TEKS

AR.6A Estimate a reasonable input value that results in a given output value for a given function, including quadratic, rational, and exponential functions.

AR.6C Solve equations arising from questions asked about functions that model real-world applications, including linear and quadratic functions, tabularly, graphically, and symbolically.

MATHEMATICAL PROCESS SPOTLIGHT

AR.1D Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

ELPS

1D Speak using learning strategies such as requesting assistance, employing non-verbal cues, and using synonyms and circumlocution (conveying ideas by defining or describing when exact English words are not known).

VOCABULARY

solution, estimate, rational equation, domain, range

MATERIALS

Engage:

- 80 beans per student group

Explore:

- graphing technology for each student
- mirror
- measuring tape (at least 6 meters long)
- meter stick
- washable marker for each student group

7.5 Solving Equations Related to Rational Functions

FOCUSING QUESTION Which representations can be used to solve equations that are related to rational functions?

LEARNING OUTCOMES

- I can use tables, graphs, and symbols to estimate solutions to rational equations modeling real-world problems.
- I can use tables, graphs, and algebraic methods to solve rational equations that model real-world problems.
- I can use graphs, symbols, diagrams, and language to communicate mathematical ideas, reasoning, and their implications.

ENGAGE

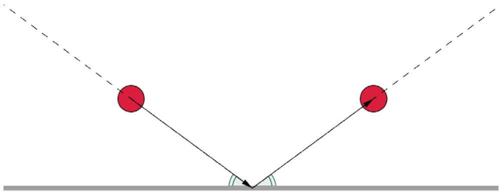

Lee Park, Dallas, Texas

A local city park has a flock of 80 ducks. The ducks like to form equal-sized groups. Work with a small group and use 80 beans or other counters to simulate the groups that the ducks might form. Record the number of groups and the number of ducks in each group in a table, and write a function that shows how the number of ducks in a group, $f(x)$, is a function of the number of groups, x.

See margin.

EXPLORE

In physics, you learn that when a moving object bounces off a flat surface, such as a wall or the floor, the angle of reflection is equal to the angle of incidence. In other words, the object bounces off the surface at the same angle that it struck the surface.



ACTIVITY SETUP

Work with a small group. Obtain a mirror, measuring tape, meter stick, and washable marker.

- Tape the meter stick on the wall so that the 0 mark is against the floor. Make sure that every 10 centimeters is visible.
- Designate one group member as the spotter. Measure the height of the spotter's eyes above the ground in centimeters.

ENGAGE ANSWER:

NUMBER OF GROUPS, x	1	2	4	5	8	10	16	20	40	80
NUMBER OF DUCKS, $f(x)$	80	40	20	16	10	8	5	4	2	1

$f(x) = \frac{80}{x}$

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- Lay the measuring tape along the floor so that the 0 mark is touching the base of the meter stick and the tape is perpendicular to the wall.
- Carefully mark a 2-centimeter by 2-centimeter square on the mirror with the washable marker.
- Place the mirror about 30 centimeters from a wall directly in front of the meter stick. The square should be centered along the 30-centimeter mark of the measuring tape.

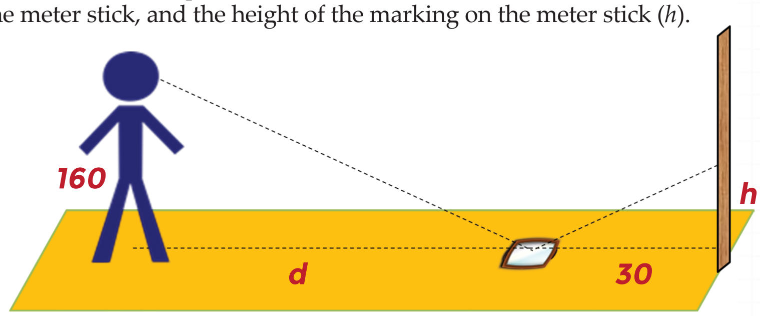
DATA COLLECTION

You will collect data to compare the distance of the spotter from the wall required to see a 10-centimeter marking on the meter stick.

1. Create a table like the one shown to collect and organize your data.

HEIGHT OF MARK (CM), h	10	20	30	40	50	60	70	80	90	100
DISTANCE FROM MIRROR (CM), d										

2. The diagram represents the setup with the spotter, the mirror, and the meter stick along the wall. Label the parts of the diagram with the height of the spotter's eyes, the distance of the spotter from the mirror (d), the distance between the mirror and the meter stick, and the height of the marking on the meter stick (h).



3. Have the spotter stand along the measuring tape at a point where the reflection of the 10-centimeter mark of the meter stick is inside the marked square on the mirror. Record the spotter's distance from the mirror in the table (remember to subtract the 30 centimeters between the mirror and the wall from the distance indicated by the measuring tape).

4. Repeat this process for the remaining marks along the meter stick.
See margin.

DATA ANALYSIS

5. The two triangles in the figure are similar. Use the diagram to explain your reasoning.
See margin.

6. Use the similar triangles to write a function, $d(h)$, representing the distance of the spotter from the mirror, d, as a function of the height of the meter stick marking, h. Include any domain or range restrictions.
See margin.

