Factoring Polynomials with Algebraic Methods

5.7

FOCUSING QUESTION How can you use algebraic methods to identify the linear factors of polynomial functions?

LEARNING OUTCOMES

- I can factor polynomials using algebraic methods.
- I can use symbols to communicate mathematical ideas and their implications.

ENGAGE

Use algebra tiles to represent and factor the polynomial $x^2 - 8x + 15$. Sketch a diagram of your tiles.

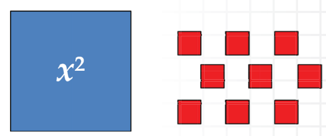
See margin.

EXPLORE

Quadratic polynomials (degree two) and cubic polynomials (degree three) can be factored algebraically in many ways. Certain factoring formulas can be written based on patterns found in the terms of the polynomials.

SUMS AND DIFFERENCES OF SQUARES

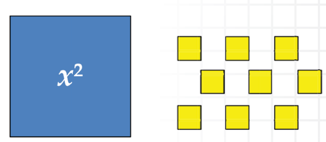
1. The algebra tiles shown represent the polynomial $x^2 - 9$. If possible, add zero pairs of x-tiles so that you can arrange the set of tiles to make a square. Sketch a diagram of your tiles. If it is not possible, explain why.
See margin.



2. Based on your tile model, what are the factors of $x^2 - 9$? Sketch a diagram of your tiles. Explain your reasoning.
The factors are $(x - 3)$ and $(x + 3)$ because those are the dimensions of the square created by the algebra tiles.

3. Use algebra tiles to model the factors of $x^2 - 16$ and $x^2 - 25$. Sketch a diagram of your tiles. What patterns do you notice between the factors of the polynomials and the original polynomials?
See margin.

4. The algebra tiles shown represent the polynomial $x^2 + 9$. If possible, add zero pairs of x-tiles so that you can arrange the set of tiles to make a square. Sketch a diagram of your tiles. If it is not possible, explain why.
See margin.



TEKS

AR.4D Determine the linear factors of a polynomial function of degree two and of degree three when represented symbolically and tabularly and graphically where appropriate.

MATHEMATICAL PROCESS SPOTLIGHT

AR.1D Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

ELPS

2D Monitor understanding of spoken language during classroom instruction and interactions and seek clarification as needed.

VOCABULARY

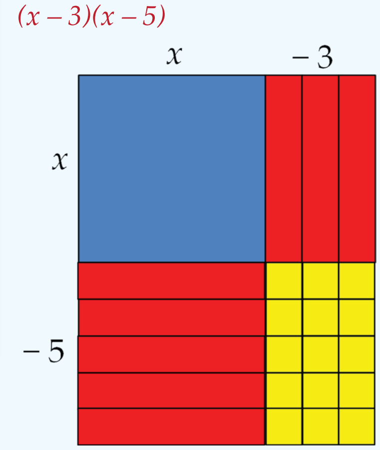
linear, quadratic, cubic, zero, factor, product

MATERIALS

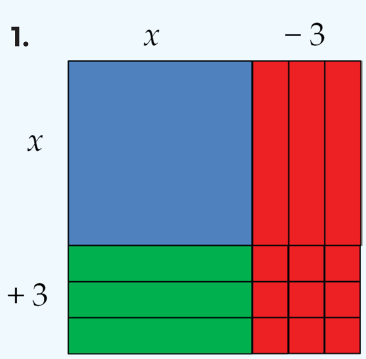
- graphing technology

ENGAGE ANSWER:

