SOLVING EQUATIONS RELATED TO LOGARITHMIC FUNCTIONS

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- A logarithmic function is the inverse of an exponential function. Some logarithms that are used the most often are base-10, base-2, or base-e. An equation that is related to a given function, f (x), is one in which the value of the dependent variable is known and you need to determine the value(s) of the independent variable that generates it. For a logarithmic function, there will only be one pair of values for which this is true.

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- Graphically, locate a point on the graph of f(x) that has a y-coordinate equal to the given function value. The x-coordinate of this point is the x-value paired with that function value. This x-value is the solution to the equation.

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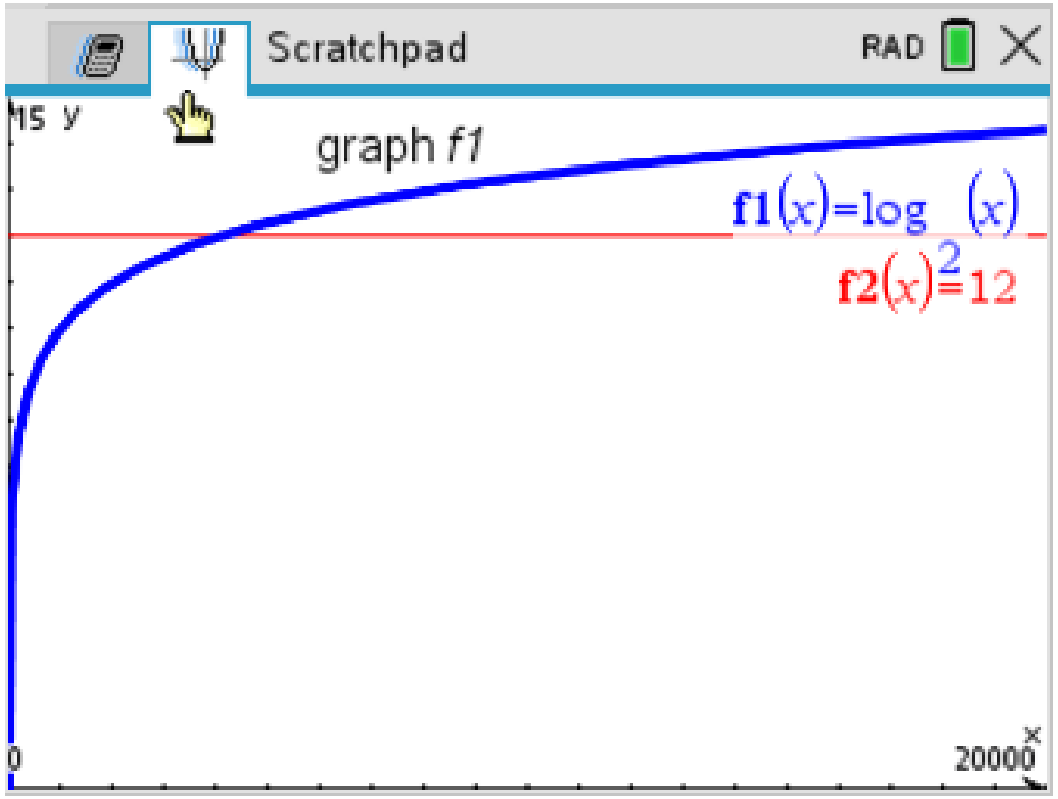
- Tabularly, locate the function value in the dependent variable column or row. The value in the independent variable column or row associated with this function value is the solution to the equation.

EXAMPLES

- Some students used the function $f(x) = \log_2 x$ to test the theory that you can find a name in a phone book in a certain number of steps $f(x)$ based on the number of names, x, contained in the phone book. After several trials, the students came up with an average of 12 steps to find some randomly selected names in their hometown phone book.
- Graph the function and then write an equation to determine a function value of 12 on the graph of $f(x) = \log_2 x$. Using the intersection point, determine approximately how many names the phone book contains.

