TEKS

AR.6A Estimate a reasonable input value that results in a given output value for a given function, including quadratic, rational, and exponential functions.

AR.6C Approximate solutions to equations arising from questions asked about exponential, logarithmic, square root, and cubic functions that model real-world applications tabularly and graphically.

MATHEMATICAL PROCESS SPOTLIGHT

AR.1D Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

ELPS

2I Demonstrate listening comprehension of increasingly complex spoken English by following directions, retelling or summarizing spoken messages, responding to questions and requests, collaborating with peers, and taking notes commensurate with content and grade-level needs.

3F Ask and give information ranging from using a very limited bank of high-frequency, high-need, concrete vocabulary, including key words and expressions needed for basic communication in academic and social contexts, to using abstract and content-based vocabulary during extended speaking assignments.

VOCABULARY

solution, estimate, function, equation

MATERIALS

- graphing technology

7.4 Estimating Solutions from Graphs and Tables

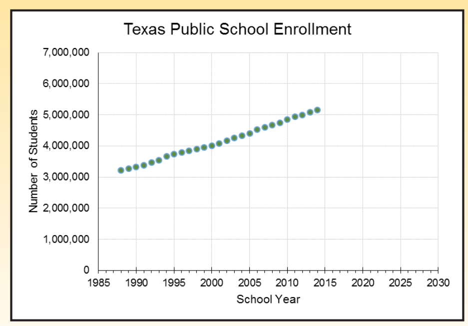
FOCUSING QUESTION How can you use graphs or tables to estimate solutions to higher-order equations?

LEARNING OUTCOMES

- I can estimate a solution to an equation related to a function that models real-world applications when the function is presented in a graph or a table.
- I can use graphs, symbols, diagrams, and language to communicate mathematical ideas, reasoning, and their implications.

ENGAGE

The graph shows the number of students enrolled in Texas public schools from the 1987-88 school year to the 2013-14 school year. In the graph, the year marked is the spring semester of the school year (e.g., 1995 along the horizontal axis is the 1994-95 school year). If these enrollment trends continue, in what school year will Texas have 6,000,000 students enrolled in public schools? Explain your answer.



Data Source: Texas Education Agency

See margin.

EXPLORE

In 2015, during the first week of the release of Adele's album, *25*, there were 1,640,000 downloads of the album in the United States. The album, *25*, set the record for the number of digital downloads of an album in one week. Suppose that for each week after the album debuted, the number of downloads decreases by 40% from the previous week.

1. What type of function do you think best represents this situation? Communicate your idea using mathematically appropriate language. ***See margin.***

ENGAGE ANSWER:

2024-25

Possible explanation: Draw a trend line through the data points and look to see where the trend line intersects the horizontal line for 6,000,000 students. From the intersection point, draw a vertical line to the horizontal axis. The vertical line meets the horizontal axis near 2025, which represents the 2024-25 school year.

1. *Possible response: An exponential decay function best represents this situation because each week the number of downloads decreases by a constant multiplier of 40%. When there is a constant multiplier for a situation, the situation is represented by an exponential function. When that multiplier is less than 1 (or 100%), then the function is exponential decay.*

2. If x represents the number of weeks since the album debuted for sale, the function $d(x) = 1.64(0.6)^{x-1}$ represents the number of downloads, in millions, for week x. Use technology to graph the function $d(x)$. Include any domain restrictions for $d(x)$ so that the function and its graph make sense in the context of the problem.
See margin.

3. Write an equation that you could use to determine the week when approximately 200,000 downloads of 25 were made. Remember that for $d(x)$, the function values are in millions of downloads.
$0.2 = 1.64(0.6)^{x-1}$

4. Estimate the solution of this equation using your graph. Explain the process that you used to determine your solution.
See margin.

The pH of a solution is a measure of how acidic the solution is. A solution with a pH of 7 is neutral, anywhere between 1 and 7 is acidic, and anywhere between 7 and 14 is basic. The pH of a solution is calculated as the negative base-10 logarithm of the concentration of hydrogen ions that are present in the solution. The function $p(x) = -\log(x)$ can be used to calculate the pH of a solution if the concentration of hydrogen ions, x, is known.

5. Create a table of values like the one shown. Use $p(x)$ to calculate the function values for the given input values. Round to the nearest thousandth.

CONCENTRATION OF HYDROGEN IONS, x	1×10^{-9}	1×10^{-8}	2.5×10^{-8}	4×10^{-8}	5×10^{-8}	8×10^{-8}	1×10^{-7}	4×10^{-7}	5.5×10^{-7}	8×10^{-7}	1×10^{-6}
pH, $p(x)$	9	8	7.602	7.398	7.301	7.097	7	6.398	6.260	6.097	6

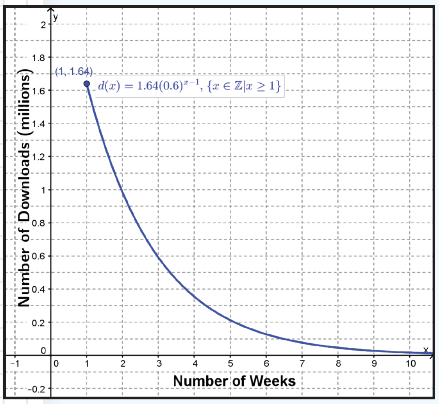
6. Examine your table of values. What patterns do you notice in the relationship between the x-values and the function values?
See margin.

