TEKS

AR.5C Multiply matrices by a scalar.

MATHEMATICAL PROCESS SPOTLIGHT

AR.1A Apply mathematics to problems arising in everyday life, society, and the workplace.

ELPS

4F Use visual and contextual support and support from peers and teachers to read grade-appropriate content area text, enhance and confirm understanding, and develop vocabulary, grasp of language structures, and background knowledge needed to comprehend increasingly challenging language.

VOCABULARY

matrix, distributive property, scalar

MATERIALS

- graphing technology

1. *Matrix A*

$$\begin{bmatrix} 8.50 & 10.50 & 12.50 \\ 7.95 & 9.95 & 11.95 \\ 8.25 & 10.25 & 12.25 \\ 9.15 & 11.15 & 13.15 \\ 8.90 & 10.90 & 12.90 \end{bmatrix}$$

6.3 Scalar Multiplication of Matrices

FOCUSING QUESTION How do I multiply a matrix by a scalar quantity?

LEARNING OUTCOMES

- I can multiply a data set that is represented in a matrix by a scalar.
- I can apply mathematics to solve problems arising in the workplace.

ENGAGE

Merrily was shopping for gifts for her family. She purchased a video game that cost $39.95, a purse that cost $19.95, and 3 T-shirts that cost $14.95 each. She had a coupon for 20% off her entire purchase and she had to pay 8.25% sales tax on her subtotal, after the discount. What was the final cost of Merrily's purchases, including the discount and sales tax?

$90.71

EXPLORE

Nicholas manages a popcorn store at a local mall. The store sells popcorn mixes in several flavors. Each flavor is available in a small, medium, or large package. Nicholas made a table, which is shown, with the prices for the five most popular flavors of popcorn.

ITEM	SMALL	MEDIUM	LARGE
CHEDDAR CHAMPION	$8.50	$10.50	$12.50
PEPPERMINT BARK	$7.95	$9.95	$11.95
CHOCOLATE CRUNCH	$8.25	$10.25	$12.25
PEANUT BUTTER DELIGHT	$9.15	$11.15	$13.15
MIDNIGHT MOCHA	$8.90	$10.90	$12.90

1. Construct matrix *A* to display the prices.
See margin.

2. Where Nicholas lives, the popcorn sales are taxable and the sales tax rate is 6.5%. Write an expression that you can use to calculate the final price of a package of popcorn, including sales tax. Let x represent the price of one package of popcorn. Simplify your expression completely.
$x + 0.065x = 1.065x$

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3. Construct matrix P showing the price of each package of popcorn including sales tax. Use either paper and pencil or technology to perform the computations. Round to the nearest cent if necessary.
See margin.

4. Use the matrices to identify the values of $a_{3,1}$ and $p_{3,1}$.
$a_{3,1}$ = 8.25 and $p_{3,1}$ = 8.79

5. Use the matrices to identify the values of $a_{4,2}$ and $p_{4,2}$.
$a_{4,2}$ = 11.15 and $p_{4,2}$ = 11.87

6. What is the ratio of $\frac{p_{3,1}}{a_{3,1}}$? The ratio of $\frac{p_{4,2}}{a_{4,2}}$?
Both ratios are approximately equal to 1.065.

7. What do these ratios represent?
The ratios are the scale factor used to calculate the amount of sales tax, 6.5% = 0.065, added to the original price, 100% = 1.

8. How are the entries in matrix P, $p_{R,C}$, related to their corresponding entries in matrix A, $a_{R,C}$?
Each entry in matrix P is 1.065 times its corresponding entry in matrix A.

9. Use graphing technology such as a graphing calculator or app to enter the data from matrix A. Remember that matrices are defined by the number of rows by the number of columns.
See margin.

10. Use matrix operations to multiply matrix A by 1.065. On some devices, this will appear on the home screen as 1.065 × [A] and the product will appear as a new matrix. How does this matrix compare to matrix P, the one you calculated in a previous question?
See margin.

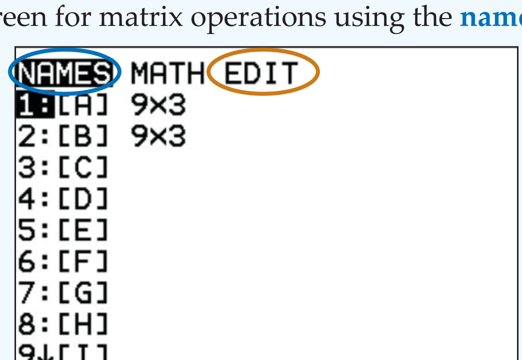
11. For online orders that Nicholas ships in-state, customers must pay both sales tax and a 9% shipping charge. What number would you multiply by matrix A, the original prices of packages of popcorn without sales tax, in order to generate matrix S, the cost of each package of popcorn with sales tax and shipping included?
6.5% + 9% = 15.5%, so multiply matrix A by 1.155.

12. Use your graphing technology to generate matrix S from the scale factor you identified and the entries in matrix A. Round to the nearest cent if necessary.
See margin.