STRATEGIES FOR SUCCESS

It is always best to have students collect data. The process of data collection provides an experience through which students can relate the variables (height of the marking and distance between the spotter and the mirror). Collecting their own data also personalizes the activity for students and increases both the rigor and the relevance of the learning experience. However, if it is not possible to collect the data, provide students with the sample data for a spotter that is 5 feet 7 inches tall. Then, continue with the data analysis and questioning.

4. *Answers will vary depending on the height of the spotter. Sample answers are given for a spotter with a height of 5 feet 7 inches, or 170 centimeters, whose eyes are 160 centimeters above the ground.*

HEIGHT OF MARK (CM),	10	20	30	40	50	60	70	80	90	100
DISTANCE FROM MIRROR (CM)	480	240	160	120	96	80	69	60	53	48

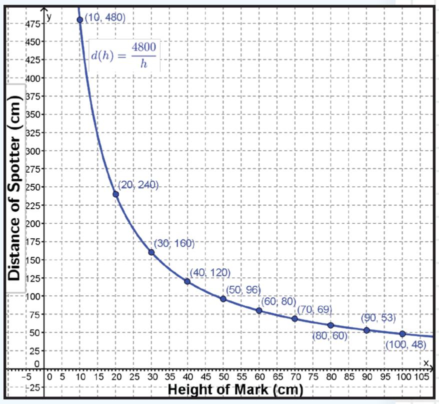
5. *Possible response: Both the spotter and the meter stick are perpendicular to the floor, so those angles are both right angles and are congruent. The angle of incidence and the angle of reflection are also congruent. If two pairs of corresponding angles in a triangle are congruent, then the triangles are similar (AA similarity theorem for triangles).*

6. *Possible response based on sample data:*

$$\frac{d(h)}{30} = \frac{160}{h}$$

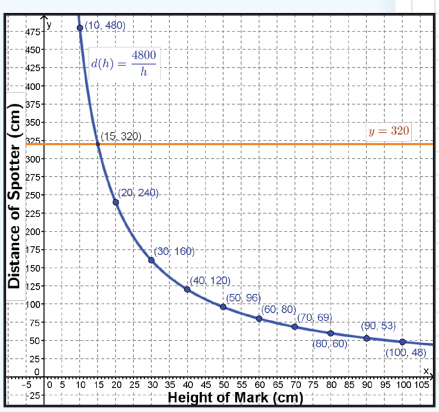
$$d(h) = 30\left(\frac{160}{h}\right) = \frac{4800}{h}, \text{ where } h \neq 0$$

7. *Possible response: The function connects or comes near each data point, so the function appears to model the data reasonably well.*



9. *$h = 15$*

Possible process: The coordinate plane uses x as the independent variable and y as the dependent variable. Thus, $d(h)$ becomes $d(x)$. Graph both $y = d(x)$ and $y = 320$ on a graphing calculator. Use the calculator to determine the coordinates of the point of intersection to be (15, 320). These coordinates mean that when $x = 15$, $y = 320$, so the solution to the equation is 15.



7. Use either paper and pencil or technology to create a scatterplot of your data. Graph your function from the previous question along with the scatterplot. Describe how well your function models your data.
See margin.

8. Use your function to write an equation you could solve to determine the height of a mark that a spotter would see if she or he stood 320 centimeters from the mirror.
$320 = \frac{4800}{h}$

9. Use a graph to solve the equation. Explain how you used your graph to solve the equation.
See margin.

10. Write an equation you could solve to determine the height of a mark that a spotter would see if she or he stood 100 centimeters from the mirror. Use a table of function values to solve the equation. Explain the process that you used to determine your solution.
See margin.

11. Write an equation you could solve to determine the height of a mark that a spotter would see if she or he stood 150 centimeters from the mirror. Use inverse operations to solve the equation. Explain the process that you used to determine your solution.
See margin.

12. **ELPS Connection** Work with a partner. Tell your partner how you can solve an equation from a rational function using a graph, table, or inverse operations. If you need to, use synonyms for key words like "solution." If you aren't sure of the exact word to use, you can convey your idea by defining what you mean or describing what you mean.
See margin.

REFLECT

- **Suppose you have the function, $g(x) = \frac{4}{3(x-5)} + 10$. What values of x would be excluded from the function? Use those values to write the domain and range of $g(x)$ to include those restrictions.**
  See margin.
- **How could you use inverse operations to solve the equation $15 = \frac{4}{3(x-5)} + 10$?**
  See margin.

10. *$100 = \frac{4800}{h}$, $h = 48$*
Possible process: Use a graphing calculator to generate a table of function values for $d(h)$. Enter $Y1 = 4800/x$ into the function editor and view the table. Look for the function value $d(h) = 100$. The row where $h = 48$ and $d(h) = 100$ shows the solution to the equation. Since $d(48) = 100$, the solution to the equation is 48.

h	$d(h)$
45	106.67
46	104.35
47	102.13
48	100
49	97.959
50	96
51	94.118

11. *$150 = \frac{4800}{h}$, $h = 32$*

Possible process: The equation involves division, and the inverse operation of division is multiplication. Multiply both members of the equation by h and simplify. Then, 150 is being multiplied by h in the left member of the equation so divide both members of the equation by 150 and simplify.

$$150 = \frac{4800}{h}$$
$$150h = \frac{4800}{h}\left(\frac{h}{1}\right)$$
$$150h = 4800$$
$$\frac{150h}{150} = \frac{4800}{150}$$
$$h = 32$$

12. *See margin page 785.*

REFLECT ANSWERS:
See margin page 785.