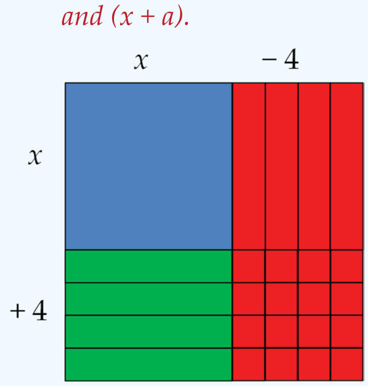
$(x - 3)(x - 5)$

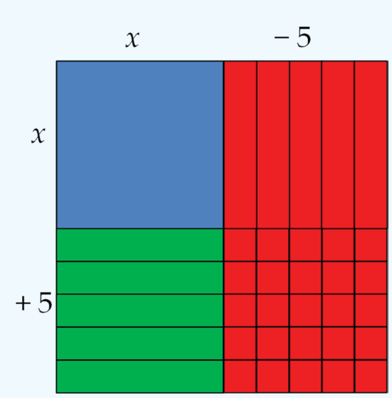


1.

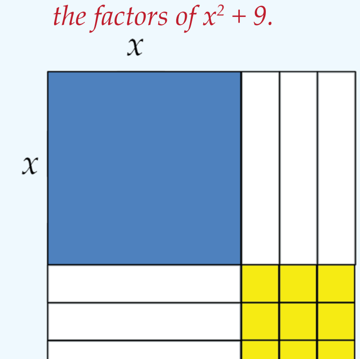


3. *The factors of the original polynomial, $x^2 - a^2$, are $(x - a)$ and $(x + a)$.*





4. *It is not possible to use algebra tiles to represent the factors of $x^2 + 9$.*



5. *There are no real factors of $x^2 + 9$ because if you take factors of +9 to represent the algebra tiles, you cannot get the x-tiles to cancel out. In order to have x-tiles cancel out through zero pairs, the signs must be different, which would generate a product of −9, not +9.*

INTEGRATING TECHNOLOGY

Graph quadratic functions such as $y = x^2 - 1$, $y = x^2 - 4$, $y = x^2 - 9$, or $y = (2x)^2 - 9$. Ask students what patterns they notice in the graphs. Responses should include that each graph has two x-intercepts that are the same distance from the origin (e.g., (1, 0) and (−1, 0), (2, 0) and (−2, 0), etc.).

Graph quadratic functions such as $y = x^2 + 1$, $y = x^2 + 4$, or $y = (2x)^2 + 9$. Ask students what patterns they notice in the graphs. Responses should include that each graph does not have an x-intercept because the range values are all greater than 0.

Ask students to use graphs to explain why binomials of the form $a^2 - b^2$ factor but binomials of the form $a^2 + b^2$ do not factor.

7. *Purple prism:*
$V = a(a)(a - b) = a^2(a - b)$

Green prism:
$V = b(a - b)(a) = ab(a - b)$

Orange prism:
$V = b(a - b)(b) = b^2(a - b)$

8. *Total volume = purple volume + green volume + orange volume*

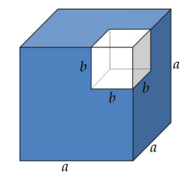
Total volume =
$a^2(a - b) + ab(a - b) + b^2(a - b)$

Total volume =
$(a - b)(a^2 + ab + b^2)$

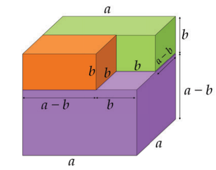
5. Based on your tile model, what are the factors of $x^2 + 9$? Explain your reasoning.
See margin.

SUMS AND DIFFERENCES OF CUBES

6. The figure shows a cube with side length a and a volume of a^3. One corner of this cube has a cube with the side length b and a volume of b^3 removed. Write an expression for the volume of the resulting composite figure.
$a^3 - b^3$



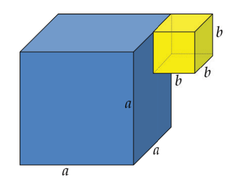
7. The resulting composite figure can be broken into three rectangular prisms. The dimensions of each prism are shown. Use the dimensions to write an expression for the volume of each prism.
See margin.



