TEKS

AR.4C Determine the quotient of a polynomial function of degree three and of degree four when divided by a polynomial function of degree one and of degree two when represented tabularly and symbolically.

MATHEMATICAL PROCESS SPOTLIGHT

AR.1G Display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

ELPS

3H Narrate, describe, and explain with increasing specificity and detail as more English is acquired.

VOCABULARY

quotient, dividend, divisor, quartic function

MATERIALS

- graphing technology

5.9 Dividing Polynomial Functions with Tables

FOCUSING QUESTION What types of functions do you get when you divide a cubic or quartic function by a linear or quadratic function?

LEARNING OUTCOMES

- I can use tables to divide two polynomial functions.
- I can explain and justify mathematical arguments using precise mathematical language in written communication.

ENGAGE

Hooke's Law states that the force, F, required to stretch or compress a spring by a distance x is directly proportional to the distance. In other words, $F(x) = kx$, where k represents the spring constant for the particular spring being stretched or compressed.

Source: pixabay.com

Make a table of values to show the force, $F(x)$, for a spring that is being stretched x millimeters. The spring constant for this spring is 0.17.

See margin.

EXPLORE

The amount of displacement, or distance that a spring is stretched or compressed, can be represented with the variable, x. When a spring is stretched, two things happen. Force, F, is required to pull the spring. The spring also gains elastic potential energy, P, which is a type of energy that would be converted to kinetic energy if the spring were released.

Both force, F, and elastic potential energy, P, are functions of displacement, x. If the spring is already stretched 0.5 centimeters, then the following functions describe both force and elastic potential energy in terms of additional displacement, x. In these functions, k represents a spring constant.

- $F(x) = k(x + 0.5)$
- $P(x) = \frac{1}{2}k(x + 0.5)^2$

For this particular spring, the spring constant, k, is $1.3\,\frac{\text{kg} \cdot \text{m}}{\text{cm} \cdot \text{s}^2}$ and x is measured in centimeters.

In metric units, the unit for force is a Newton (N), which is 1 kilogram $\cdot \frac{\text{meters}}{(\text{seconds})^2}$. The unit for energy is a Joule (J), which is one Newton-meter. The spring constant, k, for Hooke's law is usually $\frac{\text{kg}}{\text{s}^2}$. However, the spring constant can also include any conversion factors for units of length to meters.

ENGAGE ANSWER:

x	0	2	4	6	8	10
$F(x)$	0	0.34	0.68	1.02	1.36	1.70

1. Use the functions to complete a table like the one shown.

DISPLACEMENT, x (CENTIMETERS)	FORCE, $F(x)$ (NEWTONS)	ELASTIC POTENTIAL ENERGY, $P(x)$ (JOULES)
0	***0.65***	***0.1625***
1	***1.95***	***1.4625***
2	***3.25***	***4.0625***
3	***4.55***	***7.9625***
4	***5.85***	***13.1625***
5	***7.15***	***19.6625***
6	***8.45***	***27.4625***

One way to determine the efficiency of a spring is to calculate the ratio of elastic potential energy to the force generated for a particular amount of displacement.

2. Add a column to the table to calculate the ratio $\frac{P(x)}{F(x)}$.
See margin.

3. Use finite differences to determine a function for $c(x)$, the ratio of $\frac{P(x)}{F(x)}$, in polynomial form.
See margin.

4. What type of function is $c(x)$? How does its degree compare to the degrees of the dividend, $P(x)$, and the divisor, $F(x)$?
The function c(x) is a linear function that has degree one. The dividend is a function of degree two and the divisor is a function of degree one.

5. Write $c(x)$ in factored form.
$c(x) = 0.5(x + 0.5) = \frac{1}{2}(x + 0.5)$

6. Write $c(x)$ as the ratio of $P(x)$ and $F(x)$ written in symbolic forms. What common factors in the numerator and denominator do you notice?
$\frac{P(x)}{F(x)} = \frac{\frac{1}{2}k(x + 0.5)^2}{k(x + 0.5)}$ ***Common factors include k and (x + 0.5).***

7. Divide to 1 the common factors in the numerator and denominator. Simplify the resulting expression.
$\frac{P(x)}{F(x)} = \frac{\frac{1}{2}k(x + 0.5)^2}{k(x + 0.5)} = \frac{\frac{1}{2}k(x + 0.5)(x + 0.5)}{k(x + 0.5)} = \frac{k}{k} \cdot \frac{(x + 0.5)}{(x + 0.5)} \cdot \frac{\frac{1}{2}(x + 0.5)}{1} = \frac{1}{2}(x + 0.5)$

8. How does the simplified expression from the previous question compare to the symbolic representation of $c(x)$?
They are equivalent.