EXPLAIN

There are many real-world applications of rational functions. From these functions, you can write and solve rational equations. Recall that in general, a rational function is a ratio of two polynomial functions where the degree of the denominator is at least one. You can set up and solve rational equations of any degree. However, in this book, we will focus on solving rational equations where the numerator and denominator are either degree one or degree two.

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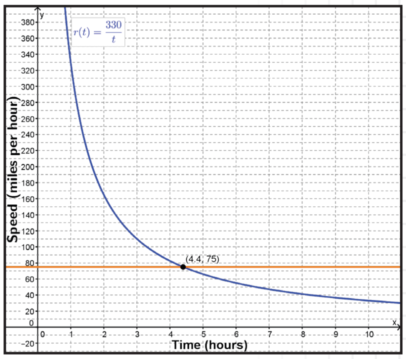
SOLVING RATIONAL EQUATIONS GRAPHICALLY

The distance between El Paso, Texas, and Santa Fe, New Mexico, is about 330 miles. The distance, driving speed, and travel time required to drive from El Paso to Santa Fe are related by the equation $d = rt$. The function $r(t) = \frac{330}{t}$ shows the driving speed, r, in miles per hour as a function of time, t, in hours.

The posted speed limit on the interstate highway between El Paso and Santa Fe is 75 miles per hour. At this rate, about how long will it take to drive from El Paso to Santa Fe? To solve this problem, use $r(t)$ to write an equation.

$$75 = \frac{330}{t}$$

Use graphing technology to graph $r(t)$ and the line $y = 75$ to determine the point on the graph of $r(t)$ with a function value of 75.



The point (4.4, 75) is the intersection point of $r(t)$ and $y = 75$. So, $r(4.4) = 75$ and when $t = 4.4$ hours, $r(t) = 75$ miles per hour.

Most people do not think of time as a decimal quantity; they think of time in terms of hours and minutes. Convert 4.4 hours to hours and minutes by separating the whole number and the fractional part of the decimal.

$$0.4 \text{ hours} \times \frac{60 \text{ minutes}}{1 \text{ hour}} = 24 \text{ minutes}$$

The drive will take 4 hours and 24 minutes.

SOLVING RATIONAL EQUATIONS TABULARLY

The wavelength, frequency, and speed of a radio wave are related by the equation $s = fw$, where s represents the speed of the wave, f represents the frequency of the wave, and w represents the wavelength of the wave. For radio waves in the Earth's atmosphere, the speed of the wave is approximately constant at 3×10^8 meters per second. The function $f(w) = \frac{3 \times 10^8}{w}$ can be used to determine the frequency of a radio wave if you know its wavelength.

12. *Possible response: Equations are solved by determining the input value that generates a given output value. In a graph, the input and output values are paired as coordinates of a point. In a table, the input and output values are paired by being in either the same row or column, depending on the orientation of the table. In a symbolic representation, inverse operations are how you "peel the onion" of the equation so that you can determine the value of the input variable from the given value of the output variable.*

SUPPORTING ENGLISH LANGUAGE LEARNERS

A learning strategy that supports English language learners is that they can speak using specific learning strategies such as requesting assistance, using non-verbal cues, using synonyms, or using circumlocution (**ELPS 1D**).

INTEGRATING TECHNOLOGY

You can use a graphing calculator or app to solve a rational equation graphically. Graph the function in $Y1$ and then graph $Y2 = n$, where n is the given output value or function value. Use the calculator's or app's features to determine where the two graphs intersect. The x-coordinate of the point of intersection is the solution to the equation.

REFLECT ANSWERS:

For $g(x)$, $x \neq 5$ since the denominator of the rational expression would be equal to 0. Division by 0 is undefined. The vertical translation of 10 also shifts the horizontal asymptote upward 10 units. The domain of $g(x)$ is $\{x \mid x \in \mathbb{R}, x \neq 5\}$ and the range of $g(x)$ is $\{g(x) \mid g(x) \in \mathbb{R}, g(x) \neq 10\}$.

Work backwards to undo the operations performed on x, the independent variable, in the right member of the equation. All operations listed must be performed to both members of the equation.

• *Subtract 10.*	$15 = \frac{4}{3(x-5)} + 10$
• *Multiply by 3(x – 5).*	$5 = \frac{4}{3(x-5)}$
• *Apply the distributive property to 3(x – 5).*	$5(3(x-5)) = 4$
• *Apply the distributive property to 5(3x – 15).*	$5(3x-15) = 4$
• *Add 75.*	$15x - 75 = 4$
• *Divide by 15.*	$15x = 79$
	$x = \frac{79}{15}$

INTEGRATING TECHNOLOGY

You can use a graphing calculator or app to solve an equation tabularly. Enter the function into $Y1$ of the function editor. In the table feature, scroll up and down until you see the given output value or function value. If you do not see the exact function value, then change the x-interval to a smaller number (e.g., 0.1 instead of 1), and look again. The x-value in the same row as the function value is the solution to the equation.

