Estimating Solutions from Cubic Functions

7.9

FOCUSING QUESTION How can you approximate a solution to a cubic equation using tables or graphs?

LEARNING OUTCOMES

- I can use tables and graphs to approximate solutions to equations involving cubic functions that model real-world problems.
- I can create and use tables and graphs to organize, record, and communicate ideas about cubic functions and their related equations.

ENGAGE

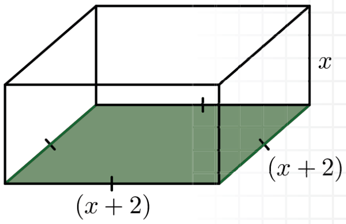
Ysleta Mission, El Paso, Texas

The annual Ysleta Mission Festival in El Paso, Texas, includes a stage on which parishioners demonstrate folkloric dances. The stage is elevated and in the shape of a rectangular prism. The stage floor is 15 meters by 12 meters and is 1.5 meters above the ground. How much space is contained beneath the stage floor?

15(12)(1.5) = 270 cubic meters

EXPLORE

A set of storage chests are designed to hold animal feed for urban agriculture. Each storage chest is in the shape of a square prism with a base that has an edge length of $(x + 2)$ feet and a height of x feet.



1. Write a function that could be used to describe the volume of the chest, $V(x)$, in terms of its dimensions. Remember, the volume of a prism can be calculated using the formula $V = Bh$.

$V(x) = (x + 2)^2x = x^3 + 4x^2 + 4x$

2. A popular storage chest has a volume of $18\frac{3}{8}$ cubic feet. Write an equation from $V(x)$ that you could use to determine the height, h, of this particular chest.

$18\frac{3}{8} = x^3 + 4x^2 + 4x$

TEKS

AR.6A Estimate a reasonable input value that results in a given output value for a given function, including quadratic, rational, and exponential functions.

AR.6C Approximate solutions to equations arising from questions asked about exponential, logarithmic, square root, and cubic functions that model real-world applications tabularly and graphically.

MATHEMATICAL PROCESS SPOTLIGHT

AR.1E Create and use representations to organize, record, and communicate mathematical ideas.

ELPS

4F Use visual and contextual support and support from peers and teachers to read grade-appropriate content area text, enhance and confirm understanding, and develop vocabulary, grasp of language structures, and background knowledge needed to comprehend increasingly challenging language.

VOCABULARY

cube root, cubic function, frustum

MATERIALS FOR THE EXPLORE

- graphing technology

RELECT ANSWERS:

Both methods use paired input-output values, where the input value is the height of the prism, and the output value is the volume of the prism. A table lists those paired values in rows or columns, and a graph shows the paired values as ordered pairs on a coordinate plane.

A cubic equation could have up to three solutions, depending on the number of real-number factors. The factors are represented by points where the graph intersects the line with the desired output value.

For example, in the graph shown, $f(x) = 8$ has one solution, $g(x) = 8$ has two solutions, and $h(x) = 8$ has three solutions. This tells us that $f(x)$ has 1 real-number factor, $g(x)$ has 2 real-number factors, and $h(x)$ has 3 real-number factors.

3. Use graphing technology or paper and pencil to generate a table of values like the one shown.

HEIGHT (FT)	0.5	1	1.5	2	2.5	3
VOLUME (FT³)	***3.125***	***9***	***18.375***	***32***	***50.625***	***75***

4. Use the table to determine a solution to your equation for the height of a storage chest with a volume of $18\frac{3}{8}$ cubic feet.
See margin.

5. What are all three dimensions of the storage chest with a volume of $18\frac{3}{8}$ cubic feet?
See margin.

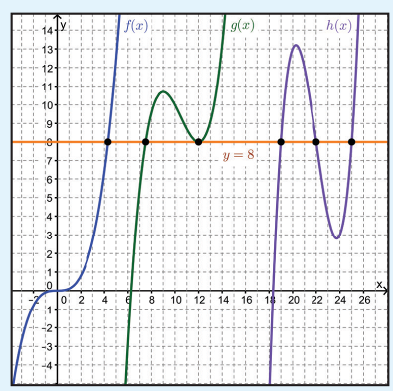
6. Nicole needs a storage chest from this series to store 100 cubic feet of chicken feed. Write an equation that she can use to determine the height of this storage chest.
$100 = x^3 + 4x^2 + 4x$

7. Use a graph to approximate a solution to the equation. Explain how you used your graph to determine your estimate.
See margin.