13. How are the entries in matrix S, $s_{R,C}$, related to their corresponding entries in matrix A, $a_{R,C}$ and the scale factor representing the combined sales tax and shipping rates?
Each entry in matrix S is 1.155 times its corresponding entry in matrix A.

INTEGRATING TECHNOLOGY

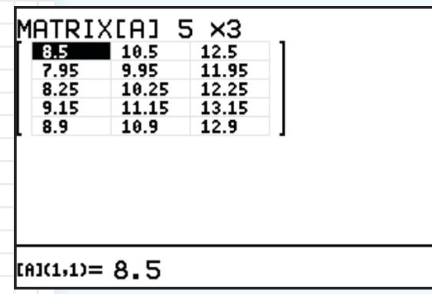
Use graphing calculators to create, add, and subtract matrices. Depending on the technology, matrices can be created through the **edit** menu and can be used on the home screen for matrix operations using the **names** menu.



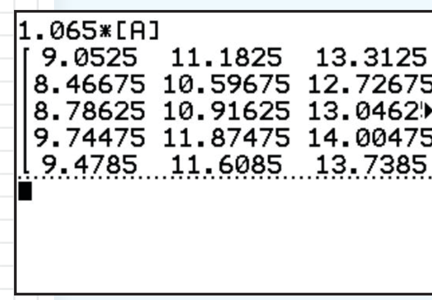
2. *Matrix P*

$$\begin{bmatrix} 9.05 & 11.18 & 13.31 \\ 8.47 & 10.60 & 12.73 \\ 8.79 & 10.92 & 13.05 \\ 9.74 & 11.87 & 14.00 \\ 9.48 & 11.61 & 13.74 \end{bmatrix}$$

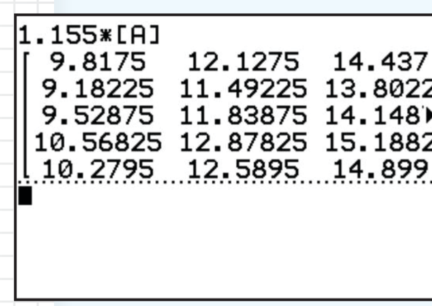
9.



10. *The matrix for 1.065 × [A] is the same as matrix P.*



12.



Matrix S

$$\begin{bmatrix} 9.82 & 12.13 & 14.44 \\ 9.18 & 11.49 & 13.80 \\ 9.53 & 11.84 & 14.15 \\ 10.57 & 12.88 & 15.19 \\ 10.28 & 12.59 & 14.90 \end{bmatrix}$$

REFLECT

- **When you multiply a matrix by a scale factor, what happens to the values of each entry in the matrix?**
  The values of the entries in the matrix change by the same scale factor by which the matrix is being multiplied.
- **How does multiplying a matrix by a number with paper and pencil compare to multiplying a matrix by a number using technology?**
  See margin.

REFLECT ANSWER:

Technology performs the same computations to generate the same product matrix as we generated by hand. Technology performs these computations much more quickly.

EXPLAIN

ELPS ACTIVITY

Read the passage below along with a peer. Ask each other questions like those below to enhance and confirm your understanding.

- What does this remind you of?
- How could you say that differently?
- How is the operation with the matrix similar to the operation with numbers?

If you encounter words that you and your peer do not know, ask your teacher for additional support.

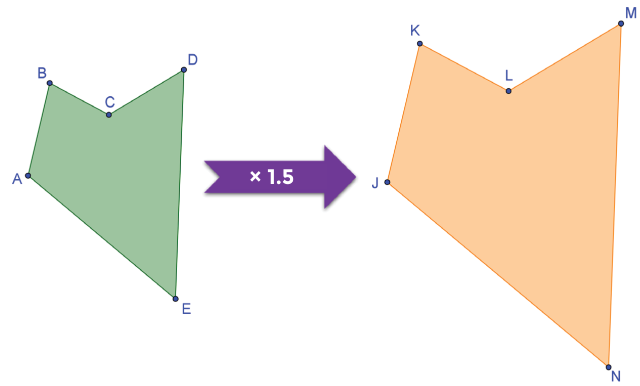
Watch Explain and You Try It Videos

or click here

SUPPORTING ENGLISH LANGUAGE LEARNERS

Students need the opportunity to use support from both their peers and their teachers as they read so that they can enhance and confirm understanding. Support can include questions or think-alouds as they read a passage. Students need the opportunity to clarify their understanding of the material and to verify that what they understand from a passage they are reading is, in fact, what the author of the passage meant. Both classmates (peers) and the teacher can provide those opportunities.

In mathematics, a **scalar** is a real number that is used to indicate the size, or scale, of something. Scalars can also be used as scale factors that indicate a proportional enlargement or reduction of an object. When you multiply the side lengths of a polygon by the same scale factor, or scalar, and keep the angle measures the same, you generate a similar figure.



Likewise, when you multiply a matrix by a scalar, you are proportionally enlarging or reducing the values of the entries in the matrix. The dimensions of the matrix do not change. Only the values of the entries in the matrix are changed proportionally.

