Multiplying Matrices

6.4

FOCUSING QUESTION How do I multiply two matrices together?

LEARNING OUTCOMES

- I can multiply one matrix by another matrix.
- I can communicate mathematical reasoning about matrix multiplication using symbols, diagrams, and language.

ENGAGE

The table below represents the type and number of homes that were for sale in two neighborhoods in a recent month.

	HOUSES	CONDOMINIUMS	TOWNHOMES
OAK GROVE	25	20	30
FERN MEADOWS	20	15	35

If 20% of each type of home sells in both neighborhoods and no additional homes are placed for sale, how many homes will be for sale the next month?

See margin.

EXPLORE

The tables below show the crop yields and average wholesale prices for different crops in Texas and Georgia during 2014.

	CORN (NUMBER OF BUSHELS)	COTTON (NUMBER OF BALES)	SOYBEANS (NUMBER OF BUSHELS)
TEXAS	294,520,000	6,203,000	5,198,000
GEORGIA	52,700,000	2,570,000	11,600,000

CORN	$4 PER BUSHEL
COTTON	$325.50 PER BALE
SOYBEANS	$10.50 PER BUSHEL

Source: U.S. Department of Agriculture

TEKS

AR.5B Multiply matrices.

MATHEMATICAL PROCESS SPOTLIGHT

AR.1D Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

ELPS

1F Use accessible language and learn new and essential language in the process.

VOCABULARY

matrix, commutative property, left-multiplication, right-multiplication

MATERIALS

- graphing technology

ENGAGE ANSWER:

	HOUSES	CONDOMINIUMS	TOWNHOMES
OAK GROVE	*20*	*16*	*24*
FERN MEADOWS	*16*	*12*	*28*

1. Create matrix Y to show the crop yields for both Texas and Georgia and matrix P to show the price for each crop. Identify the dimensions of each matrix.
See margin.

2. Determine the amount of revenue generated for each crop (corn, cotton, and soybeans) for each state (Texas and Georgia). Explain your reasoning using mathematically-appropriate language.
See margin.

3. Determine the total amount of revenue generated from the sale of these crops for both Texas and Georgia. Explain your reasoning using mathematically-appropriate language.
See margin.

4. Create matrix T to show the total revenue generated from the sale of all crops for both states. Identify the dimensions of this matrix.
See margin.

5. Compare the number of rows in the total matrix, T, to the number of rows in the crop yield matrix, Y. What do you notice?
Matrix T has the same number of rows as matrix Y.

6. Compare the number of columns in the total matrix, T, to the number of columns in the price matrix, P. What do you notice?
Matrix T has the same number of columns as matrix P.

7. Use graphing technology such as a graphing calculator or app to enter the data from matrix Y and matrix P. Remember that matrices are defined by the number of rows by the number of columns.
See margin.

8. Use matrix operations to multiply matrix Y by matrix P, $[Y] \times [P]$. (*Note: If you used other matrix names in your graphing technology such as matrix A and matrix B, you will use those matrix names in the calculation.*) How does this matrix compare to matrix T, the one you calculated in a previous question?
See margin.

9. Test to see if matrix multiplication is commutative by reversing the order of the two factor matrices. In other words, use your graphing technology to multiply matrix P by matrix Y, or $[P] \times [Y]$. What do you notice?
The graphing technology generated an error such as "Error: Dimension Mismatch."

1. *Matrix Y*

$$\begin{bmatrix} 294{,}520{,}000 & 6{,}203{,}000 & 5{,}198{,}000 \\ 52{,}700{,}000 & 2{,}570{,}000 & 11{,}600{,}000 \end{bmatrix}$$

Matrix P

$$\begin{bmatrix} 4 \\ 325.50 \\ 10.50 \end{bmatrix}$$

Matrix Y is a 2 × 3 matrix with 2 rows and 3 columns. Matrix P is a 3 × 1 matrix with 3 rows and 1 column.

2. *Texas:*
Corn: $1,178,080,000
Cotton: $2,019,076,500
Soybeans: $54,579,000

Georgia:
Corn: $210,800,000
Cotton: $836,535,000
Soybeans: $121,800,000

Possible explanation: Multiply the crop yield from matrix Y by the price from matrix P for each crop. Repeat this for both Texas and Georgia.