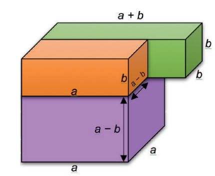
8. Use the expressions for each rectangular prism to write an expression for the total volume of the composite figure. Factor out any common factors and simplify your expression.
See margin.

9. Write an equation that shows how your expressions from Question 6 and Question 8 are related. This equation represents the difference between the volumes of two cubes.
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

10. The figure shows a cube with side length a and a volume of a^3. A cube with the side length b and a volume of b^3 has been attached to the first cube so that their vertices coincide. Write an expression for the volume of the resulting composite figure.
$a^3 + b^3$



11. The resulting composite figure can be broken into three rectangular prisms. The dimensions of each prism are shown. Use the dimensions to write an expression for the volume of each prism.
See margin.



12. Combine your expressions for the purple prism and orange prism. Factor out the common factor $(a - b)$ then factor out any remaining common factors from the resulting binomial.
See margin.

13. Combine your expression for the purple and orange prisms with the green prism. Factor out any common factors among the terms and simplify the expression. This expression represents the combined volume of all three prisms, which is the volume of the composite figure.
See margin.

11. *Purple prism:*
$V = a(a)(a - b) = a^2(a - b)$

Orange prism:
$V = a(a - b)(b) = ab(a - b)$

Green prism:
$V = b(b)(a + b) = b^2(a + b)$

12. *Purple prism + orange prism*
$= a^2(a - b) + ab(a - b)$
$= (a^2 + ab)(a - b)$
$= a(a + b)(a - b)$

13. *Total volume = purple volume + orange volume + green volume*

Total volume =
$a(a + b)(a - b) + b^2(a + b)$

Total volume =
$[a(a - b) + b^2](a + b)$

Total volume =
$[a^2 - ab + b^2](a + b)$

Total volume =
$(a + b)(a^2 - ab + b^2)$

SUPPORTING ENGLISH LANGUAGE LEARNERS

Encourage students to work in pairs and explain to each other how they are calculating the volume of each prism. Monitor students (**ELPS 2D**) to make sure that they are demonstrating understanding of spoken language, including academic vocabulary such as length, width, height, base of the prism, volume, etc.

14. Write an equation that shows how your expressions from Question 10 and Question 13 are related. This equation represents the sum of the volumes of two cubes.
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

REFLECT

- **How are the formulas for factoring the sums of cubes and the differences of cubes alike? How are they different?**
  See margin.

- **How is the formula for factoring the differences of cubes similar to the formula for factoring the differences of squares? How is it different?**
  See margin.

EXPLAIN

Not all quadratic or cubic polynomials can be factored. But for those that can, there are special patterns that you can use to determine linear factors of quadratic (degree two) or cubic (degree three) polynomial functions.

Watch Explain and You Try It Videos

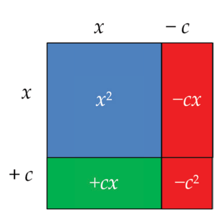
or click here

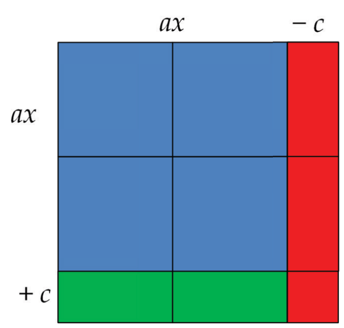
General Form		Factored Form
$f(x) = 3x^2 + 2x - 8$	$\leftrightarrow$	$f(x) = (3x - 4)(x + 2)$
Polynomial Form		

QUADRATIC FUNCTIONS: DIFFERENCES OF SQUARES

If a quadratic function is of the form $f(x) = (ax)^2 - c^2$, then it is a difference of two squared quantities: ax and c. In factored form, the function can be written as $f(x) = (ax - c)(ax + c)$.

$$f(x) = (ax)^2 - c^2 = (ax - c)(ax + c)$$





REFLECT ANSWERS:

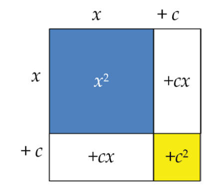
Both formulas include a binomial and a related trinomial. The terms (without their signs) of both formulas are the same. The operation between a and b in the binomial factor is the same as the operation between a^3 and b^3 in the original polynomial. The sign of the ab term in the trinomial factor is the opposite of the operation between a^3 and b^3 in the original polynomial.