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MULTIPLYING A MATRIX BY A SCALAR

In general, to multiply matrix A by a scalar, k, you multiply each entry of matrix A by k.

$$k\begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \\ a_{3,1} & a_{3,2} \end{bmatrix} = \begin{bmatrix} ka_{1,1} & ka_{1,2} \\ ka_{2,1} & ka_{2,2} \\ ka_{3,1} & ka_{3,2} \end{bmatrix}$$

For example, the table shows the price of three different size packages of popcorn for each of five flavors in Nicholas's popcorn store.

ITEM	SMALL	MEDIUM	LARGE
CHEDDAR CHAMPION	\$8.50	\$10.50	\$12.50
PEPPERMINT BARK	\$7.95	\$9.95	\$11.95
CHOCOLATE CRUNCH	\$8.25	\$10.25	\$12.25
PEANUT BUTTER DELIGHT	\$9.15	\$11.15	\$13.15
MIDNIGHT MOCHA	\$8.90	\$10.90	\$12.90

The data can be entered into a matrix.

$$\begin{bmatrix} 8.50 & 10.50 & 12.50 \\ 7.95 & 9.95 & 11.95 \\ 8.25 & 10.25 & 12.25 \\ 9.15 & 11.15 & 13.15 \\ 8.90 & 10.90 & 12.90 \end{bmatrix}$$

If Nicholas decides to have a sale where all popcorn is 20% off, then he can multiply the matrix by the scalar 80% = 0.8 in order to calculate the discounted prices.

$$B = 0.8A = \begin{bmatrix} 0.8(8.50) & 0.8(10.50) & 0.8(12.50) \\ 0.8(7.95) & 0.8(9.95) & 0.8(11.95) \\ 0.8(8.25) & 0.8(10.25) & 0.8(12.25) \\ 0.8(9.15) & 0.8(11.15) & 0.8(13.15) \\ 0.8(8.90) & 0.8(10.90) & 0.8(12.90) \end{bmatrix} = \begin{bmatrix} 6.80 & 8.40 & 10.00 \\ 6.36 & 7.96 & 9.56 \\ 6.60 & 8.20 & 9.80 \\ 7.32 & 8.92 & 10.52 \\ 7.12 & 8.72 & 10.32 \end{bmatrix}$$

Nicholas can now use the rows and columns in the product matrix B to determine the sale price of each bag of popcorn. For example, $b_{4,1}$ = \$7.32, which means that a small bag of Peanut Butter Delight will cost \$7.32 during the 20% off sale.

DISTRIBUTIVE PROPERTY WITH MATRICES

With real numbers, and algebraic expressions containing real numbers, the distributive property states that:

$$a(b + c) = ab + ac$$

$$a(b - c) = ab - ac$$

In other words, if you have a number that you're multiplying by a sum (or difference) of two numbers, you can multiply the number by each addend (or subtrahend and minuend) then add (or subtract) and get the same answer.

Let's see if this property works with matrices.

QUESTIONING STRATEGY

Ask students why Nicholas is multiplying by 0.80 instead of 0.20. Why might he multiply by 0.20 instead?

Suppose that a group of people with 4 adults, 4 kids, and 4 seniors plans to have a meal together and see a movie. The restaurant where they will be dining offers *prix fixe* meals, which is an appetizer, main course, and dessert for a set price.

Restaurant Prices

- Lunch: \$10 for kids, \$15 for adults, \$12 for seniors
- Dinner: \$12 for kids, \$25 for adults, \$20 for seniors

Movie Prices

- Matinee: \$5 for kids, \$8 for adults, \$6 for seniors
- Evening: \$7 for kids, \$10 for adults, \$7 for seniors

Each set of prices can be placed into a 2 × 3 matrix.

Matrix *R*: Restaurant Prices

$$\begin{array}{c} \begin{matrix} \text{K} & \text{A} & \text{S} \end{matrix} \\ \begin{bmatrix} 10 & 15 & 12 \\ 12 & 25 & 20 \end{bmatrix} \end{array} \begin{matrix} \text{Lunch} \\ \text{Dinner} \end{matrix}$$

Matrix *M*: Movie Prices

$$\begin{array}{c} \begin{matrix} \text{K} & \text{A} & \text{S} \end{matrix} \\ \begin{bmatrix} 5 & 8 & 6 \\ 7 & 10 & 7 \end{bmatrix} \end{array} \begin{matrix} \text{Matinee} \\ \text{Evening} \end{matrix}$$

The combined cost of the meal and movie for 4 adults, 4 kids, and 4 seniors can be represented with the matrix equation shown. This matrix expression can be simplified using the order of operations. Then, we can use the distributive property to see if the answer is the same.

$$4\left(\begin{bmatrix} 10 & 15 & 12 \\ 12 & 25 & 20 \end{bmatrix} + \begin{bmatrix} 5 & 8 & 6 \\ 7 & 10 & 7 \end{bmatrix}\right)$$

Method 1: According to the order of operations, add the matrices inside the grouping symbols (parentheses) first. Then, multiply the sum by 4.

