify radical expressions, you may use rational exponents to simplify, but your answer should be in radical form. Use the smallest index possible.

Example 1 **Simplify $y^{\frac{2}{3}} \cdot y^{\frac{3}{8}}$.**

$$y^{\frac{2}{3}} \cdot y^{\frac{3}{8}} = y^{\frac{2}{3}+\frac{3}{8}} = y^{\frac{25}{24}}$$

Example 2 **Simplify $\sqrt[4]{144x^6}$.**

$$\sqrt[4]{144x^6} = (144x^6)^{\frac{1}{4}} = (2^4 \cdot 3^2 \cdot x^6)^{\frac{1}{4}} = (2^4)^{\frac{1}{4}} \cdot (3^2)^{\frac{1}{4}} \cdot (x^6)^{\frac{1}{4}} = 2 \cdot 3^{\frac{1}{2}} \cdot x^{\frac{3}{2}} = 2x \cdot (3x)^{\frac{1}{2}} = 2x\sqrt{3x}$$

Exercises

Simplify each expression.

1. $x^{\frac{4}{5}} \cdot x^{\frac{6}{5}}$ **x^2**
2. $\left(y^{\frac{2}{3}}\right)^{\frac{3}{4}}$ **$y^{\frac{1}{2}}$**
3. $p^{\frac{4}{5}} \cdot p^{\frac{7}{10}}$ **$p^{\frac{3}{2}}$**
4. $\left(m^{-\frac{6}{5}}\right)^{\frac{2}{5}}$ **$\frac{1}{m^{\frac{12}{25}}}$ or $\frac{m^{\frac{13}{25}}}{m}$**
5. $x^{-\frac{3}{8}} \cdot x^{\frac{4}{3}}$ **$x^{\frac{23}{24}}$**
6. $\left(s^{-\frac{1}{6}}\right)^{-\frac{4}{3}}$ **$s^{\frac{2}{9}}$**

$\frac{p}{p^{\frac{1}{3}}}$

$p^{\frac{2}{3}}$

7. **a^2**
8. $\left(a^{\frac{2}{3}}\right)^{\frac{6}{5}} \cdot \left(a^{\frac{2}{5}}\right)^3$
9. $\frac{x^{-\frac{1}{2}}}{x^{-\frac{1}{3}}}$ **$\frac{1}{x^{\frac{1}{6}}}$ or $\frac{x^{\frac{5}{6}}}{x}$**
10. $\sqrt[6]{128}$ **$2\sqrt[6]{2}$**
11. $\sqrt[4]{49}$ **$\sqrt{7}$**
12. $\sqrt[5]{288}$ **$2\sqrt[5]{9}$**
13. $\sqrt{32} \cdot 3\sqrt{16}$ **$48\sqrt{2}$**
14. $\sqrt[3]{25} \cdot \sqrt{125}$ **$25\sqrt[6]{5}$**
15. $\sqrt[6]{16}$ **$\sqrt[3]{4}$**
16. $\frac{x - \sqrt[3]{3}}{\sqrt{12}}$ **$\frac{x\sqrt{3} - \sqrt[6]{3^5}}{6}$**
17. $\sqrt{\sqrt[3]{48}}$ **$\sqrt[6]{48}$**
18. $\frac{a\sqrt[3]{b^4}}{\sqrt{ab^3}}$ **$\frac{\sqrt{a}\sqrt[6]{b^5}}{b}$**

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7-6 Skills Practice

Rational Exponents

Write each expression in radical form.

1. $3^{\frac{1}{6}}$ $\sqrt[6]{3}$

2. $8^{\frac{1}{5}}$ $\sqrt[5]{8}$

3. $12^{\frac{2}{3}}$ $\sqrt[3]{12^2}$ or $(\sqrt[3]{12})^2$ or $2\sqrt[3]{18}$

4. $(s^3)^{\frac{3}{5}}$ $s\sqrt[5]{s^4}$

Write each radical using rational exponents.

5. $\sqrt{51}$ $51^{\frac{1}{2}}$

6. $\sqrt[3]{37}$ $37^{\frac{1}{3}}$

7. $\sqrt[4]{15^3}$ $15^{\frac{3}{4}}$

8. $\sqrt[3]{6xy^2}$ $6^{\frac{1}{3}}x^{\frac{1}{3}}y^{\frac{2}{3}}$

Evaluate each expression.

9. $32^{\frac{1}{5}}$ 2

10. $81^{\frac{1}{4}}$ 3

11. $27^{-\frac{1}{3}}$ $\frac{1}{3}$

12. $4^{-\frac{1}{2}}$ $\frac{1}{2}$

13. $16^{\frac{3}{2}}$ 64

14. $(-243)^{\frac{4}{5}}$ 81

15. $27^{\frac{1}{3}} \cdot 27^{\frac{5}{3}}$ 729

16. $\left(\frac{4}{9}\right)^{\frac{3}{2}}$ $\frac{8}{27}$

Simplify each expression.

17. $c^{\frac{12}{5}} \cdot c^{\frac{3}{5}}$ c^3

18. $m^{\frac{2}{9}} \cdot m^{\frac{16}{9}}$ m^2

19. $\left(q^{\frac{1}{2}}\right)^3$ $q^{\frac{3}{2}}$

20. $p^{-\frac{1}{5}}$ $\frac{1}{p^{\frac{1}{5}}}$ or $\frac{p^{\frac{4}{5}}}{p}$

21. $x^{-\frac{6}{11}}$ $\frac{1}{x^{\frac{6}{11}}}$ or $\frac{x^{\frac{5}{11}}}{x}$

22. $\frac{x^{\frac{2}{3}}}{x^{\frac{1}{4}}}$ $x^{\frac{5}{12}}$

23. $\frac{y^{-\frac{1}{2}}}{y^{\frac{1}{4}}}$ $\frac{1}{y^{\frac{3}{4}}}$ or $\frac{y^{\frac{1}{4}}}{y}$

24. $\frac{n^{\frac{1}{3}}}{n^{\frac{1}{6}} \cdot n^{\frac{1}{2}}}$ $\frac{1}{n^{\frac{1}{3}}}$ or $\frac{n^{\frac{2}{3}}}{n}$

25. $\sqrt[12]{64}$ $\sqrt{2}$

26. $\sqrt[8]{49a^8b^2}$ $|a|\sqrt[4]{7b}$

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7-6 Practice

Rational Exponents

Write each expression in radical form.