3. *Texas: $3,251,735,500; Georgia: $1,169,135,000*

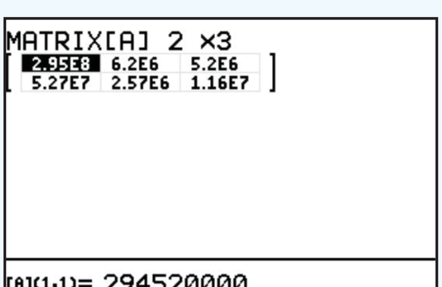
Possible explanation: For each state, multiply the crop yield by the wholesale price for corn, cotton, and soybeans. Add the revenue for each crop together.

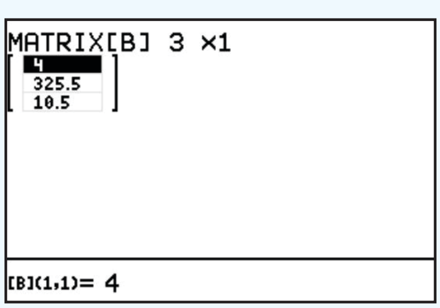
4. *Matrix T*

$$\begin{bmatrix} 3{,}251{,}735{,}500 \\ 1{,}169{,}135{,}000 \end{bmatrix}$$

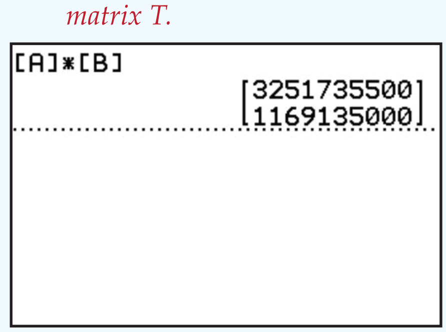
Matrix T is a 2 × 1 matrix with 2 rows and 1 column.

7.





8. *The product of [Y] × [P] is the same as matrix T.*



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REFLECT

- **When you multiply a matrix by another matrix, how do you multiply the entries in each matrix in order to determine the number to place in the product matrix?**
  See margin.
- **Is matrix multiplication commutative? Why or why not?**
  See margin.

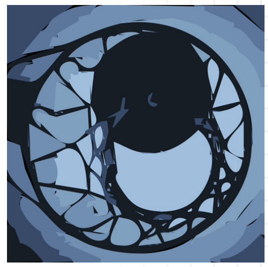
EXPLAIN

Watch Explain and You Try It Videos

or click here

Previously, you multiplied a matrix by a scalar quantity. In doing so, you multiplied the scalar by each entry in the matrix. The product matrix has the same dimensions as the original factor matrix.

When you multiply a matrix by another matrix, you have to take pairs of entries from each factor matrix into consideration as you generate the entries for the product matrix.



Source: openclipart.org

For example, there are three ways to score points in basketball: 1 point from a free throw, 2 points from a field goal, and 3 points from a long-range basket outside of a given curve. If you know how many of each that a team scored in a game, you can calculate the total points for that team. The table below shows the number of each type of score by two teams in a recent game.

	FREE THROWS	FIELD GOALS	3-POINTERS
SPURS	18	42	7
MAVERICKS	23	37	10

FREE THROWS	1 POINT
FIELD GOALS	2 POINTS
3-POINTERS	3 POINTS

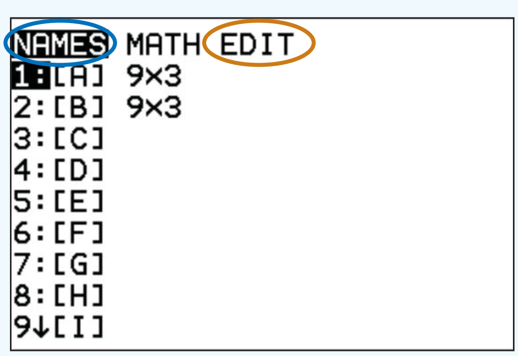
From these tables, you can write two matrices.

Matrix *S* $\begin{bmatrix} 18 & 42 & 7 \\ 23 & 37 & 10 \end{bmatrix}$ **Matrix *P*** $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

INTEGRATING TECHNOLOGY

Use graphing calculators to multiply two matrices together. Depending on the technology, matrices can be created through the **edit** menu and can be used on the home screen for matrix operations using the **names** menu. Matrix operations can be performed on the calculator's home screen using the matrix names. To multiply [*A*] by [*B*], use the Matrix-Names list to call up [*A*], the multiplication symbol, then the Matrix-Names list to call up [*B*].

[*A*] × [*B*]



REFLECT ANSWERS:

Multiply the entries from a row of the first factor matrix by the entries from a column in the second factor matrix. Add these products together, and place the sum in the product matrix based on the row number of the first factor matrix and the column number of the second factor matrix.