$$\underbrace{4\left(\begin{bmatrix} 10 & 15 & 12 \\ 12 & 25 & 20 \end{bmatrix} + \begin{bmatrix} 5 & 8 & 6 \\ 7 & 10 & 7 \end{bmatrix}\right)}_{\text{4(restaurant + movie)}} = \underbrace{4\left(\begin{bmatrix} 15 & 23 & 18 \\ 19 & 35 & 27 \end{bmatrix}\right)}_{\text{4(combined price)}} = \underbrace{\begin{bmatrix} 60 & 92 & 72 \\ 76 & 140 & 108 \end{bmatrix}}_{\text{final price}}$$

Method 2: Apply the distributive property.

$$\underbrace{4\left(\begin{bmatrix} 10 & 15 & 12 \\ 12 & 25 & 20 \end{bmatrix} + \begin{bmatrix} 5 & 8 & 6 \\ 7 & 10 & 7 \end{bmatrix}\right)}_{\text{4(restaurant + movie)}} = \underbrace{4\left(\begin{bmatrix} 10 & 15 & 12 \\ 12 & 25 & 20 \end{bmatrix}\right)}_{\text{4(restaurant)}} + \underbrace{4\left(\begin{bmatrix} 5 & 8 & 6 \\ 7 & 10 & 7 \end{bmatrix}\right)}_{\text{4(movie)}}$$

$$= \underbrace{\begin{bmatrix} 40 & 60 & 48 \\ 48 & 100 & 80 \end{bmatrix} + \begin{bmatrix} 20 & 32 & 24 \\ 28 & 40 & 28 \end{bmatrix}}_{\text{total restaurant + total movie}} = \underbrace{\begin{bmatrix} 60 & 92 & 72 \\ 76 & 140 & 108 \end{bmatrix}}_{\text{total price}}$$

The distributive property appears to work for matrices. It can be algebraically shown that the distributive property will work for both scalar multiplication over addition and scalar multiplication over subtraction with matrices.

GOOD TO KNOW!

To show algebraically that the distributive property works with scalar multiplication over addition, use two generic matrices and a generic scalar, k.

$$k\left(\begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \\ a_{3,1} & a_{3,2} \end{bmatrix} + \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \\ b_{3,1} & b_{3,2} \end{bmatrix}\right) = \begin{bmatrix} ka_{1,1} & ka_{1,2} \\ ka_{2,1} & ka_{2,2} \\ ka_{3,1} & ka_{3,2} \end{bmatrix} + \begin{bmatrix} kb_{1,1} & kb_{1,2} \\ kb_{2,1} & kb_{2,2} \\ kb_{3,1} & kb_{3,2} \end{bmatrix} = \begin{bmatrix} ka_{1,1} + kb_{1,1} & ka_{1,2} + kb_{1,2} \\ ka_{2,1} + kb_{2,1} & ka_{2,2} + kb_{2,2} \\ ka_{3,1} + kb_{3,1} & ka_{3,2} + kb_{3,2} \end{bmatrix} = \begin{bmatrix} k(a_{1,1} + b_{1,1}) & k(a_{1,2} + b_{1,2}) \\ k(a_{2,1} + b_{2,1}) & k(a_{2,2} + b_{2,2}) \\ k(a_{3,1} + b_{3,1}) & k(a_{3,2} + b_{3,2}) \end{bmatrix}$$

Likewise, to show algebraically that the distributive property works with scalar multiplication over subtraction, use two generic matrices and a generic scalar, k.

$$k\left(\begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \\ a_{3,1} & a_{3,2} \end{bmatrix} - \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \\ b_{3,1} & b_{3,2} \end{bmatrix}\right) = \begin{bmatrix} ka_{1,1} & ka_{1,2} \\ ka_{2,1} & ka_{2,2} \\ ka_{3,1} & ka_{3,2} \end{bmatrix} - \begin{bmatrix} kb_{1,1} & kb_{1,2} \\ kb_{2,1} & kb_{2,2} \\ kb_{3,1} & kb_{3,2} \end{bmatrix} = \begin{bmatrix} ka_{1,1} - kb_{1,1} & ka_{1,2} - kb_{1,2} \\ ka_{2,1} - kb_{2,1} & ka_{2,2} - kb_{2,2} \\ ka_{3,1} - kb_{3,1} & ka_{3,2} - kb_{3,2} \end{bmatrix} = \begin{bmatrix} k(a_{1,1} - b_{1,1}) & k(a_{1,2} - b_{1,2}) \\ k(a_{2,1} - b_{2,1}) & k(a_{2,2} - b_{2,2}) \\ k(a_{3,1} - b_{3,1}) & k(a_{3,2} - b_{3,2}) \end{bmatrix}$$

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SCALAR MULTIPLICATION OF MATRICES

A matrix can be multiplied by a scalar quantity.

- If the scalar is k, multiply each entry in the matrix by k. Place the product in the corresponding entry of the product matrix.
- The distributive property of scalar multiplication over addition applies to matrices.
- The distributive property of scalar multiplication over subtraction applies to matrices.

