Factoring Polynomials with Graphs and Tables

5.6

FOCUSING QUESTION How can you use tables and graphs to identify the linear factors of polynomial functions?

LEARNING OUTCOMES

- I can factor polynomials using graphs and tables.
- I can use graphs and tables to communicate mathematical ideas and their implications.

ENGAGE

The area of a rectangular painting is 72 square inches. What are some possible dimensions of the painting?

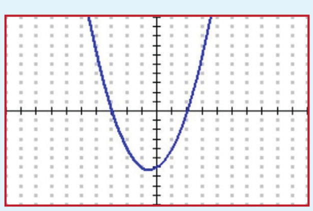
See margin.

Across the Delaware, Robert Spencer, Wikimedia Commons

EXPLORE

There are relationships among the x-intercepts and factors of polynomial functions. In this section, you will investigate those relationships using graphs and tables.

1. For each of the functions below, use your graphing technology to generate a graph and table. Sketch the graph and record some of the function values in a table like the one shown for each function.

FUNCTION	GRAPH	TABLE: x	TABLE: $f(x)$	x-INTERCEPT(S)
$f(x) = x^2 + x - 6$		−4	6	(−3, 0) (2, 0)
		−3	0	
		−2	−4	
		−1	−6	
		0	−6	
		1	−4	
		2	0	
		3	6	

TEKS

AR.4D Determine the linear factors of a polynomial function of degree two and of degree three when represented symbolically and tabularly and graphically where appropriate.

MATHEMATICAL PROCESS SPOTLIGHT

AR.1D Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

ELPS

4G Demonstrate comprehension of increasingly complex English by participating in shared reading, retelling or summarizing material, responding to questions, and taking notes commensurate with content area and grade level needs.

VOCABULARY

linear, quadratic, cubic, zero, factor, product

MATERIALS

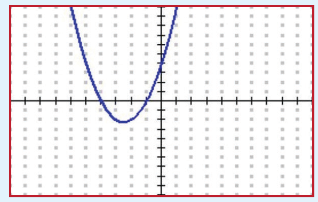
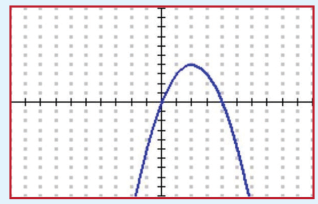
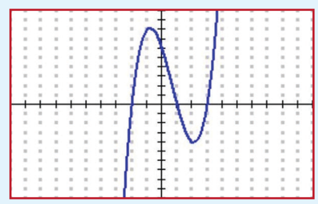
- graphing technology

ENGAGE ANSWER:

Possible dimensions include:
1 inch by 72 inches
2 inches by 36 inches
3 inches by 24 inches
4 inches by 18 inches
6 inches by 12 inches
7.2 inches by 10 inches
8 inches by 9 inches

INTEGRATING TECHNOLOGY

Use graphing technology, such as a graphing calculator, to graph the function. Then use the trace feature to identify x-intercepts. For example, for $f(x)$, it appears that the graph has an x-intercept that is close to (2, 0), so enter an x-value of 2 to confirm that $f(x) = 0$. Some graphing technologies also have a feature to calculate the zero of a function, which is the x-coordinate of the x-intercept.

FUNCTION	GRAPH	TABLE		x-INTERCEPT(S)
$g(x) = x^2 + 5x + 4$		x	$g(x)$	(−4, 0) (−1, 0)
		−6	10	
		−5	4	
		−4	0	
		−3	−2	
		−2	−2	
		−1	0	
		0	4	
		1	10	
$h(x) = -x^2 + 4x$		x	$h(x)$	(0, 0) (4, 0)
		−2	−12	
		−1	−5	
		0	0	
		1	3	
		2	4	
		3	3	
		4	0	
		5	−5	
$j(x) = x^3 - 2x^2 - 5x + 6$		x	$j(x)$	(−2, 0) (1, 0) (3, 0)
		−2	0	
		−1	8	
		0	6	
		1	0	
		2	−4	
		3	0	
		4	18	
		5	56	

Use your graphs and tables to answer the following questions.

2. How did you identify the x-intercepts from the graph or table?
The x-intercept in the graph is where the graph crosses the x-axis. The x-intercept in the table is where the function value equals 0.

3. For this set of functions, how does the number of x-intercepts compare with the degree of the function?
For this set of functions, the number of x-intercepts is the same as the degree of the function.

4. For each x-intercept, $(b, 0)$, of each function, write the binomial $x - b$. Multiply the binomials for each function together and compare the product with the original function.
See margin.

5. How can you use the x-intercepts from graphs or tables to write the linear factors of a polynomial function?
Each factor will be x – a, where a is the x-coordinate of an x-intercept.

4.

$f(x)$:	$g(x)$:	$h(x)$:	$j(x)$:
$(x-(-3))(x-2)$	$(x-(-4))(x-(-1))$	$(x-0)(x-4)$	$(x-(-2))(x-1)(x-3)$
$(x+3)(x-2)$	$(x+4)(x+1)$	$x(x-4)$	$(x+2)(x-1)(x-3)$
x^2+x-6	x^2+5x+4	x^2-4x	x^3-2x^2-5x+6

When the coefficient of x^2 is 1 in the original function, the product of the binomial factors is the same as the original function. When the coefficient of x^2 is −1 in the original function, the product of binomials must also be multiplied by −1 in order to preserve the reflection of the graph across the x-axis. The factors of h(x) would need to be multiplied by -1 in order to generate the given function.