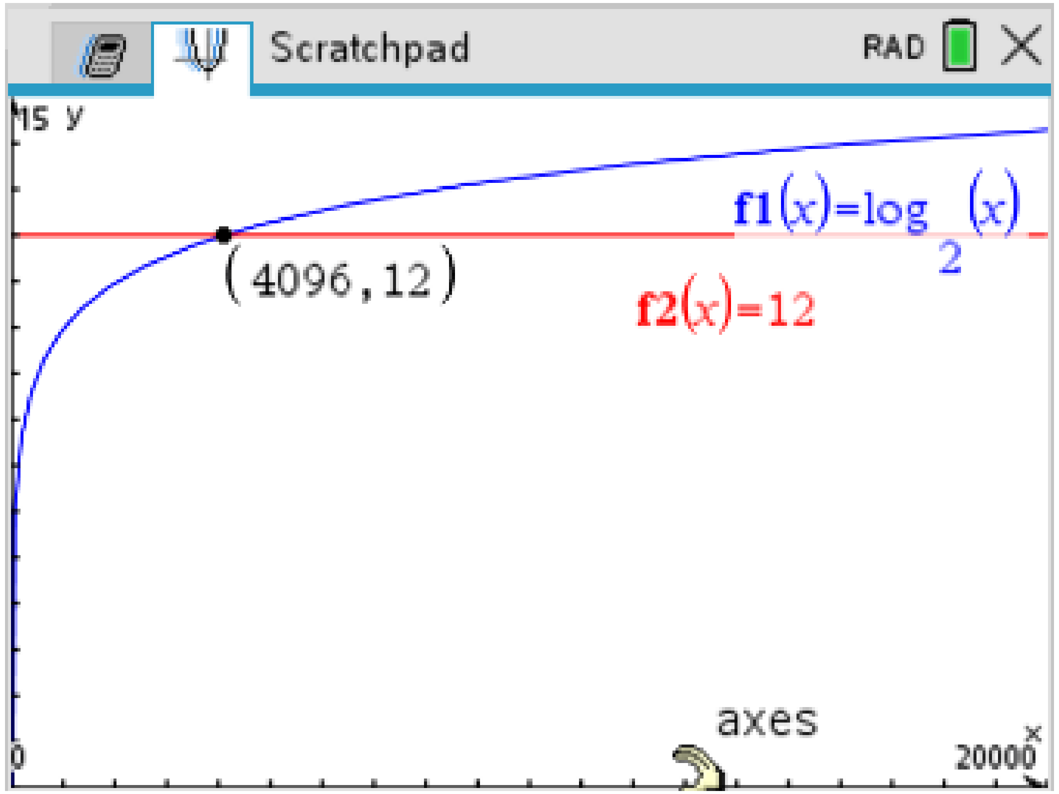
EXAMPLES

- **STEP 1** Write an equation for the average of 12 steps; then graph it and the function *f*(*x*) on the same grid.



EXAMPLES

- **STEP 2** Find the coordinates of point *of* intersection of the graphs for $f(x)$ and $y = 12$.



EXAMPLES

- **STEP 3** Interpret the meaning of the coordinates.
- You know that the *y*-coordinate is the number of steps, on average, that it took the students to find the names. The *x*-coordinate indicates the total number of names in the phone book, which is slightly greater than 4,000. There are slightly more than 4,000 names in the phone book.

EXAMPLES

- The level of sound, D, in decibels is defined as the function $D(I) = 10\log(\frac{I}{10^{-16}})$ where I is the sound intensity in watts per square centimeter. Determine the sound intensity of a hair dryer advertised as "quiet" with a sound level of 64 decibels. Write an equation for 64 decibels related to the function $D(I)$ and use the table to approximate the sound intensity of the hair dryer.

SOUND INTENSITY $\left(\frac{W}{cm^2}\right)$	3.162×10^{-11}	1×10^{-10}	3.162×10^{-10}	1×10^{-9}	3.162×10^{-9}	1×10^{-8}
SOUND LEVEL (dB)	55	60	65	70	75	80

EXAMPLES

- **STEP 1** Write an equation for the hair dryer with an advertised sound level of 64 decibels.
- The equation $64 = 10\log(\frac{I}{10^{-16}})$ will give you the answer to the question, "What is the sound intensity of a hair dryer advertised as 'quiet' with a sound level of 64 decibels?"

EXAMPLES

- **STEP 2** Find the output value of 64 in the table of values.
- The sound level of 64 decibels is not shown in the table. However, a very close value, 65 decibels, is included.

EXAMPLES

- **STEP 3** Find a sound intensity related with 65 decibels in the table.
- The input value 3.162×10^{-10} is associated with the output value of 65.