7. Write an equation that you could use to determine the concentration of hydrogen ions for a solution with a pH of 6.398.
$6.398 = -\log(x)$

8. Use your table to solve the equation. Explain the process that you used to determine your solution.
See margin.

Scientific notation is used to represent very small or very large numbers as a decimal number less than 10 times a power of 10. Large numbers have a positive power of 10 and small numbers have a negative power of 10. The number 18,735 can be represented as 1.8735×10^4 and the number 0.00057 can be represented as 5.7×10^{-4}.

2.



4. *$x \approx 5$*

Since x represents the number of weeks since the album debuted for sale, after about 5 weeks, 200,000 more copies of the album were downloaded.

Possible process: Using technology, graph both d(x) and y = 0.2, which is the number of downloads made during week x. Identify the point of intersection and then read down to the x-axis.

INTEGRATING TECHNOLOGY

Use a graphing calculator to graph $Y1 = d(x)$. To solve the equation using a graph, add the graph of $Y2 = n$, where n is the given output value (in question 3, $n = 0.2$). Then, calculate the coordinates of the intersection point of $Y1$ and $Y2$. The x-coordinate is the input value, or x-value, that generates the given output value, or y-value.

6. *Possible responses:*

- *If the coefficient of the number in scientific notation is 1, then the pH is the opposite of the exponent of 10.*
- *If the coefficient of the number in scientific notation is not 1, then the pH has a ones digit that is 1 less than the opposite of the exponent of 10.*
- *The tenths, hundredths, and thousandths follow the same pattern if the multiplier, k in $k \times 10^{-n}$, of the number in scientific notation is the same.*

8. *$x = 4 \times 10^{-7}$*

CONCENTRATION OF HYDROGEN IONS, x	1×10^{-9}	1×10^{-8}	2.5×10^{-8}	4×10^{-8}	5×10^{-8}	8×10^{-8}	1×10^{-7}	4×10^{-7}	5.5×10^{-7}	8×10^{-7}	1×10^{-6}
pH, $p(x)$	9	8	7.602	7.398	7.301	7.097	7	6.398	6.260	6.097	6

Possible process: Look at the function values in the row for concentration of hydrogen ions, p(x). When the function value is 6.398, then p(x) = 6.398. Read up the column to determine the x-value that is paired with this function value. According to the table, $p(4 \times 10^{-7}) = 6.398$, so $x = 4 \times 10^{-7}$.

9. *Possible response: Finding a solution to an equation related to a given function in both a graph and a table means that you need to find the x-value that matches a given function value. The graph and table are both representations showing a set of matched x-values and function values. In a graph, these paired values are ordered pairs that are plotted on a coordinate plane. In a table, these paired values are in the same row or column.*

SUPPORTING ENGLISH LANGUAGE LEARNERS

Students should be able to use their speaking skills to ask for information that may include using a limited bank of high-frequency, high-need concrete vocabulary terms including key words and expressions needed for basic academic and social communication. Asking for information may also include using abstract and content-based vocabulary during extended speaking assignments (**ELPS 3F**). Students should also be able to demonstrate their listening skills by taking notes commensurate to the content and grade-level needs; in this case, they will be taking notes from what their partner says about solving equations using graphs and tables.

9. **ELPS Connection** Work with a partner. Explain to your partner how finding solutions to equations using graphs and tables are similar and how they are different. Ask your partner for information ranging from the bank of concrete vocabulary provided to additional abstract and concrete-based vocabulary. Be sure to use abstract and content-based vocabulary terms. Then, listen as your partner explains what he or she notices about the similarities and differences. Demonstrate your listening comprehension by taking notes in your math journal or math notebook about what your partner says.
See margin.

WORD BANK
function
equation
solution
function value
x-value

REFLECT

- **How can you use a graph to approximate a solution to an equation?**
  See margin.
- **How can you use a table to approximate a solution to an equation?**
  See margin.

EXPLAIN

A graph or a table shows specific paired input and output values that satisfy a functional relationship. If you know an output value for a function, then you can use tables or graphs to determine the input value that generated the given output value.

Watch Explain and You Try It Videos

or click here

SOLVING EQUATIONS GRAPHICALLY

A tsunami is a large ocean wave generated by underwater earthquakes. On December 26, 2004, a magnitude 9.1 earthquake off the coast of Indonesia generated a tsunami that traveled the entire globe and killed more than 200,000 people. Closest to the epicenter of the earthquake, the tsunami was about 80 feet high. Farther away, along the Pacific coast of North America, a tsunami of 15 inches was detected.

The speed of a tsunami in kilometers per hour can be calculated if you know the depth, x, of the ocean in meters using the function $s(x) = 11.276\sqrt{x}$.

If a tsunami is measured traveling at a speed of 450 kilometers per hour, what is the depth of the ocean at that point?

You can write a related equation and solve it graphically. 450 kilometers per hour is the speed of the wave, or the function value (output value) for $s(x)$. Use this function (output) value to write an equation related to $s(x)$.

$$450 = 11.276\sqrt{x}$$

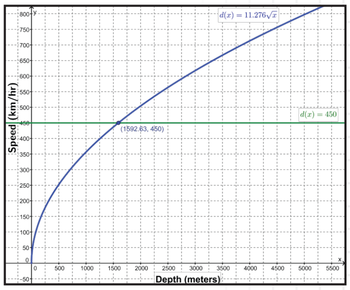
REFLECT ANSWERS:

Graph the function, f(x). For an equation of the form f(x) = b, locate the function value(s) of b that lie on the graph of the function. Read down to identify the x-value that corresponds with this function value. If you use technology, you can graph f(x) and y = b then calculate the coordinates of any intersection points. The estimated x-coordinate of an intersection point is the estimated solution to the equation.