1. $5^{\frac{1}{3}}$ $\sqrt[3]{5}$

2. $6^{\frac{2}{5}}$ $\sqrt[5]{6^2}$ or $(\sqrt[5]{6})^2$

3. $m^{\frac{4}{7}}$ $\sqrt[7]{m^4}$ or $(\sqrt[7]{m})^4$

4. $(n^3)^{\frac{2}{5}}$ $n\sqrt[5]{n}$

Write each radical using rational exponents.

5. $\sqrt{79}$ $79^{\frac{1}{2}}$

6. $\sqrt[4]{153}$ $153^{\frac{1}{4}}$

7. $\sqrt[3]{27m^6n^4}$ $3m^2n^{\frac{4}{3}}$

8. $5\sqrt{2a^{10}b}$ $5 \cdot 2^{\frac{1}{2}}|a^5|b^{\frac{1}{2}}$

Evaluate each expression.

9. $81^{\frac{1}{4}}$ 3

10. $1024^{-\frac{1}{5}}$ $\frac{1}{4}$

11. $8^{-\frac{5}{3}}$ $\frac{1}{32}$

12. $-256^{-\frac{3}{4}}$ $-\frac{1}{64}$

13. $(-64)^{-\frac{2}{3}}$ $\frac{1}{16}$

14. $27^{\frac{1}{3}} \cdot 27^{\frac{4}{3}}$ 243

15. $\left(\frac{125}{216}\right)^{\frac{2}{3}}$ $\frac{25}{36}$

16. $\frac{64^{\frac{2}{3}}}{343^{\frac{2}{3}}}$ $\frac{16}{49}$

17. $\left(25^{\frac{1}{2}}\right)\left(-64^{-\frac{1}{3}}\right)$ $-\frac{5}{4}$

Simplify each expression.

18. $g^{\frac{4}{7}} \cdot g^{\frac{3}{7}}$ g

19. $s^{\frac{3}{4}} \cdot s^{\frac{13}{4}}$ s^4

20. $\left(u^{-\frac{1}{3}}\right)^{-\frac{4}{5}}$ $u^{\frac{4}{15}}$

21. $y^{-\frac{1}{2}}$ $\frac{1}{y^{\frac{1}{2}}}$ or $\frac{y^{\frac{1}{2}}}{y}$

22. $b^{-\frac{3}{5}}$ $\frac{1}{b^{\frac{3}{5}}}$ or $\frac{b^{\frac{2}{5}}}{b}$

23. $\frac{q^{\frac{3}{5}}}{q^{\frac{2}{5}}}$ $q^{\frac{1}{5}}$

24. $\frac{t^{\frac{2}{3}}}{5t^{\frac{1}{2}} \cdot t^{-\frac{3}{4}}}$ $\frac{t^{\frac{11}{12}}}{5}$

25. $\frac{2z^{\frac{1}{2}}}{z^{\frac{1}{2}} - 1}$ $\frac{2z + 2z^{\frac{1}{2}}}{z - 1}$

26. $\sqrt[10]{8^5}$ $2\sqrt{2}$

27. $\sqrt{12} \cdot \sqrt[5]{12^3}$ $12\sqrt[10]{12}$

28. $\sqrt[4]{6} \cdot 3\sqrt[4]{6}$ $3\sqrt{6}$

29. $\frac{a}{\sqrt{3b}}$ $\frac{a\sqrt{3b}}{3b}$

30. ELECTRICITY The amount of current in amperes I that an appliance uses can be calculated using the formula $I = \left(\frac{P}{R}\right)^{\frac{1}{2}}$, where P is the power in watts and R is the resistance in ohms. How much current does an appliance use if $P = 500$ watts and $R = 10$ ohms? Round your answer to the nearest tenth. **7.1 amps**

31. BUSINESS A company that produces DVDs uses the formula $C = 88n^{\frac{1}{3}} + 330$ to calculate the cost C in dollars of producing n DVDs per day. What is the company's cost to produce 150 DVDs per day? Round your answer to the nearest dollar. **$798**

Lesson 7-6

Answers

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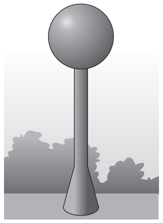
7-6 Word Problem Practice

Rational Exponents

1. SQUARING THE CUBE A cube has side length s. What is the side length of the square that has an area equal to the volume of this cube? Write your answer using rational exponents.

$s^{\frac{3}{2}}$

2. WATER TOWER A large water tower stores drinking water in a big spherical tank. The mayor of the town decides that the water tower must be replaced with a larger tank. Town residents insist that the new tower be a sphere. If the new tank will hold 10 times as much water as the old tank, how many times long should the radius of the new tank be compared to the old tank? Write your answer using rational exponents.



$10^{\frac{1}{3}}$

3. BALLOONS A spherical balloon is being inflated faster and faster. The volume of the balloon as a function of time is $9\pi t^2$. What is the radius of the balloon as a function of time? Write your answer using rational exponents.

$3\left(\frac{t}{2}\right)^{\frac{2}{3}}$

4. INTEREST Rita opened a bank account that accumulated interest at the rate of 1% compounded annually. Her money accumulated interest in that account for 8 years. She then took all of her money out of that account and placed it into another account that paid 5% interest compounded annually. After 4 years, she took all of her money out of that account. What single interest rate when compounded annually would give her the same outcome for those 12 years? Round your answer to the nearest hundredth of a percent.

2.32%

CELLS For Exercises 5-7, use the following information.

The number of cells in a cell culture grows exponentially. The number of cells in the culture as a function of time is given by the expression $N\left(\frac{6}{5}\right)^t$ where t is measured in hours and N is the initial size of the culture.

5. After 3 hours, there were 1728 cells in the culture. What is N?

1000

6. How many cells were in the culture after 20 minutes? Express your answer in simplest form.

$200 \cdot 150^{\frac{1}{3}}$

7. How many cells were in the culture after 2.5 hours? Express your answer in simplest form.

$288 \cdot 30^{\frac{1}{2}}$

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7-6 Enrichment

Lesser-Known Geometric Formulas

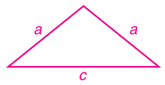
Many geometric formulas involve radical expressions.

Make a drawing to illustrate each of the formulas given on this page. Then evaluate the formula for the given value of the variable. Round answers to the nearest hundredth.

