Estimating Function Values

7.1

FOCUSING QUESTION Which numbers would make reasonable input values for a given function with a given output value?

LEARNING OUTCOMES

- I can estimate a reasonable input value that would generate a given output value for a particular function.
- I can apply mathematics to solve problems arising in everyday life, society, and the workplace.

ENGAGE

Mission San Francisco de la Espada was founded in San Antonio, Texas, in 1731. Friars at the mission built an extensive irrigation network to grow crops. Part of the network, the Espada Acequia, survives today.

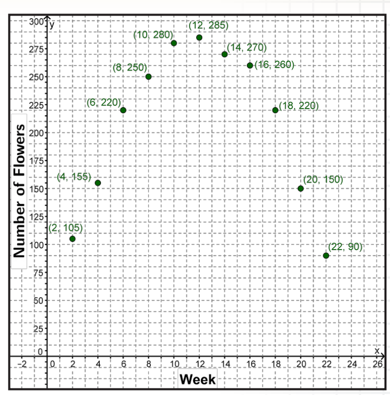
The amount of water, w, in gallons, required to grow b bushels of corn is related by the equation $w = 4{,}000b$. If the friars at Mission San Francisco de la Espada had access to 54,000 gallons of water during a growing season, how many bushels of corn could they grow? Explain how you determined your answer.

See margin.

EXPLORE

PART 1: USING GRAPHS

Marcus operates a nursery that produces flowers that he sells to local businesses and organizations. One small greenhouse contains carnations that Marcus supplies to a local Girl Scout troop for them to use with fundraisers. The graph shows the number of flowers grown and donated to the scout troop as a function of the week number since the growing season started in March.



Use the graph to answer the questions that follow.

1. For which week(s) did Marcus donate 220 flowers to the Girl Scout troop?
Weeks 6 and 18

TEKS

AR.6A Estimate a reasonable input value that results in a given output value for a given function, including quadratic, rational, and exponential functions.

MATHEMATICAL PROCESS SPOTLIGHT

AR.1F Create and use representations to organize, record, and communicate mathematical ideas.

ELPS

2I Demonstrate listening comprehension of increasingly complex spoken English by following directions, retelling or summarizing spoken messages, responding to questions and requests, collaborating with peers, and taking notes commensurate with content and grade-level needs.

VOCABULARY

function, input value, output value, estimate, compatible numbers

MATERIALS

- graphing technology

ENGAGE ANSWER:

13.5 bushels of corn

Possible explanation: Substitute 54,000 gallons for w and solve for b. Divide 54,000 by 4,000.

2. Using the patterns you see in the graph, for which week(s) could Marcus have donated 190 flowers to the Girl Scout troop?
Answers may vary. Possible answer: weeks 5 and 19

3. For which week(s) could Marcus have donated 120 flowers to the Girl Scout troop?
Answers may vary. Possible answer: weeks 3 and 21

4. **ELPS Connections:** Work with a partner to practice speaking and listening skills.
   - **Partner 1:** Explain to your partner, using increasingly complex spoken English, how you determined your answers to the previous questions from the information presented in the graph.
   - **Partner 2:** After your partner finishes their explanation, ask them one or more of the following questions:
     - ✔ What variable is represented on the vertical axis?
     - ✔ What variable is represented on the horizontal axis?
     - ✔ Which variable is the input variable? How do you know?
     - ✔ Which variable is the output variable? How do you know?
   - **Partner 1:** Listen carefully as your partner asks you one of the questions. Respond using appropriate mathematical vocabulary.
   - Reverse roles so that Partner 2 communicates their mathematical reasoning using the graph while Partner 1 listens. Partner 1 will then ask Partner 2 one or more of the provided questions while Partner 2 listens and responds.

PART 2: USING TABLES

Dametra invested $1000 into a high-yield annuity that earns 15% interest every year, compounded annually. The table shows the amount of money that Dametra has at the end of each year, beginning with her initial investment in 2010.

Use the table to answer the questions that follow.

YEAR	2010	2011	2012	2013	2014	2015
ACCOUNT BALANCE (DOLLARS)	1000	1150	1322.50	1520.88	1749	2011.36

5. Let x represent the year number and 2010 be year 0. Let y represent the account balance in dollars at the end of year x. Use successive ratios to determine a function rule for y in terms of x.
$y = 1000(1.15)^x$

6. Use your equation to extend the table of values, selecting either paper and pencil or technology. In what year will Dametra have about $2,660 in her investment account?
Year 7, which is 2017

7. Dametra has a savings goal of $5,000 with this investment. In what year will the balance in Dametra's investment account first exceed $5,000?
Year 12, which is 2022.