Generate a table of values for the function, f(x). If you are looking for a solution to the equation f(x) = b, locate the function value b in the table. If it is not present, locate a function value close to b or add a row or column to your table to include this value. The x-value that corresponds with the given function value, b, is the solution or approximate solution to the equation.

764 CHAPTER 7: SOLUTIONS OF EQUATIONS

Graph $s(x)$. Then graph the line $y = 450$ to determine the points on the graph of $s(x)$ with a y-coordinate of 450 since the y-axis represents the function values. The intersection of the graph of $s(x)$ and the line $y = 450$ will reveal those points.



Some square root equations can also be solved symbolically using inverse operations.

$$450 = 11.276\sqrt{x}$$
$$\frac{450}{11.276} = \frac{11.276\sqrt{x}}{11.276}$$
$$39.9078 \approx \sqrt{x}$$
$$(39.9078)^2 \approx (\sqrt{x})^2$$
$$1592.63 \approx x$$

The point (1592.63, 450) is the intersection point of $s(x)$ and $y = 450$. That means that an input value, or depth, of 1592.63 meters will generate an output value, or speed, of 450 kilometers per hour.

If a tsunami has a measured speed of 450 kilometers per hour, then the depth of the ocean at that point is 1592.63 meters.

SOLVING EQUATIONS TABULARLY

The air through which an airplane flies creates a resistance force called drag. The drag of an airplane is a function of the speed of the airplane. For a typical commercial aircraft, the amount of power an airplane must produce to overcome the drag can be written as a function, $p(x)$, where x is the speed of the airplane. If the speed of the airplane is in miles per hour, then the function $p(x) = 1.993x^3$ can be used to calculate the power, in watts, required to overcome the drag at a speed of x miles per hour.

If an airplane requires 52,811,000 watts of power to overcome the drag due to air resistance, how fast is the airplane flying?

You can write a related equation and solve it tabularly. 52,811,000 is the power required to overcome drag, or the function value (output value) for $p(x)$. Use this function (output) value to write an equation related to $p(x)$.

$$52{,}811{,}000 = 1.993x^3$$

Make a table of values for $p(x)$. Look in the column for the dependent variable for a value of 52,811,000. If you are using technology to generate your table, you may need to refine the interval for Δx to see additional rational number values. The velocity that is in the same row as 52,811,000 watts is the input value, or x-value, that generates the output value, or function value, of 52,811,000.

VELOCITY (MILES PER HOUR), x	POWER (WATTS), $p(x)$
100	1,993,000
150	6,726,375
200	15,944,000
250	31,140,625
300	52,811,000
350	85,449,875
400	127,552,000

An airplane requiring 52,811,000 watts of power to overcome the drag force is flying at a velocity of 300 miles per hour.

INTEGRATING TECHNOLOGY

You can use a graphing calculator or app to solve a square root equation graphically. Graph the function in $Y1$ and then graph $Y2 = b$, where b is the given output value or function value. Use the calculator's or app's features to determine where the two graphs intersect. The x-coordinate of the point of intersection is the solution to the equation.

INTEGRATING TECHNOLOGY

You can use a graphing calculator or app to solve an equation tabularly. Enter the function into $Y1$ of the function editor. In the table feature, scroll up and down until you see the given output value or function value. If you do not see the exact function value, then change the x-interval to a smaller number (e.g., 0.1 instead of 1) and look again. The x-value in the same row as the function value is the solution to the equation.

APPROXIMATING SOLUTIONS TO EQUATIONS USING GRAPHS AND TABLES

You can use a graph or a table to approximate solutions to equations that are related to a function, $f(x)$. If you have an equation, $f(x) = b$, then the following statements are true.

- Graphically, locate a point on the graph of $f(x)$ that has a y-coordinate equal to the given function value. The x-coordinate of this point is the x-value paired with that function value. This x-value is a solution to the equation. Depending on the function type, there may be zero, one, or more than one x-value that generates a particular function value.
- Tabularly, locate the function value in the dependent variable column. The value in the independent variable column corresponding with this function value is the solution to the equation. Depending on the function type, there may be zero, one, or more than one x-value that generates a particular function value.

EXAMPLE 1

Ryan purchases a new boat for $68,000. The function $v(n) = 68{,}000(1 - 0.1)^n$ represents the value of his boat that has an annual depreciation rate of ten percent in terms of n years since Ryan purchased the new boat. Ryan calculates the value over the first few years of ownership and records the results in a table.

TIME SINCE PURCHASE, n (YEARS)	BOAT VALUE, $v(n)$ (DOLLARS)
0	$68,000.00
1	$61,200.00
2	$55,080.00
3	$49,572.00
4	$44,614.80

By how much will the value of the boat will have depreciated after five years? Write an equation related to the exponential function, solve it using the table and techniques such as mental math, estimation, or number sense, and use the result to answer the question.

STEP 1 Write an equation related to $v(n)$ whose solution will answer the question, "By how much will the value of the boat will have depreciated after five years?"

766 CHAPTER 7: SOLUTIONS OF EQUATIONS

Since $v(n)$ represents the value of the boat n years after purchase, the amount the value of the boat has depreciated after five years is its initial value minus its value after five years. Let x represent the amount the value of the boat has depreciated. The equation $x = 68{,}000 - v(5)$ will yield the amount by which the value of the boat depreciates after five years.

STEP 2 **Use the table and techniques such as mental math, estimation, and number sense to solve the equation.**

Determine the first differences between successive n-values and between successive $v(n)$-values.

n	$v(n)$
0	$68,000.00
1	$61,200.00
2	$55,080.00
3	$49,572.00
4	$44,614.80
5	?