EXAMPLE 1

Multiply matrix M by the scalar $\frac{7}{8}$.

$$M = \begin{bmatrix} 16 & -24 \\ -40 & 72 \end{bmatrix}$$

STEP 1 **Decide whether to use the fractional or decimal form of $\frac{7}{8}$.**

Notice that each of the entries in matrix M is a multiple of 8. Choosing the fractional form would be easier and can be accomplished without technology.

STEP 2 **Multiply each entry in matrix M by $\frac{7}{8}$ to create matrix S.**

$$S = \frac{7}{8}M = \frac{7}{8} \cdot \begin{bmatrix} 16 & -24 \\ -40 & 72 \end{bmatrix} = \begin{bmatrix} \frac{7}{8} \cdot 16 & \frac{7}{8} \cdot -24 \\ \frac{7}{8} \cdot -40 & \frac{7}{8} \cdot 72 \end{bmatrix} = \begin{bmatrix} 14 & -21 \\ -35 & 63 \end{bmatrix}$$

STEP 3 **How are the entries in matrix S related to their corresponding entries in matrix M?**

Choose $s_{2,1}$ and $m_{2,1}$ to compare: The ratio of –35 to –40 is 7 to 8.

YOU TRY IT! #1

Using the scalar –0.189, multiply matrix M to create matrix S. Choosing technology might be helpful with the multiplication by a decimal scalar.

See margin.

$$M = \begin{bmatrix} 8 & 10 & 12 \\ -7 & -9 & 11 \\ 8 & 15 & 13 \\ 9 & 19 & 14 \\ -6 & 16 & 17 \end{bmatrix}$$

YOU TRY IT! #1 ANSWER:

$$S = \begin{bmatrix} -1.512 & -1.89 & -2.268 \\ 1.323 & 1.701 & -2.079 \\ -1.512 & -2.835 & -2.457 \\ -1.701 & -3.591 & -2.646 \\ 1.134 & -3.024 & -3.213 \end{bmatrix}$$

ADDITIONAL EXAMPLES

Multiply the given matrix M by the given scalar k to create matrix S.

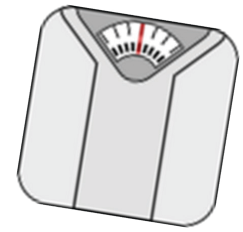
1. $k = -4$; $M = \begin{bmatrix} 0 & 2.1 & -5.3 \\ -7.5 & -1 & 8.2 \\ -12 & -0.25 & 9.7 \\ 1 & 0 & 4 \end{bmatrix}$ $S = \begin{bmatrix} 0 & -8.4 & 21.2 \\ 30 & 4 & -32.8 \\ 48 & 1 & -38.8 \\ -4 & 0 & -16 \end{bmatrix}$

2. $k = 0.375$; $M = \begin{bmatrix} 9 & 13 & 3 \\ -5 & -7 & -2 \\ 0 & 21 & 1 \\ -16 & -24 & -11 \end{bmatrix}$ $S = \begin{bmatrix} 3.375 & 4.875 & 1.125 \\ -1.875 & -2.625 & -0.75 \\ 0 & 7.875 & 0.375 \\ -6 & -9 & -4.125 \end{bmatrix}$

3. $k = -\frac{2}{5}$; $M = \begin{bmatrix} -20 & -15 & 24 \\ 35 & 1 & -12 \\ -12 & -20 & 9 \\ -9 & 80 & 75 \end{bmatrix}$ $S = \begin{bmatrix} 8 & 6 & -\frac{48}{5} \\ -14 & -\frac{2}{5} & \frac{24}{5} \\ \frac{24}{5} & 8 & -\frac{18}{5} \\ \frac{18}{5} & -32 & -30 \end{bmatrix}$

EXAMPLE 2

A group of three girls and three boys decided to lose weight and get in shape. They set a goal of losing a fourth of their body weight over the course of a year. Matrix W shows their initial weights, with the boys' weights in row 1 and the girls' weights in row 2. How much weight must each student lose to meet their target goal? What are some other uses of the matrix here?



$$W = \begin{bmatrix} 228 & 198 & 229 \\ 179 & 199 & 182 \end{bmatrix}$$

STEP 1 **Multiply matrix *W* by the scalar –0.25 to create matrix *L*, the amount of weight each intends to lose.**

$$\text{Matrix } L = -0.25\,W = -0.25\begin{bmatrix} 228 & 198 & 229 \\ 179 & 199 & 182 \end{bmatrix} = \begin{bmatrix} -57 & -49.5 & -57.25 \\ -44.75 & -49.75 & -45.5 \end{bmatrix}$$

STEP 2 **Interpret the results in Matrix *L* and describe other uses of the matrix.**

The negative sign indicates a loss or negative change in weight. The more the initial weight, the more weight will be lost. The girl whose weight is entered in $w_{2,3}$ as 182 pounds has a goal of losing 45.5 pounds.

Finding their target weight would be easy with technology by adding matrix L from matrix W. Finding their midway weight loss could be found by multiplying matrix W by the scalar, –0.125. So once the matrix is constructed in technology, there are many purposes that can be accomplished easily.