1. The area of an isosceles triangle. Two sides have length a; the other side has length c. Find A when $a = 6$ and $c = 7$.

$A = \frac{c}{4}\sqrt{4a^2 - c^2}$

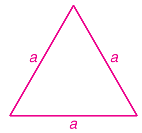
$A \approx 17.06$ units^2



2. The area of an equilateral triangle with a side of length a. Find A when $a = 8$.

$A = \frac{a^2}{4}\sqrt{3}$

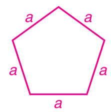
$A \approx 27.71$ units^2



3. The area of a regular pentagon with a side of length a. Find A when $a = 4$.

$A = \frac{a^2}{4}\sqrt{25 + 10\sqrt{5}}$

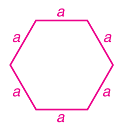
$A \approx 27.53$ units^2



4. The area of a regular hexagon with a side of length a. Find A when $a = 9$.

$A = \frac{3a^2}{2}\sqrt{3}$

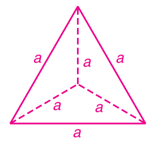
$A \approx 210.44$ units^2



5. The volume of a regular tetrahedron with an edge of length a. Find V when $a = 2$.

$V = \frac{a^3}{12}\sqrt{2}$

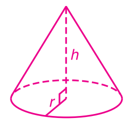
$V \approx 0.94$ units^3



6. The area of the curved surface of a right cone with an altitude of h and radius of base r. Find S when $r = 3$ and $h = 6$.

$S = \pi r\sqrt{r^2 + h^2}$

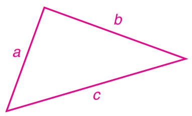
$S \approx 63.22$ units^2



7. Heron's Formula for the area of a triangle uses the semi-perimeter s, where $s = \frac{a + b + c}{2}$. The sides of the triangle have lengths a, b, and c. Find A when $a = 3$, $b = 4$, and $c = 5$.

$A = \sqrt{s(s - a)(s - b)(s - c)}$

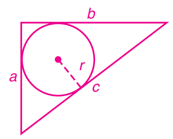
$A = 6$ units^2



8. The radius of a circle inscribed in a given triangle also uses the semi-perimeter. Find r when $a = 6$, $b = 7$, and $c = 9$.

$r = \frac{\sqrt{s(s - a)(s - b)(s - c)}}{s}$

$r \approx 1.91$ units



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7-6 Spreadsheet Activity

Appreciation and Depreciation

When an asset such as a house increases in value over time, it is said to *appreciate*. If the value increases by a fixed percent each year, or other period of time, the amount y of that quantity after t years is given by

$$y = a(1 + r)^t,$$

where a is the initial amount and r is the percent of increase expressed as a decimal. You can use a spreadsheet to investigate future values of an asset.

Example **Michael Blackstock is considering buying a piece of investment property for $95,000. The homes in the area are appreciating at an average rate of 4% per year. Find the expected value of the home in 1 year, 1 year and 6 months, 4 years, and 6 years and 9 months.**

Use rows 1 and 2 to enter the initial amount and the rate of increase. Then use Column A to enter the amounts of time. Enter the numbers of months as a fraction of a year since t is measured in years. Column B contains the formulas for the value of the home.

Format the cells containing the values as currency so that they are displayed as dollars and cents. The expected value of the home after each amount of time is shown in the spreadsheet.

Appreciation.xls

	A	B
1	Initial value =	$95,000.00
2	Rate =	0.04
3		
4	Years	Value
5	1	$98,800.00
6	1.5	$100,756.63
7	4	$111,136.56
8	6.75	$123,793.73

Sheet 1 / Sheet 2

Exercises

1. If Mr. Blackstock chooses another property in the neighborhood that costs $99,900, what are the expected values of that home in the same periods of time? **$103,896.00, $105,953.55, $116,868.87, $130,178.88**

2. What would Mr. Blackstock's profit be on the $99,900 home if he sold it after 9 years and 3 months? **$43,689.89**

3. If an antique chair worth $165.00 increases in value an average of $3\frac{1}{2}$% every year, how much will it be worth next year? **$170.78**

4. Often assets like cars decrease in value over time. This asset is said to *depreciate*. If the value decreases by a fixed percent each year, or other period of time, the amount y of that quantity after t years is given by $y = a(1 - r)^t$, where a is the initial amount and r is the percent of decrease expressed as a decimal. Use a spreadsheet to find the value of a car purchased for $18,500 after 2 years, 2 years and 6 months, and 4 years and 3 months if the car depreciates at a rate of 12% per year.

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7-7 Lesson Reading Guide

Solving Radical Equations and Inequalities

Get Ready for the lesson

Read the introduction to Lesson 7-7 in your textbook.

Explain how you would use the formula in your textbook to find the cost of producing 125,000 computer chips. (Describe the steps of the calculation in the order in which you would perform them, but do not actually do the calculation.)

Sample answer: Raise 125,000 to the $\frac{2}{3}$ power by taking the cube root of 125,000 and squaring the result (or raise 125,000 to the $\frac{2}{3}$ power by entering 125,000 ^ (2/3) on a calculator). Multiply the number you get by 10 and then add 1500.

Read the Lesson

1. a. What is an *extraneous solution* of a radical equation? **Sample answer: a number that satisfies an equation obtained by raising both sides of the original equation to a higher power but does not satisfy the original equation**

   b. Describe two ways you can check the proposed solutions of a radical equation in order to determine whether any of them are extraneous solutions. **Sample answer: One way is to check each proposed solution by substituting it into the original equation. Another way is to use a graphing calculator to graph both sides of the original equation. See where the graphs intersect. This can help you identify solutions that may be extraneous.**

2. Complete the steps that should be followed in order to solve a radical inequality.

   Step 1 If the **index** of the root is **even**, identify the values of the variable for which the **radicand** is **nonnegative**.

   Step 2 Solve the **inequality** algebraically.

   Step 3 Test **values** to check your solution.

Remember What You Learned

3. One way to remember something is to explain it to another person. Suppose that your friend Leora thinks that she does not need to check her solutions to radical equations by substitution because she knows she is very careful and seldom makes mistakes in her work. How can you explain to her that she should nevertheless check every proposed solution in the original equation? **Sample answer: Squaring both sides of an equation can produce an equation that is not equivalent to the original one. For example, the only solution of $x = 5$ is 5, but the squared equation $x^2 = 25$ has two solutions, 5 and −5.**

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7-7 Study Guide and Intervention

Solving Radical Equations and Inequalities

Solve Radical Equations The following steps are used in solving equations that have variables in the radicand. Some algebraic procedures may be needed before you use these steps.