$\Delta n = 1 - 0 = 1$ $\quad \Delta v = 61{,}200 - 68{,}000 = $ **-6,800**

$\Delta n = 2 - 1 = 1$ $\quad \Delta v = 55{,}080 - 61{,}200 = $ **-6,120**

$\Delta n = 3 - 2 = 1$ $\quad \Delta v = 49{,}572 - 55{,}080 = $ **-5,508**

$\Delta n = 4 - 3 = 1$ $\quad \Delta v = 44{,}614.8 - 49{,}572 = $ **-4,957.2**

Notice that each difference between successive values of $v(n)$ equals negative 10% of the subtrahend of each difference, which is a loss of value. This pattern will continue to determine the value of $v(5)$. Using mental math, negative 10% of 44,614.80 is -4,461.48. Add this value to 44,614.80, which number sense tells you is the same as subtracting 4,461.48 from 44,614.80. The result is $40,153.32. Estimation confirms this since approximately 44,600 minus approximately 4,500 is approximately 40,100.

The result is the value of $v(5)$. Therefore to solve the equation $x = 68{,}000 - v(5)$, simply substitute the result for $v(5)$ in the equation. $68{,}000 - 40{,}153.32 = 27{,}846.68$

STEP 3 **Interpret the answer in terms of the situation.**

The value of the boat will have depreciated by $27,846.68 after five years of ownership.

The equation $x = 68{,}000 - v(5)$ yields the amount by which the value of the boat depreciates after five years. The value of the boat will have depreciated by $27,846.68 after five years of ownership.

ADDITIONAL EXAMPLE

Alonzo invested some money that will double in value every six years. He initially invested $10,000. The value of his investment, $f(x)$, over a period of x years is shown in the table.

The function $f(x) = 10000(2)^{\frac{x}{6}}$ represents Alonzo's investment value where x is the number of years since his initial investment. How many years will it take for Alonzo's investment to reach $15,000 in value? Write an equation related to the exponential function, and solve it using the table.

YEARS INVESTED, x	INVESTMENT VALUE, $f(x)$
0	10,000.00
1	11,224.62
2	12,599.21
3	14,142.14
4	15,874.01
5	17,817.97
6	20,000.00

According to the table, Alonzo's investment will reach a value of $15,000 between years three and four. The equation $15000 = 10000(2)^{\frac{x}{6}}$ will yield the exact solution. Using the table features of graphing technology, changing the step between independent values to hundredths, you can determine that after 3.51 years, the value of his investment was about $15,000.39. Therefore, the investment reached a value of $15,000 about 3.5 years after Alonzo made his initial investment.

YOU TRY IT! #1 ANSWER:

Since the initial sheep population is 8 bighorn sheep, the population would have doubled when there were 16 sheep. The equation $16 = 8(1.384)^x$ will yield the solution. The table shows that this occurred somewhere between two and three years after their re-introduction into the national park. Since 16 is closer to 15 than it is to 21, the population likely doubled closer to 2 years after re-introduction than 3 years after re-introduction. Using table features of graphing technology, changing the step between independent values to hundredths, you can determine that after 2.13 years, the sheep population was approximately 15.985. Therefore, the bighorn sheep population doubled approximately 2 years, 2 months after they were re-introduced to the national park.

YOU TRY IT! #1

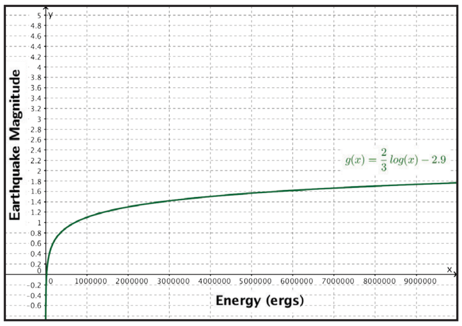
Biologists re-introduce a species of bighorn sheep into a national park and study the growth in the population of bighorn sheep since they reintroduced the species to the national park. The population $p(x)$ of bighorn sheep over a period of x years is shown in the table.

TIME IN NATIONAL PARK, x (YEARS)	SHEEP POPULATION, $p(x)$ (NUMBER OF SHEEP)
0	8
1	11
2	15
3	21
4	29
5	41
6	56
7	78
8	108
9	150

The function $p(x) = 8(1.384)^x$ represents the sheep population in the national park where x is the time since the biologists re-introduced the species of bighorn sheep. How long did it take for the sheep population to double (use number sense to determine the doubled population) after their re-introduction into the national park? Write an equation related to the exponential function, and use the table to estimate the solution.

See margin.

EXAMPLE 2



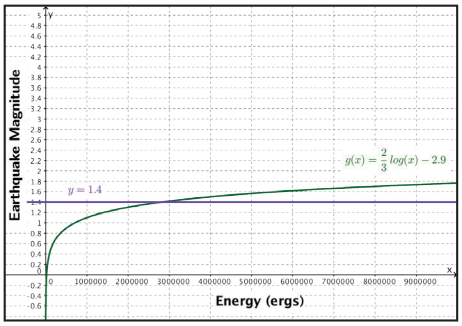
According to the United States Geological Service, seismologists use several different magnitudes to describe earthquakes. For example, the formula $g(x) = \frac{2}{3}\log x - 2.9$ calculates the magnitude $g(x)$, a measure of seismic potential for damage, as a function of x, the energy that radiates from the earthquake's epicenter as measured in ergs. If a seismologist uses this magnitude formula to report an instance of a magnitude 1.4 earthquake, how much energy radiated from the epicenter of this earthquake? Write an equation whose solution answers the question, and use the graph to solve the problem.

