Solving Equations Related To Cubic Functions

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- A cubic function is a polynomial function with degree three. An equation that is related to a given function, f(x), is one in which the value of the dependent variable is known and you need to determine the value(s) of the independent variable that generates it.
- For a cubic function, there will be either three, two, or one pair of values for which this is true. Some of these solutions will have meaning within the context, but some may not. Evaluate each solution for reasonableness using the context of the problem as a guide.

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- Graphically, locate a point on the graph of f(x) that has a y-coordinate equal to the given function value. The x-coordinate of this point is the x-value paired with that function value. This x-value is the solution to the equation.

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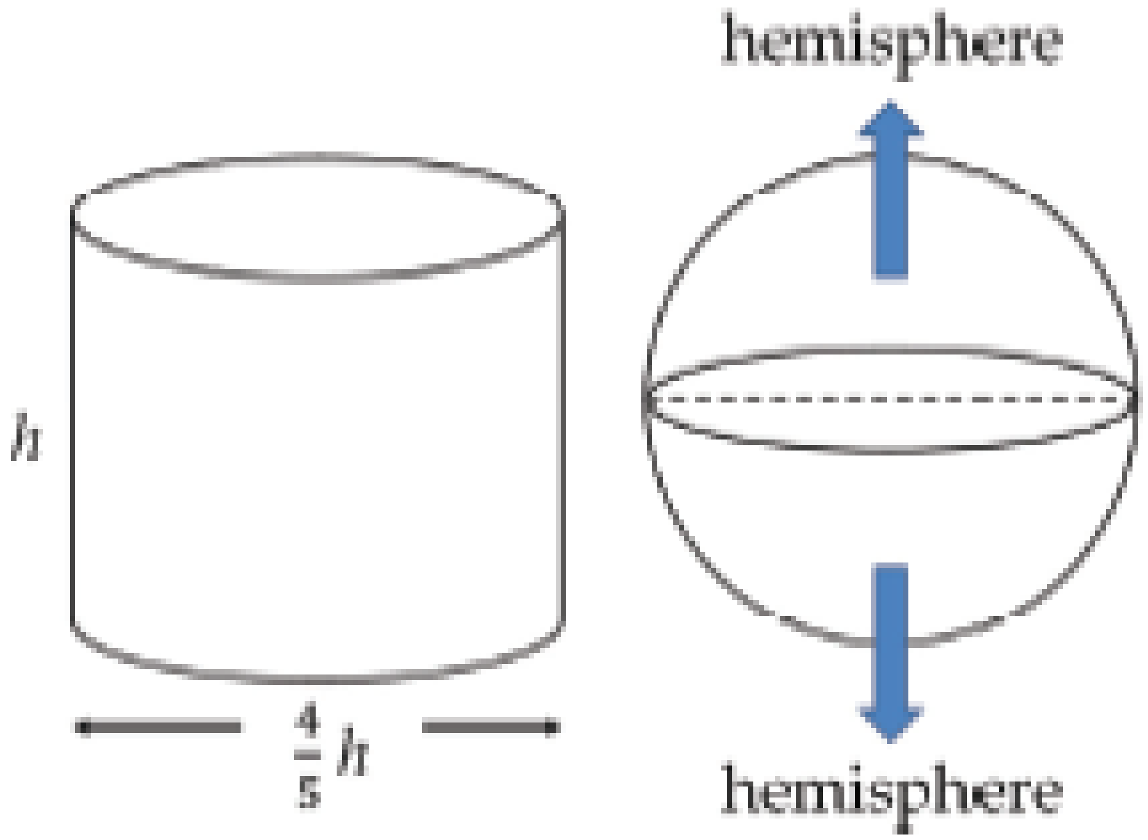
- Tabularly, locate the function value in the dependent variable column or row. The value in the independent variable column or row associated with this function value is the solution to the equation.

Examples

- The shape of a propane tank is a cylinder composed with a hemisphere on top and on bottom. The volume of a cylinder is $V = \pi r^2 h$ and the volume of a sphere is $\frac{4}{3}\pi r^3$, where r represents the radius and h represents the height.
- Propane tanks are usually filled to 80% capacity for safety and 1 gallon is equivalent to 231 cubic inches. The valve and protective collar at the top and the foot ring at the base add 6 inches to the overall height of the tank. The diameter of the cylindrical part of the tank is $\frac{4}{5}$ of its height. Will a tank with a capacity of 20 gallons of propane fit into a space that is 60 inches tall? Explain, using precise mathematical language, why or why not.

