TEKS

AR.5A Add and subtract matrices.

MATHEMATICAL PROCESS SPOTLIGHT

AR.1A Apply mathematics to problems arising in everyday life, society, and the workplace.

ELPS

4F Use visual and contextual support and support from peers and teachers to read grade-appropriate content area text, enhance and confirm understanding, and develop vocabulary, grasp of language structures, and background knowledge needed to comprehend increasingly challenging language.

VOCABULARY

matrix, commutative property

MATERIALS

- graphing technology

6.2 Adding and Subtracting Matrices

FOCUSING QUESTION How do I add and subtract matrices?

LEARNING OUTCOMES

- I can add and subtract data sets that are represented in matrices.
- I can apply mathematics to solve problems arising in everyday life, society, and the workplace.

ENGAGE

An art museum charges non-members a general admission entry fee.

- Adults: \$15, Senior/Military: \$10, Youth/Student: \$7.50

Members receive discounted general admission entry fee.

- Adults: \$7.50, Senior/Military: \$2.50, Youth/Student: free

How would you determine the amount of money that members save on admission?

Possible answer: Subtract the members' admission fee from the non-members admission fee for each category.

EXPLORE

Statistics for the volleyball teams in the Big 12 athletic conference for two recent years are shown in the tables below.

	2014			2015		
UNIVERSITY	ASSISTS	KILLS	SERVICE ACES	ASSISTS	KILLS	SERVICE ACES
BAYLOR	1517	1625	113	1303	1413	130
IOWA STATE	1404	1482	123	1389	1491	159
KANSAS	1513	1627	131	1601	1706	149
KANSAS STATE	1469	1568	126	1392	1510	110
OKLAHOMA	1453	1598	157	1324	1443	108
TEXAS	1324	1426	135	1525	1636	136
TEXAS CHRISTIAN	1450	1542	155	1214	1316	114
TEXAS TECH	1317	1392	110	1227	1308	89
WEST VIRGINIA	1327	1427	97	1091	1178	82

Source: Big 12 Sports

1.

Matrix A *Matrix B*

$$\begin{bmatrix} 1517 & 1625 & 113 \\ 1404 & 1482 & 123 \\ 1513 & 1627 & 131 \\ 1469 & 1568 & 126 \\ 1453 & 1598 & 157 \\ 1324 & 1426 & 135 \\ 1450 & 1542 & 155 \\ 1317 & 1392 & 110 \\ 1327 & 1427 & 97 \end{bmatrix} \begin{bmatrix} 1303 & 1413 & 130 \\ 1389 & 1491 & 159 \\ 1601 & 1706 & 149 \\ 1392 & 1510 & 110 \\ 1324 & 1443 & 108 \\ 1525 & 1636 & 136 \\ 1214 & 1316 & 114 \\ 1227 & 1308 & 89 \\ 1091 & 1178 & 82 \end{bmatrix}$$

3.

Matrix S

$$\begin{bmatrix} 2820 & 3038 & 243 \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \end{bmatrix}$$

4.

Matrix S

$$\begin{bmatrix} 2820 & 3038 & 243 \\ 2793 & 2973 & 282 \\ 3114 & 3333 & 280 \\ 2861 & 3078 & 236 \\ 2777 & 3041 & 265 \\ 2849 & 3062 & 271 \\ 2664 & 2858 & 269 \\ 2544 & 2700 & 199 \\ 2418 & 2605 & 179 \end{bmatrix}$$

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1. Create matrix A to display the statistics from 2014. Create matrix B to display the statistics from 2015. Use each university as a row and each statistic as a column.
See margin.

2. For Baylor, use number sense or estimation strategies to estimate the combined number of assists, kills, and service aces for both seasons.
See margin.

3. Create a new matrix S with 9 rows and 3 columns. For the first row, use addition to combine the entries from matrix A and matrix B for Baylor's assists, kills, and service aces. Place the sums in the first row of matrix S in the appropriate columns. Use either paper and pencil or technology to perform the computations.
See margin.

4. Repeat this process for the remaining universities to complete matrix S. Use either paper and pencil or technology to perform the computations.
See margin.

5. Based on the context of the data, what do the entries in matrix S represent?
The entries in matrix S represent the combined results from the 2014 season and the 2015 together for each statistic for each volleyball team.

6. Use the matrices to identify the values of $a_{6,3}$ and $b_{6,3}$.
$a_{6,3} = 135$ and $b_{6,3} = 136$

7. Determine $s_{6,3} = a_{6,3} + b_{6,3}$.
$s_{6,3} = 271$

8. How are the entries in matrix S, $s_{R,C}$, related to their corresponding entries in matrix A, $a_{R,C}$, and matrix B, $b_{R,C}$?
Each entry in matrix S is the sum of its corresponding entries in matrix A and matrix B.