REFLECT

- **How is using a table to approximate the solution to a cubic equation similar to using graphs to approximate solutions to cubic equations?**
  See margin.

- **How many solutions will a cubic equation have? Justify your answer using appropriate mathematical terminology.**



See margin.

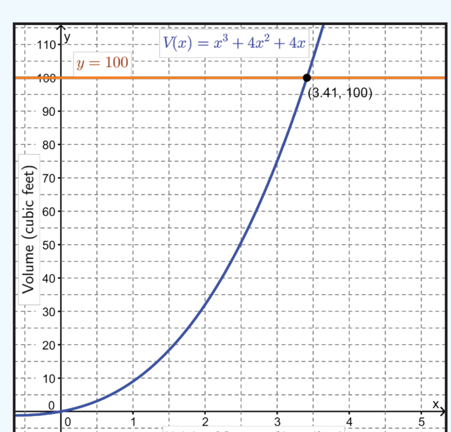
4.

HEIGHT (FT)	0.5	1	1.5	2	2.5	3
VOLUME (FT³)	***3.125***	***9***	***18.375***	***32***	***50.625***	***75***

When $x = 1.5$, $V(x) = 18.375 = 18\frac{3}{8}$, so $V(1.5) = 18\frac{3}{8}$.
A storage chest with a volume of $18\frac{3}{8}$ cubic feet has a height of 1.5 feet.

5. *height = x = 1.5 feet*
length = $x + 2 = 1.5 + 2 = 3.5$ feet
width = $x + 2 = 1.5 + 2 = 3.5$ feet

7. *$x \approx 3.41$ feet   Possible process: Graph both $V(x)$ and $y = 100$ on a graphing calculator. Adjust the window so that you can see the point of intersection. Use the calculator to determine the coordinates of the point of intersection to be approximately (3.41, 100). These coordinates mean that when $x \approx 3.41$, $V(x) = 100$, so the solution to the equation is approximately 3.41, which means that the height of the storage chest that Nicole needs would be about 3.41 feet.*



EXPLAIN

Cubic functions are polynomial functions of degree three. You can use cubic functions to write related equations, then use graphs and tables to approximate solutions to those equations for the value of the independent variable that generates a particular value of the dependent variable. There are also symbolic methods for solving cubic functions, but in this course, we will focus on approximating solutions.

Watch Explain and You Try It Videos

or click here

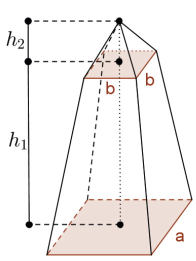
APPROXIMATING SOLUTIONS TO EQUATIONS GRAPHICALLY

ELPS Strategy
Work with a partner as you read this section. Use the diagram and photograph to provide visual support that helps develop background knowledge relating to the parts that make up the obelisk. Use the language in the narrative to provide contextual support that helps develop background knowledge relating to the obelisk, including the San Antonio Missions trail context.

An obelisk is a composite figure made up of a square pyramid on top of the frustum of a different square pyramid. A frustum of a pyramid is the base part of a pyramid formed by cutting off the top of the pyramid by a plane parallel to the base. The Mission Trail in San Antonio, Texas, has obelisks like the one shown to mark the trail from one mission to another in the San Antonio Missions UNESCO World Heritage Site.

Mission Trail
San Antonio, Texas

The volume of an obelisk with a square base can be found using the formula $V = \frac{1}{3}h_1(a^2 + ab + b^2) + \frac{1}{3}b^2h_2$, where a, b, h_1, and h_2 are as labeled in the diagram.



In this diagram, a represents the side length of the bottom square base of the frustum on the bottom of the obelisk, b represents the side length of the square base of the pyramid on top of the obelisk, h_1 represents the height of the frustum on the bottom of the obelisk, and h_2 represents the height of the square pyramid on top of the obelisk.