No, matrix multiplication is not commutative, because the number of entries in each row of the first factor matrix must match the number of entries in each column of the second factor matrix. If each matrix does not have the same number of columns and rows (that is, each matrix is not a square matrix), then the lengths of each row and column will not match up, and you will not be able to multiply each pair of entries to generate the sum of a set of products.

SUPPORTING ENGLISH LANGUAGE LEARNERS

Encourage students to produce written responses to the reflect questions in their interactive math notebooks or math journals. As they do so, make sure that students use accessible language such as terms with which they are already familiar to help them learn new and essential language about matrices in the process (**ELPS 1F**). After producing a written learning artifact, pair students up, and have them take turns explaining their responses to the reflect questions to each other. As students listen, have them pay special attention to new language structures that they hear during classroom instruction and interactions (**ELPS 2C**). As students speak, have them express opinions by communicating single words and short phrases through participation in extended discussions on grade-appropriate academic topics (**ELPS 3G**).

You can use these matrices to calculate the total number of points scored by each team in matrix T. Multiply matrix S by matrix P.

$$S \times P = T$$

$$\begin{bmatrix} 18 & 42 & 7 \\ 23 & 37 & 10 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 18 \times 1 & + & 42 \times 2 & + & 7 \times 3 \\ 23 \times 1 & + & 37 \times 2 & + & 10 \times 3 \end{bmatrix} = \begin{bmatrix} 123 \\ 127 \end{bmatrix}$$

Notice that the product matrix T has the same number of rows as the first factor matrix and the same number of columns as the second factor matrix.

$$S_{2 \times 3} \cdot P_{3 \times 1} = SP_{2 \times 1}$$

equal

In general, if you are multiplying matrix A with m rows and n columns by matrix B with n rows and p columns, the product matrix C will have m rows and p columns.

$$A_{m \times n} \cdot B_{n \times p} = C_{m \times p}$$

equal

You can also generalize multiplication of matrices. For multiplication of a 2×3 matrix by a 3×1 matrix, you can use variables for each entry.

$$\begin{bmatrix} a & b & c \\ d & f & g \end{bmatrix} \begin{bmatrix} h \\ j \\ k \end{bmatrix} = \begin{bmatrix} ah & + & bj & + & ck \\ dh & + & fj & + & gk \end{bmatrix}$$

For a $m \times n$ matrix multiplied by a $n \times p$ matrix, you can use specific paired matrix entries to generalize the product $m \times p$ matrix.

$$\begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m,1} & a_{m,2} & \cdots & a_{m,n} \end{bmatrix} \begin{bmatrix} b_{1,1} & b_{1,2} & \cdots & b_{1,p} \\ b_{2,1} & b_{2,2} & \cdots & b_{2,p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n,1} & b_{n,2} & \cdots & b_{n,p} \end{bmatrix} = \begin{bmatrix} a_{1,1}b_{1,1} + a_{1,2}b_{2,1} + \cdots + a_{1,n}b_{n,1} & \cdots & a_{1,1}b_{1,p} + a_{1,2}b_{2,p} + \cdots + a_{1,n}b_{n,p} \\ \vdots & \ddots & \vdots \\ a_{m,1}b_{1,1} + a_{m,2}b_{2,1} + \cdots + a_{m,n}b_{n,1} & \cdots & a_{m,1}b_{1,p} + a_{m,2}b_{2,p} + \cdots + a_{m,n}b_{n,p} \end{bmatrix}$$

COMMUTATIVE PROPERTY WITH MATRICES

When you multiply real numbers, the order in which you multiply does not matter; $5.5 \times 7 = 7 \times 5.5$. However, with matrices, the order of multiplication does matter. If you multiply matrix A with 2 rows and 3 columns by matrix B with 3 rows and 1 column, then you have a product matrix with 2 rows and 1 column.

$$\begin{bmatrix} a & b & c \\ d & f & g \end{bmatrix} \begin{bmatrix} h \\ j \\ k \end{bmatrix} = \begin{bmatrix} ah & + & bj & + & ck \\ dh & + & fj & + & gk \end{bmatrix}$$

Multiplying matrix A by matrix B places matrix A on the left of matrix B and is called **left multiplication**. If you were to multiply matrix A on the right of matrix B, then it would be called **right multiplication**.

$$\begin{bmatrix} h \\ j \\ k \end{bmatrix} \begin{bmatrix} a & b & c \\ d & f & g \end{bmatrix} = ?$$

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In this case the dimensions of the matrices do not match up and the multiplication is not possible. If you began with Row 1 and multiplied by Column 1, you can multiply the first pair of entries to get *ha* but there is no second entry in the first row of matrix *B* to pair with *d* in Column 1 of matrix *A*.

$$B_{3\times 1} \cdot A_{2\times 3} = ?$$

not equal

MULTIPLICATION OF MATRICES

Two matrices can be multiplied together only if the number of columns in the first factor matrix is equal to the number of rows in the second factor matrix.