9. Use graphing technology such as a graphing calculator or app to enter the data from matrix A and matrix B. Remember that matrices are defined by the number of rows by the number of columns.
See margin.

10. Use matrix operations to add matrix A to matrix B. On some devices, this will appear on the home screen as [A] + [B], and the sum of the two matrices will appear as a new matrix. How does this matrix compare to matrix S, the one you calculated in a previous question?
See margin.

11. Create a new matrix D with 9 rows and 3 columns. For each entry, use subtraction to determine the difference between the entries from matrix A and matrix B for each university's team's assists, kills, and service aces. Subtract matrix A, which represents 2014 totals, from matrix B, which represents 2015 totals. Place the differences in corresponding entry of matrix D. Use either paper and pencil or technology to perform the computations.
See margin.

1. *See bottom of page 654.*

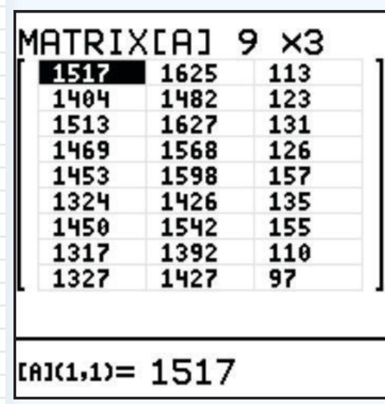
2. *Assists:*
1500 + 1300 = 2800
Kills: 1600 + 1400 = 3000
Service aces:
110 + 130 = 240

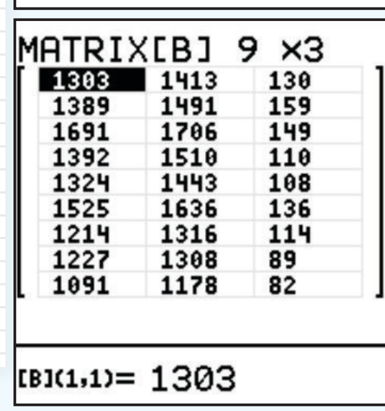
DIFFERENTIATION STRATEGY

The data set provided contains information from 9 universities. To differentiate for students who need extra time, either assign or allow students to choose 4 universities out of the 9 to build their matrices. Student work for this section will follow the same processes, but will have 4 × 3 matrices instead of 9 × 3 matrices.

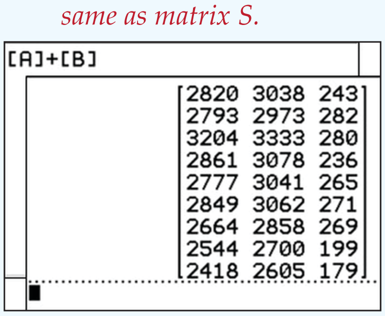
3-4. *See bottom of page 654.*

9.





10. *The matrix [A] + [B] is the same as matrix S.*



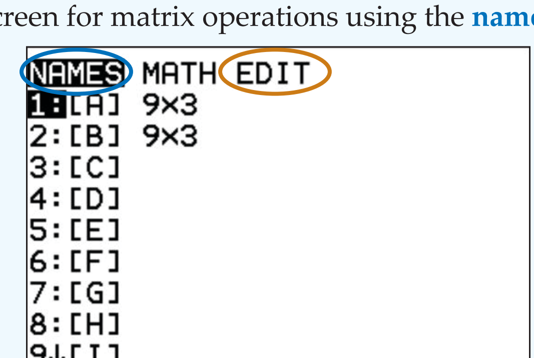
11.

Matrix D

$$\begin{bmatrix} -214 & -212 & 17 \\ -15 & 9 & 36 \\ 88 & 79 & 18 \\ -77 & -58 & -16 \\ -129 & -155 & -49 \\ 201 & 210 & 1 \\ -236 & -226 & -41 \\ -90 & -84 & -21 \\ -236 & -249 & -15 \end{bmatrix}$$

INTEGRATING TECHNOLOGY

Use graphing calculators to create, add, and subtract matrices. Depending on the technology, matrices can be created through the **edit** menu and can be used on the home screen for matrix operations using the **names** menu.



12. Based on the context of the data, what do the entries in matrix D represent?
The entries in matrix D represent the change from 2014 to 2015 in each statistic for each volleyball team.

13. Use the matrices to identify the values of $a_{7,2}$ and $b_{7,2}$.
$a_{7,2} = 1542$ and $b_{7,2} = 1316$

14. Determine $d_{7,2} = b_{7,2} - a_{7,2}$.
$d_{7,2} = -226$

15. How are the entries in matrix D, $d_{R,C}$, related to their corresponding entries in matrix A, $a_{R,C}$, and matrix B, $b_{R,C}$?
See margin.