The radio station 88.7 FM broadcasts with a frequency of 88,700,000 hertz. What is the wavelength of the radio waves used by this radio station? To solve this problem, write an equation from $f(w)$. First, write the frequency, 88,700,000 in scientific notation, 8.87×10^7.

$$8.87 \times 10^7 = \frac{3 \times 10^8}{w}$$

Use graphing technology to make a table of values for $f(w)$. Look in the column for the dependent variable for a value of 8.87×10^7. You may need to refine the interval for Δx to see additional rational number values.

WAVELENGTH (METERS), w	FREQUENCY (HERTZ), $f(w)$
3.38	88,757,396
3.381	88,731,145
3.382	88,704,908
3.383	88,678,688
3.384	88,652,482
3.385	88,626,292
3.386	88,600,118

The table shown has function values close to 88,700,000 hertz. Find two adjacent function values so that one is just less than 88,700,000 and one is just greater than 88,700,000. When w = 3.382, $f(w)$ = 88,704,908 and when w = 3.383, $f(w)$ = 88,678,688.

$$88{,}678{,}688 < 88{,}700{,}000 < 88{,}704{,}908$$
$$3.382 < w < 3.383$$

The radio station uses radio waves with a wavelength between 3.382 meters and 3.383 meters.

SOLVING RATIONAL EQUATIONS SYMBOLICALLY

To solve a rational equation symbolically, first write it as a polynomial equation.

A publisher spent $40,000 in fixed expenses prior to printing a book and each book costs $4.00 to print. The function $c(x) = \frac{40{,}000 + 4x}{x}$ can be used to determine the cost per book if x books are printed. How many books should be printed in order for each book to cost $9.00 to print?

Use $c(x)$ to write an equation. $9.00 is the cost per book, or the function value (output value) for $c(x)$. Use this function (output) value to write an equation related to $c(x)$.

$$9 = \frac{40{,}000 + 4x}{x}$$

Multiply both members of the equation by the denominator of the rational expression, and then use inverse operations and properties of algebra to solve the resulting equation for x.

$9 = \frac{40{,}000 + 4x}{x}$	Original equation
$9(x) = \frac{40{,}000 + 4x}{x}\left(\frac{x}{1}\right)$	Multiply by the denominator.
$9X = 40{,}000 + 4x$	Simplify the multiplication. Notice this is a linear equation with variables in both members of the equation.

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$9x - 4x = 40{,}000 + 4x - 4x$ $5x = 40{,}000$	Subtract $4x$ from both members (apply the additive inverse property).
$\frac{5x}{5} = \frac{40{,}000}{5}$ $x = 8{,}000$	Divide both members by 5 (apply the multiplicative inverse property).

The publisher must print 8,000 books for the cost per book to be $9.

SOLVING EQUATIONS RELATED TO RATIONAL FUNCTIONS

A rational function is a ratio of two polynomial functions. An equation that is related to a given function, $f(x)$, is one in which the value of the dependent variable is known and you need to determine the value(s) of the independent variable that generates it. For a rational function with degree one or two polynomials in the numerator or denominator there will either be zero, one, or two pairs of values for which this is true.

- Graphically, locate a point on the graph of $f(x)$ that has a y-coordinate equal to the given function value. The x-coordinate of this point is the x-value paired with that function value. This x-value is a solution to the equation if it is included in the domain of the function.
- Tabularly, locate the function value in the dependent variable column or row. The value in the independent variable column or row associated with this function value is a solution to the equation if it is included in the domain of the function.
- Symbolically, substitute the given function value for the dependent variable in the symbolic representation of $f(x)$. Use inverse operations and the properties of algebra to solve the equation symbolically.
    - ✔ If the rational function is composed of linear functions, then the resulting equation will be linear. Solve this equation as you would any linear equation with inverse operations.
    - ✔ If the rational function is composed of at least one quadratic function, then the resulting equation will be quadratic. Solve this equation as you would any quadratic equation, such as using the zero-product property, inverse operations, or the quadratic formula.

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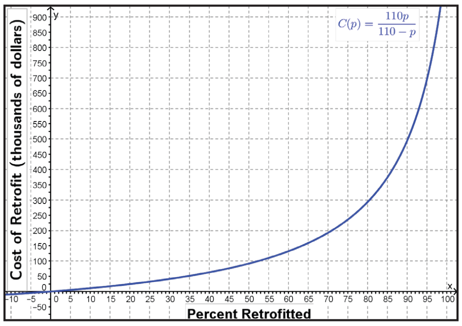
EXAMPLE 1

The cost, C, in thousands of dollars, for a utility company to retrofit p percent of its existing coal power plants in North America to reduce their emission of toxins into the atmosphere can be modeled by the rational function $C(p) = \frac{110p}{110 - p}$. A utility company has budgeted \$228,500 to retrofit its existing coal power plants. Approximately what percent of its coal power plants will the company be able to retrofit to reduce their emission of toxins into the atmosphere? Write an equation whose solution would answer the question and use the graph to find an approximate solution to the equation.