Examples

- **STEP 1** Draw a diagram of the shapes of the tank.



Examples

- **STEP 2** Write a formula for the volume of a cylinder and a sphere combined.
- $V_{\text{tank}} = V_{\text{cylinder}} + V_{\text{sphere}} = \pi r^2 h + \frac{4}{3}\pi r^3$

Examples

- **STEP 3** Modify the formula as a function for volume in terms of height h and radius $\frac{2}{5}$ h. (Remember that radius is $\frac{1}{2}$ diameter)

- $V = \pi r^2 h + \frac{4}{3}\pi r^3$
- $V = \pi(\frac{2}{5}\text{h})^2 h + \frac{4}{3}\pi(\frac{2}{5}\text{h})^3$

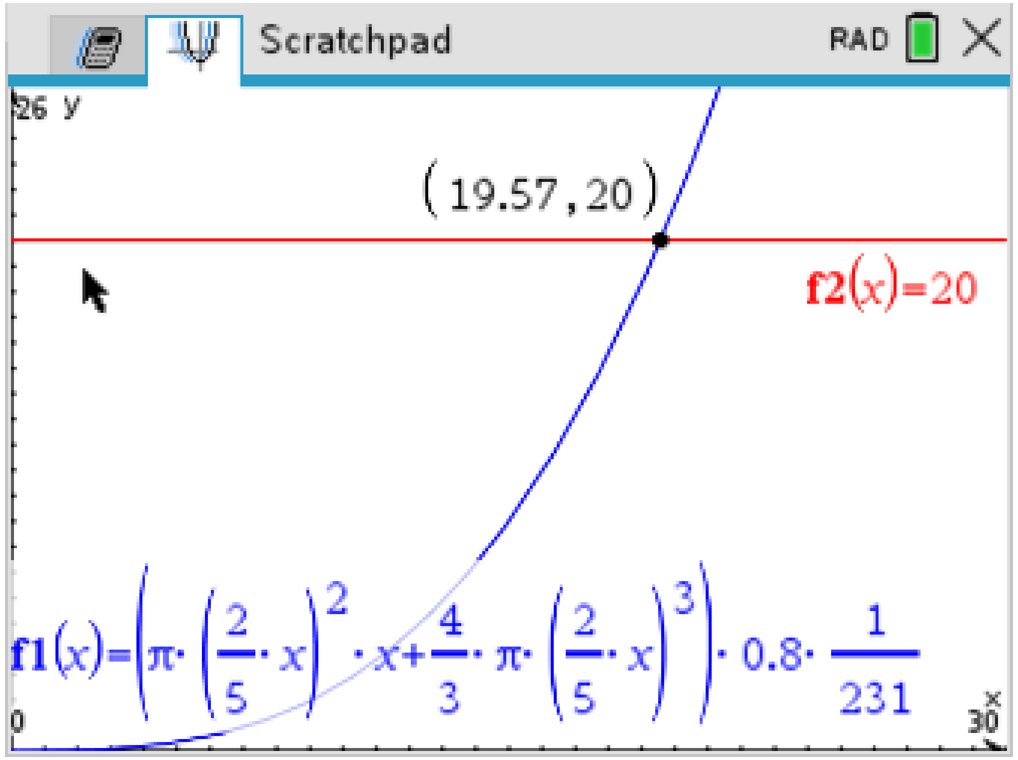
Examples

- **STEP 4** Modify the function further to show that the tanks are filled to 80% of their volume or capacity and the conversion for liquids is 231 cubic inches per gallon.

- $V = (\pi(\frac{2}{5}h)^2 h + \frac{4}{3}\pi(\frac{2}{5}h)^3)*(.8)(\frac{1}{231})$

Examples

- **STEP 5** Graph the function $V(h) = (\pi(\frac{2}{5}h)^2 h + \frac{4}{3}\pi(\frac{2}{5}h)^3)*(.8)(\frac{1}{231})$ and $y = 20$ on the same grid to find the point of intersection.