For a particular obelisk, the function $V(x) = \frac{8}{3}x^3 + \frac{70}{3}x^2$ can be used to calculate the volume if $a = 2x$, $b = x$, $h_1 = x + 10$ and $h_2 = x$.

If the obelisk has a volume of 2,000 cubic inches, what are the dimensions of the obelisk?

Use the given volume, 2,000 cubic inches, along with $V(x)$, to write the equation $2{,}000 = \frac{8}{3}x^3 + \frac{70}{3}x^2$.

SUPPORTING ENGLISH LANGUAGE LEARNERS

Visual support, such as diagrams and photographs, and contextual support, such as volume formulas and information surrounding mathematical vocabulary, are important ways for students to build the background knowledge necessary for solving meaningful mathematics problems (**ELPS 4F**). Encourage students to use diagrams and photographs for visual support and to look for context clues that surround unfamiliar language as students read information with increasingly complex English language words and structures.

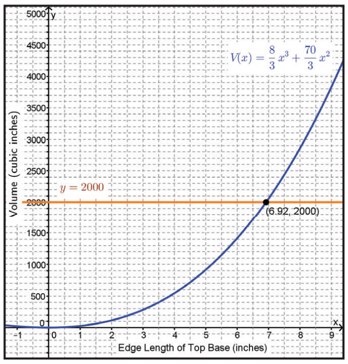
INTEGRATING TECHNOLOGY

You can use a graphing calculator or app to estimate the solution of a cubic equation graphically. Graph the function in $Y1$ and then graph $Y2 = n$, where n is the given output value or function value. Use the calculator's or app's features to determine where the two graphs intersect. Usually, for a cubic function, the x-coordinate, which represents the solution to the equation, is a rounded value. Hence, the x-coordinate of the point of intersection is an approximation for the solution to the equation.

INTEGRATING TECHNOLOGY

You can use a graphing calculator or app to solve an equation tabularly. Enter the function into $Y1$ of the function editor. In the table feature, scroll up and down until you see the given output value or function value. If you do not see the exact function value, then change the x-interval to a smaller number (e.g., 0.1 instead of 1), and look again. The x-value in the same row as the function value is the solution to the equation. For cubic functions, start out with an x-interval of about 1, and scroll until you find dependent variable values that are close to the given value for your equation. Refine the x-interval to 0.1, 0.01, or 0.001 to get a closer approximation for your solution.

Use graphing technology to graph $V(x)$ and the line $y = 2{,}000$ to determine the point on the graph of $V(x)$ with a function value of 2,000.



The point (6.92, 2000) is the intersection point of $V(x)$ and $y = 2000$. Notice that 6.92 is a rounded value for $V(x)$. You can approximate that $V(6.92) = 2000$ and estimate that when $x \approx 6.92$ inches, the volume of the obelisk will be 2,000 cubic inches.

Use the approximated value of x to determine the dimensions of the obelisk.

- $a = 2x = 2(6.92) = 13.84$ inches
- $b = x = 6.92$ inches
- $h_1 = x + 10 = 6.92 + 10 = 16.92$ inches
- $h_2 = x = 6.92$ inches

The bottom base of the obelisk is a square with edge lengths of 13.84 inches. The obelisk narrows to a square base with edge lengths of 6.92 inches at a height of the 16.92 inches above the bottom base. The top pyramid of the obelisk has a height of 6.92 inches.

APPROXIMATING SOLUTIONS TO EQUATIONS TABULARLY

If the obelisk has a volume of 4,689.75 cubic inches, what are the dimensions of the obelisk?

Use the given volume, 4,689.75 cubic inches, along with $V(x)$, to write the equation $4{,}689.75 = \frac{8}{3}x^3 + \frac{70}{3}x^2$.

Use graphing technology to make a table of values for $V(x)$. Look in the column for the dependent variable for a value of 4,689.75. You may need to refine the interval for Δx to see additional rational number values.