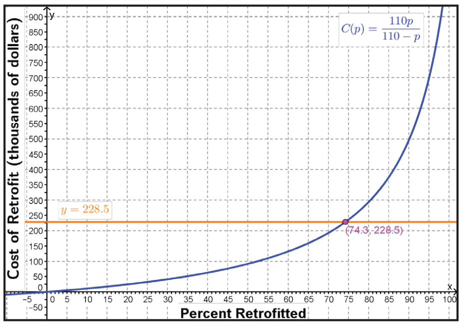


STEP 1 **Write an equation related to $C(p)$ whose solution answers the question, "Approximately what percent of its coal power plants will the company be able to retrofit to reduce their emission of toxins into the atmosphere with a budget of \$228,500?"**

Since $C(p)$ represents the cost in thousands of dollars, write the equation $C(p) = \frac{110p}{110 - p}$.

STEP 2 **Graph $y = 228.5$ on the same coordinate plane as $C(p)$.**

STEP 3 **Determine the point of intersection of $C(p)$ and $y = 228.5$.**



Using graphing technology, you can determine that the approximate point of intersection is (74,3, 228.5). Therefore, $C(74.3) \approx 228.5$.

STEP 4 **Interpret the intersection point in terms of the situation.**

The equation $228.5 = \frac{110p}{110 - p}$ will yield the approximate percentage of its existing coal power plants that the model predicts the utility company will be able to retrofit for \$228,500. The model C(p) predicts that the utility company will be able to retrofit 74.3% of its existing coal plants.

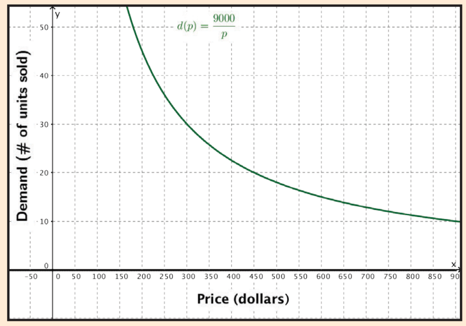
ADDITIONAL EXAMPLE

Patrick is driving from Amarillo, Texas to the state capital in Austin. The distance he needs to travel is 495 miles. Using the formula $d = rt$, he finds the rational function $t(r) = \frac{495}{r}$ will help him figure out how fast he needs to travel, r, in order to make it to Austin in a certain amount of time. Patrick would like to make it to Austin in 8 hours or less. Write an equation whose solution would answer the question. Then use graphing technology to graph the rational function, and find an approximate solution to the equation.

The model $t(r) = \frac{495}{r}$ shows that if Patrick drives about 62 miles per hour the entire way, he will make it to Austin in 8 hours or less. The equation whose solution yields an answer to the question is $8 = \frac{495}{r}$. The exact answer is 61.875 miles per hour.

YOU TRY IT! #1

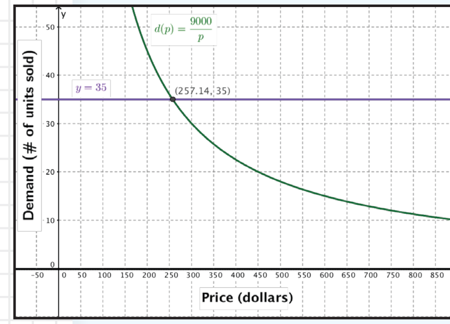
The owner of a local electronics store has noticed that when new products are released, the monthly demand, d, for them in his store can be modeled by the rational equation $d(p) = \frac{9000}{p}$, where p represents the retail price of the new product. If the local electronics storeowner wants to sell 35 of the latest tablet model this month, for what retail price should he sell them? Write an equation whose solution would answer the question and use the graph to find an approximate solution to the equation.



See margin.

YOU TRY IT! #1 ANSWER:

The model $d(p) = \frac{9000}{p}$ predicts that the storeowner will sell 35 of the latest tablet this month if he prices them at \$257.14 per unit. The equation whose solution yields an answer to the question is $35 = \frac{9000}{p}$.



EXAMPLE 2

A nurse charts the concentration, C, in milligrams per liter of ibuprofen in a patient's blood plasma over time, t, in hours using the rational function $C(t) = \frac{400t}{8t^2 + 3}$.

TIME, t (HOURS)	0	2	4	6	8	10	12
CONCENTRATION, $C(t)$ (mg/L)	0	22.86	12.21	8.25	6.21	4.98	4.16

If the ibuprofen is administered again when its concentration reaches 5 mg/L, when should the nurse administer the next dose of ibuprofen? Write an equation related to $C(t)$ that the nurse would need to solve and use the table she generated to approximate the solution.

STEP 1 Examine the dependent value row of the table.

TIME, t (HOURS)	0	2	4	6	8	10	12
CONCENTRATION, $C(t)$ (mg/L)	0	22.86	12.21	8.25	6.21	4.98	4.16

A concentration of 5 mg/L falls between these two indicated dependent values of $C(t)$.

STEP 2 Write an equation related to $C(t)$ whose solution answers the question, "If the ibuprofen is administered again when its concentration reaches 5 mg/L, when should the nurse administer the next dose of ibuprofen?"

Since $C(t)$ represents the concentration of ibuprofen in milligrams per liter, write the equation $5 = \frac{400t}{8t^2 + 3}$.