8. How did you use a table to answer questions 6 and 7? Explain your mathematical idea using precise mathematical language.
See margin.

INTEGRATING TECHNOLOGY

Use the stat editor to calculate the successive ratios. Enter the year into one list and the account balance into a second list. Copy the second list into a third list and delete the first entry ($1000 for year 0). Delete the last entry in the second list so that the second and third lists are the same length. In a fourth list, calculate the ratio of the third list to the second list.

Once the function rule has been determined, use graphing technology to generate a table of function values that can be used to answer the remaining questions. A common ratio between successive table values means that an exponential function of the form $f(x) = ab^x$ best represents the data. Enter the equation with the values for a and b into the equation editor. Then use the resulting table to answer the remaining questions.

8. *The table contains paired input (year) and output (account balance) values for the function. Once the table has been created, you can use it to identify the given output value and then locate its paired input value.*

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PART 3: USING EQUATIONS

According to Ohm's Law, within an electrical circuit, the voltage, V, is equal to the product of the current, I, and the resistance, R.

$$V = IR$$

This formula can be rearranged so that the current, I, can be found as a function of the resistance, R.

$$I = \frac{V}{R}$$

In the United States, electrical current from wall outlets has a voltage of 110 volts. Use this value to create a function, $I(R) = \frac{110}{R}$. In this function, 110 becomes a constant in a rational function.

9. Use mental math to estimate the value of R (measured in ohms) that is required to generate a current of 11 amperes.
***R* should be 10 ohms, since 110 ÷ 10 = 11.**

10. What amount of resistance is required to generate a current of 5 amperes?
22 ohms

11. An engineer is designing a circuit that requires a current of 2.2 amperes. Estimate the amount of resistance necessary to produce this current.
See margin.

12. How did you use the equation to determine the reasonable input values (amount of resistance) required to generate given output values (amount of current)?
See margin.

REFLECT

- **In a graph, how can you work backwards to estimate an input value when you are given an output value?**
  See margin.
- **In a table, how can you work backwards to estimate an input value when you are given an output value?**
  See margin.
- **In an equation, how can you work backwards to estimate an input value when you are given an output value?**
  See margin.

11. *Answers may vary. Possible answer:*

2.2 rounds to 2 and 110 ÷ 55 = 2, so the engineer needs about 55 ohms of resistance to generate a current of 2.2 amperes.

12. *Answers may vary. Possible answer:*

Look at the operations being used in the equation or symbolic representation of the function rule. Use techniques such as mental math and number sense to determine which input values, when combined with those operations, would generate an output value that is approximately equal to the given output value.

REFLECT ANSWERS:

Locate the output value along the y-axis. Read across from that value until you meet the graph. Follow a vertical line from the graph up or down to the x-axis. Estimate the value along the x-axis at this vertical line.

Locate the output value in the column or row representing the dependent variable. If the output value does not appear in the column or row, use number patterns to identify two output values, one that is less than the given output value and one that is greater. The given output value will lie between these two numbers. Read the table to determine the input values that correspond with the two output values in the table. Estimate the input value for your given output value to be between these two values in the table.

Identify the operations that are being done to the independent variable in the equation. Begin with the given output value and perform the inverse operations, in reverse order, using rounded values from the coefficients and constants.

EXPLAIN

A function is a relationship between a set of input values and a set of output values. Each input value generates only one output value. If you are given a function and a particular output value, you can work backwards to estimate the input value that generated that output value.

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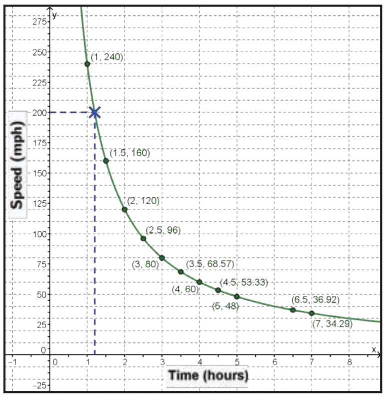
ESTIMATING INPUT VALUES FROM GRAPHS

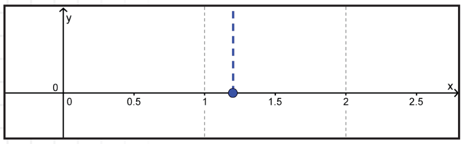
Houston and Dallas are about 240 miles apart. The travel speed required to get from one city to the other is a function of the amount of time you have to get there. This relationship is shown in the graph.