16. With graphing technology, use matrix operations to subtract matrix A from matrix B. On some devices, this will appear on the home screen as [B] – [A], and the difference between the two matrices will appear as a new matrix. How does this matrix compare to matrix D, the one you calculated in a previous question?
See margin.

17. Use technology to compare the sum of matrix A and matrix B, [A] + [B], with the sum of matrix B and matrix A, [B] + [A]. Is addition of matrices commutative (i.e., does the order of the addends matter)?
Yes, matrix addition is commutative, and the order of the addends does not matter.

18. Use technology to compare the difference between matrix A and matrix B, [A] – [B], with the difference between matrix B and matrix A, [B] – [A]. Is subtraction of matrices commutative (i.e., does the order of the two numbers being subtracted matter)?
No, matrix subtraction is not commutative, and the order of the two numbers being subtracted does matter.

REFLECT

- **When you add two matrices together, what do you do with the entries in the addend matrices in order to generate the entries for the sum matrix?**
  See margin.
- **When you subtract two matrices, what do you do with the entries in the subtrahend and minuend matrices in order to generate the entries for the difference matrix?**
  See margin.
- **How does adding or subtracting matrices with paper and pencil compare with adding or subtracting matrices using technology?**
  See margin.

15. *Each entry in matrix D is the difference between its corresponding entries in matrix A and matrix B where the value from matrix A is subtracted from the value in matrix B.*

16. *The matrix [B] – [A] is the same as matrix D.*

[B]–[A]

$$\begin{bmatrix} -214 & -212 & 17 \\ -15 & 9 & 36 \\ 178 & 79 & 18 \\ -77 & -58 & -16 \\ -129 & -155 & -49 \\ 201 & 210 & 1 \\ -236 & -226 & -41 \\ -90 & -84 & -21 \\ -236 & -249 & -15 \end{bmatrix}$$

REFLECT ANSWERS:

Each pair of corresponding entries from the addend matrices are added together. The sum is recorded in the corresponding entry of the sum matrix.

Each pair of corresponding entries from the subtrahend and minuend matrices are subtracted (minuend – subtrahend). The difference is recorded in the corresponding entry of the difference matrix.

Technology performs the same computations to generate the same sum or difference matrix as we generated by hand. Technology performs these computations much more quickly.

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EXPLAIN

Matrices are used to organize and represent sets of data. One benefit to using matrices is that once the data is presented in a matrix, you can use technology to efficiently make calculations with the data.

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ADDING MATRICES

To add two matrices together, you must first make sure that the matrices are the same size. The two addend matrices must have the same number of rows and the same number of columns.

In a matrix entry, the **subscripts** indicate which row and column in which the entry belongs. For example, $a_{3,2}$ indicates an entry from matrix A in row 3, column 2.

In general, to add matrix A and matrix B, you add each pair of corresponding elements.

$$\begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \\ a_{3,1} & a_{3,2} \end{bmatrix} + \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \\ b_{3,1} & b_{3,2} \end{bmatrix} = \begin{bmatrix} a_{1,1}+b_{1,1} & a_{1,2}+b_{1,2} \\ a_{2,1}+b_{2,1} & a_{2,2}+b_{2,2} \\ a_{3,1}+b_{3,1} & a_{3,2}+b_{3,2} \end{bmatrix}$$

For example, the table shows the number of each color of mobile phone sold in a store for each of two models of mobile phone. The same data set could also be represented in two matrices. Each row represents a month and each column represents a color.

	MODEL 5			MODEL 5c		
	PINK	GOLD	WHITE	PINK	GOLD	WHITE
LAST MONTH	12	58	37	23	83	62
THIS MONTH	17	65	44	31	92	77

$$\begin{array}{cc} & \begin{array}{ccc} \text{Pink} & \text{Gold} & \text{White} \end{array} \\ \begin{array}{c} \text{Last Month} \\ \text{This Month} \end{array} & \begin{bmatrix} 12 & 58 & 37 \\ 17 & 65 & 44 \end{bmatrix} \end{array} \qquad \begin{array}{cc} & \begin{array}{ccc} \text{Pink} & \text{Gold} & \text{White} \end{array} \\ \begin{array}{c} \text{Last Month} \\ \text{This Month} \end{array} & \begin{bmatrix} 23 & 83 & 62 \\ 31 & 92 & 77 \end{bmatrix} \end{array}$$

To determine the total number of sales for both models together, add the two matrices by adding the corresponding entries from the two addend matrices together. Place the sum of the two entries in the same row and column in the sum matrix.

$$\begin{bmatrix} 12 & 58 & 37 \\ 17 & 65 & 44 \end{bmatrix} + \begin{bmatrix} 23 & 83 & 62 \\ 31 & 92 & 77 \end{bmatrix} = \begin{bmatrix} 12+23 & 58+83 & 37+62 \\ 17+31 & 65+92 & 44+77 \end{bmatrix} = \begin{bmatrix} 35 & 141 & 99 \\ 48 & 157 & 121 \end{bmatrix}$$

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SUBTRACTING MATRICES

As with addition, to subtract two matrices, you must first make sure that the matrices are the same size. The subtrahend matrix and the minuend matrix must have the same number of rows and the same number of columns.