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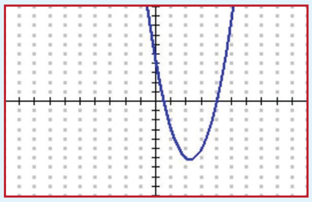
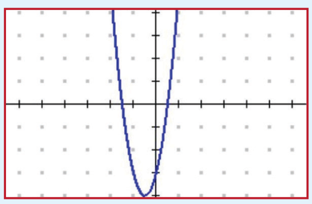
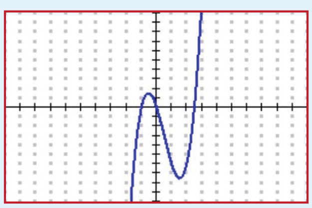
6. The x-coordinate of an x-intercept of the graph of a function is also called a zero of the function, since that is the x-value that generates a function value of zero. How could you use the zeroes of a function to identify the linear factors?
Each factor will be x – a, where a is a zero of the function.

7. For each of the functions below, use your graphing technology to generate a graph and table. Sketch the graph and record some of the function values in a table like the one shown for each function.

<table>
<tr><th>FUNCTION</th><th>GRAPH</th><th colspan="2">TABLE</th><th>x-INTERCEPT(S)</th></tr>
<tr><td rowspan="9">$p(x) = 2x^2 - 9x + 4$</td><td rowspan="9"></td><td>x</td><td>$f(x)$</td><td rowspan="9">(0.5, 0)
(4, 0)</td></tr>
<tr><td>−0.5</td><td>9</td></tr>
<tr><td>0</td><td>4</td></tr>
<tr><td>0.5</td><td>0</td></tr>
<tr><td>1</td><td>−3</td></tr>
<tr><td>1.5</td><td>−5</td></tr>
<tr><td>2</td><td>−6</td></tr>
<tr><td>3</td><td>−5</td></tr>
<tr><td>4</td><td>0</td></tr>
<tr><td rowspan="9">$q(x) = 4x^2 + 4x - 3$</td><td rowspan="9"></td><td>x</td><td>$g(x)$</td><td rowspan="9">(−1.5, 0)
(0.5, 0)</td></tr>
<tr><td>−2</td><td>5</td></tr>
<tr><td>−1.5</td><td>0</td></tr>
<tr><td>−1</td><td>−3</td></tr>
<tr><td>−0.5</td><td>−4</td></tr>
<tr><td>0</td><td>−3</td></tr>
<tr><td>0.5</td><td>0</td></tr>
<tr><td>1</td><td>5</td></tr>
<tr><td>1.5</td><td>12</td></tr>
<tr><td rowspan="9">$r(x) = 2x^3 - 3x^2 - 5x$</td><td rowspan="9"></td><td>x</td><td>$h(x)$</td><td rowspan="9">(−1, 0)
(0, 0)
(2.5, 0)</td></tr>
<tr><td>−1</td><td>0</td></tr>
<tr><td>−0.5</td><td>1.5</td></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>0.5</td><td>−3</td></tr>
<tr><td>1</td><td>−6</td></tr>
<tr><td>1.5</td><td>−7.5</td></tr>
<tr><td>2</td><td>−6</td></tr>
<tr><td>2.5</td><td>0</td></tr>
</table>

Use your graphs and tables to answer the following questions.

8. The general form of a quadratic function is $y = ax^2 + bx + c$. When $a = 1$, how does that affect the x-intercepts of the function? When $a \neq 1$, how does that affect at least one x-intercept of the function?
When a = 1, the x-intercepts occur at integer values. When a ≠ 1, x-intercepts may occur at rational values that may or may not be integers.

9. $p(x)$: $(\frac{1}{2}, 0)$ and $(4, 0)$

$q(x)$: $(-\frac{3}{2}, 0)$ and $(\frac{1}{2}, 0)$

$r(x)$: $(-1, 0)$, $(0, 0)$, and $(\frac{5}{2}, 0)$

10. $p(x)$: $(2x - 1)(x - 4)$
$2x^2 - 9x + 4$

$q(x)$: $(2x - (-3))(2x - 1)$
$(2x + 3)(2x - 1)$
$4x^2 + 4x - 3$

$r(x)$: $(x - (-1))(x - 0)(2x - 5)$
$x(x + 1)(2x - 5)$
$2x^3 - 3x^2 - 5x$

For each function, the product of the binomial factors is the same as the original function.

11. *$(3, 0)$ is the only x-intercept of $m(x)$, so the only linear factor of $m(x)$ is $x - 3$.*

Since $m(x)$ is a quadratic function, multiply $(x - 3)$ by itself, or write $(x - 3)^2$.

REFLECT ANSWERS:

If the x-intercept is $(\frac{b}{a}, 0)$, then the zero of the polynomial function will be $\frac{b}{a}$ and the binomial $ax - b$ will be a linear factor of the polynomial function. If the x-intercept is a whole number then $a = 1$, so the linear factor will be the binomial $x - b$.

The degree of a polynomial function tells you that the function could have up to that many unique linear factors. However, some factors of quadratic or cubic functions may be repeated.

Answers may vary. Possible answer: The x-coordinate of an x-intercept is the same as a zero of the polynomial function. A zero of a polynomial function tells you the coefficients of the linear factor $ax - b$.

9. Rewrite the coordinates of the non-integer x-intercepts of each function as fractions instead of decimals.
See margin.

10. For each x-intercept, $(\frac{b}{a}, 0)$, of each function, write the binomial $ax - b$. If the x-coordinate of the x-intercept is an integer, let $a = 1$. Multiply the binomials for each function together and compare the product with the original function.
See margin.