Both formulas include a binomial factor, a – b, that is the difference between the numbers being cubed or squared. However, the second factor for squares is another binomial and the second factor for cubes is a trinomial.

Differences of squares is a special pattern because when the linear factors are multiplied together, the x-terms are equal with opposite signs, so the additive inverse property of real numbers makes their sum 0. With algebra tiles, the additive inverse property is used with zero pairs that cancel out since their sum is 0.

QUADRATIC FUNCTIONS: SUMS OF SQUARES

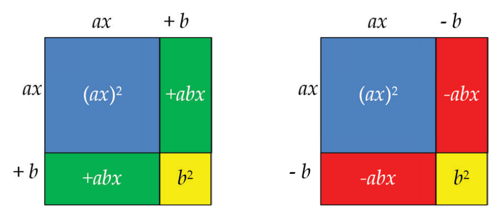
If a quadratic function is of the form $f(x) = (ax)^2 + c^2$, then it does not have linear factors whose coefficients are real numbers. Because c^2 is a positive number, the acx terms will have the same sign and will not be zero pairs that cancel to 0. There are no real numbers that can be used to generate $(ax)^2 + c^2$ from two linear factors.



QUADRATIC FUNCTIONS: PERFECT SQUARE TRINOMIALS

Another factoring pattern with quadratic functions is perfect square trinomials, which are trinomials with two terms that are perfect squares and a middle term that is 2 times the product of the square terms.

$$f(x) = (ax)^2 + 2abx + b^2 = (ax + b)^2$$
$$f(x) = (ax)^2 - 2abx + b^2 = (ax - b)^2$$



CUBIC FUNCTIONS: DIFFERENCES OF CUBES

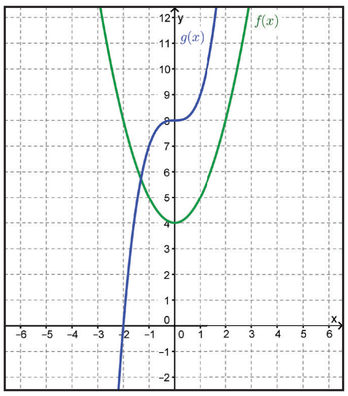
If a cubic function is of the form $f(x) = (ax)^3 - c^3$, then it can be written as two factors: a binomial linear factor and a trinomial quadratic factor.

$$f(x) = (ax)^3 - c^3 = (ax - c)[(ax)^2 + acx + c^2]$$

CUBIC FUNCTIONS: SUMS OF CUBES

If a cubic function is of the form $f(x) = (ax)^3 + c^3$, then it can be written as two factors: a binomial linear factor and a trinomial quadratic factor.

$$f(x) = (ax)^3 + c^3 = (ax + c)[(ax)^2 - acx + c^2]$$



It is worth noting that while a binomial that is the sum of two squares does not factor, a binomial that is the sum of two cubes does. To figure out why, look at the graphs of $f(x) = x^2 + 4$ and $g(x) = x^3 + 8$.

Notice that $f(x)$ does not have any x-intercepts. That means there are no real numbers for which the function is zero. Since factors are related to the zeroes of the function, there are no linear factors for $f(x)$.