9. **ELPS Strategy** Explain, with increasing specificity and detail, how you can use a table containing function values for two different functions to divide the two original functions.
See margin.

9. *Answers may vary. Possible answer: For each x-value, divide the function values for the dividend by the function values for the divisor. The resulting quotient values are the values for the quotient function. Use finite differences with these quotient function values to write a symbolic representation of the quotient function.*

INTEGRATING TECHNOLOGY

Encourage students to use technology to generate function values when given functions in symbolic form. Students can also use list operations to generate finite differences as well as divide columns of table values.

SUPPORTING ENGLISH LANGUAGE LEARNERS

Students who are learning the English language need opportunities to explain their mathematical thinking – including ideas and arguments – with increasing specificity and detail as more English is acquired (**ELPS 3H**). This ELPS standard pairs nicely with mathematical process standard **AR.1G**, which calls for students to display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

2.

DISPLACEMENT, x (CENTIMETERS)	FORCE, $F(x)$ (NEWTONS)	ELASTIC POTENTIAL ENERGY, $P(x)$ (JOULES)	$\frac{P(x)}{F(x)}$
0	0.65	0.1625	$\frac{0.1625}{0.65} = 0.25$
1	1.95	1.4625	$\frac{1.4625}{1.95} = 0.75$
2	3.25	4.0625	$\frac{4.0625}{3.25} = 1.25$
3	4.55	7.9625	$\frac{7.9625}{4.55} = 1.75$
4	5.85	13.1625	$\frac{13.1625}{5.85} = 2.25$
5	7.15	19.6625	$\frac{19.6625}{7.15} = 2.75$
6	8.45	27.4625	$\frac{27.4625}{8.45} = 3.25$

3.

DISPLACEMENT, x (CENTIMETERS)	FORCE, $F(x)$ (NEWTONS)	ELASTIC POTENTIAL ENERGY, $P(x)$ (JOULES)	$\frac{P(x)}{F(x)}$	
0	0.65	0.1625	$\frac{0.1625}{0.65} = 0.25$	
				0.5
1	1.95	1.4625	$\frac{1.4625}{1.95} = 0.75$	
				0.5
2	3.25	4.0625	$\frac{4.0625}{3.25} = 1.25$	
				0.5
3	4.55	7.9625	$\frac{7.9625}{4.55} = 1.75$	
				0.5
4	5.85	13.1625	$\frac{13.1625}{5.85} = 2.25$	
				0.5
5	7.15	19.6625	$\frac{19.6625}{7.15} = 2.75$	
				0.5
6	8.45	27.4625	$\frac{27.4625}{8.45} = 3.25$	

$c(x) = 0.5x + 0.25$

REFLECT

- **In this lesson, you divided two polynomial functions (a quadratic function divided by a linear function) and the quotient was a polynomial (linear) function. Do you think you will always have a polynomial function quotient if you divide two polynomial functions? If so, justify your mathematical argument using precise mathematical language. If not, use an example to show when dividing two polynomial functions does not generate a polynomial function quotient and justify your mathematical argument using precise mathematical language.**
  See margin.

- **If you divide a cubic function by a linear function, what type of polynomial function do you think the quotient will be?**
  Possible response: If the cubic function has a linear factor that is equivalent to the linear function divisor, then the quotient will be a quadratic function.

EXPLAIN

Watch Explain and You Try It Videos

or click here

Division is an operation that involves equal-size groups. Like numbers and polynomials, functions can also be divided.

One method you can use to divide polynomial functions is to divide paired function values from a table. Then, you can use finite differences to write a function rule.

DIVIDING A CUBIC FUNCTION (DEGREE THREE) BY A QUADRATIC FUNCTION (DEGREE TWO)

A storage box is in the shape of a rectangular prism. The volume of the box can be represented using the function $V(x) = x^3 + 2x^2 - 5x - 6$ and the area of the base can be represented using the function $B(x) = x^2 - x - 2$. Write a function, $r(x)$, that expresses the ratio $\frac{V(x)}{B(x)}$.

The function $r(x)$ is the quotient of $V(x)$ and $B(x)$. Use a table to divide $V(x)$ by $B(x)$ for particular x-values.

x	$V(x)$	$B(x)$	$r(x) = \frac{V(x)}{B(x)}$
2	0	0	UNDEFINED
3	24	4	6
4	70	10	7
5	144	18	8

REFLECT ANSWER:

Possible response: The quotient of two polynomial functions will not always be a polynomial function, because there may be a remainder. If there are no common factors between the numerator and denominator, then the function will be a rational function instead of a polynomial function. For example, if $f(x) = x(x-1)(x-2)$ and $g(x) = x+4$, then the quotient $\frac{f(x)}{g(x)} = \frac{x(x-1)(x-2)}{x+4}$ does not simplify into a polynomial function.