In general, to subtract matrix A from matrix B, you subtract each pair of corresponding elements.

$$\begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \\ b_{3,1} & b_{3,2} \end{bmatrix} - \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \\ a_{3,1} & a_{3,2} \end{bmatrix} = \begin{bmatrix} b_{1,1} - a_{1,1} & b_{1,2} - a_{1,2} \\ b_{2,1} - a_{2,1} & b_{2,2} - a_{2,2} \\ b_{3,1} - a_{3,1} & b_{3,2} - a_{3,2} \end{bmatrix}$$

Using the mobile phone data from before, you can also calculate how many more Model 5c phones were sold than Model 5 phones. Subtract the Model 5 matrix from the Model 5c matrix. Place the difference between the two entries in the same row and column in the difference matrix.

In subtraction, the subtrahend is the number being subtracted from the minuend.
Minuend – Subtrahend = Difference

$$\begin{bmatrix} 23 & 83 & 62 \\ 31 & 92 & 77 \end{bmatrix} - \begin{bmatrix} 12 & 58 & 37 \\ 17 & 65 & 44 \end{bmatrix} = \begin{bmatrix} 23-12 & 83-58 & 62-37 \\ 31-17 & 92-65 & 77-44 \end{bmatrix} = \begin{bmatrix} 11 & 25 & 25 \\ 14 & 27 & 33 \end{bmatrix}$$

ADDING AND SUBTRACTING MATRICES

In order to add or subtract two matrices, they must have the same dimensions (number of rows and number of columns).

- As with real numbers, addition of matrices is commutative, but subtraction of matrices is not.
- To add two matrices, add the corresponding entries from the two addend matrices. Place the sum in the corresponding entry of the sum matrix.
- To subtract two matrices, subtract the corresponding entries from the two matrices being subtracted, paying attention to the order of the two matrices. Place the difference in the corresponding entry of the difference matrix.
- Technology can be used to add or subtract large matrices.

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EXAMPLE 1

The Right Around the Corner Entertainment Center carries new and used electronic games, videos, and audio CDs. In Table 1, the company's end-of-year inventory is shown by new and used items. The company received a delivery of additional items, shown in Table 2. Construct a matrix for each table and then find the company's new inventory by taking the sum of the two matrices and displaying the results in the sum matrix.

TABLE 1	YEAR END INVENTORY	
	NEW	USED
GAMES	408	223
VIDEOS	383	505
AUDIO CDS	178	244

TABLE 2	NEW DELIVERY	
	NEW	USED
GAMES	120	90
VIDEOS	125	135
AUDIO CDS	110	115

STEP 1 Construct a matrix for Table 1 and 2.

The way that the table is set up suggests how we can organize the matrices, with the new and used categories for the columns and the types of products to categorize the rows.

Matrix M_1 $\quad$ Matrix M_2

$$\begin{bmatrix} 408 & 223 \\ 383 & 505 \\ 178 & 244 \end{bmatrix} \quad \begin{bmatrix} 120 & 90 \\ 125 & 135 \\ 110 & 115 \end{bmatrix}$$

STEP 2 Add M_1 and M_2 to find and display the new inventory.

$$S = \begin{bmatrix} 408+120 & 223+90 \\ 383+125 & 505+135 \\ 178+110 & 244+115 \end{bmatrix} = \begin{bmatrix} 528 & 313 \\ 508 & 640 \\ 288 & 359 \end{bmatrix}$$

The matrix $\begin{bmatrix} 528 & 313 \\ 508 & 640 \\ 288 & 359 \end{bmatrix}$ displays the store's new inventory.

ADDITIONAL EXAMPLE

Given the tables of genres of library books checked out during the fall and spring semesters of the school year, place the data into matrices A_1 and A_2. Add the matrices, $A_1 + A_2$, to find A_3, a matrix with the total books checked out from each genre for the school year.