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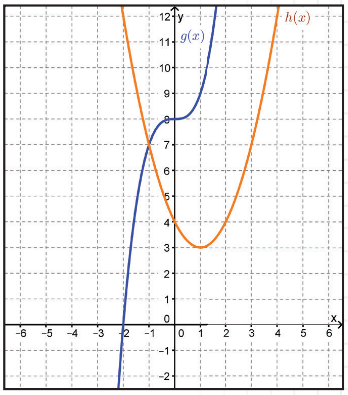
The function $g(x)$, has two components. A linear factor of $x + 2$ and a quadratic factor of $x^2 - 2x + 4$. The graph of $g(x)$ has one x-intercept at $(-2, 0)$, which makes sense knowing that $g(x)$ has one linear factor. The x-coordinate of the x-intercept matches the zero revealed by the linear factor, $x + 2$.



The second component of $g(x)$ is the quadratic trinomial. If you graph this trinomial as $h(x) = x^2 - 2x + 2$, you will notice that it has no x-intercepts. Hence, the quadratic trinomial does not further break down into additional linear factors.

POLYNOMIAL FUNCTIONS: FACTORING BY GROUPING

Factoring by grouping is a technique that extends factoring by common factors. Terms are grouped by like factors and the common factors are then factored out. Grouping is particularly useful for polynomials when $a \neq 1$.

$f(x) = 4x^2 + 7x - 15$

$f(x) = 4x^2 + 12x - 5x - 15$	**If necessary, split the middle term into two parts so that there are four terms.**	$g(x) = 2x^3 - 4x^2 + 7x - 14$
$f(x) = 4x(x + 3) - 5(x + 3)$	**Factor out the common factor in the first two terms and the last two terms.**	$g(x) = 2x^2(x - 2) + 7(x - 2)$
$f(x) = (4x - 5)(x + 3)$	**Factor out the common binomial from each pair of terms.**	$g(x) = (2x^2 + 7)(x - 2)$

FACTORING POLYNOMIAL FUNCTIONS WITH ALGEBRAIC METHODS

Polynomial functions can be factored, or written as a product of linear factors, by a variety of algebraic methods. Some types of polynomial functions have patterns that make certain algebraic methods particularly useful. Not all polynomial functions factor, but for those that do, an algebraic method can be used to identify the linear factors of the polynomial function.

- In any polynomial function, look for common factors in every term. If there are any, factor those out first.
- For quadratic functions, there are certain patterns to look for:
  - ✔ Difference of Squares: $a^2 - b^2 = (a - b)(a + b)$
  - ✔ Perfect Square Trinomials: $a^2 + 2ab + b^2 = (a + b)^2$ or $a^2 - 2ab + b^2 = (a - b)^2$

QUESTIONING STRATEGY

Students learned to factor polynomials in previous courses. Check for understanding of factoring $f(x)$ by asking "how did we decide to split $7x$ into $12x$ and $-5x$?"

FACTORING POLYNOMIAL FUNCTIONS WITH ALGEBRAIC METHODS

- For cubic functions, there are certain patterns to look for:
  - Difference of Cubes: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
  - Sum of Cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
  - Perfect Cube Polynomials (four terms):
    $a^3 + 3a^2b + 3ab^2 + b^3 = (a + b)^3$
    $a^3 - 3a^2b + 3ab^2 - b^3 = (a - b)^3$
- Polynomial functions of degree two or degree three can also be factored by grouping.
  - If necessary, split the middle term into two parts. Each part should contain factors of the first or last term.
  - For the first two terms and the last two terms, identify and factor out a common factor between the two terms.
  - Factor out the common binomial between the two pairs of factored terms.

EXAMPLE 1

Factor the quadratic function $q(x) = 9x^2 - 12x + 4$ using an algebraic method.

STEP 1 **Look for common factors in every term. If there are any, factor those out first. In this case, there is no factor that is common to every term.**

STEP 2 **Since the function is quadratic and has three terms, determine whether the function is a perfect square trinomial.**

If a quadratic function is a perfect square trinomial, it will follow the pattern $a^2 + 2ab + b^2 = (a + b)^2$ or $a^2 - 2ab + b^2 = (a - b)^2$. In this case, the coefficient of the x-term is negative, so look for the pattern $a^2 - 2ab + b^2$.