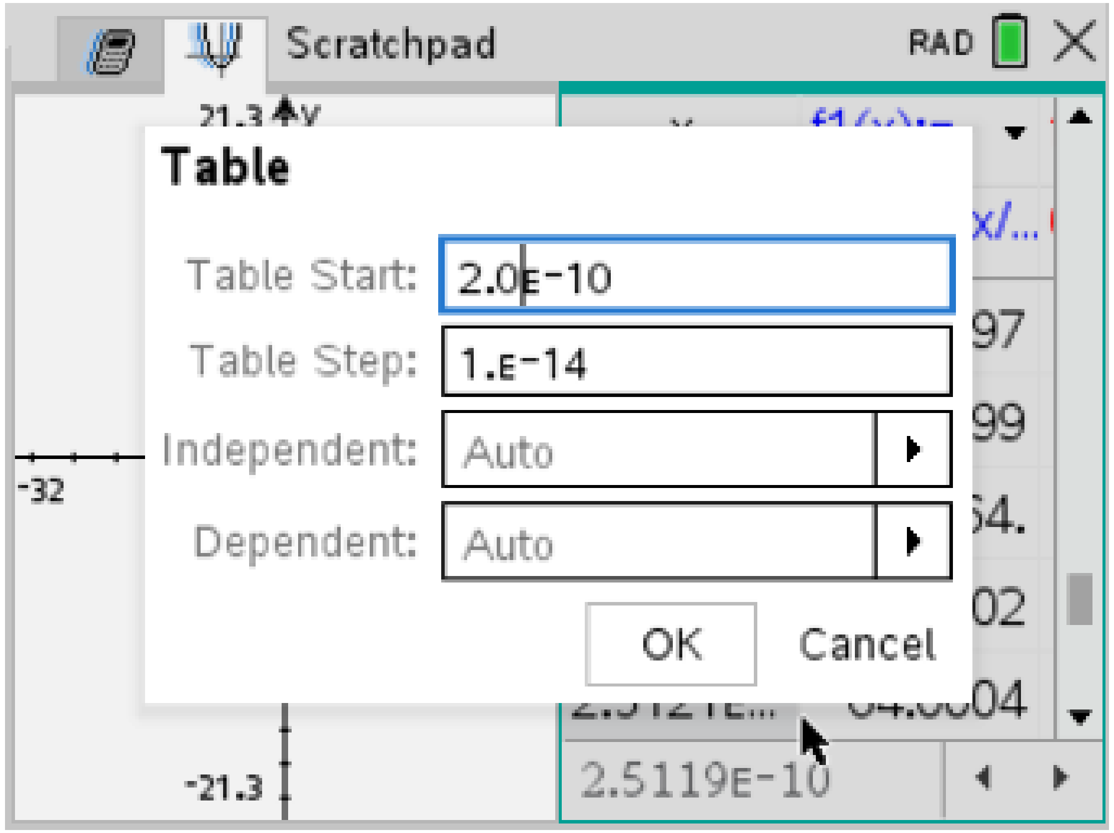
EXAMPLES

- **STEP 4** Use graphing technology with a table to determine the independent value of the function $D(I)$ that corresponds more closely with a dependent value of 64.
- Using graphing technology, you can change the intervals between successive independent values in the table to tenths to determine that the dependent value 64 corresponds to an independent function value of 2.512×10^{-10}. Therefore, $D(2.512 \times 10^{-10}) \approx 64$.
- A hair dryer advertising a sound level of 64 decibels has a sound intensity of 2.512×10^{-10} watts per square centimeter.