Examples

- **STEP 6** Interpret the point of intersection (19.57, 20).
- For a 20-gallon tank, the cylinder portion has an approximate height of 19.57 inches. Each of the hemispheres has a radius of two-fifths of the height of the cylinder, so together they have a height of four-fifths of 19.57. That adds approximately 15.66 inches to the height for a total of 35.23 inches.
- Yes, the propane tank holding 20 gallons will fit in a space 60 inches tall. The tank has a height of approximately 35.23 inches plus 6 inches for the valve, protective collar and foot ring for a total of 41.23 inches, which is less than 60 inches.

Examples

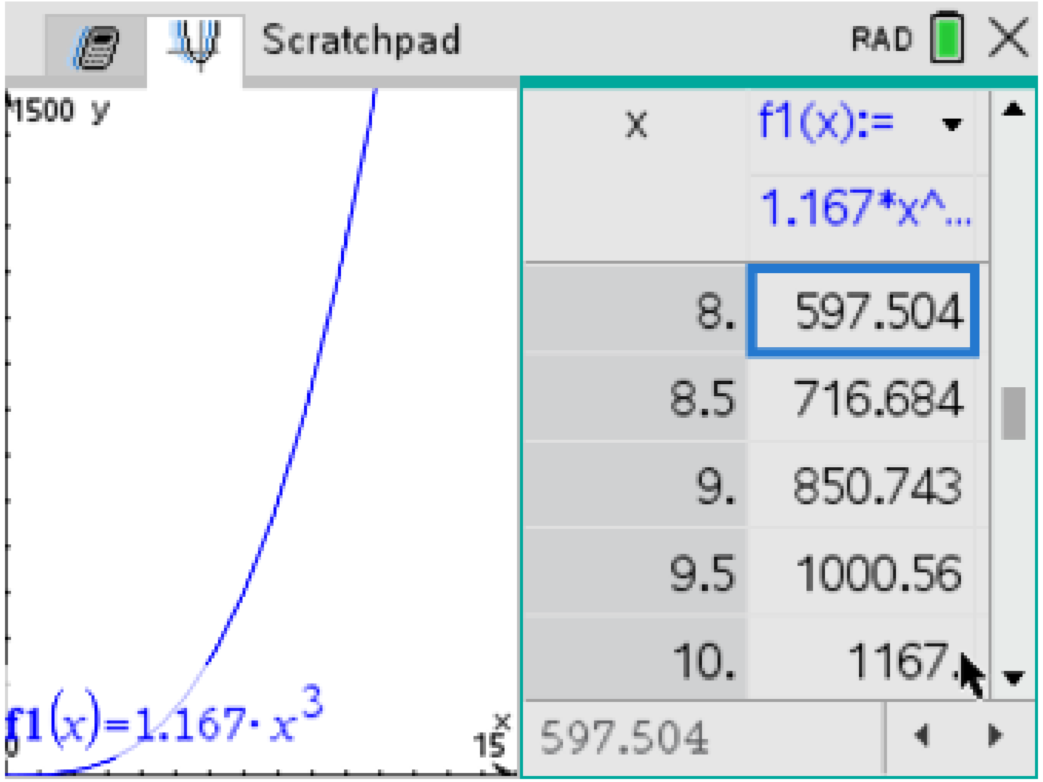
- A wind turbine produces electrical power from the kinetic energy of the wind. The electrical power, in kilowatts, produced by one of the wind turbines in a wind farm near Pecos, Texas, is $P(v) = 1.167v^3$, where v is wind velocity in meters per second. What is the wind velocity of the wind turbine when the power output is 1,000 kilowatts? Write a related equation for 1,000 kW, and create a table of values for the function to find the wind velocity required for that power output value.

Examples

- **STEP 1** Use the given power, 1,000 kW, along with $P(v)$, to write the related equation. $1000 = 1.167v^3$

Examples

- **STEP 2** Use graphing technology to make a table of values for $P(v)$. Look in the column for the dependent variable for a value of 1,000. You may need to refine the interval for Δv to see additional rational number values.
- When the interval of the independent variable is changed to 0.5, a dependent value very close to 1,000 is found.



Examples

- **STEP 3** Interpret the data found in the table.
- When v = 9.5 m/s, $P(v)$ = 1000.6 kW. Thus, $P(9.5)$ = 1000.6 kW so the value of v that generates a power of 1,000 kW is a wind velocity of slightly less than 9.5 m/s.