FALL	HISTORICAL	CURRENT EVENTS	ANIMALS
FICTION	32	12	12
NONFICTION	41	85	9

SPRING	HISTORICAL	CURRENT EVENTS	ANIMALS
FICTION	28	81	16
NONFICTION	36	102	24

The matrix A_3 is $\begin{bmatrix} 60 & 93 & 28 \\ 77 & 187 & 33 \end{bmatrix}$

YOU TRY IT! #1 ANSWER:

The matrix A_3 is

$$\begin{bmatrix} 40 & 57 \\ 57 & 83 \\ 66 & 63 \end{bmatrix}.$$

Addition of matrices is commutative since addition of real numbers is commutative.

YOU TRY IT! #1

Given the tables of coffee sales in thousands of dollars, place the data into matrices A_1 and A_2. Add the matrices, $A_1 + A_2$, to find A_3, a matrix with the sales for the first half of the year. Verify that the addition of matrices is commutative by verifying that $A_2 + A_1 = A_1 + A_2$.

TABLE 1	FIRST QUARTER SALES	
	LATTES	MOCHAS
TALL	28	38
GRANDE	35	51
VENTI	42	36

TABLE 2	SECOND QUARTER SALES	
	LATTES	MOCHAS
TALL	12	19
GRANDE	22	32
VENTI	24	27

See margin.

EXAMPLE 2

The extreme high and low temperatures for the winter months of 2015 in a town on the Canadian border are given in the list as well as the average highs and lows for the area. Construct a 2 × 3 matrix for each list, use the matrices to compare the extreme temperatures from 2015 to the average temperatures, and interpret the results.

2015 extreme temperatures: December, highest 63° and lowest 17°; January, highest 59° and lowest -4°; February, highest 28° and lowest –6°

Average temperatures from 1978 to 2014: December, high 38° and low 18°; January, high 36° and low 17°; February, high 38° and low 20°.

STEP 1 Construct matrix *E* for the 2015 extreme temperatures and matrix *A* for the average temperatures for the winter months.

$$E = \begin{array}{c} \begin{array}{ccc} \text{Dec} & \text{Jan} & \text{Feb} \end{array} \\ \begin{bmatrix} 63 & 59 & 28 \\ 17 & -4 & -6 \end{bmatrix} \end{array} \quad A = \begin{array}{c} \begin{array}{ccc} \text{Dec} & \text{Jan} & \text{Feb} \end{array} \\ \begin{bmatrix} 38 & 36 & 38 \\ 18 & 17 & 20 \end{bmatrix} \end{array}$$

Step 2 Compare the extreme temperatures to the average temperatures for each month by subtracting matrix *A* from matrix *E* to create matrix *D*.

$$D = \begin{bmatrix} 63-38 & 59-36 & 28-38 \\ 17-18 & -4-17 & -6-20 \end{bmatrix} = \begin{bmatrix} 25 & 23 & -10 \\ -1 & -21 & -26 \end{bmatrix}$$

ADDITIONAL EXAMPLE

When Selena applied to rent her first apartment, the complex gave her a list of estimated typical utilities charges. She recorded her actual charges. Place the data from the tables in a matrix for each. Find out how much more (or less) than the estimate that Selena paid for each bill each month by subtracting the matrices and showing the differences in the matrix.

TABLE 1	ACTUAL CHARGES	
	ELECTRIC	WATER
JANUARY	\$177	\$24
FEBRUARY	\$153	\$17
MARCH	\$142	\$19
APRIL	\$98	\$12

TABLE 2	ESTIMATED CHARGES	
	ELECTRIC	WATER
JANUARY	\$150	\$15
FEBRUARY	\$160	\$20
MARCH	\$130	\$20
APRIL	\$100	\$10

$$M_1 - M_2 = \begin{bmatrix} 27 & 9 \\ -7 & -3 \\ 12 & -1 \\ -2 & 2 \end{bmatrix}$$

STEP 3 Interpret the results.

The differences have specific meaning depending on the sign. In $D_{1,1}$, the difference 25 means that the extreme high temperature was 25° above the average high temperature for December. In $D_{2,3}$, the difference -26 shows that the extreme low temperature was 26° below the average low temperature for February.

YOU TRY IT! #2

Place the data from the tables in a matrix for each. Find how many more students are enrolled in athletics for each grade level by gender by subtracting the matrices and showing the differences in the matrix. Determine whether the subtraction of matrices is commutative.

TABLE 1	ATHLETICS ENROLLMENT	
	GIRLS	BOYS
9TH GR.	74	85
10TH GR.	66	78
11TH GR.	57	73
12TH GR.	43	68

TABLE 2	BAND ENROLLMENT	
	GIRLS	BOYS
9TH GR.	52	58
10TH GR.	48	51
11TH GR.	46	46
12TH GR.	39	35

See margin.