TOP BASE EDGE LENGTH (INCHES), x	VOLUME (CUBIC INCHES), $T(x)$
9	3,834
9.25	4,107
9.5	4,392.17
9.75	4,689.75
10	5,000
10.25	5,323.17

When $x = 9.75$ inches, $V(x) = 4{,}689.75$ cubic inches. Thus, $V(9.75) = 4{,}689.75$, so the value of x that generates a volume of 4,689.75 cubic inches is 9.75 inches.

Since $x = 9.75$ inches, you can use this value to determine each dimension of the obelisk.

- $a = 2x = 2(9.75) = 19.5$ inches
- $b = x = 9.75$ inches
- $h_1 = x + 10 = 9.75 + 10 = 19.75$ inches
- $h_2 = x = 9.75$ inches

The bottom base of the obelisk is a square with edge lengths of 19.5 inches. The obelisk narrows to a square base with edge lengths of 9.75 inches at a height of the 19.75 inches above the bottom base. The top pyramid of the obelisk has a height of 9.75 inches.

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APPROXIMATING SOLUTIONS TO EQUATIONS RELATED TO CUBIC FUNCTIONS

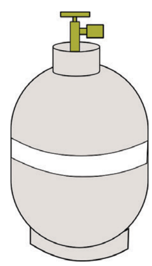
A cubic function is a polynomial function with degree three. An equation that is related to a given function, $f(x)$, is one in which the value of the dependent variable is known and you need to determine the value(s) of the independent variable that generates it.

- Graphically, locate a point on the graph of $f(x)$ that has a y-coordinate equal to the given function value. The x-coordinate of this point is the x-value paired with that function value. This x-value is the solution to the equation.
- Tabularly, locate the function value in the dependent variable column or row. The value in the independent variable column or row associated with this function value is the solution to the equation.

For a cubic function, there will be either three, two, or one pair of values for which this is true. Some of these solutions will have meaning within the context, but some may not. Evaluate each solution for reasonableness using the context of the problem as a guide.

EXAMPLE 1

The shape of a propane tank is a cylinder composed with a hemisphere on top and on bottom. The volume of a cylinder is $V = \pi r^2 h$ and the volume of a sphere is $\frac{4}{3}\pi r^3$, where r represents the radius and h represents the height. Propane tanks are usually filled to 80% capacity for safety and 1 gallon is equivalent to 231 cubic inches. The valve and protective collar at the top and the foot ring at the base add 6 inches to the overall height of the tank. The diameter of the cylindrical part of the tank is $\frac{4}{5}$ of its height. Will a tank with a capacity of 20 gallons of propane fit into a space that is 60 inches tall? Explain, using precise mathematical language, why or why not.



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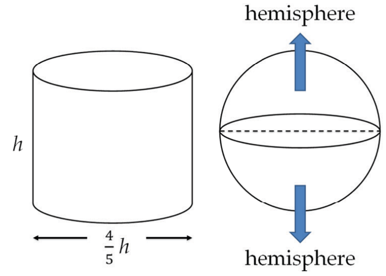
STEP 1 Draw a diagram of the shapes of the tank.

STEP 2 Write a formula for the volume of a cylinder and a sphere combined.



$$V_{\text{tank}} = V_{\text{cylinder}} + V_{\text{sphere}} = \pi r^2 h + \frac{4}{3}\pi r^3$$

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ADDITIONAL EXAMPLE

Pools-4-U installs above-ground pools shaped like rectangular prisms. While they can build pools of any size, they must follow specific formulas for the dimensions, in feet, so the pools' structural integrity is sound. The pools are always 4 times as long as they are wide. The height is always one foot less than double the width. The coming summer is Texas is going to be a real scorcher, so Gabi wants to get a pool installed. She knows she wants to the volume of the pool to be 400 cubic feet. What should the dimensions of her pool be? Write a function for the volume of Gabi's pool based on the width of the pool, and use a graph to solve a related equation for 400 cubic feet of water.

The function $V(w) = (4w)(w)(2w - 1) = 8w^3 - 4w^2$ represents the volume of Gabi's pool. The related equation is $400 = 8w^3 - 4w^2$. Using graphing technology, you can find the intersection point of (3.86, 400). That means Gabi's pool should have a length of 4(3.86) = 15.44 feet, a width of 3.86 feet, and a height of 6.72 feet.