Step 1 Isolate the radical on one side of the equation.
Step 2 To eliminate the radical, raise each side of the equation to a power equal to the index of the radical.
Step 3 Solve the resulting equation.
Step 4 Check your solution in the original equation to make sure that you have not obtained any extraneous roots.

Example 1 **Solve $2\sqrt{4x+8} - 4 = 8$.**

$2\sqrt{4x+8} - 4 = 8$ Original equation
$2\sqrt{4x+8} = 12$ Add 4 to each side.
$\sqrt{4x+8} = 6$ Isolate the radical.
$4x + 8 = 36$ Square each side.
$4x = 28$ Subtract 8 from each side.
$x = 7$ Divide each side by 4.

Check
$2\sqrt{4(7)+8} - 4 \stackrel{?}{=} 8$
$2\sqrt{36} - 4 \stackrel{?}{=} 8$
$2(6) - 4 \stackrel{?}{=} 8$
$8 = 8$
The solution $x = 7$ checks.

Example 2 **Solve $\sqrt{3x+1} = \sqrt{5x} - 1$.**

$\sqrt{3x+1} = \sqrt{5x} - 1$ Original equation
$3x + 1 = 5x - 2\sqrt{5x} + 1$ Square each side.
$2\sqrt{5x} = 2x$ Simplify.
$\sqrt{5x} = x$ Isolate the radical.
$5x = x^2$ Square each side.
$x^2 - 5x = 0$ Subtract 5x from each side.
$x(x - 5) = 0$ Factor.
$x = 0$ or $x = 5$

Check
$\sqrt{3(0)+1} = 1$, but $\sqrt{5(0)} - 1 = -1$, so 0 is not a solution.
$\sqrt{3(5)+1} = 4$, and $\sqrt{5(5)} - 1 = 4$, so the solution is $x = 5$.

Exercises

Solve each equation.

1. $3 + 2x\sqrt{3} = 5$ **$\frac{\sqrt{3}}{3}$**
2. $2\sqrt{3x+4} + 1 = 15$ **15**
3. $8 + \sqrt{x+1} = 2$ **no solution**
4. $\sqrt{5-x} - 4 = 6$ **−95**
5. $12 + \sqrt{2x-1} = 4$ **no solution**
6. $\sqrt{12-x} = 0$ **12**
7. $\sqrt{21} - \sqrt{5x-4} = 0$ **5**
8. $10 - \sqrt{2x} = 5$ **12.5**
9. $\sqrt{x^2+7x} = \sqrt{7x-9}$ **no solution**
10. $4\sqrt[3]{2x+11} - 2 = 10$ **8**
11. $2\sqrt{x+11} = \sqrt{x+2} + \sqrt{3x-6}$ **14**
12. $\sqrt{9x-11} = x + 1$ **3, 4**

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7-7 Study Guide and Intervention *(continued)*

Solving Radical Equations and Inequalities

Solve Radical Inequalities A **radical inequality** is an inequality that has a variable in a radicand. Use the following steps to solve radical inequalities.

Step 1 If the index of the root is even, identify the values of the variable for which the radicand is nonnegative.
Step 2 Solve the inequality algebraically.
Step 3 Test values to check your solution.

Example **Solve $5 - \sqrt{20x+4} \geq -3$.**

Since the radicand of a square root must be greater than or equal to zero, first solve
$20x + 4 \geq 0.$
$20x + 4 \geq 0$
$20x \geq -4$
$x \geq -\frac{1}{5}$

Now solve $5 - \sqrt{20x+4} \geq -3$.
$5 - \sqrt{20x+4} \geq -3$ Original inequality
$\sqrt{20x+4} \leq 8$ Isolate the radical.
$20x + 4 \leq 64$ Eliminate the radical by squaring each side.
$20x \leq 60$ Subtract 4 from each side.
$x \leq 3$ Divide each side by 20.

It appears that $-\frac{1}{5} \leq x \leq 3$ is the solution. Test some values.

***x* = −1**	***x* = 0**	***x* = 4**
$\sqrt{20(-1)+4}$ is not a real number, so the inequality is not satisfied.	$5 - \sqrt{20(0)+4} = 3$, so the inequality is satisfied.	$5 - \sqrt{20(4)+4} \approx -4.2$, so the inequality is not satisfied

Therefore the solution $-\frac{1}{5} \leq x \leq 3$ checks.

Exercises

Solve each inequality.

1. $\sqrt{c-2} + 4 \geq 7$ **$c \geq 11$**
2. $3\sqrt{2x-1} + 6 < 15$ **$\frac{1}{2} \leq x < 5$**
3. $\sqrt{10x+9} - 2 > 5$ **$x > 4$**
4. $5\sqrt[3]{x+2} - 8 < 2$ **$x < 6$**
5. $8 - \sqrt{3x+4} \geq 3$ **$-\frac{4}{3} \leq x \leq 7$**
6. $\sqrt{2x+8} - 4 > 2$ **$x > 14$**
7. $9 - \sqrt{6x+3} \geq 6$ **$-\frac{1}{2} \leq x \leq 1$**
8. $\frac{20}{\sqrt{3x+1}} \leq 4$ **$x \geq 8$**
9. $2\sqrt{5x-6} - 1 < 5$ **$\frac{6}{5} \leq x < 3$**
10. $\sqrt{2x+12} + 4 \geq 12$ **$x \geq 26$**
11. $\sqrt{2d+1} + \sqrt{d} \leq 5$ **$0 \leq d \leq 4$**
12. $4\sqrt{b+3} - \sqrt{b-2} \geq 10$ **$b \geq 6$**

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7-7 Skills Practice

Solving Radical Equations and Inequalities

Solve each equation or inequality.