ADDITIONAL EXAMPLE

Denise is ordering a new water trough for the horses on her ranch. It will hold 100 gallons of water which is about 14 cubic feet. The length of the trough will be three times the width. The rational function $h(w) = \frac{14}{3w^2}$ represents the height of the water trough in relation to its width, w. If Denise wants the trough to be 1.5 feet deep, what should the length and width of the trough be? Write an equation related to $h(w)$ that Denise could solve to answer the question, and use the table below to approximate the solution.

WIDTH, w	HEIGHT, $h(w)$
0.5	18.67
1	4.67
1.5	2.07
2	1.17

The equation whose solution yields an answer to the question is $1.5 = \frac{14}{3w^2}$. The table of values shows that a trough width between 1.5 and 2 feet corresponds to a trough height of 1.5 feet. Further investigation using graphing technology yields a solution 1.76 feet for the width of the trough. Since the length is three times the width, that would make the length of the through $3(1.76) = 5.28$ feet.

STEP 3 Examine the independent value row of the table to determine the independent values of $C(t)$ that correspond to the dependent function values you indicated in Step 1.

TIME, t (HOURS)	0	2	4	6	8	10	12
CONCENTRATION, $C(t)$ (mg/L)	0	22.86	12.21	8.25	6.21	4.98	4.16

A concentration of 5 mg/L occurred somewhere between 8 and 10 hours after the first dose was administered. Since 5 is closer to 4.98 than 6.21, the independent value that corresponds to a concentration of 5 mg/L will be closer to 10 than 8.

STEP 4 Use graphing technology with tables to determine the independent value of the function $C(t)$ that corresponds most closely with a dependent value of 5.

Using graphing technology, you can change the intervals between successive independent values in the table to hundredths to determine that the independent value 9.97 corresponds to a dependent function value of 5, or $C(9.97) = 5$.

STEP 5 Interpret the intersection point in terms of the situation.

The equation $5 = \frac{400t}{8t^2+3}$ will yield the time in hours when the nurse should administer the next dose of ibuprofen. The model C(t) shows that the nurse should administer the next dose of ibuprofen 9.97, or approximately 10, hours after the first dose is administered.

YOU TRY IT! #2

A land developer sells rectangular lots with lengths that are 18 feet less than three times their width. Each lot will have a 6-foot privacy fence around its entire perimeter. The developer finds that the cost of the privacy fence is $14 per linear foot for materials and installation. Since lots are sold by the square footage of their area, the developer decides to advertise the privacy fences in terms of their cost per square foot of the land on each lot rather than per linear foot of fencing. The developer uses the rational function $C(w) = \frac{14(8w-36)}{w(3w-18)}$ where w represents the width, in feet, of a rectangular lot to generate the table for cost of the privacy fence per square foot of purchased land.

WIDTH OF LOT, w (FEET)	10	15	20	25	30	35	40
COST PER SQUARE FOOT, $C(w)$ (DOLLARS/FT)	$5.13	$2.90	$2.07	$1.61	$1.32	$1.12	$0.97

A real estate agent working with the developer asks how much land a customer will buy if their fence cost $2.00 per square foot. Write an equation related to $C(w)$ that the developer could solve to answer the real estate agent's question, and use the table he generated to approximate the solution.

See margin.

YOU TRY IT! #2 ANSWER:

The equation whose solution answers the question is $2 = \frac{14(8w-36)}{w(2w-18)}$. A table of values for $C(w) = \frac{14(8w-36)}{w(2w-18)}$ shows that a lot width of 20.59 feet corresponds to a fence that costs $2.00 per square foot. Therefore, the length of the lot is $3(20.59) - 18 = 43.77$. The area of land a customer will buy if their fence cost $2.00 per square foot is $(20.59)(43.77) \approx 901.22$ square feet.

X	Y1
20.5	2.0095
20.51	2.0084
20.52	2.0073
20.53	2.0062
20.54	2.0051
20.55	2.004
20.56	2.0029
20.57	2.0018
20.58	2.0007
20.59	1.9996
20.6	1.9985

X=20.59

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EXAMPLE 3

A zoologist studies lion breeding conditions in zoos across the globe. She theorizes that for a population of lions in captivity that has four more breeding females than breeding males, the effective population, p, of lions in the zoo can be calculated using the rational function $p(x) = \frac{5x(x+4)}{2x+4}$, where x represents the number of breeding male lions in a zoo's population. If a zoo's effective lion population is 19 lions, how many breeding males does the zoologist's model predict are in the zoo's lion population? Write an equation to represent the situation and solve it algebraically.

STEP 1 **Write an equation related to $p(x)$ whose solution answers the question, "If a zoo's effective lion population is 19 lions, how many breeding males does the zoologist's model predict are in the zoo's lion population?"**

Since $p(x)$ represents the zoo's effective lion population, write the equation $19 = \frac{5x(x+4)}{2x+4}$.