STEP 3 **Modify the formula as a function for volume in terms of height *h* and radius $\frac{2}{5}h$.**

$$V = \pi r^2 h + \frac{4}{3}\pi r^3$$

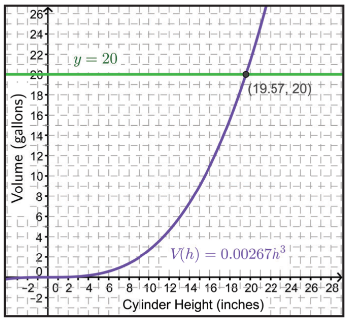
$$V(h) = \pi\left(\frac{2}{5}h\right)^2 h + \frac{4}{3}\pi\left(\frac{2}{5}h\right)^3 = \pi\left(\frac{4}{25}h^2\right)h + \frac{4}{3}\pi\left(\frac{8}{125}\right)h^3 = \pi\left(\frac{4}{25}h^3\right) + \pi\left(\frac{32}{375}h^3\right)$$

$$V(h) = \pi\left(\frac{60+32}{375}h^3\right) = \frac{92}{375}\pi h^3 \approx 0.771h^3 \text{ in.}^3$$

STEP 4 **Modify the function further to show that the tanks are filled to 80% of their volume or capacity and the conversion for liquids is 231 cubic inches per gallon.**

$$V(h) \approx (0.80)(0.771h^3 \text{ cu. in.})\left(\frac{1 \text{ gal.}}{231 \text{ cu. in.}}\right) \approx 0.00267h^3 \text{ gal}$$

STEP 5 **Write a related equation to answer the question, "Will a tank with a capacity of 20 gallons of propane fit into a space that is 60 inches tall?" Graph the function $V(h) = 0.00267h^3$ and $y = 20$ on the same grid to find the point of intersection.**



The related equation is $20 = 0.00267h^3$.

STEP 6 **Interpret the point of intersection (19.57, 20).**

For a 20-gallon tank, the cylinder portion has an approximate height of 19.57 inches. Each of the hemispheres has a radius of two-fifths of the height of the cylinder, so together they have a height of four-fifths of 19.57. That adds approximately 15.66 inches to the height for a total of 35.23 inches.

Yes, the propane tank holding 20 gallons will fit in a space 60 inches tall. The tank has a height of approximately 35.23 inches plus 6 inches for the valve, protective collar and foot ring for a total of 41.23 inches, which is less than 60 inches.

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YOU TRY IT! #1

A candy company makes caramels that are cut in small cubes, one inch on each side. Then they are wrapped, and packaged in shipping boxes of many different sizes for retail sales. The boxes are always three times as long as the width with a height of four less than the width, with all dimensions in inches. Is it possible to order a box of 1,800 caramels and, if so, what are the dimensions of the box? Draw a diagram of a box and label its dimensions in terms of x. Write a function for the number of caramels based on the width of the boxes, and use a graph to solve a related equation for 1,800 caramels.

See margin.

EXAMPLE 2

A wind turbine produces electrical power from the kinetic energy of the wind. The electrical power, in kilowatts, produced by one of the wind turbines in a wind farm near Pecos, Texas, is $P(v) = 1.167v^3$, where v is wind velocity in meters per second. What is the wind velocity of the wind turbine when the power output is 1,000 kilowatts? Write a related equation for 1,000 kW, and create a table of values for the function to find the wind velocity required for that power output value.

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STEP 1 Use the given power, 1,000 kW, along with $P(v)$, to write the related equation.

$1000 = 1.167v^3$

STEP 2 Use graphing technology to make a table of values for $P(v)$. Look in the column for the dependent variable for a value of 1,000. You may need to refine the interval for Δv to see additional rational number values.

When the interval of the independent variable is changed to 0.5, a dependent value very close to 1,000 is found.