YOU TRY IT! #2

Multiply matrix Q, containing the menu prices of three sizes of sandwiches and three sizes of drinks, by the scalar 1.0725 to determine matrix S, the cost of various sandwiches and drinks with sales tax. You might want to choose to do the multiplication with the aid of technology. Businesses usually round up charges to the next cent.

$$Q = \begin{bmatrix} 5.28 & 6.99 & 7.29 \\ 0.79 & 0.99 & 1.29 \end{bmatrix}$$

See margin.

YOU TRY IT! #2 ANSWER:

$$S = \begin{bmatrix} 5.67 & 7.50 & 7.82 \\ 0.85 & 1.07 & 1.39 \end{bmatrix}$$

As seen in $q_{1,2}$ and $s_{1,2}$, a sandwich priced at \$6.99 will cost \$7.50 when the sales tax is applied.

ADDITIONAL EXAMPLE

Multiply matrix P, containing the prices of three different dresses, three different tops, and three different pairs of pants, by the scalar -0.20 to determine matrix D, the discount on the various clothing items in a 20% sale at a local store. Then determine whether you would add or subtract the two matrices to calculate the sale prices of the items. Is there a way you could find the sales prices of the items in only one step rather than two?

$$P = \begin{bmatrix} 39.95 & 52.75 & 103.65 \\ 19.95 & 9.55 & 12.25 \\ 32.85 & 19.15 & 29.95 \end{bmatrix} \quad D = \begin{bmatrix} -7.99 & -10.55 & -20.73 \\ -3.99 & -1.91 & -2.45 \\ -6.57 & -3.83 & -5.99 \end{bmatrix}$$

Answers may vary. One might be to add matrix P and matrix D to find the sales price. Since matrix D is negative to show the price discounts, adding the two matrices will subtract the discounts from the original prices. To do this problem in one step rather than two, you could multiply matrix P, the original prices, by the scalar 0.80 to find the sales prices.

EXAMPLE 3

The PizzaPlenty pizza delivery company has two locations, one on the north side of town and one on the south side. The company tracks their sales and profits using matrices. The average profit per pizza this past year was $1.95 per pizza at both locations. Their biggest sales are on Halloween and Super Bowl Sunday. Using the matrices for the number of pizzas sold at the north (row 1) and south (row 2) locations on each of the days, find out how much more profit is generated on Super Bowl Sunday than on Halloween. What information, if any, can be found using the distributive property?

Matrix *H*: Halloween

	Cheese	Supreme	Sausage
	48	55	72
	114	89	93

$$H = \begin{bmatrix} 48 & 55 & 72 \\ 114 & 89 & 93 \end{bmatrix}$$

Matrix *H*: Super Bowl

	Cheese	Supreme	Sausage
	70	68	85
	129	110	108

$$S = \begin{bmatrix} 70 & 68 & 85 \\ 129 & 110 & 108 \end{bmatrix}$$

STEP 1 **Use the verbs in the situation to determine the operation that will determine how many more pizzas of each kind and at each location were sold on Super Bowl Sunday than on Halloween.**

When a question asks about the difference between two numbers, you should subtract matrix *H* from matrix *S*.

STEP 2 **Use what you know about profit and sales to determine the operation that will tell you how much profit was made on Super Bowl Sunday or on Halloween.**

You know the profit for one pizza is, $1.95. To determine the profit on a set of pizzas, scale up the profit for one pizza. Scaling is one meaning of multiplication, so you would need to multiply matrix *S* by the scalar for profit, $1.95. That will give you the total profit from each of the kinds of pizzas from each location for Super Bowl Sunday. For Halloween, multiply matrix *H* by $1.95.

STEP 3 **Determine the operation you need to use to calculate how much more profit was made on Super Bowl Sunday than on Halloween.**

The difference in profit for each event can be found with subtraction. You could subtract 1.95[*H*] from 1.95[*S*], giving you matrix P_1, which uses the distributive property. Or you could subtract matrix *H* from matrix *S* and then multiply the difference by 1.95, giving you matrix P_2.

$$P_1 = 1.95\begin{bmatrix} 70 & 68 & 85 \\ 129 & 110 & 108 \end{bmatrix} - 1.95\begin{bmatrix} 48 & 55 & 72 \\ 114 & 89 & 93 \end{bmatrix}$$

$$P_2 = 1.95\left(\begin{bmatrix} 70 & 68 & 85 \\ 129 & 110 & 108 \end{bmatrix} - \begin{bmatrix} 48 & 55 & 72 \\ 114 & 89 & 93 \end{bmatrix}\right)$$

STEP 4 Determine whether the profit matrices P_1 and P_2 are equal and explain the information you get from each method.

$$P_1 = \begin{bmatrix} 136.50 & 132.60 & 165.75 \\ 251.55 & 214.50 & 210.60 \end{bmatrix} - \begin{bmatrix} 93.60 & 107.25 & 140.40 \\ 222.30 & 173.55 & 181.35 \end{bmatrix}$$

$$P_1 = \begin{bmatrix} 42.90 & 25.35 & 25.35 \\ 29.25 & 40.95 & 29.25 \end{bmatrix}$$

$$P_2 = 1.95\begin{bmatrix} 22 & 13 & 13 \\ 15 & 21 & 15 \end{bmatrix} = \begin{bmatrix} 42.90 & 25.35 & 25.35 \\ 29.25 & 40.95 & 29.25 \end{bmatrix}$$

The results in matrix P_1 and in matrix P_2 are the same. If you use the first method, $1.95S - 1.95H$, you can see the profit from each type of pizza at each location. If you use the second method, $1.95(S - H)$, you can see how many more pizzas of each type were sold at each location.