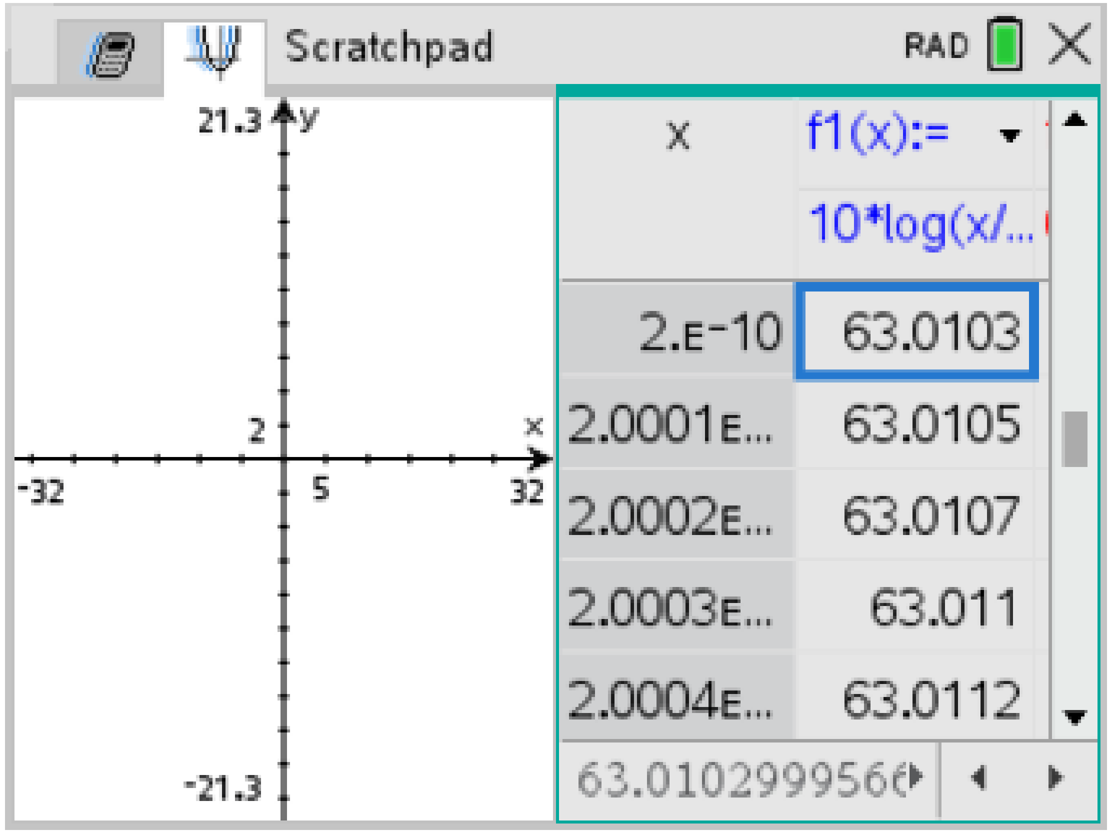
EXAMPLES

- After you get the value associated with 65 decibels, 3.162×10^{-10}, you can use that value to find the value for 64 decibels.
- Once you put the equation into the calculator and look at the table, press **menu, 2, 5** to get to the Table Settings.
- Since the table tells that the answer will be less than 3.162×10^{-10}, start around 2×10^{-10} and adjust until you get close to 64.
- Since the answer has 4 digits, 3.162, and 10 decimal places, $^{-10}$, change the table step to 1×10^{-14} to get to values that will help
- Then just scroll through until you can find a value close to 64

