TEKS

AR.6B Solve equations arising from questions asked about functions that model real-world applications, including linear and quadratic functions, tabularly, graphically, and symbolically.

MATHEMATICAL PROCESS SPOTLIGHT

AR.1A Apply mathematics to problems arising in everyday life, society, and the workplace.

ELPS

5B Write using newly acquired basic vocabulary and content-based grade-level vocabulary.

VOCABULARY

function, input value, output value, equation, domain, range, independent variable, dependent variable

MATERIALS

- graphing technology

7.3 Solving Equations Related to Quadratic Functions

FOCUSING QUESTION What does it mean to solve an equation related to a quadratic function?

LEARNING OUTCOMES

- I can write an equation that is related to a quadratic function in order to solve a real-world problem.
- I can apply mathematics to solve problems arising in everyday life, society, and the workplace.

ENGAGE

A pumpkin patch covers a rectangular pasture that has a length that is 10 yards less than three times the width of the pasture. Write an expression that describes the area of the pasture in terms of width, w.

$(3w - 10)w = 3w^2 - 10w$

EXPLORE

The population of Pittsburgh, Pennsylvania, from 1900 to 1980 can be approximately modeled using the function $p(x) = -18x^2 + 144x + 396$. In this function, x represents the decade since 1900 (e.g., for 1900, $x = 0$ and for 1910, $x = 1$), and $p(x)$ represents the population in thousands.

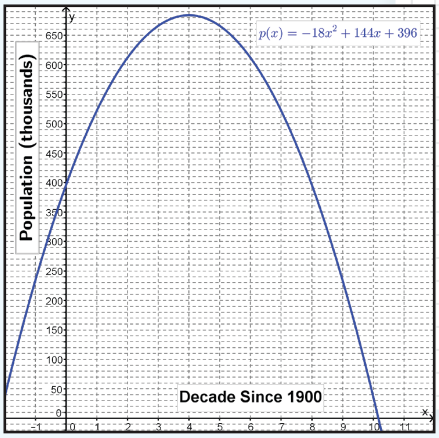
1. Graph the function, $p(x)$.
See margin.

2. Examine your graph. Approximately when does the population of Pittsburgh stop increasing and begin decreasing?
The vertex of the graph is (4, 684), so after x = 4, or the year 1940, the population of Pittsburgh stopped increasing and began decreasing.

3. Write an equation that you could use to determine the year when the population of Pittsburgh was 500,000. Remember that for $p(x)$, the function values are the population in thousands of people.
$500 = -18x^2 + 144x + 396$

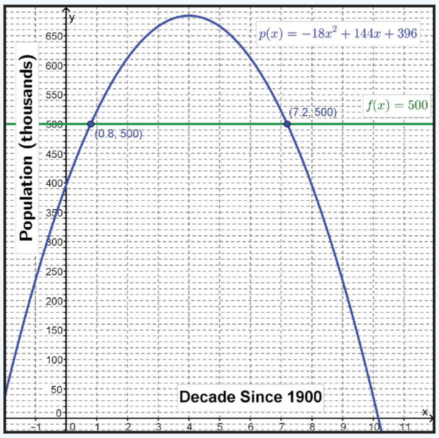
4. Solve this equation using your graph. Remember that x represents the decade since 1900, so you will need to use your x-value to calculate the year. Explain the process that you used to determine your solution.
See margin.

1.



4. *$x \approx 0.8$ and 7.2*
Since x represents the decade since 1900, multiply x by 10 and then add 1900.
$0.8(10) + 1900 = 1908$
$7.2(10) + 1900 = 1972$
The population of Pittsburgh was 500,000 in both 1908 and 1972.

Possible process: Using technology, graph both p(x) and f(x) = 500, which is the target population. Calculate the coordinates of the point of intersection, rounding to the nearest tenth. The x-coordinate of each point of intersection represents an x-value that will generate a function value of approximately 500. Because the coordinates are rounded, the solutions are approximate and not exact.



5. Create a table of values like the one shown. Use $f(x)$ to calculate the function values for the given input values.

DECADE SINCE 1900, x	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
POPULATION (THOUSANDS), $p(x)$	*396*	*463.5*	*522*	*571.5*	*612*	*643.5*	*666*	*679.5*	*684*	*679.5*	*666*

6. Examine your table of values. Approximately when does the population of Pittsburgh stop increasing and begin decreasing?
See margin.