- Begin with Row 1 in the first matrix and Column 1 of the second matrix. Multiply each pair of entries, $a_{1,1} \times b_{1,1}$, $a_{1,2} \times b_{2,1}$, $a_{1,3} \times b_{3,1}$, and so on. Record the sum of these products in Row 1, Column 1 of the product matrix.
- Repeat the process for each pair of rows and columns.
- Multiplication of matrices is not commutative.
  - Left-multiplication means to take a matrix and multiply it by another matrix from the left.
  - Right-multiplication means to take a matrix and multiply it by another matrix from the right.

EXAMPLE 1

Determine whether or not the two matrices shown below can be multiplied. Justify your answer.

$$A = \begin{bmatrix} -1 & 0 \\ 3 & -4 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & -3 \\ -8 & 1 \\ 4 & 5 \end{bmatrix}$$

STEP 1 **Determine the size of the matrices.**

Matrix *A* has two rows and two columns. $A_{2\times 2}$
Matrix *B* has three rows and two columns. $B_{3\times 2}$

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STEP 2 **Use the size of the matrices to determine whether it is possible to multiply matrix *A* by matrix *B*.**

If it is possible to multiply *AB*, then the number of columns in matrix *A* must be equal to the number of rows in matrix *B*.

$$A_{2\times 2} \cdot B_{3\times 2} = ?$$

not equal

STEP 3 **Use the size of the matrices to determine whether it is possible to multiply matrix *B* by matrix *A*.**

If it is possible to multiply *BA*, then the number of columns in matrix *B* must be equal to the number of rows in matrix *A*.

$$B_{3\times 2} \cdot A_{2\times 2} = ?$$

equal

The sizes of the matrices make it impossible to multiply AB because the number of columns in matrix A is not equal to the number of rows in matrix B. The sizes of the matrices do make it possible to multiply BA since matrix B has the same number of columns as matrix A has rows.

YOU TRY IT! #1 ANSWER:

Both matrices C and D have one row and two columns. The sizes of the matrices make it impossible to multiply CD because the number of columns in matrix C is not equal to the number of rows in matrix D. The sizes of the matrices also make it impossible to multiply DC since because the number of columns in matrix D is not equal to the number of rows in matrix C. Although it would be possible to add or subtract these two matrices, it is not possible to multiply them.

YOU TRY IT! #1

Determine whether or not the two matrices shown below can be multiplied. Justify your answer.

$$C = \begin{bmatrix} 6 & -7 \end{bmatrix} \qquad D = \begin{bmatrix} 2 & 5 \end{bmatrix}$$

See margin.

ADDITIONAL EXAMPLES

Determine whether or not the two matrices shown below can be multiplied. Justify your answer.

1. $E = \begin{bmatrix} 7 & 6 \\ -4 & 0 \end{bmatrix}$ $F = \begin{bmatrix} -3 & 0 \\ 14 & 2 \end{bmatrix}$

Both matrices E and F have two rows and two columns. It is possible to multiply both EF and FE because the number of columns in E is equal to the number of rows in F and vice versa. The product would be a 2 X 2 matrix.

2. $G = \begin{bmatrix} 12 & 5 & -3 \\ 7 & 1 & 6 \\ -4 & 8 & 0 \end{bmatrix}$ $H = \begin{bmatrix} -3 & 0 & 5 \\ 14 & 2 & -7 \end{bmatrix}$

Matrix G has three rows and three columns. Matrix H has two rows and three columns. Because the number of columns in G is not equal to the number of rows in H, it is impossible to multiply GH. However, the number of columns in H is equal to the number of rows in G, so it is possible to multiply HG. The resulting product would be a 2 X 3 matrix.

3. $J = \begin{bmatrix} 6 & 4 \\ -2 & -4 \\ 9 & 7 \end{bmatrix}$ $K = \begin{bmatrix} 0 & 1 & 2 \\ 11 & 22 & -33 \end{bmatrix}$

Matrix J has three rows and two columns. Matrix K has two rows and three columns. Because the number of columns in J is equal to the number of rows in K, it is possible to multiply JK. The resulting product would be a 3 X 3 matrix. The number of columns in K is equal to the number of rows in J, so it is also possible to multiply KJ. The resulting product would be a 2 X 2 matrix.