$q(x) = 9x^2 - 12x + 4$
$9x^2 = (3x)^2 \rightarrow a = 3x$
$4 = 2^2 \rightarrow b = 2$
$-2ab = -2(3x)(2) = -12x$

Therefore, $q(x)$ is a perfect square trinomial.

STEP 3 **Use the pattern for perfect square trinomials to rewrite the quadratic function.**

$q(x) = 9x^2 - 12x + 4$
$q(x) = (3x)^2 - 2(3x)(2) + 2^2$
$q(x) = (3x - 2)^2$

ADDITIONAL EXAMPLES

Factor the quadratic functions below using an algebraic method.

1. $f(x) = 25x^2 + 30x + 9$

Since f(x) is a perfect square trinomial, it can be rewritten as $f(x) = (5x + 3)^2$.

2. $g(x) = 16x^2 + 49$

While g(x) is the sum of two squares, it is not factorable and cannot be rewritten.

3. $h(x) = 144x^2 - 81$

Since h(x) is the difference of two squares, it can be rewritten as $h(x) = (12x - 9)(12x + 9)$.

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YOU TRY IT! # 1

Factor the quadratic function $d(x) = 64x^2 - 121$ using an algebraic method.

Since d(x) is the difference of two squares, it can be rewritten as d(x) = (8x – 11)(8x + 11).

EXAMPLE 2

Factor the cubic function $w(x) = 8x^3 - 125$ using an algebraic method.

STEP 1 Look for common factors in every term. If there are any, factor those out first. In this case, there is no factor that is common to every term.

STEP 2 Since the function is cubic, has two terms, and a negative constant term, determine whether the function is the difference of cubes.

If a cubic function is the difference of cubes, it will follow the pattern $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.

$w(x) = 8x^3 - 125$
$8x^3 = (2x)^3 \rightarrow a = 2x$
$125 = 5^3 \rightarrow b = 5$

Therefore, $w(x)$ is the difference of cubes.

STEP 3 Use the pattern for the difference of cubes to rewrite the cubic function.

$w(x) = 8x^3 - 125$
$w(x) = (2x)^3 - 5^3$
$w(x) = (2x - 5)((2x)^2 + (2x)(5) + 5^2)$
$w(x) = (2x - 5)(4x^2 + 10x + 25)$

YOU TRY IT! #2

Factor the cubic function $f(x) = 27x^3 + 1$ using an algebraic method.

Since f(x) is the sum of cubes, it can be rewritten as f(x) = (3x + 1)(9x² – 3x + 1).

ADDITIONAL EXAMPLES

Factor the cubic functions below using an algebraic method.

1. $m(x) = x^3 - 12x^2 + 48x - 64$

Since m(x) is a perfect cube polynomial, it can be rewritten as $m(x) = (x - 4)^3$.

2. $n(x) = 125x^3 + 512$

Since n(x) is the sum of cubes, it can be rewritten as $n(x) = (5x + 8)(25x^2 - 40x + 64)$.

3. $p(x) = 8x3 + 36x^2 + 54x + 27$

Since p(x) is a perfect cube polynomial, it can be rewritten as $p(x) = (2x + 3)^3$.

EXAMPLE 3

Factor the quadratic function $v(x) = 2x^2 - 11x - 21$ using grouping.

STEP 1 **Look for common factors in every term. If there are any, factor those out first. In this case, there is no factor that is common to every term.**

STEP 2 **Split the middle term into two parts that, when added, result in the middle term. Each part should contain factors of the first or last term.**

Additionally, one more relationship between the middle term and the first and last terms of a factorable trinomial will help you factor by grouping. If a quadratic function is a factorable trinomial, it can be written as $v(x) = (ax + b)(cx + d)$.