7. Write an equation that you could use to determine the year when the population of Pittsburgh was 679,500. Remember that for $p(x)$, the function values are the population in thousands of people.
$679.5 = -18x^2 + 144x + 396$

8. Use your table to solve the equation. Explain the process that you used to determine your solution.
See margin.

9. Write an equation that you could use to determine the year when the population of Pittsburgh was 594,000. Remember that for $p(x)$, the function values are the population in thousands of people.
$594 = -18x^2 + 144x + 396$

10. Use inverse operations, factoring, or the quadratic formula to solve this equation for x.
See margin.

REFLECT

- **For a linear function, you obtained one solution for each equation. For a quadratic function, there were two solutions for each equation. Why do you think that is the case?**
  See margin.
- **Solving graphically, you were able to obtain an estimated (rounded) answer. Solving symbolically, you were able to obtain an exact answer. Why do you think that is the case?**
  See margin.

INTEGRATING TECHNOLOGY

Use a graphing calculator to graph $Y1 = p(x)$. To solve the equation using a graph, add the graph of $Y2 = n$, where n is the given output value (in question 2, $n = 500$). Then, calculate the coordinates of the intersection point of $Y1$ and $Y2$. The x-coordinate is the input value, or x-value, that generates the given output value, or y-value. For quadratic functions, the coordinates that are calculated are often rounded estimates of irrational numbers (exact solutions can be obtained using the quadratic formula).

6. *The greatest function value in the table is when $x = 4$, or the year 1940. The symmetry of the data values indicates that (4, 684) is the vertex of the graph and the population in 1940, which was 684,000, is the maximum population.*

10. $x = 4 \pm \sqrt{5}$

Rewrite the equation $-18x^2 + 144x + 396 = 594$ *to* $-18x^2 + 144x - 198 = 0$.

Factor the equation to $-18(x^2 - 8x + 11) = 0$.

Notice that $x^2 - 8x + 11$ *does not factor, so use the quadratic formula.*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(11)}}{2(1)}$$

$$x = \frac{8 \pm \sqrt{64 - 44}}{2}$$

$$x = \frac{8 \pm \sqrt{20}}{2}$$

$$x = 4 \pm \sqrt{5}$$

The population of Pittsburgh was 594,000 when $x = 4 + \sqrt{5} \approx 6.2$ *and when* $x = 4 - \sqrt{5} \approx 1.8$. *When* $x = 6.2$, *the year is 1962 and when* $x = 1.8$, *the year is 1918.*

The population of Pittsburgh was approximately 594,000 in both 1918 and 1962.

8. *$x = 3.5$, so the population of Pittsburgh was 679,500 in the year 1935.*
$x = 4.5$, so the population of Pittsburgh was 679,500 in the year 1945

DECADE SINCE 1900, x	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
POPULATION (THOUSANDS), $p(x)$	396	463.5	522	571.5	612	643.5	666	679.5	684	679.5	666

Possible process: Look at the function values in the row for population, p(x). When the function value is 679.5, then p(x) = 679.5. Read up the column to determine the x-value that is paired with this function value. According to the table, p(3.5) = 679.5, so x = 3.5. Also, p(4.5) = 679.5, so x = 4.5.

SUPPORTING ENGLISH LANGUAGE LEARNERS

Encourage students to use newly acquired basic vocabulary and content-based grade-level vocabulary by recording their responses to the Reflect questions in an interactive math notebook or a math journal (**ELPS 5B**).

REFLECT ANSWERS: *See margin on page 752.*

EXPLAIN

Quadratic functions relate a set of input values (domain of the independent variable) to a set of output values (range of the dependent variable) using a relationship with a varying rate of change. Within the domain and range of a quadratic function, each input value generates only one output value so that the input value and its corresponding output value are paired numbers.