EXAMPLE 2

Multiply the two matrices to determine the product matrix *EF*.

$$E = \begin{bmatrix} 1 & -3 & 10 \end{bmatrix} \qquad F = \begin{bmatrix} 9 & -1 \\ -5 & -6 \\ 3 & 4 \end{bmatrix}$$

STEP 1 Use the sizes of the factor matrices to determine the size of the product matrix so that you know how many entries will be in the product matrix.

$$E_{1\times 3} \cdot F_{3\times 2} = EF_{1\times 2}$$

equal

dimensions of product matrix

There will be two entries in the product matrix since the product matrix has one row and two columns.

STEP 2 Multiply the entries in the row of matrix *E* by the corresponding entries in the first column of matrix *F*. Determine the sum of all these products to determine the entry in the first row and first column of the product matrix.

$$EF = \begin{bmatrix} 1 & -3 & 10 \end{bmatrix} \cdot \begin{bmatrix} 9 & -1 \\ -5 & -6 \\ 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} (1)(9) + (-3)(-5) + (10)(3) & \square \end{bmatrix} = \begin{bmatrix} 9 + 15 + 30 & \square \end{bmatrix} = \begin{bmatrix} 54 & \square \end{bmatrix}$$

STEP 3 Multiply the entries in the row of matrix *E* by the corresponding entries in the second column of matrix *F*. Calculate the sum of all these products to determine the entry in the first row and second column of the product matrix.

$$EF = \begin{bmatrix} 1 & -3 & 10 \end{bmatrix} \cdot \begin{bmatrix} 9 & -1 \\ -5 & -6 \\ 3 & 4 \end{bmatrix}$$

$$EF = \begin{bmatrix} 54 & (1)(-1) + (-3)(-6) + (10)(4) \end{bmatrix} = \begin{bmatrix} 54 & -1 + 18 + 40 \end{bmatrix} = \begin{bmatrix} 54 & 57 \end{bmatrix}$$

$$EF = \begin{bmatrix} 54 & 57 \end{bmatrix}$$

ADDITIONAL EXAMPLES

Multiply the two matrices to determine the product matrix *AB*.

1.

$$A = \begin{bmatrix} 0 & 5 & 20 \\ -3 & 7 & -3 \\ -4 & 2 & 9 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & 12 \\ -4 & 6 \\ 10 & -5 \end{bmatrix}$$

$$AB = \begin{bmatrix} 180 & -70 \\ -55 & 21 \\ 86 & -81 \end{bmatrix}$$

2.

$$A = \begin{bmatrix} 4 & -19 & 10 & 15 \\ -6 & -18 & 2 & 0 \\ -20 & 3 & -2 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 14 & 12 \\ -2 & 6 \\ 7 & 7 \end{bmatrix}$$

Matrix *A* has three rows and four columns, and Matrix *B* has three rows and two columns. Because the number of columns in *A* is not equal to the number of rows in *B*, it is impossible to multiply *AB*.

3.

$$A = \begin{bmatrix} 10 & -5 & 0 & 12 & -8 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 \\ -6 \\ 1 \\ -14 \\ -6 \end{bmatrix}$$

$$AB = \begin{bmatrix} -70 \end{bmatrix}$$

YOU TRY IT! #2

Multiply the two matrices to determine the product matrix *HG*.

$$G = \begin{bmatrix} 4 & 3 \\ 0 & -2 \end{bmatrix} \qquad H = \begin{bmatrix} 1 & -7 \\ 8 & 12 \\ -3 & 0 \\ 0 & 4 \end{bmatrix}$$

See margin.

EXAMPLE 3

Two friends both have family recipes for a traditional Spanish rice dish called paella. They compare the cost and protein content of their recipes.

	CHICKEN (POUNDS)	RICE (POUNDS)	SHELLFISH (POUNDS)
JAIME'S FAMILY RECIPE	2.5	0.75	2.25
MIA'S FAMILY RECIPE	1.5	3.5	0.5

	COST PER POUND (DOLLARS)	PROTEIN CONTENT PER POUND (GRAMS)
CHICKEN	$3.35	139 G
RICE	$0.70	12.3 G
SHELLFISH	$10.15	109 G

Whose paella recipe would you use if your goal were to eat a high-protein meal? Whose paella recipe would you use if your goal were to make a meal that cost less than $15? Justify your responses.