1. $\sqrt{x} = 5$ **25**
2. $\sqrt{x} + 3 = 7$ **16**
3. $5\sqrt{j} = 1$ **$\frac{1}{25}$**
4. $v^{\frac{1}{2}} + 1 = 0$ **no solution**
5. $18 - 3y^{\frac{1}{2}} = 25$ **no solution**
6. $\sqrt[3]{2w} = 4$ **32**
7. $\sqrt{b - 5} = 4$ **21**
8. $\sqrt{3n + 1} = 5$ **8**
9. $\sqrt[3]{3r - 6} = 3$ **11**
10. $2 + \sqrt{3p + 7} = 6$ **3**
11. $\sqrt{k - 4} - 1 = 5$ **40**
12. $(2d + 3)^{\frac{1}{3}} = 2$ **$\frac{5}{2}$**
13. $(t - 3)^{\frac{1}{3}} = 2$ **11**
14. $4 - (1 - 7u)^{\frac{1}{3}} = 0$ **−9**
15. $\sqrt{3z - 2} = \sqrt{z - 4}$ **no solution**
16. $\sqrt{g + 1} = \sqrt{2g - 7}$ **8**
17. $\sqrt{x - 1} = 4\sqrt{x + 1}$ **no solution**
18. $5 + \sqrt{s - 3} \leq 6$ **$3 \leq s \leq 4$**
19. $-2 + \sqrt{3x + 3} < 7$ **$-1 \leq x < 26$**
20. $-\sqrt{2a + 4} \geq -6$ **$-2 \leq a \leq 16$**
21. $2\sqrt{4r - 3} > 10$ **$r > 7$**
22. $4 - \sqrt{3x + 1} > 3$ **$-\frac{1}{3} \leq x < 0$**
23. $\sqrt{y + 4} - 3 \geq 3$ **$y \geq 32$**
24. $-3\sqrt{11r + 3} \geq -15$ **$-\frac{3}{11} \leq r \leq 2$**

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7-7 Practice

Solving Radical Equations and Inequalities

Solve each equation or inequality.

1. $\sqrt{x} = 8$ **64**
2. $4 - \sqrt{x} = 3$ **1**
3. $\sqrt{2p} + 3 = 10$ **$\frac{49}{2}$**
4. $4\sqrt{3h} - 2 = 0$ **$\frac{1}{12}$**
5. $c^{\frac{1}{2}} + 6 = 9$ **9**
6. $18 + 7h^{\frac{1}{2}} = 12$ **no solution**
7. $\sqrt[3]{d + 2} = 7$ **341**
8. $\sqrt[5]{w - 7} = 1$ **8**
9. $6 + \sqrt[3]{q - 4} = 9$ **31**
10. $\sqrt[4]{y - 9} + 4 = 0$ **no solution**
11. $\sqrt{2m - 6} - 16 = 0$ **131**
12. $\sqrt[3]{4m + 1} - 2 = 2$ **$\frac{63}{4}$**
13. $\sqrt{8n - 5} - 1 = 2$ **$\frac{7}{4}$**
14. $\sqrt{1 - 4t} - 8 = -6$ **$-\frac{3}{4}$**
15. $\sqrt{2t - 5} - 3 = 3$ **$\frac{41}{2}$**
16. $(7v - 2)^{\frac{1}{4}} + 12 = 7$ **no solution**
17. $(3g + 1)^{\frac{1}{2}} - 6 = 4$ **33**
18. $(6u - 5)^{\frac{1}{3}} + 2 = -3$ **−20**
19. $\sqrt{2d - 5} = \sqrt{d - 1}$ **4**
20. $\sqrt{4r - 6} = \sqrt{r}$ **2**
21. $\sqrt{6x - 4} = \sqrt{2x + 10}$ **$\frac{7}{2}$**
22. $\sqrt{2x + 5} = \sqrt{2x + 1}$ **no solution**
23. $3\sqrt{a} \geq 12$ **$a \geq 16$**
24. $\sqrt{z + 5} + 4 \leq 13$ **$-5 \leq z \leq 76$**
25. $8 + \sqrt{2q} \leq 5$ **no solution**
26. $\sqrt{2a - 3} < 5$ **$\frac{3}{2} \leq a < 14$**
27. $9 - \sqrt{c + 4} \leq 6$ **$c \geq 5$**
28. $\sqrt[3]{x - 1} < -2$ **$x < -7$**

29. **STATISTICS** Statisticians use the formula $\sigma = \sqrt{v}$ to calculate a standard deviation σ, where v is the variance of a data set. Find the variance when the standard deviation is 15. **225**

30. **GRAVITATION** Helena drops a ball from 25 feet above a lake. The formula $t = \frac{1}{4}\sqrt{25 - h}$ describes the time t in seconds that the ball is h feet above the water. How many feet above the water will the ball be after 1 second? **9 ft**

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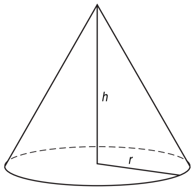
7-7 Word Problem Practice

Rational Equations and Inequalities

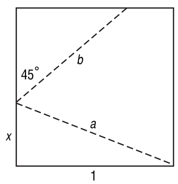
1. **SIGNS** A sign painter must spend $\$8n^{\frac{2}{3}} + 400$ to make n signs. How many signs can the painter make for \$1200?
1000

2. **LATERAL AREA** The lateral area of a cone with base radius r and height h is given by the formula $L = \pi r\sqrt{r^2 + h^2}$. A cone has a lateral area of 65π square units and a base radius of 5 units.



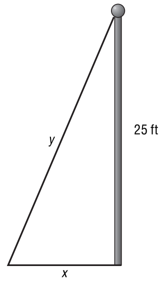
What is the height of the cone?
12 units

3. **ORIGAMI** Georgia wants to fold a square piece of paper into an equilateral triangle. She wants to locate the distance x up the side of the square where she can make the fold indicated by the dashed line in the figure so that $a = b$. From geometry class, she knows that $a = \sqrt{1 + x^2}$ and $b = \sqrt{2}(1 - x)$. So the equation she must solve is $\sqrt{1 + x^2} = \sqrt{2}(1 - x)$. What is x?



$2 - \sqrt{3}$

4. **TETHERS** A tether is being attached to a 25-foot pole in such a way that $x + y = 50$. By the Pythagorean Theorem, the distance $y = \sqrt{x^2 + 25^2}$. What must x be?



18.75 ft

RANGE For Exercises 5 and 6, use the following information.

An asteroid is passing near Earth. If Earth is located at the origin of a coordinate plane, the path that the asteroid will trace out is given by $y = \frac{17}{x}$, $x > 0$. One unit corresponds to one million miles. Carl learns that he will be able to see the asteroid with his telescope when the asteroid is within $\frac{145}{12}$ million miles of Earth.