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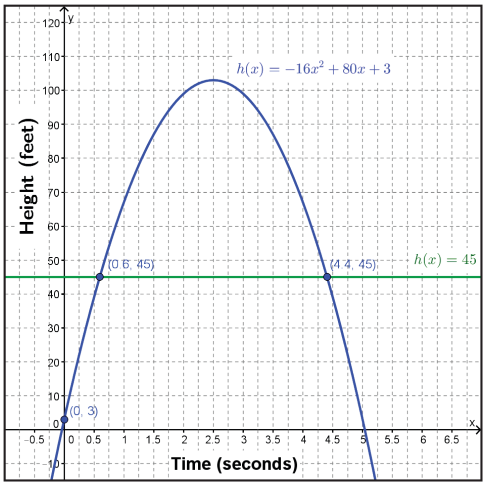
SOLVING QUADRATIC EQUATIONS GRAPHICALLY

The punter on a football team punted the ball from a height of about 3 feet above the field. The function $h(x) = -16x^2 + 80x + 3$ describes the height of the ball above the field, x seconds after the ball was punted. The graph of $h(x)$ is shown.



At what times after being punted was the ball 45 feet above the field?

You can write a related equation and solve it graphically. 45 feet is the height of the ball, or the function value (output value) for $h(x)$. Use this function (output) value to write an equation related to $h(x)$.

$$45 = -16x^2 + 80x + 3$$

Graph $h(x)$. Also, graph the line $y = 45$ to determine the points on the graph of $h(x)$ with a y-coordinate of 45. The intersection of the graph of $h(x)$ and the line $y = 45$ will reveal those points. Since $h(x)$ is a quadratic function and its graph has horizontal symmetry about the vertical line through the vertex of $h(x)$, there will be up to two points of intersection. The range of $h(x)$ is $(-\infty, 103]$, and the height of 45 falls inside this range. Thus, there will be exactly two points of intersection.

The points (0.6, 45) and (4.4, 45) are the two intersection points of $h(x)$ and $y = 45$. That means that an input value, or time, of 0.6 seconds and 4.4 seconds will generate a height of 45 feet.

The ball will be 45 feet above the field 0.6 seconds and 4.4 seconds after being punted.

SOLVING QUADRATIC EQUATIONS TABULARLY

In the football problem, at what times is the ball 87 feet above the field?

You can write a related equation and solve it tabularly. 87 is the height of the ball, or the function value (output value) for $h(x)$. Use this function (output) value to write an equation related to $h(x)$.

$$87 = -16x^2 + 80x + 3$$

REFLECT ANSWERS:

A linear function is a first degree polynomial function, and a quadratic function is a second degree polynomial function. The potential number of solutions to a polynomial equation matches the degree of the polynomial, so a linear equation generally has one solution, and a quadratic equation has as many as two solutions.

If you look at the graph of a linear function, since a line extends in both directions at a constant rate of change, any horizontal line will only intersect the line once as long as the slope of the line itself is not zero. If you look at the graph of a quadratic function, there is symmetry around the vertex so that any horizontal line within the range of the quadratic function (excluding the vertex) will intersect the graph twice.

If the solutions to the equation are rational numbers, then you can obtain an exact solution graphically as well as symbolically. However, when the solutions to the equation are irrational numbers (i.e., contain square roots), then graphically-obtained solutions are estimates because the intersection points are calculated and rounded to a decimal number.

INTEGRATING TECHNOLOGY

You can use a graphing calculator or app to solve a quadratic equation graphically. Graph the function in $Y1$ and then graph $Y2 = n$ where n is the given output value or function value. Use the device or app's features to determine where the two graphs intersect. The x-coordinate of the point of intersection is the solution to the equation.

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Make a table of values for $h(x)$. Look in the column for the dependent variable with a value of 87 as shown in the table. If you are using technology to generate your table, you may need to refine the interval for Δx to see additional rational number values. The time that is associated with 87 feet is the input value, or x-value, that generates the output value, or function value, or 87.

TIME (SEC.), x	HEIGHT (FT), $h(x)$
1.25	78
1.5	87
1.75	94
2	99
2.25	102
2.5	103
2.75	102
3	99
3.25	94
3.5	87

In this case, there are two points with a function value of 87: (1.5, 87) and (3.5, 87).

The ball was 87 feet above the field 1.5 seconds after being punted and again 3.5 seconds after being punted.

SOLVING QUADRATIC EQUATIONS SYMBOLICALLY

There are several ways to solve a quadratic equation symbolically.

- If the equation is in vertex form, $n = a(bx - c)^2 + d$, then you can use inverse operations to solve for x.
- If the equation is in factored form, $0 = k(ax - b)(cx - d)$, then you can use the zero product property to solve for x.
- If the equation is in polynomial form, $0 = ax^2 + bx + c$, then you can use the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, to solve for x. Sometimes, an equation in polynomial form can be factored or written in vertex form by completing the square.