EXAMPLES

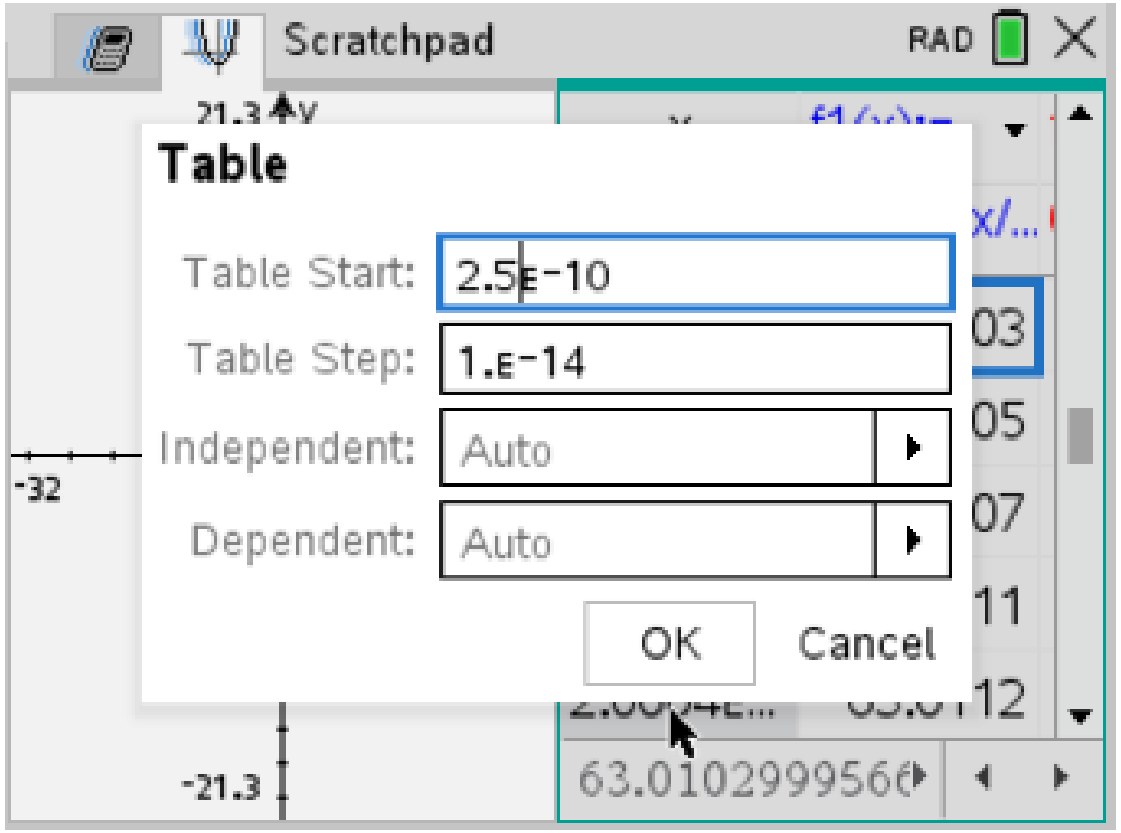


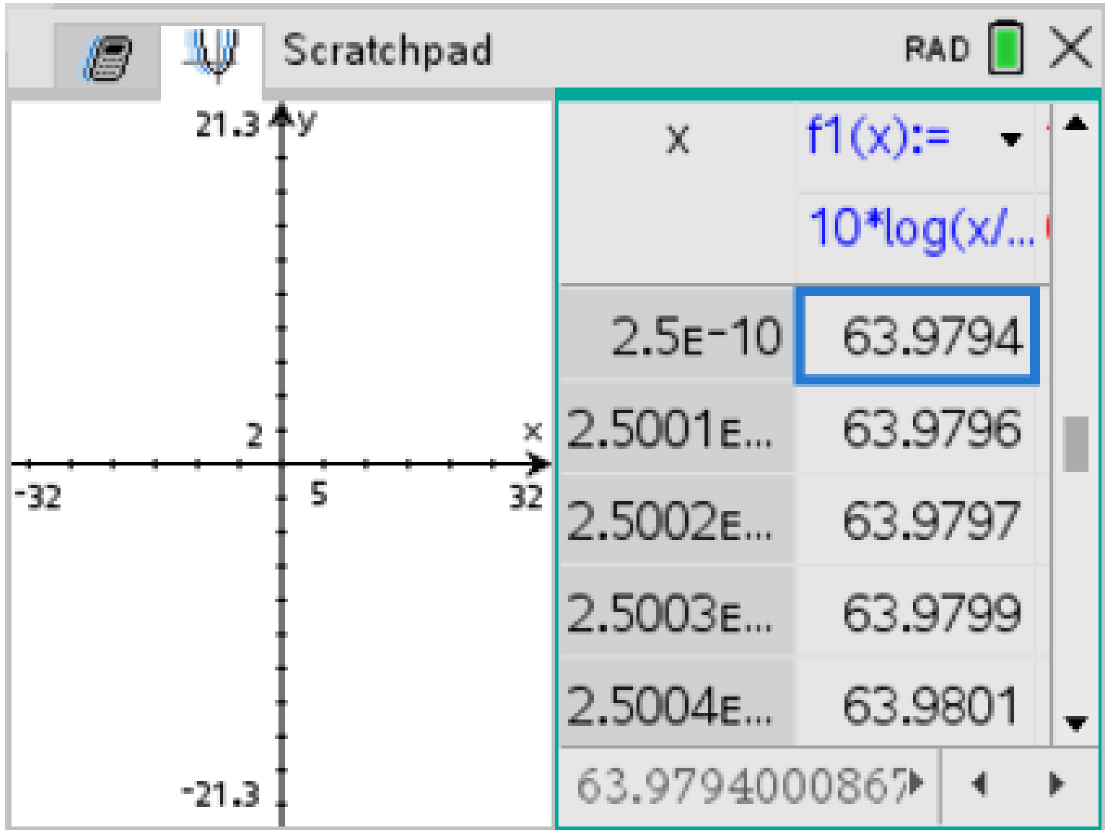
Scratchpad
RAD
Table
Table Start:
2.0E-10
Table Step:
1.E-14
Independent:
Auto
Dependent:
Auto
OK
Cancel
2.5119E-10



Scratchpad
RAD
x
f1(x):=
10*log(x/...
2.E-10
63.0103
2.0001E...
63.0105
2.0002E...
63.0107
2.0003E...
63.011
2.0004E...
63.0112
63.010299566

EXAMPLES


Scratchpad
RAD
21.3
-32
-21.3
Table
Table Start: 2.5E-10
Table Step: 1.E-14
Independent: Auto
Dependent: Auto
OK
Cancel


Scratchpad
RAD
21.3
y
2
x
-32
5
32
-21.3
x
f1(x):=
10*log(x/...
2.5E-10 63.9794
2.5001E... 63.9796
2.5002E... 63.9797
2.5003E... 63.9799
2.5004E... 63.9801