YOU TRY IT! #2 ANSWER:

No, matrix subtraction isn't commutative. The matrix that results from $M_1 - M_2$ is not the same as the matrix resulting from $M_2 - M_1$. To answer the question, $M_1 - M_2$ is needed.

$$M_1 - M_2 = \begin{array}{c} \begin{array}{cc} \textit{Girls} & \textit{Boys} \end{array} \\ \begin{bmatrix} 22 & 27 \\ 18 & 27 \\ 11 & 27 \\ 4 & 33 \end{bmatrix} \end{array}$$

EXAMPLE 3

Brothers Fernando and Ramon have a friendly contest about who can make more money during the summer. The tables show how much each of them has earned at the end of each summer month by how much they have deposited in their checking and savings accounts. Make a matrix with the data from each table, and subtract the matrices to compare their earnings. Use the difference matrix to compare their overall earnings to determine who wins the contest.

TABLE 1	FERNANDO'S EARNINGS	
	CHECKING	SAVINGS
JUNE	$425	$50
JULY	$375	$40
AUG.	$450	$30

TABLE 2	RAMON'S EARNINGS	
	CHECKING	SAVINGS
JUNE	$560	$30
JULY	$515	$40
AUG.	$410	$20

YOU TRY IT! #3 ANSWER:

$$S = \begin{bmatrix} 985 & 80 \\ 890 & 80 \\ 860 & 50 \end{bmatrix},$$

and their combined earnings in June exceed $1,000. The brothers will be able to take their parents up on their offer to match their deposited earnings of $1,065 in June.

STEP 1 Place the data from the tables into matrices.

$$F = \begin{bmatrix} 425 & 50 \\ 375 & 40 \\ 450 & 30 \end{bmatrix} \quad R = \begin{bmatrix} 560 & 30 \\ 515 & 40 \\ 410 & 20 \end{bmatrix}$$

STEP 2 Subtract the matrices, $R - F$, to compare Ramon's earnings to Fernando's.

$$D = \begin{bmatrix} 560 - 425 & 30 - 50 \\ 515 - 375 & 40 - 40 \\ 410 - 450 & 20 - 30 \end{bmatrix} = \begin{bmatrix} 135 & -20 \\ 140 & 0 \\ -40 & -10 \end{bmatrix}$$

STEP 3 Matrix D shows the comparison of Ramon's earnings to Fernando's. In June, he has earned $115 more than Fernando because he has $135 more in checking and $20 less in savings. In July, Ramon has earned $140 more if his deposits in checking and savings are combined. In August, Ramon has deposited a total of $50 less than Fernando. So, overall, Ramon is $115 + $140 – $50, or $205 ahead of Fernando, so Ramon wins their contest.

YOU TRY IT! #3

Fernando and Ramon's parents require them to combine their summer earnings to help buy a car to share. The parents say they will match what the brothers make for each month that they deposit $1,000 or more together. Use Matrices F and R to determine Matrix S, the sum of their monthly earnings and to see if they can take their parents up on their offer for any of the months.

$$F = \begin{bmatrix} 425 & 50 \\ 375 & 40 \\ 450 & 30 \end{bmatrix} \quad R = \begin{bmatrix} 560 & 30 \\ 515 & 40 \\ 410 & 20 \end{bmatrix}$$

See margin.

ADDITIONAL EXAMPLE

Joanna and Tracy challenge each other to a fitness competition to encourage each other to be able to complete more push-ups and sit-ups. The tables show their totals by week for the month. First, make a matrix with the data from each table, and subtract the matrices to determine who wins the competition.

Second, the gym decides to up the ante. For every week Tracy and Joanna's combined totals are over 350, the gym will give them a free month of membership. How many months will Tracy and Joanna get for free?

TABLE 1	TRACY	
	PUSH-UPS	SIT-UPS
WEEK 1	80	76
WEEK 2	87	80
WEEK 3	94	85
WEEK 4	102	89

TABLE 2	JOANNA	
	PUSH-UPS	SIT-UPS
WEEK 1	70	84
WEEK 2	88	85
WEEK 3	93	94
WEEK 4	113	92

Matrix S shows the comparison of Joanna's fitness exercises to Tracy's.

$$S = \begin{bmatrix} -10 & 8 \\ 1 & 5 \\ -1 & 9 \\ 11 & 3 \end{bmatrix}$$

Overall Joanna completed more repetitions of the exercises than Tracy.

Matrix T shows the total number of push-ups and sit-ups that Joanna and Tracy completed.

$$T = \begin{bmatrix} 150 & 160 \\ 175 & 165 \\ 187 & 179 \\ 215 & 181 \end{bmatrix}$$

Tracy and Joanna completed over 350 repetitions in both the 3rd and 4th weeks, so they earned 2 free months of gym membership each.