WIND VELOCITY (m/s)	POWER (KILOWATTS)
8	597.5
8.5	716.7
9	850.7
9.5	1000.6
10	1167.0
10.5	1350.9

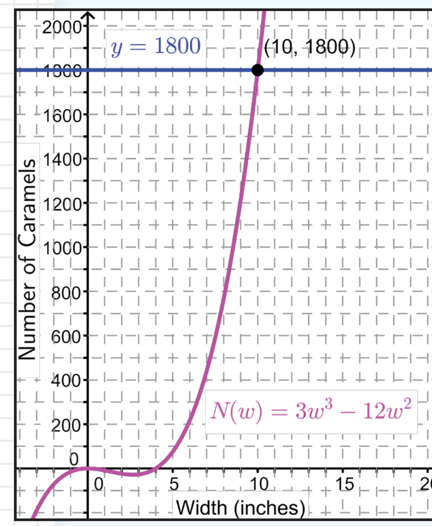
STEP 3 Interpret the data found in the table.

When v = 9.5 m/s, $P(v)$ = 1000.6 kW. Thus, $P(9.5)$ = 1000.6 kW so the value of v that generates a power of 1,000 kW is a wind velocity of slightly less than 9.5 m/s.

YOU TRY IT! #1 ANSWER:

Yes, it is possible to order 1800 caramels and the box would measure 10 inches by 30 inches by 6 inches.

Since the caramels are each 1 cubic inch, the volume of a box is the same as the number of caramels it contains. The function $N(w) = (w)(3w)(w - 4) = 3w^3 - 12w^2$ gives the volume of the box (or number of caramels). As such, it is a discrete function, but for ease of graphing, it is shown with a continuous curve. The related equation is $1800 = 3w^3 - 12w^2$. The intersection point of the graphs for $N(w)$ and $y = 1800$ is (10, 1800).



ADDITIONAL EXAMPLES

Use graphing technology to create a table of values for the functions and their related equations below in order to determine a solution.

1. If $f(x) = \frac{4}{5}x^3$, for what value of x will $f(x) = 500$?

Using a table of values, you can find that $f(8.55) \approx 500$.

2. If $f(x) = 1.45x^3 + 5$, for what value of x will $f(x) = 250$?

Using a table of values, you can find that $f(5.53) \approx 250$.

3. If $f(x) = 2.7x^3 - 3.5x^2 - 8$, for what value of x will $f(x) = 125$?

Using a table of values, you can find that $f(4.15) \approx 125$.

YOU TRY IT! #2

A pet store sells fish tanks in the shape of a hexagonal prism with square lateral faces. They advertise various size tanks, but the manufacturer will customize any size ordered. The function $V(s) = \frac{3}{2}\sqrt{3}(s^3)$ can be used to calculate the volume in cubic inches based on the side length of a face of the prism, measured in inches. Recall that 1 gallon is equivalent to 231 cubic inches. What side length would be needed for a 30-gallon fish tank? Write a function and related equation then use a table of values to determine your solution.

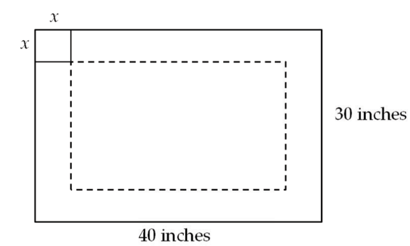
See margin.

PRACTICE/HOMEWORK

Use the scenario below to complete questions 1 – 9.

GEOMETRY

A company manufactures open storage boxes that are each made from a single piece of sheet metal that measures 40 inches by 30 inches. To manufacture the box, a machine cuts a square with a side length of x inches from each corner of the metal and then folds the four sides up (along the dotted lines in the drawing) to create an open box that is in the shape of a rectangular prism.



1. Write an expression, in terms of x, to represent the length, width, and height of the open box.
See margin

2. Write a function, $V(x)$ that could be used to represent the volume of the open box. Use the expressions you wrote in problem 1 to write the function in factored form.
$V(x) = (40 - 2x)(30 - 2x)(x)$

3. In this scenario, the maximum value of x has to be less than 15 inches. Explain why.
See margin.

4. The function for determining the volume of an open box is $V(x) = 4x^3 - 140x^2 + 1200x$. Use graphing technology to complete the table shown.