In the football problem, Jeannie and her cousin Barbara are in seats that are 67 feet above the field. They see the ball at eye level twice before it hits the ground. At what times did Jeannie and Barbara see the ball at eye level?

You can write a related equation and solve it symbolically. 67 is the height of the ball above the field, or the function value (output value) for $h(x)$. Use this function (output) value to write an equation related to $h(x)$.

$$67 = -16x^2 + 80x + 3$$

Subtract 67 from both members of the equation (i.e., apply the additive inverse) so that one member of the equation is 0.

$$67 - 67 = -16x^2 + 80x + 3 - 67$$
$$0 = -16x^2 + 80x - 64$$

INTEGRATING TECHNOLOGY

You can use a graphing calculator or app to solve a linear equation tabularly. Enter the function into $Y1$ of the function editor. In the table feature, scroll up and down until you see the given output value or function value. If you do not see the exact function value, then change the x-interval to a smaller number (e.g., 0.1 instead of 1) and look again. The x-value in the same row as the function value is the solution to the equation.

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The polynomial on the right member of the equation will factor, so you can factor the equation and then use the zero product property.

$0 = -16x^2 + 80x - 64$	
$0 = -16(x^2 - 5x + 4)$ $0 = -16(x - 4)(x - 1)$	Factor completely.
$x - 4 = 0 \qquad x - 1 = 0$	Set each factor equal to 0 (zero product property).
$x - 4 + 4 = 0 + 4 \qquad x - 1 + 1 = 0 + 1$	Add 4 to both members of the first factor and add 1 to both members of the second factor (additive inverse).
$x = 4 \qquad x = 1$	

The ball was at eye level with Jeannie and Barbara (that is, 67 feet above the field) at 1 second and 4 seconds after being punted.

SOLVING EQUATIONS RELATED TO QUADRATIC FUNCTIONS

A quadratic function is a second degree polynomial function. An equation that is related to a given function, $f(x)$, is one in which the value of the dependent variable is known and you need to determine the value(s) of the independent variable that generates it. For a quadratic function, there may be as many as two points for which this is true.

- Graphically, locate a point on the graph of $f(x)$ that has a y-coordinate equal to the given function value. The x-coordinate of this point is the x-value paired with that function value. This x-value is a solution to the equation.
- Tabularly, locate the function value in the dependent variable column or row. The value in the independent variable column or row corresponding with this function value is the solution to the equation.
- Symbolically, substitute the given function value for the dependent variable in the symbolic representation of $f(x)$. Use a method such as inverse operations, factoring and applying the zero product property, or the quadratic formula to solve for x.

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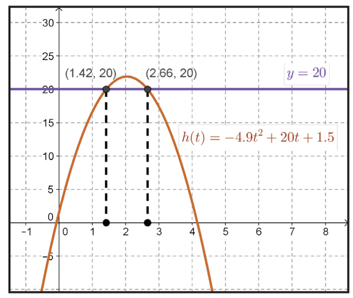
EXAMPLE 1

Several high schools in the area are having a model rocket launch. A special camera will be fixed on the top of a 20-meter tall pole to capture pictures during the rocket flights. A quadratic function can be used to represent the height above the ground, $h(t)$, in terms of the number of seconds, t, into the flight. The Science Club has calculated the function for their rocket as $h(t) = -4.9t^2 + 20t + 1.5$ when the launch pad is 1.5 meters above the ground. A related equation, $y = 20$, represents the height of the camera's aim. Graph the function and equation, then determine when the camera will capture a picture of their rocket.

STEP 1 Graph the function $h(t) = -4.9t^2 + 20t + 1.5$ and the related equation $y = 20$.



STEP 2 Determine the points on the graph of $h(t)$ with a y-coordinate or height of 20 meters.

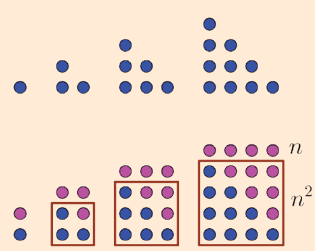
The intersection of the graph of $h(t)$ and the line $y = 20$ will show two such points since $h(x)$ is a quadratic function, unless 20 meters is the highest point of the flight or taller than the highest point of the flight. The points (1.42, 20) and (2.66, 20) both have a y-coordinate of 20.