5. Write an expression that gives the distance of the asteroid from Earth as a function of x.
$\sqrt{x^2 + \frac{289}{x^2}}$

6. For what values of x will the asteroid be in range of Carl's telescope?
$\frac{17}{12} \le x \le 12$

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7-7 Enrichment

Truth Tables

In mathematics, the basic operations are addition, subtraction, multiplication, division, finding a root, and raising to a power. In logic, the basic operations are the following: *not* ($\sim$), *and* ($\wedge$), *or* ($\vee$), and *implies* ($\rightarrow$).

If p and q are statements, then $\sim p$ means not p; $\sim q$ means not q; $p \wedge q$ means p and q; $p \vee q$ means p or q; and $p \rightarrow q$ means p implies q. The operations are defined by truth tables. On the left below is the truth table for the statement $\sim p$. Notice that there are two possible conditions for p, true (T) or false (F). If p is true, $\sim p$ is false; if p is false, $\sim p$ is true. Also shown are the truth tables for $p \wedge q$, $p \vee q$, and $p \rightarrow q$.

p	$\sim p$
T	F
F	T

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

You can use this information to find out under what conditions a complex statement is true.

Example **Under what conditions is $\sim p \vee q$ true?**

Create the truth table for the statement. Use the information from the truth table above for $p \vee q$ to complete the last column.

p	q	$\sim p$	$\sim p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

When one statement is true and one is false, the conjunction is true.

The truth table indicates that $\sim p \vee q$ is true in all cases except where p is true and q is false.

Use truth tables to determine the conditions under which each statement is true.

1. $\sim p \vee \sim q$
all except where both *p* and *q* are true

2. $\sim p \rightarrow (p \rightarrow q)$
all

3. $(p \vee q) \vee (\sim p \wedge \sim q)$
all

4. $(p \rightarrow q) \vee (q \rightarrow p)$
all

5. $(p \rightarrow q) \wedge (q \rightarrow p)$
both *p* and *q* are true; both *p* and *q* are false

6. $(\sim p \wedge \sim q) \rightarrow \sim(p \vee q)$
all

Chapter 7 Assessment Answer Key

Quiz 1 (Lessons 7–1 through 7–2)
Page 59

1. $x^2 - x + 6$; $x^2 - 5x - 2$; $2x^3 - 2x^2 - 8x + 8$; $\frac{x^2 - 3x + 2}{2x + 4}$; $x \neq -2$

2. {(2, 4), (3, 8), (4, 3), (8, 4)}; {(2, 5), (4, 2), (5, 4)}

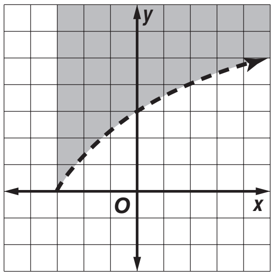
3. $x^2 - 6x + 7$; $x^2 + 2x - 5$

4. 28; 122

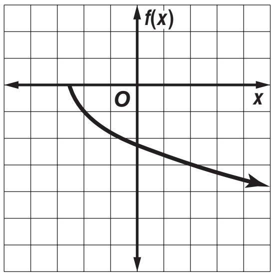
5. {(5, −2), (4, 0), (−8, 1), (7, 4)}

Quiz 2 (Lessons 7–3 and 7–4)
Page 59

1. D: $x > -3$; R: $y \geq 0$



2. D: $x \geq -\frac{5}{2}$; R: $y \leq 0$



3. $-3w^3y^2$

4. −3.826

5. $\frac{|x^3|}{5}$

Quiz 3 (Lessons 7–5 and 7–6)
Page 60

1. $\frac{\sqrt{10x}}{2x}$

2. $3m^2|n^3|\sqrt{2m}$

3. $14\sqrt{3} + 39\sqrt{2}$

4. $12 - 2\sqrt{35}$

5. $11 + 11\sqrt{5}$

6. $\frac{7 - 3\sqrt{6}}{5}$

7. $\sqrt[8]{x^5}$ or $(\sqrt[8]{x})^5$

8. $2z^{\frac{3}{5}}$

9 $\frac{1}{64}$

10. D

Quiz 4 (Lesson 7–7)
Page 60

1. no solution

2. $-\frac{1}{2}$

3. $\frac{13}{64}$

4. $x > 2$

5. no solution

Mid-Chapter Test
Page 61

1. D

2. F

3. B

4. H

5. A

6. J

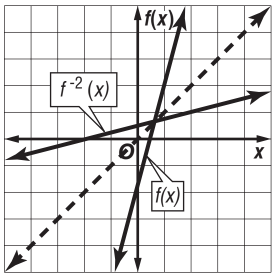
7. B

8. $-3x^2 + 4x - 9$; $12x^3 - 27x^2$

9. $p^{-1}(x) = \frac{x + 8}{4}$

10. 4.116

11. $f^{-1}(x) = \frac{x + 2}{4}$



12. yes

(continued on the next page)

Answers

Chapter 7 Assessment Answer Key

Vocabulary Test

Page 62

1. one-to-one
2. square root
3. composition of functions
4. extraneous solution
5. radical inequalities
6. conjugates
7. principal root
8. radical equations
9. inverse function
10. rationalizing the denominator
11. Sample answer: Two radical expressions are called like radical expressions if both the indices and the radicands are alike.
12. Sample answer: The inverse relation is the set of ordered pairs obtained by reversing the coordinates of each ordered pair in a relation.

Form 1

Page 63

1. A
2. H
3. B
4. J
5. D
6. F
7. D
8. F
9. A

Page 64

10. J
11. A
12. J
13. C
14. G
15. B
16. J
17. B
18. F
19. C
20. J

B: $4x + 3$

Chapter 7 Assessment Answer Key

Form 2A

Page 65

1. D
2. F
3. C
4. G
5. B
6. H
7. A
8. J
9. C
10. G

Page 66

11. D
12. H
13. A
14. H
15. D
16. J
17. D
18. J
19. A
20. G

B: -2

Form 2B

Page 67

1. C
2. J
3. B
4. H
5. A
6. J
7. D
8. H
9. A
10. J

Page 68

11. A
12. H
13. B
14. J
15. C
16. H
17. A
18. F
19. C
20. J

B: 3

Answers

Chapter 7 Assessment Answer Key

Form 2C

Page 69

1. $-x^3 + 7x^2 - 4x + 28$

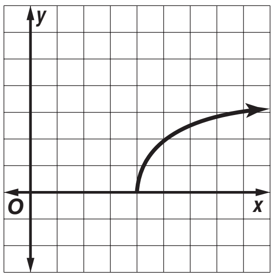
2. 2

3. $2x^2 - 1$

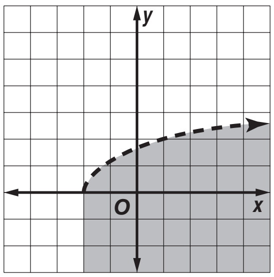
4. $f^{-1}(x) = \frac{1}{5}x - 2$

5. yes

6. D: $x \geq 4$
R: $y \geq 0$



7.