STEP 2 **Solve the equation.**

$$19 = \frac{5x(x+4)}{2x+4}$$
$$19(2x+4) = \frac{5x(x+4)}{2x+4}(2x+4)$$
$$38x + 76 = 5x(x+4)$$
$$38x + 76 = 5x^2 + 20x$$
$$38x + 76 - 38x - 76 = 5x^2 + 20x - 38x - 76$$
$$0 = 5x^2 - 18x - 76$$

You can use the quadratic formula to solve the resulting quadratic equation. $a = 5$, $b = -18$, and $c = -76$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{18 \pm \sqrt{(-18)^2 - 4(5)(-76)}}{2(5)} = \frac{18 \pm \sqrt{324 + 1520}}{10} = \frac{18 \pm \sqrt{1844}}{10}$$
$$x = \frac{18 + \sqrt{1844}}{10} \approx 6.09$$
$$x = \frac{18 - \sqrt{1844}}{10} \approx -2.49$$

STEP 3 **Interpret the solution(s) in terms of the situation.**

The answer -2.49 does not make sense in this situation since there cannot be a negative number of breeding male lions. The solution $x \approx 6.09$ can be interpreted as the model approximating 6 breeding male lions in the zoo's lion population resulting in an effective population of 19 lions.

ADDITIONAL EXAMPLE

Mrs. Lowery starts a prize candle company. She creates candles with rings inside of them. As the candle burns, the wax melts away revealing a ring for the customer to wear. The initial cost for her candle making supplies is $552. Each prize candle costs an additional $8 to create. The total cost to create a prize candle is modeled by the function $c(x) = \frac{552 + 8x}{x}$, where x represents the number of candles produced. To make a profit, Mrs. Lowery needs the candles to cost her $10 or less to produce each candle. How many candles must she mold in order to meet this cost boundary? Write an equation to represent the situation, and solve it algebraically.

The equation whose solution yields an answer to the question is $10 = \frac{552 + 8x}{x}$. Solving algebraically with inverse operations, $x = 276$. Mrs. Lowery will need to produce 276 or more prize candles to ensure their cost to create is $9 or less.

YOU TRY IT! #3 ANSWER:

The equation $6 = \frac{170t}{15t^2 + 2}$ will yield the time in hours that it takes for a healthy adult male's blood sugar level to reach 6 milligrams per 100 milliliters after consuming one serving of a sugary beverage. Solving algebraically using the quadratic formula, $t \approx 0.073$ and 1.815 hours, which is about 4 minutes and 1 hour 49 minutes after consumption.

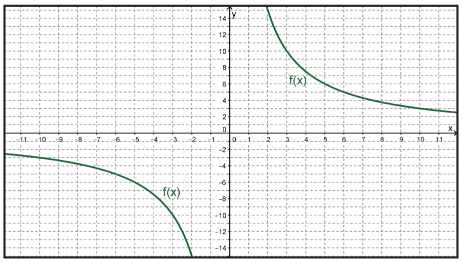
YOU TRY IT! #3

Sugar concentration in the bloodstream varies over the time passed after a sugary beverage is consumed. The blood sugar level, B, in milligrams per 100 milliliters of blood, of a healthy adult male as he metabolizes a single serving of a sugary beverage can be represented by the rational function $B(t) = \frac{170t}{15t^2 + 2}$ where t is time in hours. How long after an adult male consumes a standard serving of a sugary beverage will his blood sugar level reach 6 milligrams per 100 milliliters? Write an equation to represent the situation, and solve it algebraically.

See margin.

PRACTICE/HOMEWORK

1. The function $f(x) = \frac{30}{x}$ is graphed.



A. What is the value of x when $f(x) = 10$

$x = 3$

B. What is the value of x when $f(x) = -10$

$x = -3$

C. What is the value of x when $f(x) = 6$

$x = 5$

2. A summer camp director must decide on the number of groups she needs for various activities. This year, she has 120 campers attending camp. The table below shows some possible groupings.

NUMBER OF GROUPS, x	2	3	5	6	8	10	12	15	20	24
NUMBER OF CAMPERS IN A GROUP, $c(x)$	60	40	24	20	15	12	10	8	6	5

The function $c(x) = \frac{120}{x}$ is describes the number of campers in a group, based on the number of groups.

A. How many campers would be in a group if there are 15 groups?

8 campers per group

B. If there are 24 campers in a group, how many groups are there?

There would be 5 groups of 24.

C. If there were 160 campers, instead of 120, what would be the new function?

$c(x) = \frac{160}{x}$

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Use the situation described below to answer questions 3 – 5.

SPORTS

In the Indianapolis 500, racers compete to be the first to finish the 500 miles. The function that represents a racer's average speed during the race, $r(t)$, for a given finish time, t, is $r(t) = \frac{500}{t}$.

3. One of the race finishers had an average speed of 148 miles per hour during the race. Write an equation whose solution would give you his approximate race time.
$148 = \frac{500}{t}$

4. Reference the given graph of $r(t)$. Determine the approximate point on the graph of $r(t)$ with a function value of 148.
Answers will vary, but should be near (3.38, 148).

5. What does the intersection point mean for the situation?
See margin.

Use the situation described below to answer questions 6 – 8.

SPORTS

Colton has made 73% of his basketball free throws this year, making 37 out of 51 attempts. The function that would represent his free throw percentage, $f(x)$, for a given number of consecutive free throw shots made, x, is $f(x) = \frac{37 + x}{51 + x}$.

6. Colton wants a free throw percentage of 80%. Write an equation whose solution would give you the number of consecutive free throw shots he would need to make to achieve his goal.
$0.8 = \frac{37 + x}{51 + x}$.