STEP 3 Interpret the points of intersection of the graphs.

If all goes as planned, the camera will capture pictures of the Science Club's rocket at 1.42 and 2.66 seconds after the launch.

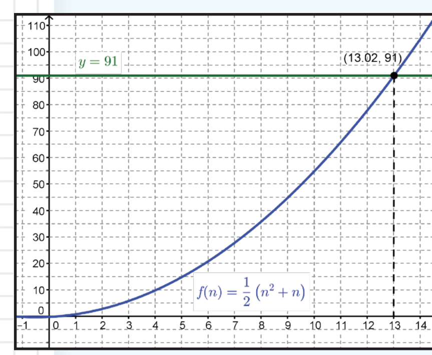
YOU TRY IT! #1

Triangular numbers can be modeled by a function suggested in the diagram. If you duplicate each triangular number on top of itself, you can partition the resulting arrangement into an n by n square and a 1 by n row. Graph the function $f(n) = \frac{1}{2}(n^2 + n)$ and $y = 91$ to determine which triangular number 91 is. Use your graph to communicate why any intersection points may be a solution to the equation, but not a reasonable solution to the problem.



See margin.

YOU TRY IT! #1 ANSWER:



The graphs of $f(n) = \frac{1}{2}(n^2 + n)$ and $y = 91$ shows an intersection point at (13, 91). So the 13^{th} triangular number is 91. There is another intersection point, where n is negative, but for triangular numbers, we only consider positive integers.

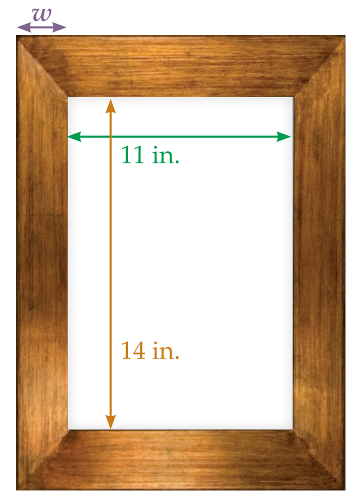
ADDITIONAL EXAMPLE

During a storm, a boat capsizes on Lake Texoma. The entire family makes it onto their life raft where they have several flares to signal for help. If the flares are fired from sea level, their path can be described by the function $h(t) = -16t^2 + 144t$, where t is the time in seconds since the flare was fired. The flare is visible from land when it is higher than 200 feet. Graph $h(t)$ and the function $y = 200$ to determine how long the flare will be visible from land.

The graphs of $h(t)$ and $y = 200$ intersect at (1.72, 200) and (7.28, 200). Each flare will be visible from land for 7.28 – 1.72 = 5.56 seconds.

EXAMPLE 2

The standard framing for an 11-inch by 14-inch picture with a frame w inches wide produces a total area $A(w)$. The total area, measured in square inches, varies according to the width of the frame. How wide should the frame be for a framed picture with a total area of about 400 square inches?



STEP 1 **Write a function for the total area, $A(w)$, in terms of w, the width of the frame.**

The width of the framed picture is $11 + 2w$. The length is $14 + 2w$. The total area is $A(w) = (11 + 2w)(14 + 2w) = 154 + 22w + 28w + 4w^2 = 4w^2 + 50w + 154$.

STEP 2 **Create a table of function values for the total area.**

WIDTH OF FRAME, w (IN.)	0	1	2	3	4	5	6
TOTAL AREA OF FRAMED PICTURE, $A(w)$ (IN.^{2})	154	208	270	340	418	504	598

STEP 3 **Write a related equation for the total area of 400 square inches.**

$400 = 4w^2 + 50w + 154$

STEP 4 **Use the table to determine the necessary width for this value of $A(w)$.**

The total area of 418 square inches is produced when the frame is 4 inches wide. To find a total area closer to 400 square inches evaluate the function for a narrower width, such as 3.75 inches.

$4(3.75)^2 + 50(3.75) + 154 = 397.75$ square inches

So the frame width of 3.75 inches will produce a total framed picture area of almost 400 square inches.

ADDITIONAL EXAMPLE

Ray is building a square pen for his piglets. He has 35 feet of fencing, and he wants the pen to have an area of at least 150 ft^2. If the sides of the pen can be represented by $2x - 4$, does Ray have enough fencing material to build a pen with 150 ft^2 or more? Use a table to determine your answer.