11. Identify the x-intercepts and factors of $m(x) = x^2 - 6x + 9$. How is the graph of $m(x)$ different from the graphs of the other quadratic functions you factored?
See margin.

REFLECT

- **If you can identify the *x*-intercepts or zeroes of a polynomial function from a graph or a table, how can you write the factors of the polynomial function?**
  See margin.
- **What does the degree of a polynomial function tell you about the number of factors to expect?**
  See margin.

ELPS FEATURE

- **Retell or summarize, using increasingly complex English, how the *x*-intercepts of a polynomial function, the zeros of the function, and the linear factors of the polynomial function are related.**
  See margin.

EXPLAIN

Factoring a polynomial is a method of rewriting a polynomial as a set of factors that can be multiplied together in order to generate the original polynomial. Writing a polynomial in factored form allows you to do several things, including divide polynomials, simplify rational or polynomial expressions, and look for solutions to polynomial equations.

Watch Explain and You Try It Videos

or click here

General Form		Factored Form
$f(x) = 3x^2 + 2x - 8$	↔	$f(x) = (3x - 4)(x + 2)$
Polynomial Form		

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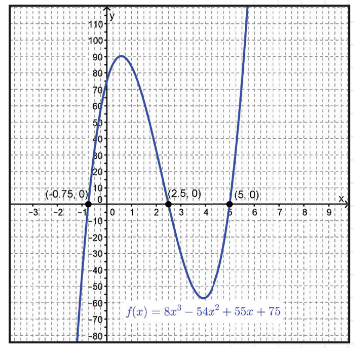
The factors of a polynomial function are related to the x-intercepts or zeroes of the function. For example, the graph of $f(x) = 8x^3 - 54x^2 + 55x + 75$ with its x-intercepts is shown.



The function $f(x)$ has three x-intercepts labeled in the graph: (−0.75, 0), (2.5, 0), and (5, 0). Write the non-integer x-intercepts as fractions: $\left(-\frac{3}{4}, 0\right)$, $\left(\frac{5}{2}, 0\right)$ and (5, 0). From here, the zeros of $f(x)$ can be identified as $-\frac{3}{4}$, $\frac{5}{2}$, and 5. The zeros can be used to identify the linear factors of $f(x)$ so that the function can be rewritten in factored form. If the zero is a fraction $\frac{b}{a}$, then the linear factor is the binomial $ax - b$.

$-\frac{3}{4} \rightarrow (4x - (-3)) = 4x + 3$ $\qquad$ $\frac{5}{2} \rightarrow (2x - 5) = 2x - 5$ $\qquad$ $5 \rightarrow (x - 5) = x - 5$

The function $f(x)$ can be rewritten as the product of these linear factors.

$$f(x) = (4x + 3)(2x - 5)(x - 5)$$

WHY DOES THIS WORK?

Recall that when you are multiplying a set of numbers together, if one of the numbers is 0, then the entire product will be zero.

$$3.5 \times 1{,}376 \times 0 \times 14\tfrac{7}{12} = 0$$

For the function $f(x)$, if any one of the three factors is equal to 0, then the entire product, or function value, will also be 0. When the function value is 0 (i.e., $f(x) = 0$), the graph shows an x-intercept. Thus, the x-intercepts identify the x-values that will generate a function value of 0.

If you have an x-intercept at $\left(\frac{b}{a}, 0\right)$, then $x = \frac{b}{a}$. Use this relationship to write a linear expression.

$$x = \frac{b}{a}$$
$$ax = b$$
$$ax - b = 0$$

With our original function, $f(x)$, we know that there is an x-intercept at $\left(\frac{5}{2}, 0\right)$. Set $x = \frac{5}{2}$ and write the linear factor associated with this x-intercept.

$$x = \frac{5}{2}$$
$$2x = 5$$
$$2x - 5 = 0$$

INSTRUCTIONAL HINT

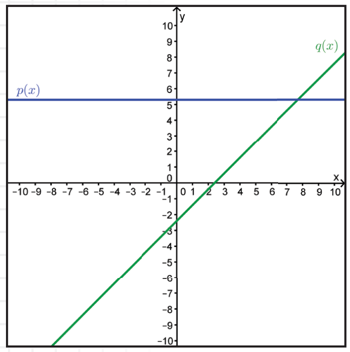
Before reading this section of the textbook, ask students the question that has been posed: "why does this work?" Allow students to brainstorm on their own, then discuss with a partner, and then continue to discuss with a larger group. Listen for students who understand why this works. Prompt students who do not understand by asking thought-provoking questions such as "when multiplying a set of numbers, what happens if one number is 0?" Then continue the lesson.

x	$f(x)$
−1	−42
−0.75	0
−0.5	33
0	75
1	84
2	33
2.5	0
3	−30
4	−57
5	0
6	189

You can also use a table of values to identify the zeros (x-intercepts) of a function. The table shows a set of function values for $f(x) = 8x^3 - 54x^2 + 55x + 75$.

The zeros of $f(x)$ are the x-values where $f(x) = 0$. In the table, there are three x-values where $f(x) = 0$: $x = -0.75$, 2.5, and 5.

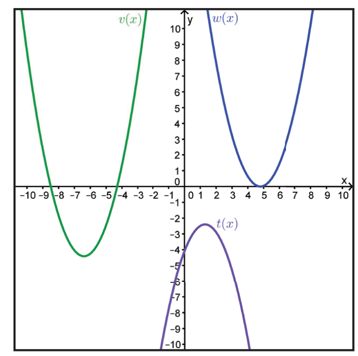
Once the zeros have been identified, write any non-integer zeros as fractions and use the fraction to write a linear factor.