608 CHAPTER 5: ALGEBRAIC METHODS

With the table of values, use finite differences to determine a function rule, including rows for $x = 0$ and $x = 1$ to work backwards using patterns from the table. Notice that one of the values of the quotient, $\frac{V(x)}{B(x)}$, is undefined.

x	$V(x)$	$B(x)$	$r(x) = \frac{V(x)}{B(x)}$	
0			3	
				1
1			4	
				?
2	0	0	UNDEFINED	
				?
3	24	4	6	
				1
4	70	10	7	
				1
5	144	18	8	

The function values for $r(x)$ have a constant first difference, indicating that $r(x)$ is a linear function. Since $r(x)$ is undefined at $x = 2$, there is a domain restriction for $r(x)$ so that the domain of $r(x)$ is $\{x \mid x \in \mathbb{R}, x \neq 2\}$. Looking more closely at the table, you can see why. When $x = 2$, $B(x) = 0$. Since $B(x)$ is the divisor for $r(x)$, it cannot have a value of 0 because division by 0 is undefined. Thus, the function $r(x)$ is not defined at $x = 2$.

Using the finite differences, $r(x) = x + 3$.

The **degree of a polynomial** function is represented by its largest exponent.

- $g(x) = ax + b$ is a linear function of degree one.
- $h(x) = ax^2 + bx + c$ is a quadratic function of degree two.
- $j(x) = ax^3 + bx^2 + cx + d$ is a cubic function of degree three.
- $k(x) = ax^4 + bx^3 + cx^2 + dx + f$ is a quartic function of degree four.

DIVIDING A CUBIC FUNCTION (DEGREE THREE) BY A LINEAR FUNCTION (DEGREE ONE)

Using the same storage box, the height of the box is represented by the function $h(x) = x + 3$. The volume of the box can be represented using the function $V(x) = x^3 + 2x^2 - 5x - 6$. Write a function, $s(x)$, that expresses the ratio $\frac{V(x)}{B(x)}$.

The function $s(x)$ is the quotient of $V(x)$ and $h(x)$. Use a table to divide $V(x)$ by $h(x)$ for particular x-values.

x	$V(x)$	$h(x)$	$s(x) = \frac{V(x)}{h(x)}$
2	0	5	0
3	24	6	4
4	70	7	10
5	144	8	18

With the table of values, use finite differences to determine a function rule, including rows for x = 0 and x = 1 to work backwards using patterns from the table.

x	$V(x)$	$h(x)$	$s(x) = \frac{V(x)}{h(x)}$
0			-2
1			-2
2	0	5	0
3	24	6	4
4	70	7	10
5	144	8	18

First differences: 0, 2, 4, 6, 8

Second differences: 2, 2, 2, 2

The function values for $s(x)$ have a constant second difference, indicating that $s(x)$ is a quadratic function.

$$2a = 2 \qquad a + b = 0 \qquad c = -2$$
$$a = 1 \qquad (1) + b = 0$$
$$b = -1$$

Using the finite differences, $s(x) = x^2 - x - 2$.

DIVIDING POLYNOMIAL FUNCTIONS USING TABLES

Functions can be divided using tables of function values.

- For a set of x-values, determine the function values that represent the dividend and the function values that represent the divisor.
- Divide each set of function values.
- Use finite differences to determine what type of function the quotient is and represent the quotient function symbolically.

EXAMPLE 1

Given the functions $f(x) = 6x^3 - 2x^2 - 51.5x - 52.5$ and $g(x) = 3x + 5$ and the ratio $r(x) = \frac{f(x)}{g(x)}$, determine what type of functions the divisor and dividend are and use the table of values to write a symbolic representation of the ratio.

610 CHAPTER 5: ALGEBRAIC METHODS

x	$f(x)$	$g(x)$	$r(x) = \frac{f(x)}{g(x)}$
-4	-262.5	-7	37.5
-3	-78	-4	19.5
-2	-5.5	-1	5.5
-1	-9	2	-4.5
0	-52.5	5	-10.5
1	-100	8	-12.5
2	-115.5	11	-10.5
3	-63	14	-4.5
4	93.5	17	5.5

STEP 1 **The dividend for the ratio, $r(x)$, is the function $f(x) = 6x^3 - 2x^2 - 51.5x - 52.5$, and the divisor is the function $g(x) = 3x + 5$. Determine the type of functions $f(x)$ and $g(x)$ are and predict the type of function that best represents the ratio $r(x)$.**

The equation for $f(x)$ is of the type $ax^3 + bx^2 + cx + d$ and so it is a cubic function of degree three. The equation for $g(x)$ is of the type $ax + b$ and so it is a linear function of degree one. You might predict that the ratio $r(x)$ will be a quadratic function.