7. Use graphing technology to graph $f(x)$ and the line $y = 0.8$. Determine the point on the graph of $f(x)$ with a function value of 0.8.
(19, 0.8)

8. What does the intersection point mean for the situation?
Colton will have a free throw percentage of 80% if he makes the next 19 free throw attempts.

5. *The racer who had an average speed of 148 mph finished the race in about 3.38 hours (which is about 3 hours and 23 minutes).*

Use the situation described below to answer questions 9 – 11.

CRITICAL THINKING

Anne is creating an enclosed garden in her backyard. She estimates that the work will take about 35 hours to complete. Some of her friends have offered to help, which will reduce the number of hours Anne will need to work. The function that represents the work time per person, $h(p)$, for a given number of people working, p, is $h(p) = \frac{35}{p}$.

9. Anne is hoping that she and her friends can complete the job in about 8 hours. Write an equation whose solution would give you the number of people who would need to work on the task to complete it in 8 hours.
$8 = \frac{35}{p}$

10. Use graphing technology to make a table of values for $h(p)$. Use the table to determine the approximate point of $h(p)$ with a function value of 8.
Answers will vary, but should be near (4.4, 8).

11. What does the intersection point mean for the situation?
Four people will not be enough to finish the job in 8 hours. It would actually take 5 people, and they would finish the job in less than 8 hours.

Use the situation described below to answer questions 12 – 14.

FINANCE

A school choir is taking a trip. Those going on the trip will each pay $300, plus their share of the $2500 deposit. The function that represents the cost per person, $c(p)$, for a given number of people going on the trip, p, is $c(p) = \frac{2500}{p} + 300$.

12. Write an equation whose solution would give you the number of people who would need to go on the trip in order for the cost per person to be $425.
$425 = \frac{2500}{p} + 300$

13. Use graphing technology to make a table of values for $c(p)$. Use the table to determine the approximate point of $c(p)$ with a function value of 425.
(20, 425)

14. What does the intersection point mean for the situation?
In order for each person to pay $425 for the trip, 20 people would need to participate.

Use the situation described below to answer questions 15 – 16.

SPORTS

Gus is planning a 25-mile bike ride in the Texas hill country. The function that represents his average speed, $r(h)$, for h hours is $r(h) = \frac{25}{h}$.

15. Write an equation whose solution would give you the average speed Gus would need to go in order to finish his ride in 2 hours.
$2 = \frac{25}{h}$

16. Solve your equation algebraically to determine the necessary average speed.
See margin.

16. $2 = \frac{25}{h}$

$2\,(h) = \frac{25}{h}(h)$

$2h = 25$

$h = 12.5$

His average speed should be about 12.5 miles per hour.

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Use the situation described below to answer questions 17 – 18.

FINANCE

A neighborhood is planning a block party. They want to rent an inflatable bounce house for $399, and they will share in the cost. They already have 5 families willing to contribute toward the bounce house, but more plan to participate. The function that represents the cost per family, $c(f)$, for the number of additional families, f, is $c(f) = \frac{399}{5+f}$.

17. Write an equation whose solution would give you the number of additional families who would need to participate in order for the cost per family to be about $40.

$40 = \frac{399}{5+f}$

18. Solve your equation algebraically to determine the number of additional families needed.

See margin.

For problems 19 – 23, solve using any method.

19. Given the function $f(x) = \frac{x}{x-4}$, determine the value of x when $f(x) = 2$.

$x = 8$

20. Given the function $f(x) = \frac{25}{x+10}$, determine the value of x when $f(x) = 2$.

$x = 2.5$

21. Given the function $f(x) = \frac{3}{x+12}$, determine the value of x when $f(x) = 9.3$.

$x = -11.68$

FINANCE

22. A club is renting a mini-bus to attend a competition. Those going on the trip will each pay a $25 entry fee for the competition, plus their share of the $600 bus cost. The function that represents the cost per person, $c(p)$, for a given number of people going to the competition, p, is $c(p) = \frac{600}{p} + 25$.

A. Write an equation whose solution would give you the number of people going to the competition if they are each paying a total of $50.

$50 = \frac{600}{p} + 25$

B. What is the solution to your equation, and what does it mean for the situation?

The solution is 24; 24 people will need to participate in order for costs to be $50 per person.

BUSINESS

23. A company spent $3,000 in fixed expenses prior to printing a set of advertisement flyers, and each flyer costs $0.15 to print. The function $c(x) = \frac{3{,}000 + 0.15x}{x}$ can be used to determine the cost per flyer if x flyers are printed. How many flyers should be printed in order for each flyer to cost $0.35 to print?

A. Write an equation whose solution would give you the number of flyers that need to be made in order for each flyer to cost $0.35 to print.

$0.35 = \frac{3{,}000 + 0.15x}{x}$

B. What is the solution to your equation, and what does it mean for the situation?

15,000 is the solution; The company will need to print 15,000 flyers if they want each flyer to cost $0.35 to print.

18. $40 = \frac{399}{5+f}$

$40(5+f) = \frac{399}{5+f}(5+f)$

$200 + 40f = 399$

$40f = 199$

$f = 4.975$

In order to get the price for the bounce house to about $40 per family, 5 more families would need to participate.