FACTORS AND POLYNOMIAL DEGREE

There is also a relationship between the degree of a polynomial and the number of factors you can expect.

A linear function is a polynomial function of degree one. A linear function has up to one x-intercept, but it may have no x-intercepts. The function $q(x) = ax + b$, where $a \neq 0$, has one x-intercept at $\left(\frac{b}{a}, 0\right)$. However, the constant function $p(x) = b$ is a horizontal line with no x-intercept.



A quadratic function is a polynomial function of degree two. A quadratic function has up to two x-intercepts, but it may have only one x-intercept or no x-intercepts.

- The function $v(x)$ has two x-intercepts because the function crosses the x-axis.
- The function $w(x)$ has only one x-intercept because the function touches the x-axis but does not cross the x-axis.
- The function $t(x)$ has no x-intercepts because it does not touch or cross the x-axis.

A cubic function is a polynomial function of degree three. A cubic function has up to three x-intercepts, but it may have fewer than three x-intercepts.

- The functions $a(x)$ and $b(x)$ each have only one x-intercept. The cubic function crosses the x-axis only once.
- The function $c(x)$ has two x-intercepts. The graph of the function touches the x-axis, decreases, and then increases to cross the x-axis.
- The function $d(x)$ has three x-intercepts. The graph of the function crosses the x-axis in three separate locations.

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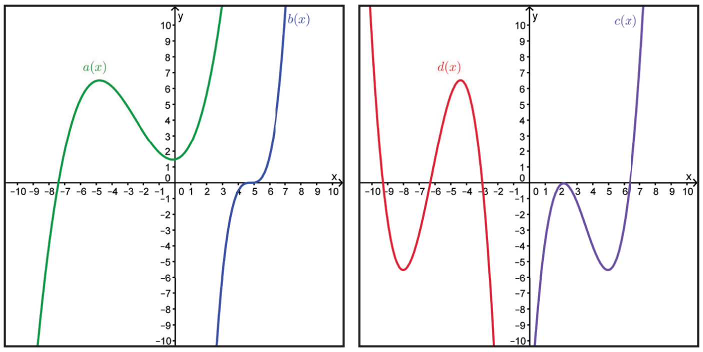


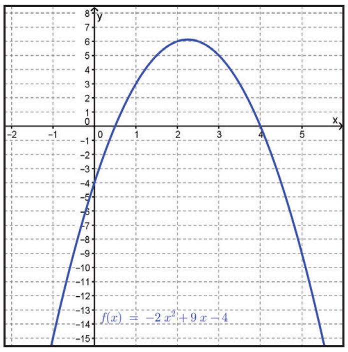
FACTORING POLYNOMIAL FUNCTIONS WITH GRAPHS AND TABLES

Polynomial functions can be factored, or written as a product of linear factors, by identifying their *x*-intercepts from a graph or their zeroes from a table.

- In a graph, locate an *x*-intercept, $\left(\frac{b}{a}, 0\right)$, by identifying the exact location where the graph touches or crosses the *x*-axis. The linear factor associated with this *x*-intercept is $ax - b$.
- In a table, locate a zero, $\frac{b}{a}$, by determining the *x*-value that generates a function value of 0. The linear factor associated with this zero is $ax - b$.

EXAMPLE 1

Factor the quadratic function $f(x) = -2x^2 + 9x - 4$ using a graph.



STEP 1 **Determine the *x*-intercepts of the quadratic function from its graph. Verify your answers by using the equation of the function.**

The graph of $f(x)$ appears to cross the *x*-axis at $\frac{1}{2}$ and 4.

$f(x) = -2x^2 + 9x - 4$

$f\left(\frac{1}{2}\right) = -2\left(\frac{1}{2}\right)^2 + 9\left(\frac{1}{2}\right) - 4 = -2\left(\frac{1}{4}\right) + \frac{9}{2} - 4 = -\frac{1}{2} + \frac{9}{2} - 4 = 4 - 4 = 0$

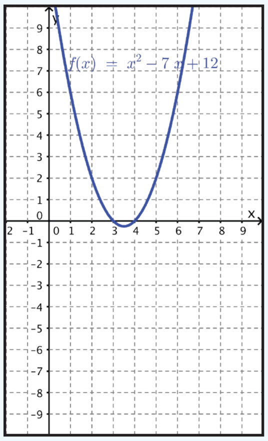
$f(4) = -2(4)^2 + 9(4) - 4 = -2(16) + 36 - 4 = -32 + 36 - 4 = 4 - 4 = 0$

Therefore, both $\frac{1}{2}$ and 4 are zeroes of $f(x)$.

ADDITIONAL EXAMPLES

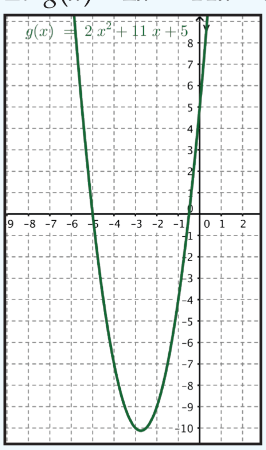
Factor the following quadratic functions using the graphs below.