STEP 2 **Find the finite differences in the table values for $r(x)$.**

x	-4	-3	-2	-1	0	1	2	3	4
$r(x) = \frac{f(x)}{g(x)}$	37.5	19.5	5.5	-4.5	-10.5	-12.5	-10.5	-4.5	5.5

-18 -14 -10 -6 -2 2 6 10

4 4 4 4 4 4 4

As predicted, the second finite differences are constant and $r(x)$ is a quadratic function of degree two.

STEP 3 **Use the patterns found in the table and the finite differences to determine a function rule.**

$2a = 4$
$a = 2$

$a + b = -2$
$2 + b = -2$
$b = -4$

$c = -10.5$

The function rule for the ratio is the quadratic polynomial $r(x) = 2x^2 - 4x - 10.5$.

ADDITIONAL EXAMPLES

Given the functions $f(x)$ and $g(x)$ and the ratio $r(x) = \frac{f(x)}{g(x)}$, create and use a table of values to write a symbolic representation of the ratio.

1. $f(x) = 5x3 - 6x^2 - 23x - 12$ and $g(x) = 5x + 4$

$r(x) = x^2 - 2x - 3$

2. $f(x) = -3x^3 - 3x^2 + 7x - 1$ and $g(x) = x - 1$

$r(x) = -3x^2 - 6x + 1$

3. $f(x) = -4x^3 - 24x^2 - 43x - 28$ and $g(x) = -2x - 7$

$r(x) = 2x^2 + 5x + 4$

YOU TRY IT! #1

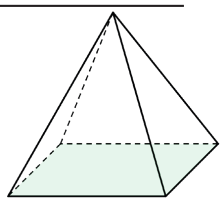
Find the ratio of the functions $q(x) = -2x^3 + x^2 + 190x - 693$ and $p(x) = 2x - 9$ by completing the table of values for $r(x) = \frac{q(x)}{p(x)}$. Determine the type of function that best represents the quotient using the table values for r(x) and write a function rule for the ratio.

x	$q(x)$	$p(x)$	$r(x) = \frac{q(x)}{p(x)}$
0	-693	-9	
1	-504	-7	
2	-325	-5	
3	-168	-3	
4	-45	-1	
5	32	1	
6	51	3	
7	0	5	
8	-133	7	

The ratio is best represented by the quadratic polynomial, r(x) = –x² –4x + 77.

EXAMPLE 2

The ratio of the volume of a square pyramid to the area of its square base is equal to the height of the pyramid. Find the ratio, the height, $h(x)$, if the volume of the pyramid is $V(x) = 48x^3 - 8x^2 - 5x + 1$ and the area of the square is $A(x) = 16x^2 - 8x + 1$. Using the table of values, determine the type of function that represents the height and write its rule.



x	$V(x) = 48x^3 - 8x^2 - 5x + 1$ (cm³)	$A(x) = 16x^2 - 8x + 1$ (cm²)	$h(x) = \frac{V(x)}{A(x)}$ (cm)
0	1	1	1
1	36	9	4
2	343	49	7
3	1210	121	10
4	2925	225	13

612 CHAPTER 5: ALGEBRAIC METHODS

STEP 1 Find the differences in the function values in the table. Notice that the differences in the x-values are a constant, 1.

x	$h(x) = \frac{V(x)}{A(x)}$, in cm
0	1
1	4
2	7
3	10
4	13

$\Delta y = 4 - 1 = 3$
$\Delta y = 7 - 4 = 3$
$\Delta y = 10 - 7 = 3$
$\Delta y = 13 - 10 = 3$

STEP 2 Determine what type of function the ratio is and write its function rule.

Since the first finite differences in the function values are constant, the ratio of the volume and the area is a linear function. The constant difference, a, is 3 and the function value b when $x = 0$ is 1, so $h(x) = ax + b = 3x + 1$.

YOU TRY IT! #2

Given the table for $m(x) = 5x^3 - x^2 - 180x + 36$ and $n(x) = x^2 - 36$, calculate the values for their quotient, $s(x) = \frac{m(x)}{n(x)}$. Use the patterns and differences in those values to determine the degree of the quotient function $s(x)$ and to write the polynomial function for it.

x	$m(x)$	$n(x)$	$s(x) = \frac{m(x)}{n(x)}$
0	36	-36	
1	-140	-35	
2	-288	-32	
3	-378	-27	
4	-380	-20	
5	-264	-11	
6	0	0	

The quotient is best represented by the first-degree linear function, s(x) = 5x – 1, with a domain restriction such that the domain of s(x) is {x | x ∈ ℝ, x ≠ 6}.