$v(x) = (ax + b)(cx + d) = acx^2 + adx + bcx + bd$

Thus, $v(x)$ represents the factorable trinomial written with four terms, that is, with its middle term split into two terms. Notice what happens when you multiply the two middle terms and when you multiply the first and last terms:

$(adx)(bcx) = abcdx^2$ and $(acx^2)(bd) = abcdx^2$

The two products are the same. Therefore, not only must the two terms into which we split the middle term add up to the middle term, they must result in the same product as the product of the first and last terms of the factorable trinomial!

In this case, the middle term is $-11x$. The two terms we split this term into must add up to $-11x$.

The first term is $2x^2$. The last term is -21. The product of these terms is $-42x^2$. The two terms we split $-11x$ into must multiply to equal $-42x^2$.

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FIRST OF TWO TERMS	SECOND OF TWO TERMS	SUM OF TERMS	DO THE TERMS ADD TO $-11x$?	PRODUCT OF TERMS	DO THE TERMS MULTIPLY TO $-42x^2$?
$-6x$	$7x$	x	NO	$-42x^2$	YES
$-9x$	$-2x$	$-11x$	YES	$18x^2$	NO
$-10x$	$-x$	$-11x$	YES	$10x^2$	NO
$-12x$	x	$-11x$	YES	$-12x^2$	NO
$-14x$	$3x$	$-11x$	YES	$-42x^2$	YES

$v(x) = 2x^2 - 11x - 21$
$v(x) = 2x^2 - 14x + 3x - 21$

STEP 3 **For the first two terms and the last two terms, identify and factor out a common factor between the two terms.**

The common factor between $2x^2$ and $-14x$ is $2x$.
The common factor between $3x$ and -21 is 3.

$v(x) = 2x^2 - 14x + 3x - 21$
$v(x) = 2x(x - 7) + 3(x - 7)$

Step 4 **Factor out the common binomial between the two pairs of factored terms.**

$v(x) = 2x(x - 7) + 3(x - 7)$
$v(x) = (x - 7)(2x + 3)$

YOU TRY IT! #3

Factor the quadratic function $h(x) = x^2 - 11x - 60$ using grouping.

$h(x) = (x + 4)(x - 15)$

ADDITIONAL EXAMPLES

Factor the quadratic functions below by grouping.

1. $q(x) = x^2 - 16x + 60$

$q(x) = (x - 1)(x - 6)$

2. $r(x) = 3x^2 + 34x + 63$

$r(x) = (3x + 7)(x + 9)$

3. $s(x) = 2x^2 - 20x + 48$

$s(x) = (2x - 8)(x - 6)$

or $s(x) = (2x - 12)(x - 4)$

ADDITIONAL EXAMPLES

Factor the cubic functions below using grouping.

1. $t(x) = 2x^3 + 2x^2 - 5x - 5$

$t(x) = (x + 1)(2x^2 - 5)$

2. $u(x) = 6x^3 + 9x^2 + 14x + 21$

$u(x) = (2x + 3)(3x^2 + 7)$

3. $v(x) = 28x^3 - 8x^2 - 7x + 2$

$v(x) = (7x - 2)(4x^2 - 1) =$
$(7x - 2)(2x - 1)(2x + 1)$

EXAMPLE 4

Factor the cubic function $p(x) = x^3 + 5x^2 - 9x - 45$ using grouping.

STEP 1 Look for common factors in every term. If there are any, factor those out first. In this case, there is no factor that is common to every term.

STEP 2 For the first two terms and the last two terms, identify and factor out a common factor between the two terms.

The common factor between x^3 and $5x^2$ is x^2.
The common factor between $-9x$ and -45 is -9.

$p(x) = x^3 + 5x^2 - 9x - 45$
$p(x) = x^2(x + 5) - 9(x + 5)$

STEP 3 Factor out the common binomial between the two pairs of factored terms.