SIDE LENGTH OF SQUARE, x	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
VOLUME OF OPEN BOX, $V(x)$	*1064*	*1872*	*2448*	*2816*	*3000*	*3024*	*2912*	*2688*	*2376*	*2000*	*1584*	*1152*	*728*	*336*	*0*

1. *Length: 40 – 2x*

Width: 30 – 2x

Height: x

3. *The smallest dimension of the sheet metal is 30 inches and x has to be less than half the length of that distance or a box cannot be formed. Therefore, x has to be less than 15 inches.*

YOU TRY IT! #2 ANSWER:

The function $V(s)=[\frac{3}{2}\sqrt{3}\,(s^3)\text{ cu.in.}] \times \left(\frac{1\text{ gal.}}{231\text{ cu.in.}}\right)$ simplifies to $V(s) \approx 0.0112(s^3)$ gallons. The related equation is $30 \approx 0.0112(s^3)$. The intervals between input values in the table can be decreased to find a function value close to 30, which occurs between 13.85 and 13.9. Thus a 30-gallon fish tank would have faces with the side length of approximately 13.875 inches.

SIDE LENGTH (IN.)	VOLUME (GAL.)
13.75	*29.238*
13.8	*29.558*
13.85	*29.881*
13.9	*30.205*
13.95	*30.533*
14	*30.862*

5. The company wants to manufacture an open box with a volume of 2,000 cubic inches. What should be the dimensions of the square removed from each corner of the sheet metal?
10 inches by 10 inches

6. The company wants to manufacture an open box with a volume of 3,000 cubic inches. What should be the dimensions of the square removed from each corner of the sheet metal?
5 inches by 5 inches

7. The company wants to manufacture an open box with a volume of 2,500 cubic inches. Write an equation that could be used to determine x, the side length of the removed square.
$2500 = 4x^3 - 140x^2 + 1200x$

8. Use a graph to approximate the 3 solutions to the equation.
See margin.

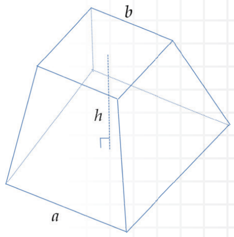
9. Which solution(s) from the previous question do NOT apply to the open box scenario and why?
See margin.

Use the scenario below to complete questions 10 – 15.

ART AND ARCHITECTURE

The volume formula of a frustum of a square pyramid was introduced by the ancient Egyptian mathematics in what is called the Moscow Mathematical Papyrus, written ca. 1850 BC. The volume formula is shown.

$$V = \frac{1}{3}h(a^2 + ab + b^2)$$



In the formula, a and b are the base and top side lengths of the truncated pyramid and h is the height. The frustum of a square pyramid is shown.

For this particular frustum, the function $V(x) = 0.5x^3 - 3x^2 + 6x$ can be used to calculate the volume when $a = x$, $b = x - 6$, and $h = \frac{1}{2}x$.

10. Use graphing technology to complete the table shown.

x	1	2	3	4	5	6	7	8	9	10
$V(x)$	3.5	4	4.5	8	17.5	36	66.5	112	175.5	260

11. What is the length of the larger base, a, if the volume is 4 cubic units?
2 units

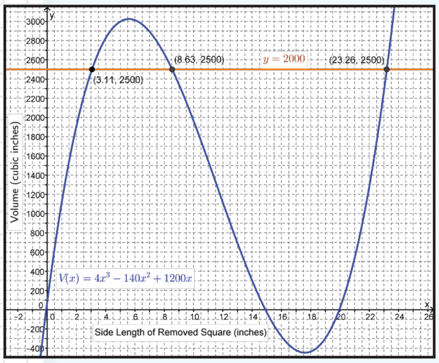
12. What is the length of the larger base, a, if the volume is 175.5 cubic units?
9 units

13. Write an equation that could be used to determine the value of the length of the larger base, x, if the volume is 100 cubic units.
$100 = 0.5x^3 - 3x^2 + 6x$