STEP 1 Write matrix *R* to represent the recipe ingredients and matrix *C* to represent the costs and protein contents per pound.

$$R = \begin{bmatrix} 2.5 & 0.75 & 2.25 \\ 1.5 & 3.5 & 0.5 \end{bmatrix} \qquad C = \begin{bmatrix} 3.35 & 139 \\ 0.7 & 12.3 \\ 10.15 & 109 \end{bmatrix}$$

STEP 2 Notice that the total cost and total protein content of each recipe can be found by multiplying the amount of each ingredient by its respective cost and protein content and adding the products. Therefore, you can multiply the two matrices to determine the total cost and total protein content of each paella recipe.

YOU TRY IT! #2 ANSWER:

$$HG = \begin{bmatrix} 1 & -7 \\ 8 & 12 \\ -3 & 0 \\ 0 & 4 \end{bmatrix} \cdot \begin{bmatrix} 4 & 3 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} (1)(4)+(-7)(0) & (1)(3)+(-7)(-2) \\ (8)(4)+(12)(0) & (8)(3)+(12)(-2) \\ (-3)(4)+(0)(0) & (-3)(3)+(0)(-2) \\ (0)(4)+(4)(0) & (0)(3)+(4)(-2) \end{bmatrix} = \begin{bmatrix} 4 & 17 \\ 32 & 0 \\ -12 & -9 \\ 0 & -8 \end{bmatrix}$$

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Multiply the entries in the first row of matrix R by the corresponding entries in the first column of matrix C. Determine the sum of all these products to determine the entry in the first row and first column of the product matrix.

$$RC = \begin{bmatrix} 2.5 & 0.75 & 2.25 \\ 1.5 & 3.5 & 0.5 \end{bmatrix} \cdot \begin{bmatrix} 3.35 & 139 \\ 0.7 & 12.3 \\ 10.15 & 109 \end{bmatrix}$$

$$\begin{bmatrix} (2.5)(3.35) + (0.75)(0.7) + (2.25)(10.15) & \square \\ \square & \square \end{bmatrix}$$

$$= \begin{bmatrix} 8.375 + 0.525 + 22.8375 & \square \\ \square & \square \end{bmatrix} = \begin{bmatrix} 31.7375 & \square \\ \square & \square \end{bmatrix}$$

Multiply the entries in the first row of matrix R by the corresponding entries in the second column of matrix C. Calculate the sum of all these products to determine the entry in the first row and second column of the product matrix.

$$RC = \begin{bmatrix} 2.5 & 0.75 & 2.25 \\ 1.5 & 3.5 & 0.5 \end{bmatrix} \cdot \begin{bmatrix} 3.35 & 139 \\ 0.7 & 12.3 \\ 10.15 & 109 \end{bmatrix}$$

$$\begin{bmatrix} 31.7375 & (2.5)(139) + (0.75)(12.3) + (2.25)(109) \\ \square & \square \end{bmatrix}$$

$$= \begin{bmatrix} 31.7375 & 347.5 + 9.225 + 245.25 \\ \square & \square \end{bmatrix} = \begin{bmatrix} 31.7375 & 601.975 \\ \square & \square \end{bmatrix}$$

Multiply the entries in the second row of matrix R by the corresponding entries in the first column of matrix C. Calculate the sum of all these products to determine the entry in the second row and first column of the product matrix.

$$RC = \begin{bmatrix} 2.5 & 0.75 & 2.25 \\ 1.5 & 3.5 & 0.5 \end{bmatrix} \cdot \begin{bmatrix} 3.35 & 139 \\ 0.7 & 12.3 \\ 10.15 & 109 \end{bmatrix}$$

$$\begin{bmatrix} 31.7375 & 601.975 \\ (1.5)(3.35) + (3.35)(0.7) + (0.5)(10.15) & \end{bmatrix}$$

$$= \begin{bmatrix} 31.7375 & 601.975 \\ 5.025 + 2.45 + 5.075 & \square \end{bmatrix} = \begin{bmatrix} 31.7375 & 601.975 \\ 12.55 & \square \end{bmatrix}$$

ADDITIONAL EXAMPLE

Alan develops apps for smartphones. These apps fall into three categories: photo editing, music downloads, and games. Each app type sells for a different price, and they are sold in two different online stores for different devices. The sales for the month are shown in the tables below. Which app store is the most profitable for Alan? Create a matrix to organize the mathematical ideas, and justify your answer.