If one side of the pen is $2x - 4$, then the perimeter of the pen is represented by $P(x) = 8x - 16$. The area of the pen can be represented by $A(x) = (2x - 4)(2x - 4) = 4x^2 - 16x + 16$. The related equation is $150 = 4x^2 - 16x + 16$.

x	0	1	2	3	4	5	6	7	8	9
$A(x)$	16	4	0	4	16	36	64	100	144	196

An area of 150 ft^2 or more occurs at a negative x value, -4.12, which does not make sense for the problem and between 8 and 9, specifically 8.12 feet. When $x = 8.12$, the perimeter of the pen is about 49 feet. Ray does not have enough fencing to make a pen that large for his piglets.

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YOU TRY IT! #2

Deon has 26 meters of wire fencing to make a rectangular pen for his two show lambs. He has learned that each lamb should have 20 m^2 of space. Does he have enough fencing for an area of 40 m^2? Use a table to determine your answer.

See margin.


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EXAMPLE 3

Mrs. Samuels want to build a store with an area of 2,000 square feet on a lot measuring 60 feet by 70 feet. She finds out that the city code restricts building any closer than 10 feet from the lot lines. Can her store have an area of 2,000 square feet and stay within legal limits?

STEP 1 **Write a function to model the building area, $A(x)$, in terms of the distance, x, to the lot lines.**

The area of the lot is 60 times 70, or 4,200 square feet. The building can be no wider than $60 - 2x$ and no longer than $70 - 2x$. The area of the building should be $A(x) = (l)(w) = (70 - 2x)(60 - 2x)$.

STEP 2 **Simplify the function rule.**

$A(x) = (70 - 2x)(60 - 2x)$
$A(x) = 70(60 - 2x) - 2x(60 - 2x)$
$A(x) = 4200 - 140x - 120x + 4x^2$
$A(x) = 4x^2 - 260x + 4200$

STEP 3 **Write a related equation with Mrs. Samuels' building area of 2,000 square feet.**

$2000 = 4x^2 - 260x + 4200$

STEP 4 **Using inverse operations, make one member of the equation equal to zero.**

$2000 - 2000 = 4x^2 - 260x + 4200 - 2000$
$0 = 4x^2 - 260x + 2200$
$\frac{1}{4}(0) = \frac{1}{4}(4x^2 - 260x + 2200)$
$0 = x^2 - 65x + 550$

YOU TRY IT! #2 ANSWER:

Let w stand for the width of the pen. Since the perimeter of 26 is twice the length plus twice the width ($26 = 2l + 2w$), twice the length is 26 meters less twice the width, $26 - 2w$, so the length is half that amount. The area of the pen, then, is $A(w) = \frac{(26 - 2w)}{2} \cdot w = (13 - w)w = 13w - w^2$. The related equation is $13w - w^2 = 40$.

WIDTH OF PEN, w (METERS)	0	1	2	3	4	5	6	7	8	9	10	11	12	13
AREA OF PEN, $A(w)$ (SQUARE METERS)	0	12	22	30	36	40	42	42	40	36	30	22	12	0

An area of 40 m^2 occurs when the pen is 5 m wide and (13 – 5), or 8 m long, or when it is 8 m wide and (13 – 8) or 5 m long. Yes, he has enough fencing.

STEP 5 Since this is a quadratic polynomial equation equal to zero, you can use the quadratic formula to solve for x. In this case, $a = 1$, $b = -65$, and $c = 550$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-65) \pm \sqrt{(-65)^2 - 4(1)(550)}}{2(1)} = \frac{65 \pm \sqrt{4225 - 220}}{2} = \frac{65 \pm \sqrt{2025}}{2} = \frac{65 \pm 45}{2}$$

$$x = \frac{65 + 45}{2} = \frac{110}{2} = 55 \text{ or } x = \frac{65 - 45}{2} = \frac{20}{2} = 10$$

STEP 6 As this is a quadratic function, there are two solutions, $x = 55$ or $x = 10$. The distance, x, from the lot lines can be no less than 10 feet. The first solution does not make sense in this problem because a building cannot be 55 feet from all sides of the 60-feet by 70-feet lot.

However, the second solution of 10 generates a building with the dimensions shown.