768 CHAPTER 7: SOLUTIONS OF EQUATIONS

STEP 1 **Write an equation related to $g(x)$ whose solution answers the question, "If a seismologist reports an instance of a magnitude 1.4 earthquake, how much energy radiated from the epicenter of this earthquake?"**

Since $g(x)$ represents the magnitude of the earthquake, write the equation $1.4 = \frac{2}{3}\log x - 2.9$.

STEP 2 **Graph $y = 1.4$ on the same coordinate plane as $g(x)$.**



STEP 3 **Determine the point of intersection of $g(x)$ and $y = 1.4$.**

Using graphing technology, you can determine that the approximate point of intersection is (2818383, 1.4). Therefore, $C(2818383) \approx 1.4$.

STEP 4 **Interpret the intersection point in terms of the situation.**

The equation $1.4 = \frac{2}{3}\log x - 2.9$ will yield the amount of energy in ergs that radiates from the epicenter of a magnitude 1.4 earthquake. The model g(x) shows that an earthquake that radiates 2,818,383 ergs of energy from its epicenter will have a magnitude of 1.4.

ADDITIONAL EXAMPLE

Saranya is using a popular "brain-training" app to improve her memory. The makers of the app claim that the average user's score will improve each month using the logarithmic function $m(t) = 15\log(t + 1)$where t in the number of months the user has been utilizing the app. Saranya is excited to train her brain, and she uses the app daily for many months. The goal is to attain a score as close to 18 points as possible. How long will it take for her reach a score of 18 points? Write an equation whose solution answers the question, create a graph, and use the graph to solve the problem.

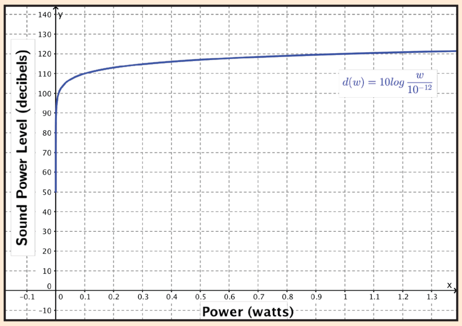
The equation $18 = 15\log(t + 1)$will yield how many months it should take Saranya to reach a score of 18 points. Using graphing technology, graph both m(t) and y = 18. The intersection of these two equations is (14.8, 18), which means that after 14.8 months, Saranya should have achieved a score of 18 points.

YOU TRY IT! #2

Construction workers building a new airport terminal receive training that includes a pamphlet about the importance of wearing protective ear coverings.

SOUND SOURCE	SOUND POWER LEVEL (DECIBELS)
WHISPER	20
BUSINESS OFFICE	70
PNEUMATIC DRILL	110
RIVETER	120
COMMERCIAL JET	140

One construction worker's job requires that he work for several hours a day with a pneumatic drill. If the power level of sound in decibels can be described by the function $d(w) = 10\log\left(\frac{w}{10^{-12}}\right)$, where w represents the power in watts required to generate the sound, then what power is required to generate the sound of the pneumatic drill the construction worker uses?



Write an equation whose solution answers the question, and use the graph to solve the problem.

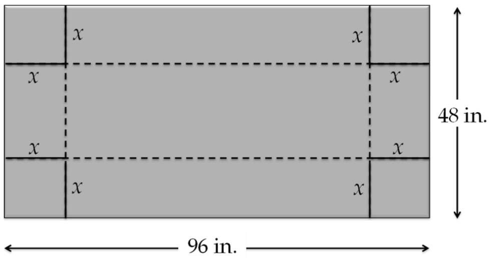
See margin.

EXAMPLE 3

A welder makes a window box planter out of a sheet of aluminum that measures ninety-six inches by forty-eight inches. He will cut a square out of each corner that is x inches on each side, and bend and weld the remaining aluminum to create a window box in the shape of a rectangular prism. What length should he cut from the corners in order to maximize the volume of the window box?

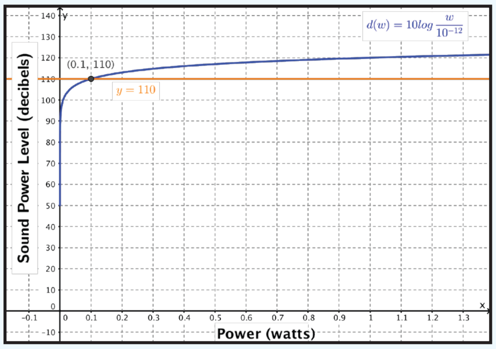
STEP 1 **Analyze the given information.**

Drawing a picture is a strategy that can help with your analysis.



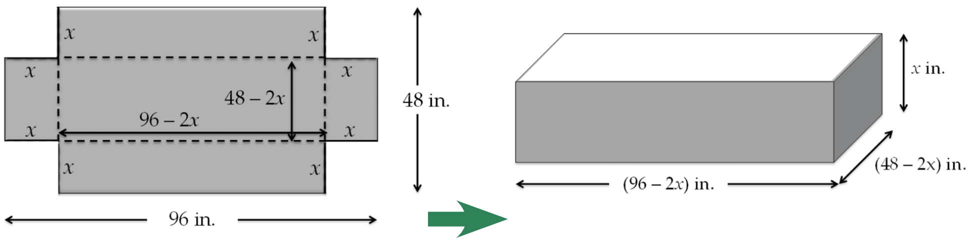
YOU TRY IT! #2 ANSWER:

The equation $110 = 10\log\left(\frac{w}{10^{-12}}\right)$ will yield the power in watts that is required to generate the 110 decibel sound of a pneumatic drill. The model $d(w)$ yields that 0.1 watts of power will generate 110 decibels of sound.