ADDITIONAL EXAMPLES

Given the functions $f(x)$ and $g(x)$ and the ratio $r(x) = \frac{f(x)}{g(x)}$, create and use a table of values to write a symbolic representation of the ratio.

1. $f(x) = 3x^3 + 20x^2 + 13x + 6$ and $g(x) = 3x^2 + 2x + 1$

$r(x) = x + 6$

2. $f(x) = -4x^3 + 21x^2 - 13x + 2$ and $g(x) = x^2 - 5x + 2$

$r(x) = -4x + 1$

3. $f(x) = -6x^3 + 11x^2 + 29x - 10$ and $g(x) = -2x^2 + 7x - 2$

$r(x) = 3x + 5$

EXAMPLE 3

Is $q(x) = x^4 + 3x^3 + 27x + 81$ divisible by $l(x) = x + 3$? To answer that, analyze the table values for the quotient $c(x)$ for a constant difference to determine if the quotient is a polynomial. If so, write a function rule for the quotient. Then, look for a pattern in the quotient to see if you can write any other factors of $q(x)$.

x	$q(x)$	$l(x)$	$c(x) = q(x) \div l(x)$
-3	0	0	ERROR
-2	19	1	19
-1	52	2	26
0	81	3	27
1	112	4	28
2	175	5	35
3	324	6	54
4	637	7	91

STEP 1 **Determine the first differences between successive *x*-values and successive *y*-values.**

x	$c(x) = q(x) \div l(x)$
-3	ERROR
-2	19
-1	26
0	27
1	28
2	35
3	54
4	91

First differences: ?, 7, 1, 1, 7, 19, 37

Second differences: ?, -6, 0, 6, 12, 18

Third differences: ?, 6, 6, 6, 6

The constant difference in the x-values is 1. The constant difference in the function values is 6 at the third finite difference. Thus the quotient function $c(x)$ is a cubic polynomial and there is no remainder. This means that $l(x)$ is a factor of $q(x)$.

ADDITIONAL EXAMPLES

Determine if f(x) is divisible by g(x). Use a table of values to write a polynomial for the quotient, $h(x) = \frac{f(x)}{g(x)}$, if possible. Look for a pattern in the quotient for additional factoring so that you can completely factor $h(x)$.

1. $f(x) = 192x^4 - 128x^3 - 3x + 2$ and $g(x) = 3x - 2$

$h(x) = 64x^3 - 1$ which is the difference of cubes and can be rewritten in factored form: $h(x) = (4x - 1)(16x^2 + 4x + 1)$.

2. $f(x) = 4x^4 + 20x^3 + 9x + 45$ and $g(x) = x + 5$

$h(x) = 4x^3 + 9$ which does not have a pattern, so it is fully factored as is.

3. $f(x) = -4x^4 + 17x^3 - 6x^2 - 52x + 56$ and $g(x) = -4x - 7$

$h(x) = x3 - 6x^2 + 12x - 8$ which can be rewritten as $h(x) = x^3 - (3)(2)x^2 + (3)(4)x - (2)^3 = (x - 2)^3$.

614 CHAPTER 5: ALGEBRAIC METHODS

STEP 2 Write the polynomial for the quotient function $c(x)$ using the table. Since $c(x)$ is a quotient, include any domain restrictions that would generate a denominator equal to 0.

$6a = 6$
$a = 1$

$6a + 2b = 6$
$6 + 2b = 6$
$2b = 0$
$b = 0$

$a + b + c = 1$
$1 + 0 + c = 1$
$c = 0$

$d = 27$

$c(x) = 1x^3 + 0x^2 + 0x + 27 = x^3 + 27, \{x \mid x \in \mathbb{R}, x \neq -3\}$

STEP 3 Because $c(x) = \frac{q(x)}{l(x)}$, $q(x) = c(x) \cdot l(x)$ and both $c(x)$ and $l(x)$ are factors of $q(x)$. Determine if there are any other factors by looking for a pattern in $c(x) = x^3 + 27$.

$c(x) = x^3 + 27$ is the sum of cubes, $(x)^3$ and $(3)^3$, which can be written in factored form.

$c(x) = x^3 + 27 = (x + 3)(x^2 - 3x + 9)$

The factors of $q(x) = x^4 + 3x^3 + 27x + 81$ are $(x + 3)$, $(x + 3)$, and $(x^2 - 3x + 9)$.

YOU TRY IT! #3

Determine if $f(x) = x^4 + 14x^3 - 686x - 2401$ is divisible by $g(x) = x - 7$. Use the table of values to write a polynomial for the quotient, $h(x)$, if possible. Look for a pattern in the quotient for additional factoring so that you can completely factor $h(x)$.