A bullet train is being considered that would travel an average of 200 miles per hour between the cities. What input value (time) would generate this output value (speed)?

In a graph, locate the output value (in this case, 200 miles per hour) along the vertical, or y-axis. Read across until the line $y = 200$ meets the graph. Once you reach the graph, read vertically down until you reach the x-axis.

The vertical line from the point on the graph with a y-value of 200 meets the x-axis between 1 and 1.5. Let's take a closer look at this intersection.



Notice that the x-axis appears like a number line. You can use estimation skills from a number line to estimate the x-value of this intersection point. The point is between 1 and 1.5 and is closer to 1 than 1.5. Halfway between 1 and 1.5 is 1.25, and the point is slightly less than halfway between 1 and 1.5. You can estimate the x-value to be about 1.2.

From the graph, it appears that an output value (speed) of 200 miles per hour would be generated by an input value (time) of about 1.2 hours.

ESTIMATING INPUT VALUES FROM TABLES

Continuing with the Houston and Dallas example, you can make a table of values. For given amounts of travel time, in hours, the average speed, in miles per hour, can be calculated.

TIME	1	2	3	4	5	6	7	8
SPEED	240	120	80	60	48	40	34.39	30

Kendra is driving from Houston to visit family in Dallas. She knows her average speed will be 70 miles per hour. About how long will it take Kendra to drive from Houston to Dallas?

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In the table, look for 70 miles per hour in the row for the output variable, speed. This table of values does not have a speed of 70, but a speed of 70 would fall between 80 miles per hour and 60 miles per hour.

TIME	1	2	3	4	5	6	7	8
SPEED	240	120	80	60	48	40	34.39	30

Notice that 70 is halfway between 80 and 60, so one estimate for the input value (time) that would generate an output value (speed) of 70 would be halfway between 3 hours and 4 hours, or 3.5 hours.

To get a more precise estimate, let's look at the number patterns in the table. Time increases by 1 hour for each pair of values, but the change in speed is not constant.

TIME	1	2	3	4	5	6	7	8
SPEED	240	120	80	60	48	40	34.39	30

-120 -40 -20 -12 -8 -5.61 -4.39

As time increases, the amount of decrease in speed decreases. So even though 70 miles per hour is halfway between 80 and 60, the time required for an average driving speed of 70 miles per hour will be less than halfway between 3 hours and 4 hours. Kendra can estimate an input value (time) of about 3.4 to 3.5 hours for an output value (speed) of 70 miles per hour ($240 \div 70 \approx 3.429$).

ESTIMATING INPUT VALUES FROM EQUATIONS

Distance, rate, and time are related by the equation $d = rt$. You can use this equation to write a function rule for the relationship between the amount of time and the average speed for travel between Houston and Dallas, a distance of 240 miles.

$$d = rt$$
$$240 = rt$$
$$r(t) = \frac{240}{t}$$

An airplane travels at an average speed of about 500 miles per hour. With this output value, what is the amount of time (input value) that the airplane would take to complete the trip?

Since you have an equation, substitute 500 for $r(t)$.

$$500 = \frac{240}{t}$$

To get an estimated input value, t, use compatible numbers. 240 is approximately equal to 250. If the input value is 1 hour, then the average speed would be $\frac{250}{1} = 250$ miles per hour. The airplane travels twice that speed. If you divide 250 by 0.5, then you get the average speed of 500. Thus, an output value of 500 miles per hour would be generated by an estimated input value of 0.5 hours.

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ESTIMATING INPUT VALUES

A function relates a set of input values to a set of output values. If you are given a function and an output value, you can work backwards to estimate an input value that generates that output value. Note that, depending on the type of function, there may be more than one input value for that output value.