PRACTICE/HOMEWORK

Use the scenario below to answer questions 1 – 5.

STATISTICS

Mr. Alvarez and Ms. Bento each recorded the grades of their students in their first period classes. The table below shows the number of students who received each letter grade during each of the first three grading periods. Both teachers have 40 students in each class.

MR. ALVAREZ

	1ST GRADING PERIOD	2ND GRADING PERIOD	3RD GRADING PERIOD
A	10	8	12
B	12	15	10
C	8	7	11
D	6	7	6
F	4	3	1

MS. BENTO

	1ST GRADING PERIOD	2ND GRADING PERIOD	3RD GRADING PERIOD
A	4	7	9
B	21	16	19
C	10	14	10
D	4	1	2
F	1	2	0

1. Construct matrix A to represent Mr. Alvarez' grades and matrix B to represent Ms. Bento's grades.
See margin.

2. What is the combined number of students receiving an A, B, C, D, and F for the two teachers' classes? Express your answer as matrix T.
See margin.

3. Using matrix T, what is the value of $T_{4,3}$ and what does it represent in the situation?
The value of $T_{4,3}$ is 8. It represents the 8 students who received a D during the 3rd grading period in Mr. Alvarez's AND Ms. Bento's classes combined.

4. Find the difference between Mr. Alvarez's and Ms. Bento's grades $[A]-[B]=[D]$.
See margin.

5. What is the value of $D_{3,2}$ and what does it represent in the situation?
The value of $D_{3,2}$ is –7. It means Mr. Alvarez has 7 fewer students making a C in the 2nd grading period than Ms. Bento.

1.

$$A = \begin{bmatrix} 10 & 8 & 12 \\ 12 & 15 & 10 \\ 8 & 7 & 11 \\ 6 & 7 & 6 \\ 4 & 3 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 4 & 7 & 9 \\ 21 & 16 & 19 \\ 10 & 14 & 10 \\ 4 & 1 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

2.

$$T = \begin{bmatrix} 14 & 15 & 21 \\ 33 & 31 & 29 \\ 18 & 21 & 21 \\ 10 & 8 & 8 \\ 5 & 5 & 1 \end{bmatrix}$$

4.

$$D = \begin{bmatrix} 6 & 1 & 3 \\ -9 & -1 & -9 \\ -2 & -7 & 1 \\ 2 & 6 & 4 \\ 3 & 1 & 1 \end{bmatrix}$$

Use the scenario below to answer questions 6 – 10.

STATISTICS

Rodrigo is preparing a report on internet usage in various countries. He gathers data for two different clusters of countries. The tables below shows the data he has collected on internet usage for the years 2000 and 2015. The values in the table are expressed in millions.

	CLUSTER A	
	2000	2015
CHINA	22.5	674
INDIA	5	375
JAPAN	47.1	114.9

	CLUSTER B	
	2000	2015
UNITED STATES	95.3	280.7
BRAZIL	5	117.7
RUSSIA	3.1	103.1

Source: www.internetworldstats.com/top20

6. Construct matrix A to represent the data from Cluster A and matrix B to represent the data from Cluster B.
See margin.

7. What are the values of $A_{1,2}$ and $B_{3,1}$ and what do they represent in the situation?
See margin.

8. Rodrigo decides to combine the data in the two clusters. Combine Cluster A with Cluster B and express your answer as matrix C.
See margin.

9. Using matrix C, what is the value of $C_{2,2}$ and what does it represent in the situation?
The value of $C_{2,2}$ is 492,700,000. It represents the combined number of internet users of India and Brazil in 2015.

10. Rodrigo wanted to find the difference in the data between the two clusters. He subtracted the data in Cluster A from the data in Cluster B and created matrix D, which is shown below.

$$D = \begin{bmatrix} 72.8 & -393.3 \\ 0 & -257.3 \\ -44 & -11.8 \end{bmatrix}$$

Which of the following conclusions can Rodrigo make about the data using matrix D? Select all conclusions that can be made.

A. Most of Cluster A has more internet users than Cluster B.
B. Most of Cluster B has more internet users than Cluster A.
C. In the year 2015, all of the countries in Cluster B had more internet users than all of the countries in Cluster A.
D. In the year 2015, all of the countries in Cluster A had more internet users than all of the countries in Cluster B.
A and D

6. $A = \begin{bmatrix} 22.5 & 674 \\ 5 & 375 \\ 47.1 & 114.9 \end{bmatrix}$

$B = \begin{bmatrix} 95.3 & 280.7 \\ 5 & 117.7 \\ 3.1 & 103.1 \end{bmatrix}$

7. *The value of $A_{1,2}$ is 674,000,000; the number of internet users in China in 2015.*

The value of $B_{3,1}$ is 3,100,000; the number of internet users in Russia in 2000.