1. $f(x) = x^2 - 7x + 12$



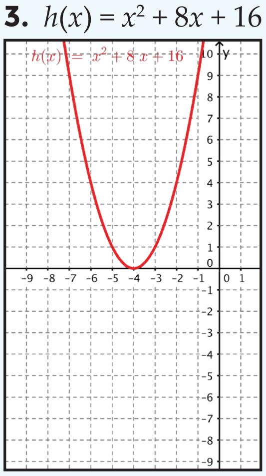
f(x) = (x – 3)(x – 4)

2. $g(x) = 2x^2 + 11x + 5$



g(x) = (2x + 1)(x + 5)

3. $h(x) = x^2 + 8x + 16$



h(x) = (x + 4)²

STEP 2 **Use each *x*-intercept to determine binomial factors.**

$$x = \frac{1}{2} \qquad\qquad\qquad$$
$$2x = 1 \qquad \text{and} \qquad x = 4$$
$$2x - 1 = 0 \qquad\qquad x - 4 = 0$$

The binomial factors of $f(x)$ are $(2x - 1)$ and $(x - 4)$.

STEP 3 **Multiply the binomial factors and compare to the symbolic representation of *f*(*x*).**

$$(2x - 1)(x - 4) = 2x^2 - 8x - x + 4 = 2x^2 - 9x + 4 = -(-2x^2 + 9x - 4) = -f(x)$$

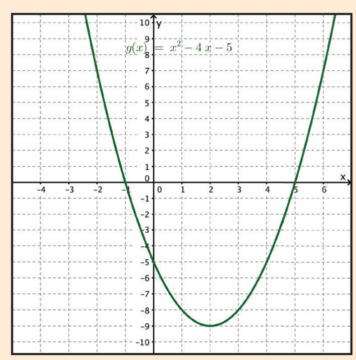
Because the parabola opens downward, the value of the parameter a is negative. The binomial factors do not take this into account. Therefore, you must multiply the function by −1 in order to ensure that $a < 0$ in the factored form of the function.

STEP 4 **Write the function as a product of its binomial factors. Since the parabola opens downward, multiply by −1.**

$$-f(x) = (2x - 1)(x - 4)$$
$$(-1)(-f(x)) = (-1)(2x - 1)(x - 4)$$
$$f(x) = -(2x - 1)(x - 4)$$

YOU TRY IT! #1

Factor the quadratic function $g(x) = x^2 - 4x - 5$ using a graph.



g(x) = (x + 1)(x – 5)

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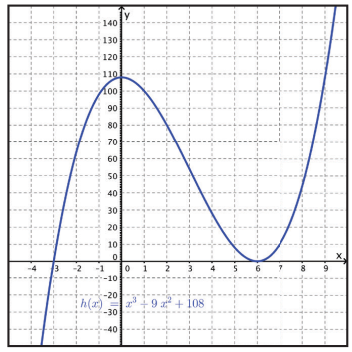
EXAMPLE 2

Factor the cubic function $h(x) = x^3 - 9x^2 + 108$ using a graph.



STEP 1 **Determine the x-intercepts of the cubic function from its graph. Verify your answers by using the equation of the function.**

The graph of $h(x)$ appears to cross the x-axis at -3 and touch the x-axis at 6.

$h(x) = x^3 - 9x^2 + 108$

$h(-3) = (-3)^3 - 9(-3)^2 + 108 = -27 - 9(9) + 108 = -27 - 81 + 108 = -108 + 108 = 0$

$h(6) = (6)^3 - 9(6)^2 + 108 = 216 - 9(36) + 108 = 216 - 324 + 108 = -108 + 108 = 0$

Therefore, both -3 and 6 are zeroes of $h(x)$.

STEP 2 **Use each x-intercept to determine binomial factor(s).**

$x = -3$	and	$x = 6$
$x + 3 = 0$		$x - 6 = 0$

The binomial factors of $h(x)$ are $(x + 3)$ and $(x - 6)$.

Because the graph touches the x-axis at 6 rather than crossing it, the binomial factor $(x - 6)$ is repeated.

STEP 3 **Multiply the binomial factors and compare to the symbolic representation of $h(x)$.**

$(x + 3)(x - 6)(x - 6) = (x^2 - 6x + 3x - 18)(x - 6) = (x^2 - 3x - 18)(x - 6)$
$= x^3 - 6x^2 - 3x^2 + 18x - 18x + 108 = x^3 - 9x^2 + 108 = h(x)$

STEP 4 **Write the function as a product of its binomial factors.**

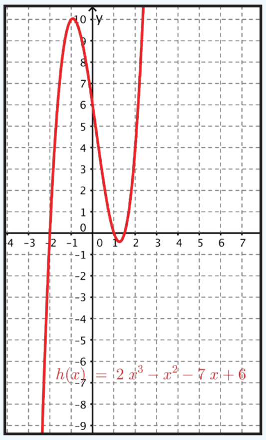
$h(x) = (x + 3)(x - 6)(x - 6)$

$h(x) = (x + 3)(x - 6)^2$

ADDITIONAL EXAMPLES

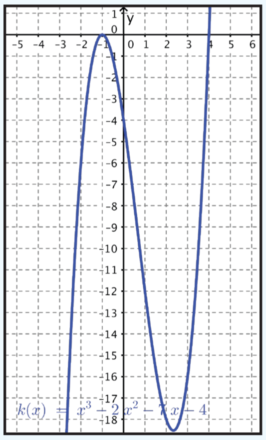
Factor the following cubic functions using the graphs below.