770 CHAPTER 7: SOLUTIONS OF EQUATIONS

The rectangle represents the sheet of aluminum out of which the welder will cut x by x inch squares. There are two lengths x inches long that are cut off each side, meaning that the resulting dimensions that will be bent and welded to make the window box are x, $96 - 2x$, and $48 - 2x$ as shown.



STEP 2 **Formulate a plan or strategy.**

You need to write an equation to solve the problem. You can use a graph to solve the equation. Since the problem asks about maximizing the volume of the box, your equation should represent the volume, v, of the window box. An equation whose solution would answer the question would represent the volume on one side of the equation and the value of the maximum volume on the other side of the equation.

Since you don't know the maximum volume, you can use graphing technology to generate a graph to approximate the maximum volume. You can also use the graph to determine the value of x, the length of the square cutout that would result in the approximate maximum volume.

STEP 3 **Determine a solution.**

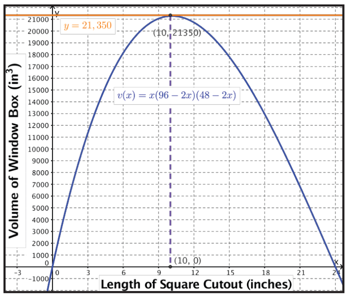
The volume of the window box can be determined using the formula for the volume of a rectangular prism, $V = Bh$. Since all the measures of the edge lengths of the window box are in terms of x, the volume will be a function in terms of x.

$V = Bh$

So $V = v(x)$ and $B = (96 - 2x)(48 - 2x)$ and $h = x$.

Therefore, $v(x) = (96 - 2x)(48 - 2x)(x)$ or $v(x) = x(96 - 2x)(48 - 2x)$.

From a graph of $v(x)$, estimate the maximum value of $v(x)$ to be a little more than 21,000 cubic inches, perhaps 21,350 in^3. This approximate maximum is reached when somewhere between 9- and 12-inch squares are cut out of the corners of the sheet of aluminum, perhaps squares whose sides measure about 10 inches.



Using the features of graphing technology, you can determine the more precise approximate maximum of the curve to be the point (10.144, 21,283.44). The window box has an approximate maximum volume of 21,283.44 cubic inches when square corners with a side length of approximately 10.144 inches are cut from the sheet of aluminum.

STEP 4 **Justify your solution.**

Continuing to use the features of graphing technology, you can generate a table of values for x and $v(x)$.

x	$v(x)$
10.11	21,283.25
10.12	21,283.35
10.13	21,283.41
10.14	21,283.44
10.15	21,283.43
10.16	21,283.40
10.17	21,283.32
10.18	21,283.22

Notice that the function values of $v(x)$ increase incrementally leading up to the input value of 10.14 and decrease incrementally after 10.15. The table verifies that the approximate maximum is 21,283.44 and that the maximum occurs at or near $x = 10.14$.

STEP 5 **Evaluate the problem solving process.**

It was necessary to draw pictures to understand the relationships in this situation and formulate a function and its related equation. Graphing technology allows you to generate a graph of the function $v(x)$. If you had used graphing technology to calculate the maximum function value immediately after determining the function that describes the volume of the window box in this situation, the problem could have been solved more rapidly without sacrificing precision. However, analyzing the graph before calculating the maximum function value enables you to predict and estimate the maximum function value and the independent value of the function associated with the maximum before applying graphing technology.

ADDITIONAL EXAMPLE

Merriam is creating a scale model of her bedroom to label for a Spanish class project. The length of her bedroom is represented by $l(x) = 3x + 2$, and the width of her bedroom is $w(x) = 9 - x$. What value of x, in inches, will yield the largest area for her model? How much material will she need to border the perimeter of her model? Write an equation whose solution answers the question. Then use graphing technology (tables and graphs) to answer the questions.

The equation $A(x) = (3x + 2)(9 - x)$ represents the area of Merriam's model bedroom. Using graphing technology, you can analyze the graph to find that the maximum value is (4.17, 70.1) which means that a value of $x = 4.17$ inches will create a model with an area with the largest possible area of 70.1 square inches. Since the length of her model is represented by $l(x) = 3x + 2$, $l(4.17) = 3(4.17) + 2 = 14.51$ inches. The width of her model is represented by $w(x) = 9 - x$, so $w(4.17) = 9\text{-}4.17 = 4.83$. Therefore, Merriam should purchase $2(14.51) + 2(4.17) = 37.36$ inches of material to border her model.

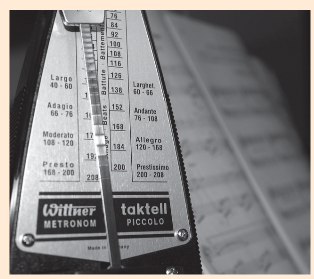
772 CHAPTER 7: SOLUTIONS OF EQUATIONS

These close estimates provide greater confidence in the answer you ultimately obtain through graphing technology. Moreover, when using the table feature of graphing technology to justify your answer, you are able to zoom in to function values very close to the maximum to show that for very close input values to your answer, the function increased leading up to the maximum and decreased thereafter. Therefore, graphing technology is a useful tool, and when combined with sound mathematical reasoning through the problem solving process, it is even more powerful.

The equation 21,283.44 = x(96 – 2x)(48 – 2x) will yield the approximate maximum volume of a window box made from cutting square corners out of a sheet of aluminum. The model v(x) shows that a window box with a maximum volume of approximately 21,283.44 in^3 will result when it is made by cutting out square corners with approximately 10.144 inch sides from a sheet of aluminum that measures 96 inches by 48 inches.