	PHOTO	MUSIC	GAMES
APP STORE #1	1075	2055	3872
APP STORE #2	2044	3095	2105

	COST PER DOWNLOAD
PHOTO	$1.99
MUSIC	$2.99
GAMES	$0.99

$$\begin{bmatrix} 1075 & 2055 & 3872 \\ 2044 & 3095 & 2105 \end{bmatrix} \begin{bmatrix} 1.99 \\ 2.99 \\ 0.99 \end{bmatrix} = \begin{bmatrix} 12116.98 \\ 15405.56 \end{bmatrix}$$

App Store #1 sells $12,116.98 of Alan's apps, which is less than App Store #2's sales of $15,405.56 worth of apps. The product of the matrix representing the number of apps downloaded and the matrix representing the price of each download describes the total sales for each app store.

Multiply the entries in the second row of matrix R by the corresponding entries in the second column of matrix C. Calculate the sum of all these products to determine the entry in the second row and second column of the product matrix.

$$RC = \begin{bmatrix} 2.5 & 0.75 & 2.25 \\ 1.5 & 3.5 & 0.5 \end{bmatrix} \cdot \begin{bmatrix} 3.35 & 139 \\ 0.7 & 12.3 \\ 10.15 & 109 \end{bmatrix}$$

$$\begin{bmatrix} 31.7375 & 601.975 \\ 12.55 & (1.5)(139) + (3.5)(12.3) + (0.5)(109) \end{bmatrix}$$

$$RC = \begin{bmatrix} 31.7375 & 601.975 \\ 12.55 & 208.5 + 43.05 + 54.5 \end{bmatrix} = \begin{bmatrix} 31.7375 & 601.975 \\ 12.55 & 306.05 \end{bmatrix}$$

STEP 3 **Interpret the resulting product matrix.**

Jaime's family recipe for paella costs approximately $31.74 to make and contains about 602 grams of protein. Mia's family recipe for paella costs $12.55 to make and contains approximately 306 grams of protein. If my goal were to eat a high protein meal, I would use Jaime's family recipe for paella, but if my goal were to make a meal that cost less than $15 to make, I would use Mia's family recipe for paella.

YOU TRY IT! #3

Aimee and Christopher work at a local movie theater. They both sold tickets in the ticket booth on Tuesday. Which employee will have the higher receipt total at the end of their shifts? Create a matrix to organize the mathematical ideas, and justify your answer.

	NUMBER OF CHILD TICKETS SOLD	NUMBER OF STUDENT TICKETS SOLD	NUMBER OF ADULT TICKETS SOLD	NUMBER OF SENIOR TICKETS SOLD
AIMEE	60	48	144	30
CHRISTOPHER	102	27	108	60

	COST PER TICKET (DOLLARS)
CHILD	$7.25
STUDENT	$10.00
ADULT	$12.50
SENIOR	$8.00

See margin.

YOU TRY IT! #3 ANSWER:

$$\begin{bmatrix} 60 & 48 & 144 & 30 \\ 102 & 27 & 108 & 60 \end{bmatrix} \begin{bmatrix} 7.25 \\ 10 \\ 12.5 \\ 8 \end{bmatrix} = \begin{bmatrix} (60)(7.25) + (48)(10) + (144)(12.5) + (30)(8) \\ (102)(7.25) + (27)(10) + (108)(12.5) + (60)(8) \end{bmatrix} = \begin{bmatrix} 2955 \\ 2839.5 \end{bmatrix}$$

Aimee will have a total of $2,955.00 in receipts, which is more than Christopher's total of $2,839.50. The product of the matrix representing the number of tickets sold and the matrix representing the price of each ticket describes the total sales for each employee.

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PRACTICE/HOMEWORK

Use the scenario below to complete questions 1 – 4.

BUSINESS

Marcia owns a bakery where she bakes and sells bread, muffins, cakes, and pies. The table below represents the items she sells and the number of each item she sold for two weeks.

MARCIA'S BAKERY

	BREAD	MUFFINS	CAKES	PIES
WEEK 1	60	154	20	17
WEEK 2	68	147	23	15

The price she changes for each item is shown in the table below.

MARCIA'S PRICES

ITEM	PRICE
BREAD	$5.00
MUFFIN	$1.25
CAKE	$32.50
PIE	$14.00

1. Create matrix A to show the number of items Marcia sold in week 1 and week 2.
See margin.

2. Create matrix B to show the price for each item sold in the bakery.
See margin.

3. How should the matrices be multiplied in order to generate a matrix representing the total revenue for each week? Choose one and justify your answer.
A. $[A] \cdot [B]$ **B.** $[B] \cdot [A]$
Choice A; when multiplied $A_{2x4} \cdot B_{4x1}$, the number of columns in matrix A equal the number of rows in matrix B.