1. $h(x) = 2x^3 - x^2 - 7x + 6$



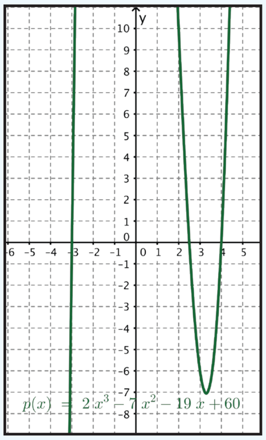
$h(x) = (x + 2)(x - 1)(2x - 3)$

2. $k(x) = x^3 - 2x^2 - 7x - 4$



$k(x) = (x + 1)^2(x - 4)$

3. $p(x) = 2x^3 - 7x^2 - 19x + 60$



$p(x) = (x - 3)(2x - 5)(x - 4)$

ADDITIONAL EXAMPLES

Factor the following quadratic functions using the tables below.

1. $m(x) = 8x^2 - 6x - 9$

x	$m(x)$
-1	5
-0.75	0
-0.5	-4
-0.25	-7
0	-9
0.25	-10
0.5	-10
0.75	-9
1	-7
1.25	-4
1.5	0

$m(x) = (4x + 3)(2x - 3)$

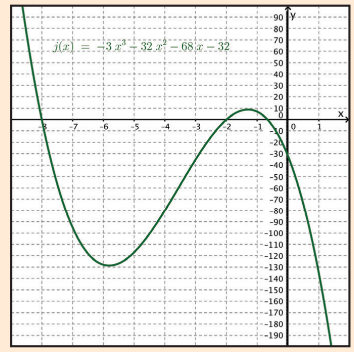
2. $n(x) = 2x^2 - 19x + 45$

x	$m(x)$
0	45
0.5	36
1	28
1.5	21
2	15
2.5	10
3	6
3.5	3
4	1
4.5	0
5	0
5.5	1
6	3

$n(x) = (2x - 9)(x - 5)$

YOU TRY IT! #2

Factor the cubic function $j(x) = -3x^3 - 32x^2 - 68x - 32$ using a graph.



j(x) = –(x + 8)(x + 2)(3x + 2)

EXAMPLE 3

Factor the quadratic function $p(x) = 16x^2 - 56x + 49$ using a table.

x	$p(x)$
1	9
1.25	4
1.5	1
1.75	0
2	1
2.25	4
2.5	9
2.75	16
3	25

STEP 1 **Determine the x-intercepts of the quadratic function from its table.**

The table for $p(x)$ has a function value of zero at 1.75, or $\frac{7}{4}$. Therefore, $\frac{7}{4}$ is a zero of $p(x)$.

STEP 2 **Use each x-intercept to determine binomial factor(s).**

$$x = \frac{7}{4}$$
$$4x = 7$$
$$4x - 7 = 0$$

The binomial factor of $p(x)$ is $(4x - 7)$.

As can be seen from the values in the table, the point (1.75, 0) is the vertex of the parabola, since it is the minimum value in the table and the values around it demonstrate that $x = 1.75$ is the line of symmetry of the parabola.

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MINIMUM → 1.75

x	$p(x)$
1	9
1.25	4
1.5	1
1.75	0
2	1
2.25	4
2.5	9
2.75	16
3	25

Because the parabola touches the x-axis at $\frac{7}{4}$ rather than crossing it, the binomial factor $(4x - 7)$ is repeated.

STEP 3 **Multiply the binomial factors and compare to the symbolic representation of $p(x)$.**

$$(4x - 7)(4x - 7) = 16x^2 - 28x - 28x + 49 = 16x^2 - 56x + 49 = p(x)$$

STEP 4 **Write the function as a product of its binomial factors.**

$$p(x) = (4x - 7)(4x - 7)$$
$$p(x) = (4x - 7)^2$$

YOU TRY IT! #3

Factor the quadratic function $r(x) = x^2 + 3x + 4$ using a table.

x	$r(x)$
-3	4
-2.5	2.75
-2	2
-1.5	1.75
-1	2
-0.5	2.75
0	4
0.5	5.75
1	8

See margin.

YOU TRY IT! #3 ANSWER:

Since $r(x)$ does not have any zeroes, as can be seen by the table that contains function values decreasing to its vertex at $(-1.5, 1.75)$ and then increasing without any negative function values, the function is not factorable. The quadratic function $r(x) = x^2 + 3x + 4$ cannot be written as the product of binomial factors using real numbers.

ADDITIONAL EXAMPLES

Factor the following cubic functions using the tables below.

1. $p(x) = x^3 + 4x^2 - 15x - 18$

x	$p(x)$
-7	-60
-6	0
-5	32
-4	42
-3	36
-2	20
-1	0
0	-18
1	-28
2	-24
3	0

$p(x) = (x + 6)(x + 1)(x - 3)$

2. $q(x) = 4x^3 + 8x^2 + 3x$

x	$n(x)$
-2	-6
-1.5	0
-1	1
-0.5	0
0	0
0.5	4
1	15
1.5	36
2	70

$q(x) = x(2x + 3)(2x + 1)$

EXAMPLE 4

Factor the cubic function $t(x) = -5x^3 - 22x^2 - 9x - 4$ using a table. The table of values shows all x-intercepts of $t(x)$.

x	$t(x)$
-5	116
-4	0
-3	-40
-2	-34
-1	-12
-0.5	-4.375
0	-4
0.5	-14.625
1	-40

STEP 1 **Determine the x-intercepts of the cubic function from the table. Verify your answers by using the equation of the function.**

The graph of $t(x)$ appears to cross the x-axis at -4. There does not appear to be any other zero since the function values decrease to approximately -40, then increase to approximately -4 and then decrease again without crossing the x-axis again.