YOU TRY IT! #3

Mechanical metronomes use a weight that is adjusted along a rod to change their tempo. The time in seconds that it takes for the rod of a mechanical metronome to swing can be described by the function $t(x) = 2\pi\sqrt{\frac{x}{9.8}}$, where x represents the distance in meters between the weight and the fulcrum in the base of the metronome along its rod. Metronome speeds are described in beats per minute, or the number of swings per minute. A piece of music is to be played at 70 beats per minute. How far from the fulcrum in the base of the metronome should the weight be placed along the rod to time this piece of music? Use a problem-solving model that includes analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process to write an equation whose solution answers the question, and use the graph to solve the problem.



pixabay.com

See margin.

YOU TRY IT! #3 ANSWER:

A piece of music played at 70 beats per minute takes 1 minute to play 70 beats. Since t is measured in seconds, you must convert this measure to determine the number of seconds a single beat or swing of the metronome's rod takes.

$\frac{1 \text{ min}}{70 \text{ beats}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} = \frac{60 \text{ sec}}{70 \text{ beats}} = \frac{60}{70}\text{sec/beat} = \frac{6}{7}\text{sec/beat}$

Therefore, the equation $\frac{6}{7} = 2\pi\sqrt{\frac{x}{9.8}}$ *will determine how far from the fulcrum the weight should be placed in order for the rod to swing at a rate of 70 beats per minute. Using graphing technology, you can determine that the weight should be placed approximately 0.182 meters, or 18.2 centimeters, from the fulcrum to time a piece of music that is to be played at 70 beats per minute.*

PRACTICE/HOMEWORK

Use the scenario and table below to answer questions 1 – 5.

SCIENCE

The Texas Department of Public Safety can use the length of a skid marks to help determine the speed of a vehicle before the brakes were applied. The quadratic function that best models the data is $f(x) = \frac{x^2}{24}$ where x represents the speed of the vehicle, and $f(x)$ is the length of the skid mark. The speeds of a vehicle and the length of the corresponding skid mark are shown in the table below.

SPEED OF A VEHICLE IN MILES PER HOURS, x	DISTANCE OF THE SKID IN FEET, $f(x)$
30	37.5
36	54
42	73.5
48	96
54	121.5

1. Write an equation and use the table of data to determine the length of a skid mark of a vehicle that was traveling at a speed of 60 miles when it applied the brakes.

$f(x) = \frac{60^2}{24}$ 150 feet

2. Write an equation and use the table of data to determine the length of a skid mark of a vehicle that was traveling at a speed of 20 miles when it applied the brakes.

$f(x) = \frac{20^2}{24}$ 16.7 feet

3. Write an equation and use the table of data to determine how fast a vehicle was traveling if the length of the skid mark was 26 feet.

$26 = \frac{x^2}{24}$ 25 miles per hour

4. Write an equation and use the table of data to determine how fast a vehicle was traveling if the length of the skid mark was 216 feet.

$216 = \frac{x^2}{24}$ 72 miles per hour

5. Phyllis was driving on a road with a posted speed limit of 65 miles an hour. She had to apply the brakes suddenly and left a skid mark on the road. When the highway patrol arrived, she said she was driving less than the posted speed limit. The officers measured the skid mark and found the mark to be 170 feet. Was Phyllis speeding? Justify your answer?

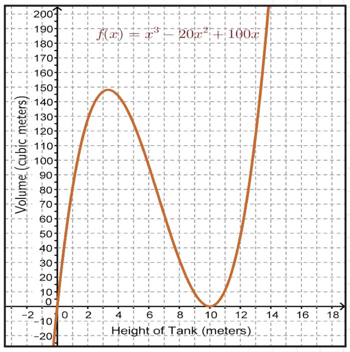
No. $f(x) = \frac{65^2}{24}$, $f(x) = 176.04$, or $176 = \frac{x^2}{24}$; $x \approx 63.87$

774 CHAPTER 7: SOLUTIONS OF EQUATIONS

Use the scenario and the graph below to answer questions 6 – 9.

GEOMETRY

Watershed Tanks builds storage tanks in the shape of a square prism with a height of x meters and a base edge length of (x – 10) meters. The volume of the tank is found using the formula $V = Bh$, where B represents the area of the base and h represents the height of the prism. The graph of $f(x)$, which can be used to calculate the volume of the prism with a height of x meters, is shown.



6. If the height of a tank is 14 meters, what is the volume of the tank? Write an equation related to $f(x)$ and use the graph and equation to determine the volume.
$f(x) = (14)^3 - 20(14)^2 + 100(14)$
$f(x) = 224$ cubic meters

7. A customer wants to have a tank with a height of 12 meters. Write an equation related to $f(x)$ and use the graph and equation to determine the volume.
$f(x) = (12)^3 - 20(12)^2 + 100(12)$
$f(x) = 48$ cubic meters

8. A customer needs a tank with a volume of 100 cubic meters. Write an equation related to $f(x)$ and use the graph to approximate solutions to the equation.
See margin.

9. Of the possible heights generated by your equation, which height(s) could actually be used to construct a tank? Justify your reasoning.
See margin

Use the scenario and table below to answer questions 10 – 12.