4. Multiply the matrices to generate a matrix representing the total revenue for each week.
See margin.

Use the scenario below to complete questions 5 – 8.

SPORTS

There are several ways to score points in professional football: touchdown, field goal, safety, extra point after a touchdown, and conversion after a touchdown. The table below shows the number of each type of score by two teams.

FOOTBALL TEAMS

	TOUCHDOWN	FIELD GOAL	SAFETY	EXTRA POINT	CONVERSION
TEAM C	3	2	0	2	1
TEAM B	4	1	1	3	0

1. $A = \begin{bmatrix} 60 & 154 & 20 & 17 \\ 68 & 147 & 23 & 15 \end{bmatrix}$

2. $B = \begin{bmatrix} 5 \\ 1.25 \\ 32.5 \\ 14 \end{bmatrix}$

4. $AB = \begin{bmatrix} 1380.50 \\ 1481.25 \end{bmatrix}$

The table below shows the points awarded for each type of score.

POINTS

TYPE	POINTS
TOUCHDOWN	6
FIELD GOAL	3
SAFETY	2
EXTRA POINT	1
CONVERSION	2

5. Create matrix C to show the number of each type of score made by both teams.
See margin.

6. Create matrix B to show the points awarded for each type of score.
See margin.

7. How should the matrices be multiplied in order to generate a matrix representing the total points for each team? Choose one and justify your answer.
A. $[B] \cdot [C]$ **B.** $[C] \cdot [B]$
Choice B; when multiplied $C_{2x5} \cdot B_{5x1}$, the number of columns in matrix C = the number of rows in matrix B.

8. Multiply the matrices to generate a matrix representing the total points for each team.
See margin.

For questions 9 – 14, write 'yes' if the matrices can be multiplied in the order listed or 'no' if they cannot. If 'yes', indicate the size of the product matrix.

9. $X_{4x3} \cdot Y_{3x3}$ ***Yes; 4 x 3***
10. $R_{5x1} \cdot T_{5x7}$ ***No***
11. $K_{6x2} \cdot L_{6x4}$ ***No***
12. $B_{7x4} \cdot D_{4x7}$ ***Yes; 7 x 7***

13. $A_{1x3} \cdot C_{1x2}$ ***No***
14. $F_{5x6} \cdot G_{6x8}$ ***Yes; 5 x 8***

Use the matrices below for questions 15 – 20.

$$A = \begin{bmatrix} -3 & 1 \\ 7 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -6 & 10 \\ -9 & 0 & -5 \\ 5 & -1 & 4 \end{bmatrix} \quad C = \begin{bmatrix} 0 & -4 \\ 6 & -5 \\ 8 & -10 \end{bmatrix} \quad D = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 3 & 9 \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & 3 & -2 & 5 \\ 6 & -4 & 8 & 0 \end{bmatrix} \quad F = \begin{bmatrix} 6 \\ -4 \end{bmatrix} \quad G = \begin{bmatrix} 7 & -7 \\ 2 & 0 \\ -3 & 2 \\ 5 & 6 \end{bmatrix} \quad H = \begin{bmatrix} 5 & -6 & 2 \end{bmatrix}$$

Perform each multiplication, if possible, and record the product matrix. If it is not possible to multiply, write 'no solution'.

15. BC ***See margin.***
16. EG ***See margin.***
17. AH ***No solution.***
18. DC ***See margin.***

19. AC ***No solution.***
20. FH ***See margin.***

5. $C = \begin{bmatrix} 3 & 2 & 0 & 2 & 1 \\ 4 & 1 & 1 & 3 & 0 \end{bmatrix}$

6. $B = \begin{bmatrix} 6 \\ 3 \\ 2 \\ 1 \\ 2 \end{bmatrix}$

8. $CB = \begin{bmatrix} 28 \\ 32 \end{bmatrix}$

15. $\begin{bmatrix} 44 & -78 \\ -40 & 86 \\ 26 & -55 \end{bmatrix}$

16. $\begin{bmatrix} 44 & 19 \\ 10 & -26 \end{bmatrix}$

18. $\begin{bmatrix} 0 & -8 \\ 90 & -101 \end{bmatrix}$

20. $\begin{bmatrix} 30 & -36 & 12 \\ -20 & 24 & -8 \end{bmatrix}$

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