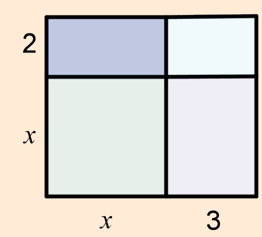
length = $70 - 2x$	width = $60 - 2x$
length = $70 - 2(10)$	width = $60 - 2(10)$
length = 50 feet	width = 40 feet

The building will have an area of 2,000 square feet and remain within the city code.

YOU TRY IT! #3

The diagram represents a large Mondrian-style painting. Different sizes of the painting and different colors can be ordered. Determine the value of x, in feet, that will make the area of the painting 56 square feet, and then determine the dimensions of the painting.

See margin.



YOU TRY IT! #3 ANSWER:

$A(x) = x^2 + 2x + 3x + 6$, or $A(x) = x^2 + 5x + 6$

The related equation is $x^2 + 5x + 6 = 56$ or $x^2 + 5x - 50 = 0$.

The factored form of this quadratic equation is $(x + 10)(x - 5) = 0$.

Using the zero product property, $x + 10 = 0$ or $x - 5 = 0$.

The first case generates $x = -10$, which is not a possible measurement.

The second case generates $x = 5$, which is a valid measurement.

The dimensions of the painting would be 5 + 2, or 7 feet, by 5 + 3, or 8 feet.

ADDITIONAL EXAMPLE

LaRae is completing a project for her geography class. One of the tasks includes creating a flag for a country. She makes her flag in the shape of a right triangle with legs represented by $x + 6$ and $x + 2$. Determine the value of x, in inches, that will make the area of the flag 30 in^2, and then determine the dimensions of the flag.

$A(x) = \frac{1}{2}(x^2 + 6x + 2x + 12)$, or

$A(x) = \frac{1}{2}(x^2 + 8x + 12)$

The related equation is $\frac{1}{2}(x^2 + 8x + 12) = 30$ or $x^2 + 8x - 48 = 0$.

The factored form of this quadratic equation is $(x + 12)(x - 4) = 0$.

Using the zero product property, $x + 12 = 0$ or $x - 4 = 0$.

The first case generates $x = -12$, which is not a possible measurement.

The second case generates $x = 4$, which is a valid measurement.

The dimensions of the flag would be 4 + 6, or 10 feet, by 4 + 2, 6 feet.

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PRACTICE/HOMEWORK

Use the scenario below to answer questions 1 – 5.

SPORTS

A football is punted up into the air. The football reaches a maximum altitude, and then comes back to the ground. The relationship between x, the number of seconds since the ball has been punted, and $f(x)$, the height of the ball in feet, is represented by the function $f(x) = -16x^2 + 40x + 4$.

1. Use a graphing calculator to complete the table below.

NUMBER OF SECONDS, x	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5
HEIGHT OF BALL, y	*4*	*13*	*20*	*25*	*28*	*29*	*28*	*25*	*20*	*13*	*4*

2. Based on the table values, when does the ball stop increasing in height and start decreasing in height?
At 1.25 seconds

3. What is the maximum height of the ball?
29 feet

4. When is the ball 20 feet above the ground?
0.5 seconds and 2 seconds

5. What do the points (0, 4) and (2.5, 4) represent in the scenario?
(0, 4); The ball was at a height of 4 feet from the ground when it was kicked.
(2.5, 4); After 2.5 seconds, the ball reached a height of 4 feet again.

Use the scenario below to answer questions 6 – 11.

ART AND ARCHITECTURE

A small town is building a park in the center of town. The city has a rectangular plot of land measuring 150 feet by 300 feet. The city plans on building a sidewalk x feet wide around the perimeter of the lot. The area bounded by the sidewalk, $f(x)$, is represented by the function $f(x) = (150 - 2x)(300 - 2x)$ or $f(x) = 4x^2 - 900x + 45{,}000$.

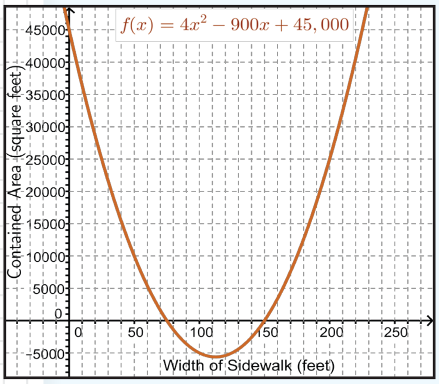
6. Using a graphing calculator, plot the graph of the function.
See margin.