- In a graph, locate a point that has a *y*-coordinate equal to the output value. The *x*-coordinate of this point is the input value paired with that output value.
- In a table, locate the output value in the dependent variable column or row. If the output value is not present in the table, locate dependent variable values that are close to the given output value. Use the input values that correspond to the nearby output values to estimate the input value for the given output value.
- In an equation, begin with the given output value. Work backwards using inverse operations and estimation strategies such as rounding and compatible numbers to estimate an input value for the given output value.

EXAMPLE 1

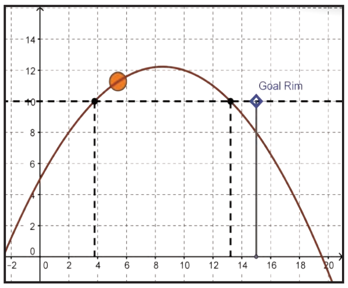
The arc of an attempted basketball free throw followed a path of the graph of a quadratic function. The equation for this particular graph is $h(x) = -0.1x^2 + 1.7x + 5$, where x is the distance from the free throw line and $h(x)$ is the height of the basketball from the ground. The basketball hoop is 15 feet from the free throw line and the height of the rim is 10 ft. above the ground. Is it possible that this free throw attempt scored a point?

Image source: openclipart.org

STEP 1 **Determine the desired value of $h(x)$ in this problem.**

$h(x)$ needs to be 10 feet, the height of the rim of the basketball hoop.

STEP 2 **Use a graph to estimate the distance from the free throw line when the ball was at a height of 10 feet.**



According to the graph, the ball was at a height of 10 feet when it was slightly less than 4 feet from the free throw line and slightly greater than 13 feet from the free throw line.

ADDITIONAL EXAMPLE

In the 7th inning in a Texas Rangers baseball game, the star hitter pops a foul ball. His bat strikes the ball 2.5 feet from the ground. In order to be called out, a player from the other team must catch the ball before it hits the ground. How long does the team have to get under the ball to catch it? The chart shows the height of the ball, in feet, over time, in seconds. Use the chart to determine how long the opponent has to catch the baseball.

TIME	0	3	6	9	12
HEIGHT	2.5	338.5	386.5	146.5	-381.5

The baseball will drop to the ground between 9 and 12 seconds. Looking at the difference in the height between 9 and 12 seconds, it would appear the ball would hit the ground close to 9 seconds than 12. A valid estimate would be that the opponent would have about 10 seconds to get under the baseball in time to catch it to call the player out.

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STEP 3 **Compare the distances from the free throw line at a height of 10 feet to the distance of the goal rim from the free throw line.**

The goal rim is 15 feet from the free throw line and the ball would have to be at a height of 10 feet for it to score a point. The ball reached a height of 10 feet when it was slightly less than 4 feet or slightly greater than 13 feet. The ball was not at a height of 10 feet when it was 15 feet from the free throw line.

The basketball is only 10 feet in height when it is about 3.8 feet or about 13 feet from the free throw line, suggesting that the free throw attempt did not score a point, since the ball would need to be at a height of 10 feet when it is 15 feet from the free throw line.

YOU TRY IT! #1

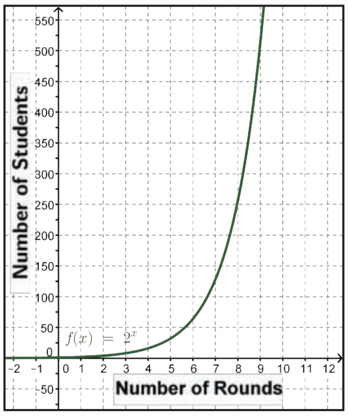
The Student Council is planning to make a helium balloon arch, shaped like a parabola, for the entrance into the gym for an upcoming dance. The students will anchor the ends of the arch so that the height near the top of the arch is approximately two feet from the ceiling of the gym. If the height of the gym's ceiling is 30 feet, how far apart must the ends of the arch be placed? The chart shows the arch's height above the floor at a distance along the floor from where the first end is anchored. Use the chart to determine the distance the second end should be anchored from the first end.

DISTANCE	0	5	10	15	20	25	30	35
HEIGHT	0	15.6	25	28.3	25.4	16.3	1.0	-20.2

See margin.