See margin.

x	$f(x)$	$g(x)$	$h(x) = f(x) \div g(x)$
0	-2401	-7	343
1	-3072	-6	512
2	-3645	-5	729
3	-4000	-4	1000
4	-3993	-3	1331
5	-3456	-2	1728
6	-2197	-1	2197
7	0	0	ERROR

YOU TRY IT! #3 ANSWER:

The quotient is $h(x) = x^3 + 21x^2 + 147x + 343$ which can be written as $h(x) = x^3 + (3)(7)x^2 + (3)(49)x + (7)^3 = (x + 7)^3$.

Since $h(x) = f(x) \div g(x)$, So $f(x) = h(x) \cdot g(x) = (x + 7)^3(x - 7)$, with a domain restriction such that the domain of $f(x)$ is $\{x \mid x \in \mathbb{R}, x \neq 7\}$.

ADDITIONAL EXAMPLES

For the functions $f(x)$ and $g(x)$, determine a symbolic representation for the quotient $h(x) = \frac{f(x)}{g(x)}$ using a table of values for each.

1. $f(x) = x^4 + 3x^3 - 32x^2 - 108x - 144$ and $g(x) = x^2 - 36$

$h(x) = x^2 + 3x + 4$

2. $f(x) = x^4 - 81$ and $g(x) = x^2 - 9$

$h(x) = x^2 + 9$

3. $f(x) = 2x^4 - 10x^3 - x^2 + 5x - 1$ and $g(x) = 2x^2 - 5x + 1$

$h(x) = x^2 - 1$ which can be rewritten as $h(x) = (x + 1)(x - 1)$.

EXAMPLE 4

Use the table to determine the function rule for the quotient $k(x)$ when the quartic function $j(x) = 0.12x^4 + 1.1x^3 + 0.5x^2 - 10x$ is divided by the quadratic function $m(x) = 0.4x^2 + x - 5$.

x	$j(x)$	$m(x)$	$k(x)$
-3	14.52	-4.4	-3.3
-2	15.12	-5.4	-2.8
-1	9.52	-5.6	-1.7
0	0	-5	0
1	-8.28	-3.6	2.3
2	-7.28	-1.4	5.2
3	13.92	1.6	8.7

STEP 1 **Determine the first differences between successive x-values and the differences between successive y-values.**

x	$j(x)$	$m(x)$	$k(x)$
-3	14.52	-4.4	-3.3
-2	15.12	-5.4	-2.8
-1	9.52	-5.6	-1.7
0	0	-5	0
1	-8.28	-3.6	2.3
2	-7.28	-1.4	5.2
3	13.92	1.6	8.7

1, 1, 1, 1, 1, 1

0.5, 1.1, 1.7, 2.3, 2.9, 3.5

0.6, 0.6, 0.6, 0.6, 0.6

The differences in the x-values are 1 while the second finite differences in the table values for $k(x)$ are constant, 0.6. This means that the quotient is a quadratic polynomial function.

STEP 2 **Using the patterns and differences in the table for $k(x)$, write a function rule.**

$2a = 0.6$
$a = 0.3$

$a + b = 2.3$
$(0.3) + b = 2.3$
$b = 2$

$c = 0$

STEP 3 **Write the function rule for $k(x)$ using the parameters a, b, and c.**

$k(x) = 0.3x^2 + 2x + 0 = 0.3x^2 + 2x$

STEP 4 **Determine any domain restrictions for $k(x)$ by determining values of x that make the divisor, $m(x)$, equal to 0.**

$m(x) = 0.4x^2 + x - 5 = 0$

$5[0.4x^2 + x - 5] = 5[0]$
$2x^2 + 5x - 25 = 0$
$(2x - 5)(x + 5) = 0$
$2x - 5 = 0$ or $x + 5 = 0$
$x = 2.5$ or $x = -5$

$\{x \mid x \in \mathbb{R}, x \neq -5, 2.5\}$

The quotient function $k(x) = 0.3x^2 + 2x$, $\{x \mid x \in \mathbb{R}, x \neq -5, 2.5\}$

616 CHAPTER 5: ALGEBRAIC METHODS

YOU TRY IT! #4

For the functions $f(x) = x^4 + 5x^3 - 125x - 625$ and $g(x) = x^2 - 25$, determine a symbolic representation for the quotient $h(x) = f(x) \div g(x)$ using the table of values for each.

See margin.

x	$f(x)$	$g(x)$	$h(x)$
-6	341	11	31
-5	0	0	ERROR
-4	-189	-9	21
-3	-304	-16	19
-2	-399	-21	19
-1	-504	-24	21
0	-625	-25	25
1	-744	-24	31

PRACTICE/HOMEWORK

For questions 1 – 3, use the following information.