YOU TRY IT! #3

To apply the sales tax to a purchase price, you find the tax by multiplying the purchase price by the percent of the tax and then add the tax to the purchase price to find the total cost. You can also add the percent of the tax to the percent of the purchase price, which is 100%, and then multiply the purchase price by the combined percent. To verify this, find the total cost with a 7.5% sales tax included on the purchase prices of a pair of jeans, a shirt, and a belt shown in matrix P. Employ the method without using the distributive property (matrix T_1) and with using it (matrix T_2).

$$P = \begin{bmatrix} 45.79 & 23.59 & 10.99 \end{bmatrix}$$

See margin.

ADDITIONAL EXAMPLE

Below are two matrices that compare prices of specific items at competing grocery stores. The scalar of 3 represents how many of each item the customer purchased at the stores. To find the customer's total costs for each item, complete the problem without using the distributive property and with using it. Compare both answers, and share your findings.

$$3\left(\begin{bmatrix} 2.05 & 3.12 & 0.87 \\ 5.52 & 1.25 & 0.18 \end{bmatrix} + \begin{bmatrix} 3.10 & 2.99 & 0.79 \\ 5.75 & 1.05 & 0.20 \end{bmatrix}\right)$$

Using and without using the distributive property, the solution is the same: $\begin{bmatrix} 15.45 & 18.33 & 4.98 \\ 33.81 & 6.90 & 1.14 \end{bmatrix}$

PRACTICE/HOMEWORK

Use the following matrices to answer questions 1 – 8.

$$A = \begin{bmatrix} 3 & -1 & 5 \\ 2 & 1 & -4 \\ -6 & 3 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & -6 & -8 \\ -2 & 10 & 4 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & -3 \\ 5 & -4 \\ -7 & 5 \end{bmatrix} \qquad D = \begin{bmatrix} 1.375 & 0.875 & -2.5 \\ -3.625 & 2.125 & -4.75 \end{bmatrix}$$

1. Using the scalar 3, multiply matrix A to create matrix M.
See margin.

2. Using the scalar $-\frac{1}{2}$, multiply matrix B to create matrix N.
See margin.

3. Using the scalar 1.085, multiply matrix C to create matrix P.
See margin.

4. Using the scalar -2, multiply matrix D to create matrix R.
See margin.

5. Using the scalar -1.65, multiply matrix A to create matrix S.
See margin.

6. Using the scalar -0.8, multiply matrix B to create matrix T.
See margin.

7. Using the scalar 3.64, multiply matrix C to create matrix X.
See margin.

8. Using the scalar 4, multiply matrix D to create matrix Y.
See margin.

Use the following scenario for questions 9 - 12.

FINANCE

Maria bought a jacket, a shirt, and a pair of jeans at two different stores. Matrix M shows the cost of the jacket, shirt, and jeans at store A on row 1 and store B on row 2.

$$M = \begin{bmatrix} 42.80 & 24.30 & 38.90 \\ 36.60 & 28.80 & 36.40 \end{bmatrix}$$

9. Both stores offer a discount of 40% off any purchase. Create matrix D to show the amount of discount on each item at each store.
See margin.

10. Create matrix C to show the amount each item will cost Maria with the discount.
See margin.

1. $$M = \begin{bmatrix} 9 & -3 & 15 \\ 6 & 3 & -12 \\ -18 & 9 & 6 \end{bmatrix}$$

2. $$N = \begin{bmatrix} -2 & 3 & 4 \\ 1 & -5 & -2 \end{bmatrix}$$

3. $$P = \begin{bmatrix} 1.085 & -3.255 \\ 5.425 & -4.34 \\ -7.595 & 5.425 \end{bmatrix}$$

4. $$R = \begin{bmatrix} -2.75 & -1.75 & 5 \\ 7.25 & -4.25 & 9.5 \end{bmatrix}$$

5. $$P = \begin{bmatrix} -4.95 & 1.65 & -8.25 \\ -3.3 & -1.65 & 6.6 \\ 9.9 & -4.95 & -3.3 \end{bmatrix}$$

6. $$T = \begin{bmatrix} -3.2 & 4.8 & 6.4 \\ 1.6 & -8 & -3.2 \end{bmatrix}$$

7. $$X = \begin{bmatrix} 3.64 & -10.92 \\ 18.2 & -14.56 \\ -25.48 & 18.2 \end{bmatrix}$$

8. $$Y = \begin{bmatrix} 5.5 & 3.5 & -10 \\ -14.5 & 8.5 & -19 \end{bmatrix}$$

9. $$D = \begin{bmatrix} 17.12 & 9.72 & 15.56 \\ 14.64 & 11.52 & 14.56 \end{bmatrix}$$

10. $$C = \begin{bmatrix} 25.68 & 14.58 & 23.34 \\ 21.96 & 17.28 & 21.84 \end{bmatrix}$$

YOU TRY IT! #3 ANSWER:

Without the distributive property, there are fewer steps to finding the total cost of the items.