$p(x) = x^2(x + 5) - 9(x + 5)$
$p(x) = (x + 5)(x^2 - 9)$

STEP 4 If it is possible to do so, completely factor the remaining quadratic term.

In this case, the factor $x^2 - 9$ is a difference of squares.
$p(x) = (x + 5)(x^2 - 9)$
$p(x) = (x + 5)(x - 3)(x + 3)$

YOU TRY IT! #4

Factor the cubic function $g(x) = 3x^3 - 12x^2 + 8x - 32$ using grouping.

$g(x) = (x - 4)(3x^2 + 8)$

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PRACTICE/HOMEWORK

For questions 1 – 5, factor the quadratic function using an algebraic method. For questions 1 – 2, draw a diagram of algebra tiles that could be used to model the factored function.

1. $f(x) = x^2 + 8x + 16$
$f(x) = (x + 4)^2$

2. $g(x) = 4x^2 - 25$
$g(x) = (2x + 5)(2x - 5)$

3. $h(x) = 9x^2 + 30x + 25$
$h(x) = (3x + 5)^2$

4. $p(x) = \frac{1}{4}x^2 + 7x + 49$
$p(x) = (\frac{1}{2}x + 7)^2$

5. $q(x) = \frac{1}{16}x^2 - 9$
$q(x) = (\frac{1}{4}x + 3)(\frac{1}{4}x - 3)$

For questions 6 – 10, factor the cubic function using an algebraic method.

6. $f(x) = 8x^3 + 1$
$f(x) = (2x + 1)(4x^2 - 2x + 1)$

7. $g(x) = 27x^3 - 8$
$g(x) = (3x - 2)(9x^2 + 6x + 4)$

8. $h(x) = x^3 + 64$
$h(x) = (x + 4)(x^2 - 4x + 16)$

9. $p(x) = \frac{1}{8}x^3 - 1$
$p(x) = (\frac{1}{2}x - 1)(\frac{1}{4}x^2 + \frac{1}{2}x + 1)$

10. $q(x) = 8x^3 + \frac{1}{27}$
$q(x) = (2x + \frac{1}{3})(4x^2 - \frac{2}{3}x + \frac{1}{9})$

For questions 11 –15, factor the quadratic function using grouping.

11. $f(x) = 2x^2 + 3x + 1$
$f(x) = (2x + 1)(x + 1)$

12. $g(x) = 3x^2 + 5x + 2$
$g(x) = (3x + 2)(x + 1)$

13. $h(x) = 4x^2 + x - 5$
$h(x) = (4x + 5)(x - 1)$

14. $p(x) = 6x^2 + 13x + 6$
$p(x) = (2x + 3)(3x + 2)$

15. $q(x) = 20x^2 + 31x - 7$
$q(x) = (4x + 7)(5x - 1)$

For questions 16 –20, factor the cubic function using grouping.

16. $f(x) = 4x^3 + 12x^2 - 9x - 27$
$f(x) = (2x + 3)(2x - 3)(x + 3)$

17. $g(x) = 8x^3 + 4x^2 + 6x + 3$
$g(x) = (2x + 1)(4x^2 + 3)$

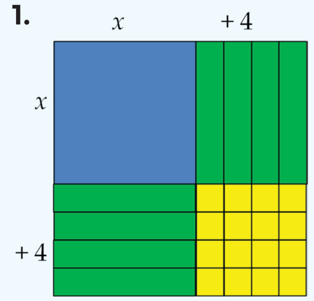
18. $h(x) = 4x^3 - 5x^2 - 100x + 125$
$h(x) = (4x - 5)(x + 5)(x - 5)$

19. $p(x) = 18x^3 + 12x^2 + 15x + 10$
$p(x) = (3x + 2)(6x^2 + 5)$

20. $q(x) = 45x^3 - 9x^2 - 80x + 16$
$q(x) = (5x - 1)(3x + 4)(3x - 4)$

ADDITIONAL ANSWERS:

1.



2.