7. The city wants to build a sidewalk wide enough to leave an area of land 36,400 square feet contained by the sidewalk. Write an equation that could be used to determine the width, x, of the sidewalk.
36,400 = 4x² – 450x + 45,000

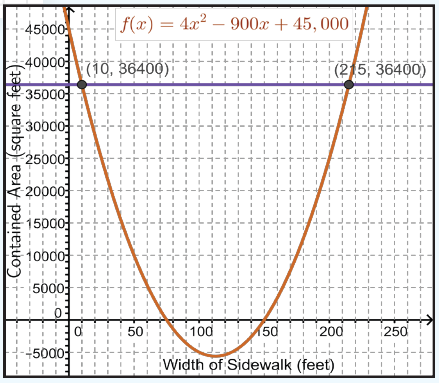
8. Plot the line $y = 36{,}400$ over the function graphed in problem 6.
See margin.

9. What are the coordinates of the points of intersection on the graph in problem 8?
(10, 36,400) and (215, 36,400)

6.



8.



10. What do these points represent in the scenario?
A sidewalk 10 feet wide or 215 feet wide result in an enclosed area of 36,400 square feet.

11. Do both points make sense in the scenario?
See margin.

Use the scenario below to answer questions 12 – 16.

SCIENCE

A model rocket is launched at ground level with an initial velocity of 80 feet per second. The function $f(x)$ represents the relationship between x, the number of seconds since the rocket was launched, and $f(x)$, the height of the rocket in feet. The function representing this relationship is $f(x) = -16x^2 + 80x$.

12. Write an equation that can be used to determine when the rocket will be at a height of 64 feet.
$64 = -16x^2 + 80x$

13. Solve the equation you wrote in problem 12.
$x = 1$ **and** $x = 4$

14. Write and solve an equation to determine the time(s) the rocket will be at a height of 84 feet.
$84 = -16x^2 + 80x$
$x = 1.5, 3.5$

15. Write and solve an equation to determine the time(s) the rocket will be at a height of 100 feet.
$100 = -16x^2 + 80x$
$x = 2.5$

16. How is the solution(s) to the equation in problem 15 different from the solution(s) to the equation in problem 14? Why is it different?
See margin.

Use the scenario below to answer questions 17 – 22.

CITY PLANNING

A large city conducted a traffic study at one of their busiest intersections. A machine is set up to record the number of vehicles that pass through the intersection at 15-minute intervals. The machine reports the number of vehicles at the end of each interval. The function $f(x)$ shown below represents the relationship between x, the number of 15-minute intervals, and $f(x)$, the number of cars that pass through the intersection.

$$f(x) = -12x^2 + 88x + 100$$

17. Write an equation that could be used to determine at the end of what interval did 260 cars pass through the intersection.
$260 = -12x^2 + 88x + 100$

18. Solve the equation in problem 17 algebraically.
$x = 4, 3\frac{1}{3}$

11. *No. The point (215, 36,400) is not reasonable for the scenario. The plot of land is 150 feet by 300 feet, so it is impossible to build a sidewalk on the property that is 215 feet wide.*

16. *The solution to the equation in problem 15 has only 1 solution, the equation in problem 14 has 2 solutions. The solution to the equation in problem 15 represents the time the rocket is at its maximum height.*

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19. Do both solutions make sense in the scenario? Why or why not?
See margin.

20. Write an equation that could be used to determine at what interval did 128 cars pass through the intersection.
$128 = -12x^2 + 88x + 100$

21. Solve the equation in problem 20 algebraically.
$x = 7, \frac{1}{3}$

22. Do both solutions make sense in the scenario? Why or why not?
See margin.

For questions 23 – 25, choose a method for solving the equation (graph, table, or algebraically), solve the equation, and record the solutions.

23. $6x^2 + 42x + 72 = 36$
$x = -6$ and $x = -1$

24. $0 = 2x^2 + 13x - 7$
$x = 0.5$ and $x = -7$

25. $86 = -16x^2 + 70x + 10$
$x = 2.375$ and $x = 2$

19. *Only the 4 makes sense in the scenario. The machine reports the number of cars at the end of an interval, not during an interval. Therefore, the only solutions that make sense are whole numbers.*

22. *Only the 7 makes sense in the scenario. The machine reports the number of cars at the end of an interval, not during an interval. Therefore, the only solutions that make sense are whole numbers.*