EXAMPLE 2

A calling chart is used to alert students when an event like a game or concert is canceled due to bad weather or other reasons. To begin the process, the school principal calls the coach or director. On the first round, the coach or director calls two students. On the second round, each of those students calls two other students on the list and so on until all the students are contacted. The graph shows a function, $f(x) = 2^x$, which can be used to represent the number of calls made on each round. Assuming that there are 500 students participating, by which round will they all have been called?



YOU TRY IT! #1 ANSWER:

The tallest point in the arch is approximately 28.3 feet above the floor. At this point, the arch is 15 feet from the first end, measured along the floor.

The first end has a height of zero feet. The other pair of distance and height where the height is also zero feet occurs a little more than 30 feet from the first end. So the second end should be anchored about 30 feet away from the first end. That will place the top of the arch in the middle of that floor distance, at about 15 feet from either end.

STEP 1 **Determine how many students have been called by each round.**

Looking at the graph, by the 3rd round, it looks like there have been just a few of the students called. By the 4th round, it appears that about 20 more have been called. On the 5th round, 30 or so more have been called. So after the 5th round, that would be a total of around 50 to 55 students.

STEP 2 **Continue the process of estimating and adding students for the next few rounds until the total is close to 500. Use number sense to estimate the total number of students that have been called after each round.**

- 6th round: Approximately 60 more students are called, plus the 50 to 55 students previously called, for a total of about 110 students called.
- 7th round: Approximately 125 more students are called, plus the 110 students previously called, for a total of about 235 students called.
- 8th round: Approximately 250 more students are called, plus the 235 students previously called, for a total of about 485 students called.

After the 8th round of calls, you can estimate that most, if not all, of the 500 students would have been called.

STEP 3 **Verify the conclusion using a different strategy.**

To verify this estimate, you could use the function $f(x) = 2^x$ to create a table of values.

The table also shows that after the 8th round, all of the 500 students will have been called.

ROUND	CALLS MADE ON THAT ROUND	TOTAL STUDENTS CALLED
0	1 (COACH OR DIRECTOR)	0
1	2	2
2	4	6
3	8	14
4	16	30
5	32	62
6	64	126
7	128	254
8	256	510

ADDITIONAL EXAMPLE

Suzanne opened a clothing store in 2000. In her first year, she had a $10,000 profit. Every year after that, her profits increased 8%. Suppose the function rule $p(x) = 10000(1.08)^x$, where x is the number of years after Suzanne's initial year of business, can be used to determine her store's profit. Use the function values shown in the table to find out what year her business would make $20,000 in profits.

YEARS	0	2	4	6	8	10	12
PROFIT	10,000	11,664	13,604	15,868	18,509	21,589	25,182

Between 8 and 10 years after Suzanne opened her store, the profit was $20,000. A good estimate would be just under 9 years for Suzanne's store to profit $20,000. That would be around 2009.

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YOU TRY IT! #2

The loss of crispness of French fries happens rather quickly with the passage of time. Suppose the function rule $f(x) = 100(0.90)^x$, where x is the number of minutes since the French fries were taken out of the fryer, can be used to determine percent of crispness. Use the function values shown in the table to find out how many minutes it would take the French fries to be at 50% of their original 100% crispness.

MINUTES	0	1	2	3	4	5	6	7	8
PERCENT OF CRISPNESS	100	90	81	72.9	65.61	59.05	53.14	47.83	43.05

See margin.

YOU TRY IT! #2 ANSWER:

Fifty percent of crispness happens between 6 and 7 minutes. Because this is an exponential decay function and not a linear function, the decrease does not occur at a constant rate, so the time will be closer to 7 minutes. Using the table feature on a graphing calculator can give you a more exact answer.

EXAMPLE 3

An output value of 7 is given for the function $f(x) = \frac{45}{4x-3} + 2$. Estimate the input value for this output value. Solve the equation algebraically for the value of x.

STEP 1 Write an equation for this function and output value.

$7 = \frac{45}{4x-3} + 2$

STEP 2 Use the opposite operations to determine a possible value of x for the value of y.

$7 - 2 = 5$, so $\frac{45}{4x-3} = 5$.

$45 \div 9 = 5$, so $4x - 3 = 9$.

$12 - 3 = 9$, so $4x = 12$ and $x = 3$.