8. $\frac{2}{7}$

9. $7|x^3|y^2$

10. $2a^2b\sqrt[3]{3b^2}$

11. $18\sqrt{2} + 5\sqrt{3}$

Page 70

12. 47.693 in.

13. $2m^{\frac{3}{5}}$

14. $x^{\frac{1}{6}}$ or $\sqrt[6]{x}$

15. 21

16. $y \geq 7$

17. 20 ft/s

18. $\frac{\sqrt{6}}{3}$

19. $24rs^{\frac{3}{2}}$ units^2

20. 2.5 cm

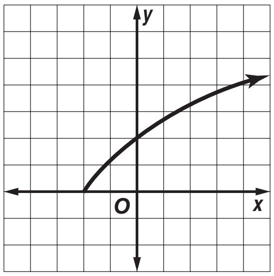
B: 2

Chapter 7 Assessment Answer Key

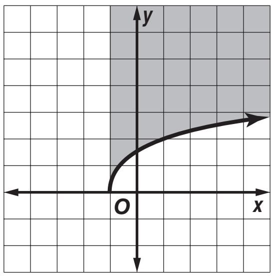
Form 2D

Page 71

1. $-x^3 + 6x^2 + 4x - 24$
2. 4
3. $x^2 - 6x + 5$
4. $g^{-1}(x) = -\frac{1}{2}x + 2$
5. yes
6. D: $x \geq -2$
   R: $y \geq 0$



7.



8. $\frac{3}{5}$
9. $2|x|y^2$
10. $-4a^2b^2\sqrt[3]{b}$

Page 72

12. 40.406 in.
13. $-5x^{\frac{2}{3}}$
14. $x^{\frac{1}{10}}$ or $\sqrt[10]{x}$
15. 8
16. $t > 1$
17. 7 ft/s
18. $\frac{\sqrt{15}}{5}$
19. $10rs^{\frac{4}{5}}$ units^2
20. 3.30 cm

B: -4

Answers

Chapter 7 Assessment Answer Key

Form 3

Page 73

1. $x^2 + 2x$

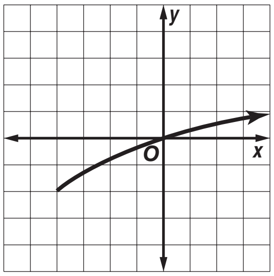
2. $27x^3 - 9x^2 + 3x - 1$

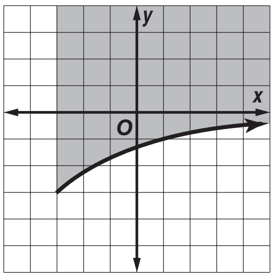
3. 960

4. $h^{-1}(x) = \frac{5x - 6}{2}$

5. yes

6. D: $x \geq -4$
R: $y \geq -2$



7. 

8. $|2x-5|$

9. $-3x^2y$

10. $xy^2\sqrt[3]{x^2y}$

11. $4\sqrt{15} - 9\sqrt{5}$

Page 74

12. 5.760 m

13. $2x^{\frac{9}{4}}y$

14. $-5 + 3\sqrt{3}$

15. 565

16. $-2 \leq x < 2$

17. $\sqrt{x} - 3$

18. $\frac{45\sqrt{7}}{4}$ units^2

19. 10 units

20. 147 pounds

B: $-28\sqrt{3} + 96$

Chapter 7 Assessment Answer Key

Page 75, Extended-Response Test
Scoring Rubric

Score	General Description	Specific Criteria
4	**Superior** A correct solution that is supported by well-developed, accurate explanations	• Shows thorough understanding of the concepts of *operations with functions; finding the inverse of a function; operations with radical expressions;* and *solving equations and inequalities containing radicals.* • Uses appropriate strategies to solve problems. • Computations are correct. • Written explanations are exemplary. • Diagrams are accurate and appropriate. • Goes beyond requirements of some or all problems.
3	**Satisfactory** A generally correct solution, but may contain minor flaws in reasoning or computation	• Shows an understanding of the concepts of *operations with functions; finding the inverse of a function; operations with radical expressions;* and *solving equations and inequalities containing radicals*. • Uses appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are effective. • Diagrams are mostly accurate and appropriate. • Satisfies all requirements of problems.
2	**Nearly Satisfactory** A partially correct interpretation and/or solution to the problem	• Shows an understanding of most of the concepts of *operations with polynomials; operations with radical expressions;* and *solving equations and inequalities containing radicals.* • May not use appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are satisfactory. • Diagrams are mostly accurate. • Satisfies the requirements of most of the problems.
1	**Nearly Unsatisfactory** A correct solution with no supporting evidence or explanation	• Final computation is correct. • No written explanations or work is shown to substantiate the final computation. • Diagrams may be accurate but lack detail or explanation. • Satisfies minimal requirements of some of the problems.
0	**Unsatisfactory** An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given	• Shows little or no understanding of most of the concepts of *operations with functions; finding the inverse of a function; operations with radical expressions;* and *solving equations and inequalities containing radicals.* • Does not use appropriate strategies to solve problems. • Computations are incorrect. • Written explanations are unsatisfactory. • Diagrams are inaccurate or inappropriate. • Does not satisfy requirements of problems. • No answer may be given.

Chapter 7 Assessment Answer Key

Page 75, Extended-Response Test
Sample Answers

In addition to the scoring rubric found on page A33, the following sample answers may be used as guidance in evaluating open-ended assessment items.

1a. Student responses should indicate that the monthly profit for each company depends on the number of sleds sold; one company may have a greater profit for a given number of sleds, but the other company may have the greater profit for a different number of sleds.