$T_1 = (0.075 + 1.00) \cdot \begin{bmatrix} 45.79 & 23.59 & 10.99 \end{bmatrix}$

$T_1 = 1.075 \cdot \begin{bmatrix} 45.79 & 23.59 & 10.99 \end{bmatrix}$

$T_1 = \begin{bmatrix} 49.23 & 25.36 & 11.82 \end{bmatrix}$

With the distributive property, you can see the tax on the individual items.

$T_2 = 0.075 \cdot \begin{bmatrix} 45.79 & 23.59 & 10.99 \end{bmatrix} + 1.00 \cdot \begin{bmatrix} 45.79 & 23.59 & 10.99 \end{bmatrix}$

$T_2 = \begin{bmatrix} 3.44 & 1.77 & 0.83 \end{bmatrix} + \begin{bmatrix} 45.79 & 23.59 & 10.99 \end{bmatrix}$

$T_2 = \begin{bmatrix} 49.23 & 25.36 & 11.82 \end{bmatrix}$

Either way, the total costs are the same.

11. $S = \begin{bmatrix} 2.12 & 1.21 & 1.93 \\ 1.82 & 1.43 & 1.81 \end{bmatrix}$

12. $T = \begin{bmatrix} 27.80 & 15.79 & 25.27 \\ 23.78 & 18.71 & 23.65 \end{bmatrix}$

13. $E = \begin{bmatrix} 561 & 660 & 429 & 627 \\ 396 & 627 & 660 & 594 \end{bmatrix}$

14. $D = \begin{bmatrix} -100.98 & -118.80 & -77.22 & -112.86 \\ -71.28 & -112.86 & -118.80 & -106.92 \end{bmatrix}$

15. $I = \begin{bmatrix} 460.02 & 541.20 & 351.78 & 514.14 \\ 324.72 & 514.14 & 541.20 & 487.08 \end{bmatrix}$

16. $S = \begin{bmatrix} 27.61 & 32.48 & 21.11 & 30.85 \\ 19.49 & 30.85 & 32.48 & 29.23 \end{bmatrix}$

17. $W = \begin{bmatrix} 12 & 16 & 20 \\ 8 & 14 & 18 \\ 24 & 24 & 28 \end{bmatrix}$

18. $X = \begin{bmatrix} 4.5 & 6 & 7.5 \\ 3 & 5.25 & 6.75 \\ 9 & 9 & 10.5 \end{bmatrix}$

19. $Y = \begin{bmatrix} 21 & 28 & 35 \\ 14 & 24.5 & 31.5 \\ 42 & 42 & 49 \end{bmatrix}$

20. $Z = \begin{bmatrix} 3.6 & 4.8 & 6 \\ 2.4 & 4.2 & 5.4 \\ 7.2 & 7.2 & 8.4 \end{bmatrix}$

11. The sales tax Maria must pay is 8.25%. Create matrix S to show the amount of tax Maria must pay for each item.
See margin.

12. Create matrix T to show the final cost of each item after the discount and sales tax.
See margin.

Use the following scenario for questions 13 - 16.

FINANCE

Andrea and Tony work at a sporting goods store. Matrix N shows the number of hours that Andrea worked for four different weeks on row 1 and the number of hours that Tony worked for the same four weeks on row 2.

$$N = \begin{bmatrix} 34 & 40 & 26 & 38 \\ 24 & 38 & 40 & 36 \end{bmatrix}$$

13. Andrea and Tony earn $16.50 an hour. Create matrix E to show the gross income for each of them for each week.
See margin.

14. Andrea and Tony both pay 18% of their gross income in deductions. Create matrix D to show the amount of deductions that each of them will pay for each week.
See margin.

15. Create matrix I to show Andrea's and Tony's net income for each week.
See margin.

16. Andrea and Tony both save 6% of their net income each week. Create matrix S to show the amount of money each of them will same for each week.
See margin.

Use the following scenario for questions 17 - 20.

GEOMETRY

Matrix T shows the lengths of the sides of three different triangles. The lengths of the sides or triangle A are on row 1, the lengths of the sides of triangle B are on row 2, and the lengths of the sides of triangle C are on row 3.

$$T = \begin{bmatrix} 6 & 8 & 10 \\ 4 & 7 & 9 \\ 12 & 12 & 14 \end{bmatrix}$$

17. The three triangles are dilated by a scale factor of 2. Create matrix W to show the lengths of the sides of the triangles after they are dilated.
See margin.

18. The three original triangles are dilated by 75%. Create matrix X to show the lengths of the sides of the triangles after they are dilated.
See margin.

19. The three original triangles are dilated by a scale factor of 3.5. Create matrix Y to show the lengths of the sides of the triangles after they are dilated.
See margin.

20. The three original triangles are dilated by 60%. Create matrix Z to show the lengths of the sides of the triangles after they are dilated.
See margin.

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