STEP 3 Show the algebraic operations needed to solve the equation.

$$7 = \frac{45}{4x-3} + 2$$
$$7 - 2 = \frac{45}{4x-3} + 2 - 2$$
$$5 = \frac{45}{4x-3}$$
$$5(4x-3) = \frac{45}{4x-3}(4x-3)$$
$$20x - 15 = 45$$
$$20x - 15 + 15 = 45 + 15$$
$$20x = 60$$
$$\frac{20x}{20} = \frac{60}{30}$$
$$x = 3$$

ADDITIONAL EXAMPLE

Find the input value of x when the output value of the function $h(x) = \frac{46}{-5x+7} - 9$ is –11. Write an equation with the output value. Estimate the input value for this output value. Solve the equation algebraically for the value of x.

The equation is -11 = $\frac{46}{-5x+7} - 9$. To estimate, $\frac{46}{-5x+7}$ is –2, then -5x + 7 is -23, and -5x is -30. So x is 6. Solving the equation gives the same value for x.

STEP 4 Verify that the value of x is 3 when $f(x)$ is 7.

$$\text{Is } \frac{45}{4(3)-3} + 2 = 7\text{? Yes, } \frac{45}{12-3} + 2 = \frac{45}{9} + 2 = 5 + 2 = 7$$

YOU TRY IT! #3

Find the input value of x when the output value of the function $g(x) = \frac{14}{-2x-4} + 3$ is 10. Write an equation with the output value. Estimate the input value for this output value. Solve the equation algebraically for the value of x.

See margin.

YOU TRY IT! #3 ANSWER:

The equation is $10 = \frac{14}{-2x-4} + 3$. *To estimate,* $\frac{14}{-2x-4}$ *is 7, then* $-2x - 4$ *is 2, and* $-2x$ *is 6. So* x *is -3. Solving the equation gives the same value for* x.

PRACTICE/HOMEWORK

Use the scenario below to answer questions 1 – 5.

SPORTS

Dennis is standing on the football field and shot an arrow upward with an initial velocity of 48 feet per second. The height of the arrow $h(t)$, in terms of time since the arrow was released is $h(t) = 48t - 16t^2$.

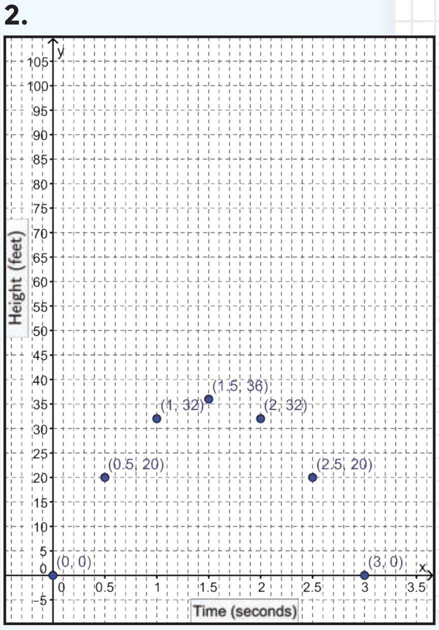
1. Complete the table to determine the height of the arrow at different times.

TIME, t	HEIGHT, $h(t)$
0	***0***
0.5	***20***
1	***32***
1.5	***36***
2	***32***
2.5	***20***
3	***0***

2. Sketch a graph to represent the height of the arrow at different times.

See margin.

2.



3. What is the maximum height of the arrow? How many seconds does it take for the arrow to reach its maximum height?

36 feet and it reaches this height at 1.5 seconds.

4. What is the height of the arrow at 1 second?

32 feet

5. When will the arrow be at a height of 10 feet?

Between 0 and 0.5 seconds and between 2.5 and 3 seconds

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Use the scenario below to answer questions 6 – 8.

SPORTS

Dennis decides to go up in the stands of the stadium and shoot the arrow. If he is standing in the stands 64 feet above the ground and shoots the arrow upward, the height of the arrow $h(t)$, in terms of time since the arrow was released is $h(t) = 64 + 48t - 16t^2$.

6. Complete the table to determine the height of the arrow at different times now that Dennis is standing in the stands of the stadium.

TIME, t	HEIGHT, $h(t)$
0	***64***
0.5	***84***
1	***96***
1.5	***100***
2	***96***
2.5	***84***
3	***64***
3.5	***36***
4	***0***

7. Sketch a graph to represent the height of the arrow at different times.
See margin.

8. What do you notice about the heights and times in the table and graph in the first scenario (problems 1 and 2) and the table and graph in the second scenario (problems 6 and 7)?
See margin.