Therefore, -4 is a zero of $t(x)$.

STEP 2 **Use each x-intercept to determine binomial factor(s).**

$x = -4$

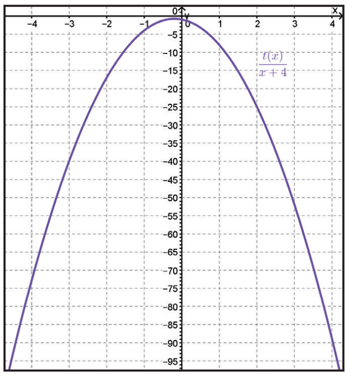
$x + 4 = 0$

The binomial factor of $t(x)$ is $(x + 4)$. Since $t(x)$ is a cubic function (function of degree three) and it has only one binomial factor that is not repeated, then its other factor must be quadratic (function of degree two).

STEP 3 **Use graphing technology to graph or generate a table for the function $\frac{t(x)}{(x+4)}$.**

Use the finite differences in the table or your knowledge of function transformations in the graph to determine the quadratic factor of $t(x)$.

Using the patterns in the finite differences in the table, you can determine that the quadratic factor is $(-5x^2 - 2x - 1)$.



x	$\frac{t(x)}{x+4}$
-3	-40
-2	-17
-1	-4
0	-1
1	-8
2	-25
3	-52

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Notice that this function, $y = -5x^2 - 2x - 1$ has no real number linear factors. The graph of the function does not touch or cross the x-axis and the table values show no place where the function values change from positive to negative or vice versa.

STEP 3 **Multiply the binomial and quadratic factors and compare to the symbolic representation of $t(x)$.**

$(x + 4)(-5x^2 - 2x - 1) = -5x^3 - 2x^2 - x - 20x^2 - 8x - 4$
$= -5x^3 - 2x^2 - 20x^2 - x - 8x - 4 = -5x^3 - 22x^2 - 9x - 4 = t(x)$

STEP 4 **Write the function as a product of its binomial and quadratic factors.**

$t(x) = (x + 4)(-5x^2 - 2x - 1)$ or $t(x) = -(x + 4)(5x^2 + 2x + 1)$
Either is correct.

YOU TRY IT! #4

Factor the cubic function $w(x) = x^3 - 6x^2 - 9x + 14$ using a table.

x	$w(x)$
-4	-110
-3	-40
– 2	0
-1	16
1	0
2	-20
3	-40
4	-54
5	-56
6	-40
7	0
8	70

$w(x) = (x + 2)(x - 1)(x - 7)$

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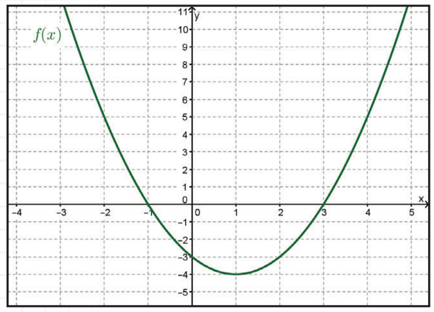
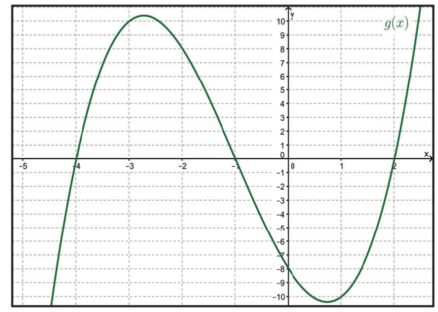
PRACTICE/HOMEWORK

1. Determine the x-intercepts of $f(x)$.



(-1, 0)(3, 0)

2. Determine the zeros of $g(x)$.



-4, -1, and 2

For questions 3 – 6, determine the factors of the polynomial function given the x-intercepts of the function.

3. The x-intercepts of a quadratic function, $f(x)$, are (5, 0) and (1, 0). Determine the factors of the function.

(x – 5)(x – 1)

4. The x-intercepts of a quadratic function, $p(x)$, are (-2.5, 0) and (3, 0). Determine the factors of the function.

(2x + 5)(x – 3)

5. The x-intercepts of a cubic function, $f(x)$, are (6, 0), (-2, 0) and (0.5, 0). Determine the factors of the function.

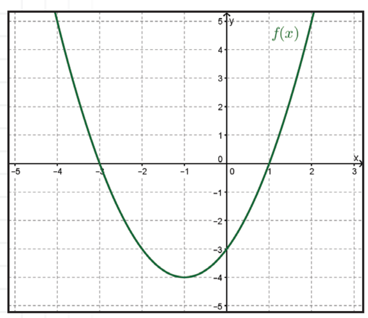
(x – 6)(x + 2)(2x – 1)

6. The x-intercepts of a cubic function, $p(x)$, are (-3, 0), and (1, 0) and (3, 0). Determine the factors of the function.

(x + 3)(x – 1)(x – 3)

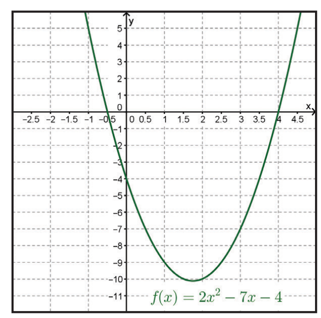
For questions 7 – 10, identify the x-intercept(s) of each function, and then write the quadratic function in factored form.