1b. Student responses may vary but must be between 2 and 50. For a response of $x = 10$ sleds, the A-Glide Company would earn a profit of $\sqrt{3(10) + 19} = 7$ hundred dollars, or \$700, while SnowFun would earn a profit of $3 + \sqrt{2(10)} \approx 7.47$ hundred dollars, or \$747.

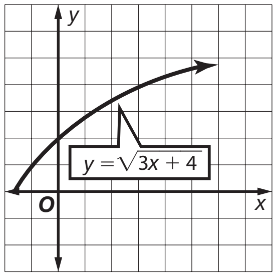
1c. Students should indicate that Mark's decision to work for A-Glide means that A-Glide has the greater monthly profit for the number of sleds sold by each company, so $\sqrt{3x + 19} > 3 + \sqrt{2x}$. The solution of this inequality is $\{x | x < 2 \text{ or } x > 50\}$ which means that A-Glide's profits are greater than SnowFun's profits during a month that one sled or more than 50 sleds are sold.

2a. Sample answers: For $g(x) = x + 1$ and $h(x) = x^2$, the answers would be: $2x + 4$; $9a^2$; $x^2 + x + 1$; $1 + x - x^2$; $x^3 + x^2$; $\frac{x^2}{x+1}$ *for* $x \neq -1$; $x^2 + 2x + 1$; $x^2 + 1$; 16; $x - 1$.

2b. Students should show that, for their functions $g(x)$ and $g^{-1}(x)$, $g^{-1} \circ g(x) = x$. Students should then indicate that the graphs of inverse functions are reflections of one another over the line $y = x$.

3a. Students should replace a and b with positive integers.
Sample answer: $y = \sqrt{3x + 4}$.

3b. Check that students' graphs are the graphs of the square root function from part **a**. For the sample function in **a**, the graph is shown below.



3c. Students should state the domain and range for the function they wrote in part **a**. For the sample function, the domain is $x \geq -\frac{4}{3}$, the range is $y \geq 0$. The y-intercept is 2 and the x-intercept is $-\frac{4}{3}$.

3d. Students should replace the equals sign in the function they wrote in part **a** with one of the following: $>$, $<$, $\geq$, $\leq$. For the sample function in **a**: $y \leq \sqrt{3x + 4}$.

3e. Students should indicate that the curve in the graph of their inequality will be the same as the curve in the graph of their function, if the inequality is $\geq$ or $\leq$, the curve will be a solid line, if the inequality is $>$ or $<$ the curve will be a broken line. The graph of the inequality will also be shaded either above or below the curve. For the sample inequality in part **d**: the graph will be a solid line and shaded below the curve and to the right of $x = -\frac{4}{3}$.

Chapter 7 Assessment Answer Key

Standardized Test Practice

Page 76

1. A **B** C D

2. F G **H** J

3. **A** B C D

4. **F** G H J

5. A B C **D**

6. **F** G H J

7. A **B** C D

8. F **G** H J

9. A B C **D**

10. F **G** H J

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11. A **B** C D

12. F G **H** J

13. **A** B C D

14. F **G** H J

15. **A** B C D

16. F G **H** J

17.

			9	.			

18.

				.	0	5	

Answers

Chapter 7 Assessment Answer Key

Standardized Test Practice
Page 78

19. s = amount invested in stock: $0.07s + 0.05(10{,}000 - s) \geq 550$; at least \$2500

20. consistent and dependent

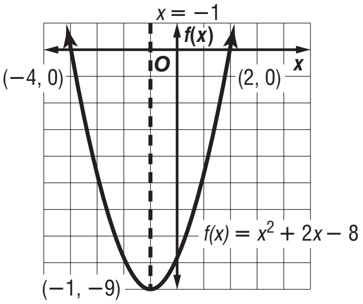
21. $A'(4, -6)$, $B'(7, 0)$, $C'(1, -2)$

22. $(-3, 5)$

23. $(2n + 5)^2$

24. $x > 13$

25.



26. $x^2 - x - 6 = 0$

27. 278

28a. $3x^8y^2$

28b. $x^4|y|\sqrt{13}$

28c. $5x^4y + x^4|y|\sqrt{13}$

Chapter 7 Assessment Answer Key

Unit 2 Test

Page 79

1. $8x^2 - 5x - 6$

2. $245x^5$

3. $4x^2 - 12x + 9$

4. $\dfrac{4y^2 - 6y + 9}{x - 5}$

5. $4|x|y^2$

6. $7\sqrt{3} + 12\sqrt{2}$

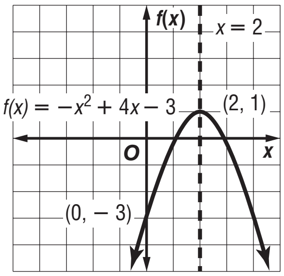
7. $-\dfrac{1}{10} + \dfrac{7}{10}i$

8. $2x^2 - 3x + 4 + \dfrac{3}{x - 1}$

9. $\sqrt[9]{m^7}$

10. $-2 \leq x \leq 1$

11.



12. 75 m

13. between −1 and 0; 2

14. $\{-4, 6\}$

15. $4x^2 - 13x - 12 = 0$

16. $\dfrac{-1 \pm i\sqrt{95}}{12}$

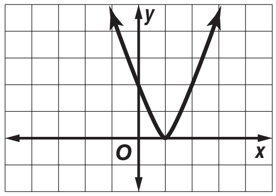
17. 0; 1 real root

Page 80

18. $(-3, -5)$; $x = -3$; up

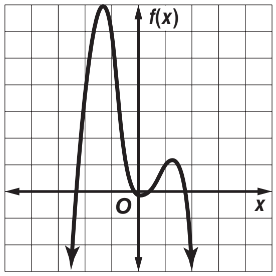
19. $y = -4(x - 1)^2 + 3$

20.



21. -216

22.



Sample answer: rel. max at $x = -2$ and $x = 1$, rel. min. at $x = 0$

23. $10, -10; \sqrt{2}, -\sqrt{2}$

24. -86

25. $x + 2$; $x - 5$

26. 3 or 1; 0; 2 or 0

27. $\pm 1, \pm 3, \pm 5, \pm 15, \pm\frac{1}{3}, \pm\frac{5}{3}$

28. 51; 57

29. $f^{-1}(x) = \dfrac{x + 2}{7}$

30.