Use the scenario below to answer questions 9 – 12.

GEOMETRY

The height of a triangle is 4 centimeters longer than the length of the base, x. The area of the triangle in terms of the length of the base is $h(x) = \frac{x^2 + 4x}{2}$.

9. Complete the table to determine the area of the triangle for different lengths of the base.

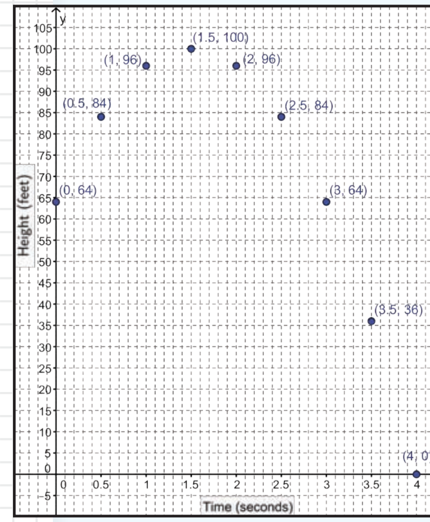
LENGTH OF THE BASE, x	AREA OF THE TRIANGLE, $h(x)$
0	***0***
0.5	***1.125***
1	***2.5***
1.5	***4.125***
2	***6***
2.5	***8.125***
3	***10.5***

10. Sketch a graph to represent the area of the triangle for different lengths of the base.
See margin.

11. What is the area of the triangle when the base is 7 cm long?
38.5 cm²

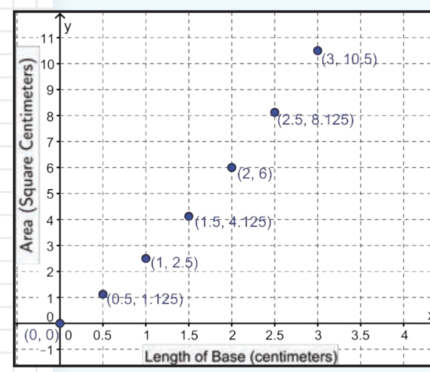
12. What is the length of the base if the area of the triangle is 82.5 cm²?
11 cm

7.



8. *Answers will vary. The maximum height is 64 feet higher than the original scenario. The arrow is in the air one second longer.*

10.



Use the scenario below to answer questions 13 – 16.

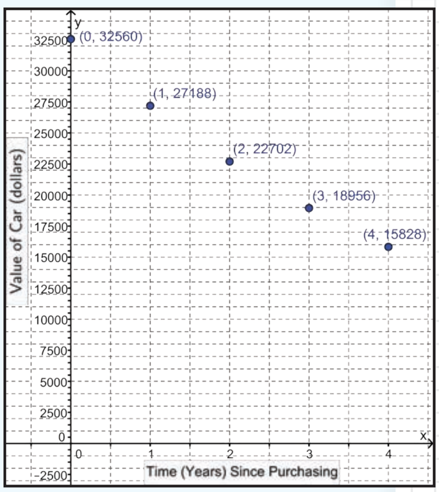
FINANCE

Kent bought a car for $32,560. He knows the car will decrease in value over time. The function rule $f(x) = 32{,}560(0.835)^x$, where x is the number of years Kent owned the car, can be used to determine the value of the car in any given year.

13. Complete the table to determine the value of the car, rounded to the nearest dollar, in different years.

TIME (YEARS) SINCE THE CAR WAS PURCHASED, x	YEAR THE CAR WAS PURCHASED	VALUE OF THE CAR, $f(x)$
0	2009	***$32,560***
1	2010	***$27,188***
2	2011	***$22,702***
3	2012	***$18,956***
4	2013	***$15,828***

14.



14. Sketch a graph to represent the value of the car over time.
See margin.

15. In what year will the car be worth $6000?
Approximately 10 years or 2019

16. How much will the car be worth in 2016?
Approximately $9215

Use the function below to answer questions 17 – 20.

A function $f(x) = \frac{24}{2x-1} + 3$ is used to determine output values when certain input values are given.

17. What is the output value for an input value of 0?
−21

18. What is the output value for an input value of 3?
7.8

19. What is the input value for an output value of 4.6?
8

20. What is the input value for an output value of 1.4?
−7

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