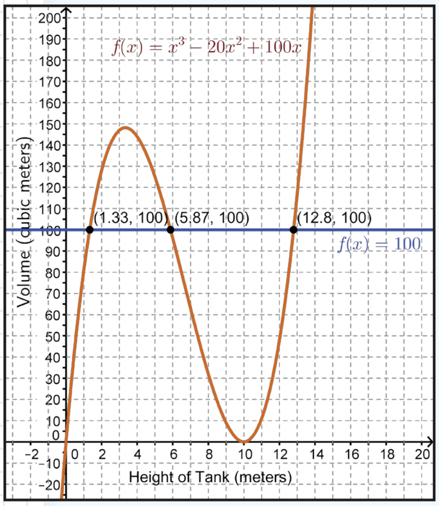
TRAVEL

This summer, Amber is going to drive to her vacation destination which is 500 miles from her home. The driving time it takes Amber to travel the 500 miles, $f(x)$, is represented by the function $f(x) = \frac{500}{x}$, for any driving speed, x. The average speeds Amber can drive and the length of time it will take her to drive to her vacation destination are shown in the table below.

AVERAGE SPEED (MILES PER HOUR)	DRIVING TIME (HOURS)
50	10
55	$9\frac{1}{11}$
60	$8\frac{1}{3}$
65	$7\frac{9}{13}$
70	$7\frac{1}{7}$

10. Write an equation, and use the table of data to determine how long it will take Amber to drive to her vacation destination if she drives an average of 62.5 miles per hour.
$f(x) = \frac{500}{62.5}$ ***8 hours***

8. *$100 = x^3 - 30x^2 + 100x$*
$x \approx 1.33, 5.87, 12.8$



9. *Only 12.8 meters could be used to construct a tank, because the edge length of the square base is x – 10 meters. If the height of the tank is less than 10 meters, then the edge length will be negative and you cannot have a negative edge length of a square or prism.*

11. Write an equation and use the table of data to determine the average speed Amber would need to drive if she wants to arrive at her vacation destination in 12.5 hours.

$12.5 = \frac{500}{x}$ 40 miles per hour

12. Amber knows that there is road construction along her path and determines that she should add an additional hour to the driving time to allow for delays. The driving time it takes Amber to travel the 500 miles allowing for construction delays, $f(x)$, is represented by the function $f(x) = \frac{500}{x} + 1$, for any driving speed, x. Write an equation and use the table of data to determine what average speed Amber will need to drive if she wants to make it to her vacation destination in approximately 9.5 hours?

$9.5 = \frac{500}{x} + 1$ $x \approx 58.8$ or about 59 miles per hour

Use the scenario below to answer questions 13 – 15.

SPORTS

Robert enjoys running for exercise. He has started a training plan that will gradually increase his weekly mileage as he prepares for a marathon. The miles he will run per week, $f(x)$, is represented by the function $f(x) = 16(1.1)^x$, for the number of weeks he plans to run, x. The number of weeks he plans to run and the miles he will run in each week are shown in the table below.

# OF WEEKS, x	MILES PER WEEK, $f(x)$
0	16
1	18
2	19
3	21
4	23
5	26

13. Write an equation and use the table of data to determine about how many miles Robert will run in week 7 of his training program.

$f(x) = 16(1.1)^7$ 31 miles

14. Write an equation and use the table of data to determine during which week Robert will run about 38 miles.

$38 = 16(1.1)^x$ during week 9

15. The race Robert is training for is in 12 weeks and he set a goal to be running approximately 60 miles a week by that time. Will Robert meet his training goal? Justify your answer.

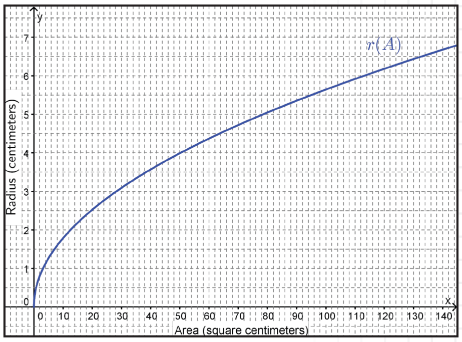
No. $f(x) = 16(1.1)^{12}$ $x \approx 50.2$ or about 50 miles per week

776 CHAPTER 7: SOLUTIONS OF EQUATIONS

Use the scenario and the graph below to answer questions 16 – 17.

GEOMETRY

The radius of a circle, $r(A)$, is represented by the function $r(A) = \sqrt{\frac{A}{\pi}}$ with A representing the area of the circle. The graph of this function is shown.



16. Write an equation and use the graph to determine radius of a circle with an area that is about 78.5 cm.

$r(A) = \sqrt{\frac{78.5}{3.14}}$ ***$r \approx 5$cm***

17. Write an equation, and use the graph to determine the area of a circle with a radius of 6 cm.

$6 = \sqrt{\frac{A}{3.14}}$ ***$A \approx 113.04$cm^2***

Use the scenario below to answer questions 18 – 20.

GEOMETRY

The volume of a rectangular prism, $f(x)$, is represented by the function $f(x) = 2.4x^3$, for a prism with a length of the base that is x inches. The length of the base and the volume of the prism are shown in the table below.

LENGTH OF BASE, x (INCHES)	VOLUME, $f(x)$ (CUBIC INCHES)
0	0
1	2.4
2	19.2
3	64.8
4	153.6

18. Write an equation and use the table of data to determine the volume of a rectangular prism with a length of the base 6 inches.

$f(x) = 2.4(6)^3$ ***518.4 cubic inches***

19. Write an equation and use the table of data to determine the length of the base of the rectangular prism that has a volume of 1228.8 cubic inches.

$1228.8 = 2.4(x)^3$ ***8 inches***

20. A shipping company wants to use one of the rectangular prisms as a shipping box. They want a box that will hold 3100 cubic inches of product. What is the length of the base of the rectangular prism that will be closest to the size they need?

$3100 = 2.4(x)^3$ ***$x \approx 11$ inches***