8. *(3.11, 2,500)*

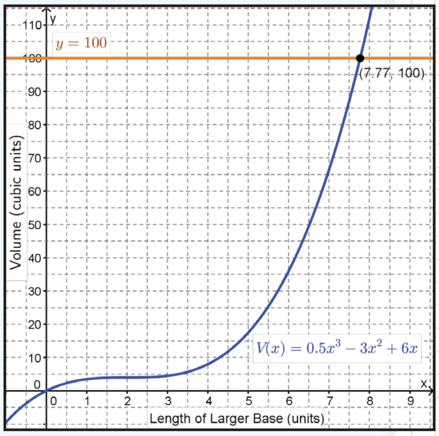
(8.63, 2,500)

(23.26, 2,500)



9. *(23.3, 2,500) does not apply because the sheet of metal is only 30 inches wide. The largest square that can be cut from each corner has a side length of 15 inches because after that, there is no metal remaining in the width. Since 23.3 > 15, it does not make sense in this situation.*

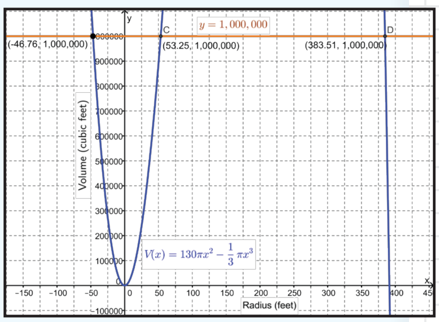
14. *(7.77, 100)*



19. *(-46.76, 1,000,000)*

(53.25, 1,000,000)

(383.51, 1,000,000



14. Use graphing technology to graph $V(x)$ and the line $y = 100$ to determine the point on the graph of $V(x)$ with a function value of 100.
See margin.

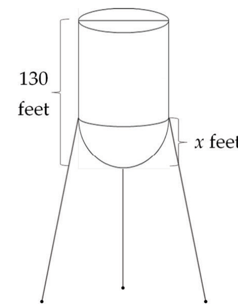
15. Use the value of x to determine the dimensions of a frustum with a volume of 100 cubic units.

- Length of larger base (a): ***x = 7.769 units***
- Length of smaller base (b): ***x – 6 = 1.769 units***
- Height: ***$\frac{1}{2}x$ = 3.885 units***

Use the scenario below to complete problems 16 – 22.

GEOMETRY

A town has a water tower that is composed of a cylinder and a hemisphere. The dimensions, in terms of the radius x, are shown.



16. Using the diagram, express each dimension in terms of x.

- Radius of base of cylinder: ***x feet***
- Height of cylinder: ***130 – x feet***
- Radius of hemisphere: ***x feet***

The function $130\pi x^2 - \frac{1}{3}\pi x^3$ can be used to calculate the volume of the water tower. Use graphing technology to complete the table below.

x	5	10	15	20	25	30	35	40	45	50
$V(x)$	*10079.28*	*39793.51*	*88357.29*	*154985.24*	*238891.94*	*339292.01*	*455400.04*	*586430.63*	*731598.39*	*890117.92*

17. What is the radius of the cylinder if the volume of the water tower is approximately 455,400 cubic feet?
About 35 feet

18. If a water tower has to have a minimum diameter of 20 feet and a maximum diameter of 40 feet, what is the minimum and maximum volume of water the tower could hold?
A minimum of 39,793.51 cubic feet and a maximum of 154,985.24 cubic feet

19. Use graphing technology to graph $V(x)$ and the line $y = 1{,}000{,}000$ to determine the three points of intersection or solutions to the equation $1{,}000{,}000 = 130\pi x^2 - \frac{1}{3}\pi x^3$.
See margin.

20. Use the approximate value of x in all three points to determine the possible dimensions of the water tower.

Point A	**Point B**	**Point C**
radius: ***-46.76 ft***	radius: ***53.25 ft***	radius: ***383.51 ft***
height: ***176.76 ft***	height: ***76.75 ft***	height: ***-253.51 ft***

21. Which x-value makes sense in the situation and why?
53.25 because one of the x-values is negative, and another x value results in a negative dimension.

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