8. $C = \begin{bmatrix} 117.8 & 954.7 \\ 10 & 492.7 \\ 50.2 & 218 \end{bmatrix}$

Use the scenario below to answer questions 11 – 15.

STATISTICS

On a day in December, two movie theaters recorded the number and type of tickets they sold for four different movies. The data is shown in the tables below.

AVALON THEATER

	ADULT	STUDENT	CHILD	SENIOR
STAR WARS THE FORCE AWAKENS	650	525	460	275
THE BIG SHORT	309	188	17	198
CONCUSSION	485	301	22	214
THE REVENANT	517	480	73	165
TOTAL	**1961**	**1494**	**572**	**852**

BIJOU THEATER

	ADULT	STUDENT	CHILD	SENIOR
STAR WARS THE FORCE AWAKENS	704	611	398	149
THE BIG SHORT	436	102	10	211
CONCUSSION	517	376	37	155
THE REVENANT	684	310	82	227
TOTAL	**2341**	**1399**	**527**	**742**

11. Construct matrix A to represent the ticket sales at Avalon Theater and matrix B to represent the ticket sales at Bijou Theater.
See margin.

12. The same company owns both theaters and want to see their combined ticket sales. Express their combined ticket sales as matrix C.
See margin.

13. Using matrix C, what is the value of $C_{2,3}$ and $C_{5,4}$ and what do they represent in the situation?
See margin.

14. The company subtracted the data from the Bijou Theater from the data from the Avalon Theater. Find the difference in the two data sets and express the difference as matrix D.
See margin.

15. Based on the data shown in matrix D, which theater sold the most adult tickets and by how much?
Bijou Theater sold 380 more tickets than the Avalon Theater.

Use the matrices below to answer questions 16 – 20.

$$A = \begin{bmatrix} 2 & 7 & -5 \\ 9 & 0 & 4 \\ -4 & 6 & 11 \end{bmatrix} \quad B = \begin{bmatrix} 0 & -7 & 12 \\ 3 & -6 & -8 \\ 15 & 2 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 18 & -9 & -3 \\ 1 & 5 & -5 \\ 4 & 14 & -7 \end{bmatrix}$$

Perform the indicated operations. Express your answer as a matrix.

16. $[B] + [C]$
See margin.

17. $[C] - [A]$
See margin.

18. $[B] + [A]$
See margin

19. $[A] - [C]$
See margin.

20. $[A] + [C]$
See margin.

11.
$$A = \begin{bmatrix} 650 & 525 & 460 & 275 \\ 309 & 188 & 17 & 198 \\ 485 & 301 & 22 & 214 \\ 517 & 480 & 73 & 165 \\ 1961 & 1494 & 572 & 852 \end{bmatrix}$$

$$B = \begin{bmatrix} 704 & 611 & 398 & 149 \\ 436 & 102 & 10 & 211 \\ 517 & 376 & 37 & 155 \\ 684 & 310 & 82 & 227 \\ 2341 & 1399 & 527 & 742 \end{bmatrix}$$

12.
$$C = \begin{bmatrix} 1354 & 1136 & 858 & 424 \\ 745 & 290 & 27 & 409 \\ 1002 & 677 & 59 & 369 \\ 1201 & 790 & 155 & 392 \\ 4302 & 2893 & 1099 & 1594 \end{bmatrix}$$

13. *The value of $C_{2,3}$ is 27; it represents the combined total of child tickets sold for The Big Short.*

The value of $C_{5,4}$ is 1594; it represents the combined total of senior tickets sold for all four movies.

14.
$$C = \begin{bmatrix} -54 & -86 & 62 & 126 \\ -127 & 86 & 7 & -13 \\ -32 & -75 & -15 & 59 \\ -167 & 170 & -9 & -62 \\ -380 & 95 & 45 & 110 \end{bmatrix}$$

16.
$$\begin{bmatrix} 18 & -16 & 9 \\ 4 & -1 & -13 \\ 19 & 16 & -7 \end{bmatrix}$$

17.
$$\begin{bmatrix} 16 & -16 & 2 \\ -8 & 5 & -9 \\ 8 & 8 & -18 \end{bmatrix}$$

18.
$$\begin{bmatrix} 2 & 0 & 7 \\ 12 & -6 & -4 \\ 11 & 8 & 11 \end{bmatrix}$$

19.
$$\begin{bmatrix} -16 & 16 & -2 \\ 8 & -5 & 9 \\ -8 & -8 & 18 \end{bmatrix}$$

20.
$$\begin{bmatrix} 20 & -2 & -8 \\ 10 & 5 & -1 \\ 0 & 20 & 4 \end{bmatrix}$$