The functions $f(x) = 2x^3 - 5x^2 - 18x + 45$, $g(x) = 2x - 5$, and the ratio $r(x) = \frac{f(x)}{g(x)}$ are represented in the table of values shown.

x	$f(x)$	$g(x)$	$r(x) = \frac{f(x)}{g(x)}$
-5	-240	-15	16
-4	-91	-13	7
-3	0	-11	0
-2	45	-9	-5
-1	56	-7	-8
0	45	-5	-9
1	24	-3	-8
2	5	-1	-5
3	0	1	0
4	21	3	7

1. What type of functions are the divisor, $g(x)$, and the dividend, $f(x)$?
g(x) is linear, and f(x) is cubic.

2. What type of function will the ratio of the two functions produce?
The quotient of a cubic and linear function will produce a quadratic function.

3. Use finite differences to determine a function rule for $r(x)$. Include any domain restrictions.
$r(x) = x^2 - 9$, $\{x \mid x \in \mathbb{R}, x \neq 2.5\}$

YOU TRY IT! #4 ANSWER:

Since $h(x)$ is undefined at $x = -5$ and $x = 5$, there is a domain restriction for $h(x)$ so that the domain of $h(x)$ is $\{x \mid x \in \mathbb{R}, x \neq -5, 5\}$. The quadratic polynomial function is $h(x) = x^2 + 5x + 25$ with a domain of $\{x \mid x \in \mathbb{R}, x \neq -5, 5\}$.

4.

x	$q(x)$	$p(x)$	$r(x) = \frac{q(x)}{p(x)}$
-2	-3	-3	***1***
-1	-2	-2	***1***
0	-9	-1	***9***
1	0	0	***undefined***
2	49	1	***49***
3	162	2	***81***
4	363	3	***121***
5	676	4	***169***
6	1125	5	***225***

For questions 4 – 6, use the following information.

The functions $q(x) = 4x^3 + 8x^2 - 3x - 9$ and $p(x) = x - 1$ are represented in the table of values shown.

x	$q(x)$	$p(x)$	$r(x) = \frac{q(x)}{p(x)}$
-2	-3	-3	
-1	-2	-2	
0	-9	-1	
1	0	0	
2	49	1	
3	162	2	
4	363	3	
5	676	4	
6	1125	5	

4. Complete the table of values for $r(x) = \frac{q(x)}{p(x)}$.
See margin.

5. Determine the type of function that best represents the ratio of $q(x)$ and $p(x)$.
The function of the ratio will be quadratic.

6. Use the table of values to write a symbolic representation of the ratio. Include any domain restrictions.
The ratio is best represented by the quadratic polynomial $r(x) = 4x^2 + 12x + 9$, $\{x \mid x \in \mathbb{R}, x \neq 1\}$

For questions 7 – 10, use the following information.

GEOMETRY

The ratio of the volume of a rectangular prism to the area of its base is equal to the height of the rectangular prism.

The table shows values of a rectangular prism with a volume of $V(x) = x^3 + 2x^2 - x - 2$. The area of its base is represented by $B(x) = x^2 - 1$.

x	$V(x) = x^3 + 2x^2 - x - 2$ (cm³)	$B(x) = x^2 - 1$ (cm²)	$h(x) = \frac{V(x)}{B(x)}$ (cm)
1	0	0	undefined
2	12	3	4
3	40	8	5
4	90	15	6
5	168	24	7
6	280	35	8

7. Determine the type of function that best represents the prism's height, $h(x)$.
The height is best represented by a linear function.

8. What is the function rule for $h(x)$? Include any domain restrictions.
$h(x) = x + 2$, $\{x \mid x \in \mathbb{R}, x \neq 1\}$

9. If the ratio of volume to area of the base is 15, what is the height of the prism?
Since the height of the prism is equivalent to the ratio of volume to area, the height is 15 cm.

10. If the height of the prism is 18 cm, what is its volume?
A height of 18 gives a value of 16 for x. Substitute 16 for x in the function for the volume, and the result is a volume of 4590 cm³.

618 CHAPTER 5: ALGEBRAIC METHODS

For questions 11 – 13, use the following information.

The functions $m(x) = 3x^3 - 11x^2 - 56x - 48$ and $n(x) = x^2 - 5x - 12$ are represented in the table of values below.

x	$m(x)$	$n(x)$	$s(x) = \frac{m(x)}{n(x)}$
-4	-192	24	
-3	-60	12	
-2	-4	2	
-1	-6	-6	
0	-48	-12	
1	-112	-16	
2	-180	-18	

11. Determine the table values for the quotient function, $s(x) = \frac{m(x)}{n(x)}$.
See margin.

12. Determine the degree of the quotient function, and write a polynomial function for it. Include any domain restrictions.
See margin.