7. $f(x) = x^2 + 2x - 3$



The x-intercepts are (-3, 0) and (1, 0); f(x)= (x + 3)(x – 1)

8. $f(x) = 2x^2 - 7x - 4$



The x-intercepts are (-0.5, 0) and (4, 0); f(x)= (2x + 1)(x – 4)

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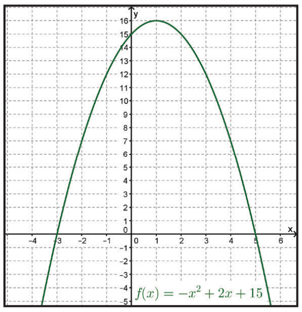
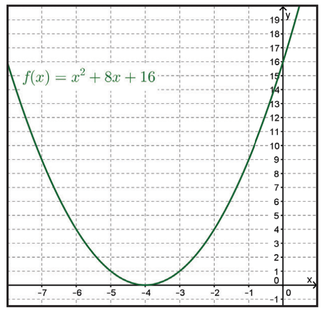
9. $f(x) = -x^2 + 2x + 15$



The x-intercepts are (-3, 0) and (5, 0); f(x)= -(x + 3)(x – 5)

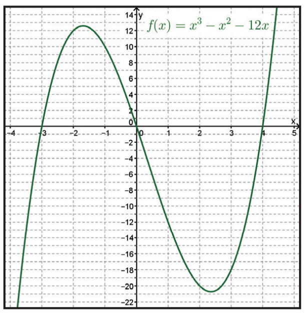
10. $f(x) = x^2 + 8x + 16$



The x-intercept is (-4, 0); f(x)= (x + 4)^2

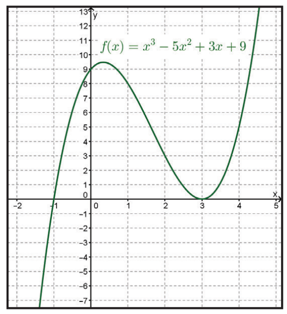
For questions 11 – 14, identify the x-intercept(s) of each function, and then write the cubic function in factored form.

11. $f(x) = x^3 - x^2 - 12x$



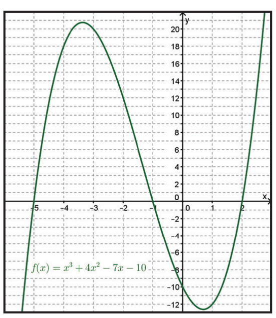
The x-intercepts are (-3, 0), (0, 0) and (4, 0); f(x)= x(x + 3)(x – 4)

12. $f(x) = x^3 - 5x^2 + 3x + 9$



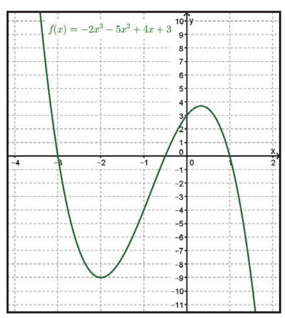
The x-intercepts are (-1, 0) and (3, 0); f(x)= (x + 1)(x – 3)^2

13. $f(x) = x^3 + 4x^2 - 7x - 10$



The x-intercepts are (-5, 0), (-1, 0) and (2, 0); f(x)= (x + 5)(x + 1)(x – 2)

14. $f(x) = -2x^3 - 5x^2 + 4x + 3$



The x-intercepts are (-3, 0), (-0.5, 0) and (1, 0); f(x)= -(x + 3)(2x + 1)(x – 1)

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17. *The zero shown in the table is -2, and the other zero is 0.5;* $p(x) = (x + 2)(2x - 1)$

For questions 15 – 17 identify the zeros of each function, and then write the quadratic function in factored form.

15.

x	$p(x)$
−6	9
−5	4
−4	1
−3	0
−2	1
−1	4

There is only one zero at -3; $p(x) = (x + 3)^2$

16.

x	$p(x)$
−2	0
−1	4
0	6
1	6
2	4
3	0
4	−6

The zeros are -2 and 3; $p(x) = -(x + 2)(x - 3)$

17.

x	$p(x)$
−3	7
−2.5	3
−2	0
−1.5	−2
−1	−3
−0.5	−3
0	−2
0.5	0
1	3

See margin.

For questions 18 – 19 identify the zeros of each function, then write the cubic function in factored form.

18.

x	$q(x)$
-4	-10
-3	0
-2	0
-1	-4
0	-6
1	0

The zeros are -3, -2, and 1; $q(x) = (x + 3)(x + 2)(x - 1)$

19.

x	$q(x)$
-4	15
-3	0
-2	-3
-1	0
0	3
1	0
2	-15

The zeros are -3, -1 and 1; $q(x) = -(x + 3)(x + 1)(x - 1)$

For questions 20 – 23, use your graphing technology to generate a table or a graph of the given function. Identify what type of function it is (quadratic or cubic) and the x-intercepts of the function. Write the function as a product of its factors.

20. $g(x) = -x^3 - 3x^2 + 6x + 8$
Cubic function; (−4, 0), (−1, 0), (2, 0); $g(x) = -(x + 4)(x + 1)(x - 2)$

21. $g(x) = -x^2 - 4x - 4$
Quadratic function; (−2, 0); $g(x) = -(x + 2)^2$

22. $g(x) = x^2 - 10x + 21$
Quadratic function; (3, 0), (7, 0); $g(x) = (x - 3)(x - 7)$

23. $q(x) = 4x^3 - 13x^2 + 3x$
Cubic function; (0, 0), (0.25, 0), (3, 0); $g(x) = x(4x - 1)(x - 3)$

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