13. What is the value of $s(x)$ when $x = 10$?
When x = 10, s(x) = 34.

For questions 14 – 15, use the following information.

Values from three functions are shown in the table.

x	$j(x)$	$m(x)$	$k(x) = \frac{j(x)}{m(x)}$
2	-35	-35	1
3	-144	-36	4
4	-297	-33	9
5	-416	-26	16
6	-375	-15	25
7	0	0	ERROR
8	931	19	49

14. Use the values in the table to determine the function rule for the quotient $k(x)$ when the quartic function $j(x) = 2x^4 - 15x^3 + 3x^2 + 31x - 21$ is divided by the quadratic function $m(x) = 2x^2 - 11x - 21$. Include any domain restrictions.
$k(x) = x^2 - 2x + 1, \{x \mid x \in \mathbb{R}, x \neq -\frac{3}{2}, 7\}$

15. What is the value of $k(x)$, when $x = 10$?
When x = 10, k(x) = 81.

11.

x	$s(x) = \frac{m(x)}{n(x)}$
-4	-8
-3	-5
-2	-2
-1	1
0	4
1	7
2	10

12. *The quotient is best represented by the first-degree linear function $s(x) = 3x + 4$, $\{x \mid x \in \mathbb{R}, x \neq -1.77, 6.77\}$.*

For question 16, use the following information.

The functions $f(x) = x^4 + 9x^2 + 20$ and $g(x) = x^2 + 5$ are represented in the table shown.

x	$f(x)$	$g(x)$	$h(x)$
-2	72	9	8
-1	30	6	5
0	20	5	4
1	30	6	5
2	72	9	8
3	182	14	13
4	420	21	20

16. Use the table of values to determine a symbolic representation for the quotient $h(x) = f(x) \div g(x)$.
$h(x) = x^2 + 4$

For questions 17 – 19, use the following information.

The functions $q(x) = 3x^4 + 14x^3 - 32x^2 - 126x + 45$, $m(x) = 3x^2 + 14x - 5$, and $c(x) = q(x) \div m(x)$ are represented in the table.

x	$q(x)$	$m(x)$	$c(x) = q(x) \div m(x)$
-2	105	-21	-5
-1	128	-16	-8
0	45	-5	-9
1	-96	12	-8
2	-175	35	-5
3	0	64	0
4	693	99	7
5	2240	140	16

17. What types of functions are $q(x)$, $m(x)$, and $c(x)$?
q(x) is quartic; m(x) is quadratic, and c(x) is also quadratic.

18. If $q(x)$ is divisible by $m(x)$, their quotient is a polynomial. Write the polynomial for the quotient $c(x)$ using the values in the table. Include any domain restrictions.
$c(x) = x^2 - 9$, $\{x \mid x \in \mathbb{R}, x \neq \frac{1}{3}, -5\}$

19. Look for a pattern in the quotient to see if you can write any other factors of $q(x)$.
The quotient $c(x) = x^2 - 9$. This can be written as $c(x) = (x + 3)(x - 3)$. So, $q(x) = (x + 3)(x - 3)(3x - 1)(x + 5)$.

For questions 20 – 23, use the following information.

The functions $f(x) = x^3 + 7x^2 - 5x - 75$, $g(x) = x - 3$, and $h(x) = f(x) \div g(x)$ are represented in the table below.

x	$f(x)$	$g(x)$	$h(x) = f(x) \div g(x)$
0	-75	-3	25
1	-72	-2	36
2	-49	-1	49
3	0	0	ERROR
4	81	1	81
5	200	2	100
6	363	3	121

20. What types of functions are $f(x)$, $g(x)$, and $h(x)$?
f(x) is cubic; g(x) is linear, and h(x) is quadratic.

21. If $f(x)$ is divisible by $g(x)$, their quotient is a polynomial. Write the polynomial for the quotient $h(x)$ using the values in the table.
See margin.

21. *$h(x) = x^2 + 10x + 25$, with a domain restriction such that the domain of $h(x)$ is $\{x \mid x \in \mathbb{R}, x \neq 3\}$.*

22. So, is $f(x)$ divisible by $g(x)$?
Yes. There is no remainder when dividing f(x) by g(x).

23. Look for a pattern in the quotient to see if you can write any other factors of $f(x)$.
The quotient is $h(x) = x^2 + 10x + 25$. This can also be written as $h(x) = (x + 5)^2$. So, $f(x) = (x - 3)(x + 5)^2$.

620 CHAPTER 5: ALGEBRAIC METHODS