Multiplying Polynomial Functions

5.3

FOCUSING QUESTION How do you solve problems involving multiplication of linear, quadratic, or cubic functions?

LEARNING OUTCOMES

- I can use tables and equations to multiply linear, quadratic, or cubic functions.
- I can apply mathematics to solve problems arising in everyday life, society, and the workplace.

ENGAGE

A rectangular tomato garden has dimensions $(2x + 5)$ and $(3x - 7)$. Isabella wrote the expression $(2x + 5)(3x - 7)$ to show the area of the garden. Cooper wrote the expression $6x^2 + x - 35$ to show the area of the garden. Which student is correct? Use graphs and tables to explain your reasoning.

See margin.

EXPLORE

Rita manages a clothing department at a local store. Rita has sales data for a particular clothing line for six recent years. She made a table of the data.

YEAR	2010	2011	2012	2013	2014	2015	2016
YEAR SINCE 2010	0	1	2	3	4	5	6
NUMBER OF ITEMS SOLD (THOUSANDS)	3	3.75	4.2	4.35	4.2	3.75	3

1. Let x represent the year since 2010 and $f(x)$ represent the number of items sold, in thousands. Use finite differences or successive ratios to look for patterns in the table and write a symbolic function rule for $f(x)$.
See margin.

2. Rita also knows that the function $p(x) = 1.25x + 20$ describes the average price per item when x is the number of years since 2010. Add a row to the original table for the average price per item and use the function $p(x)$ to enter the data into the row.

TEKS

AR.4A Connect tabular representations to symbolic representations when adding, subtracting, and multiplying polynomial functions arising from mathematical and real-world situations, such as applications involving surface area and volume.

MATHEMATICAL PROCESS SPOTLIGHT

AR.1A Apply mathematics to problems arising in everyday life, society, and the workplace.

ELPS

5G Narrate, describe, and explain with increasing specificity and detail to fulfill content area writing needs as more English is acquired.

VOCABULARY

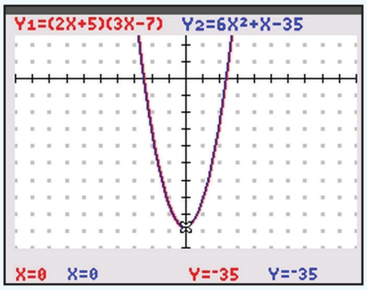
polynomial, factor, product

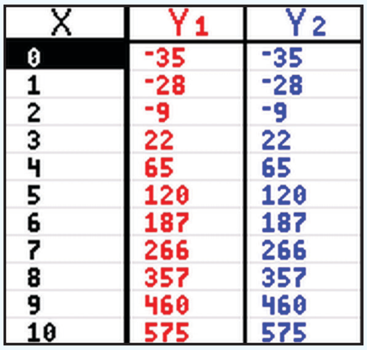
MATERIALS

- graphing technology

EXPLORE ANSWER:

Both students are correct. Let Y1 = $(2x + 5)(3x - 7)$ and Y2 = $6x^2 + x - 35$. Using a graphing calculator, compare the graphs and tables. The graphs of Y1 and Y2 coincide. In the table, for each x-value, the function values for Y1 and Y2 are equal. Thus, Y1 is equivalent to Y2 and both students correctly wrote an expression for the area of the tomato garden.





X	Y1	Y2
0	-35	-35
1	-28	-28
2	-9	-9
3	22	22
4	65	65
5	120	120
6	187	187
7	266	266
8	357	357
9	460	460
10	575	575

1.

+1 +1 +1 +1 +1 +1

YEAR	2010	2011	2012	2013	2014	2015	2016
YEAR SINCE 2010	0	1	2	3	4	5	6
NUMBER OF ITEMS SOLD (THOUSANDS)	3	3.75	4.2	4.35	4.2	3.75	3

+0.75 +0.45 +0.15 -0.15 -0.45 -0.75

-0.3 -0.3 -0.3 -0.3 -0.3

$2a = -0.3$
$a = -0.15$

$a + b = 0.75$
$(-0.15) + b = 0.75$
$b = 0.9$

$c = 3$

$f(x) = -0.15x^2 + 0.9x + 3$

INTEGRATING TECHNOLOGY

The table feature of a graphing calculator or a spreadsheet program or app can be used to calculate the first, second, and third differences for any data set. Technology is particularly useful for this data set where the numbers are realistic but include decimal numbers to the ten-thousandths place. In this lesson, technology alleviates the burden of number crunching or manually entering subtraction problems into a calculator so that students can more readily see the patterns within the data set.

4. *See margin below.*

5. $r(x) = f(x) \cdot p(x)$

$r(x) = (-0.15x^2 + 0.9x + 3) \cdot (1.25x + 20)$

$r(x) = -0.1875x^3 - 1.875x^2 + 21.75x + 60$

6. *The function rules are equivalent. If you make a graph or a table of each function rule, the graphs coincide and the table shows equal function values for each independent variable value.*

YEAR SINCE 2010	0	1	2	3	4	5	6
NUMBER OF ITEMS SOLD (THOUSANDS)	3	3.75	4.2	4.35	4.2	3.75	3
AVERAGE PRICE PER ITEM, $p(x)$	***$20.00***	***$21.25***	***$22.50***	***$23.75***	***$25.00***	***$26.25***	***$27.50***

3. The amount of revenue generated in Rita's department for this particular clothing line is the product of the number of items sold and the average price per item. Calculate the amount of revenue generated each year for this particular clothing line.

YEAR SINCE 2010	0	1	2	3	4	5	6
NUMBER OF ITEMS SOLD (THOUSANDS)	3	3.75	4.2	4.35	4.2	3.75	3
AVERAGE PRICE PER ITEM, $p(x)$	***$20.00***	***$21.25***	***$22.50***	***$23.75***	***$25.00***	***$26.25***	***$27.50***
REVENUE GENERATED (THOUSANDS OF DOLLARS)	***60***	***79.6875***	***94.5***	***103.3125***	***105***	***98.4375***	***82.50***

4. Let $r(x)$ be the function that describes the revenue generated, in thousands of dollars, when x represents the number of years since 2010. Use finite differences or successive ratios in the table to write a symbolic function rule for $r(x)$.
See margin.

5. Rita used a table to calculate the function describing the annual revenue, $r(x)$, in her department. Multiply the items sold function, $f(x)$, by the average price function, $p(x)$.
See margin.

6. How does the function rule for $r(x)$ you wrote from number patterns in the table compare to the function rule you wrote for $r(x)$ from the function rules for $f(x)$ and $p(x)$? Use graphing technology to explain your answer.
See margin.

7. What are the degrees of the functions that are factors, $f(x)$ and $p(x)$, and the function that is the product, $r(x)$?
See margin.

8. How do the degrees of the polynomial factors relate to the degree of the polynomial product? Explain the implications of your mathematical reasoning.
See margin.

4.

+1 +1 +1 +1 +1 +1

YEAR SINCE 2010	0	1	2	3	4	5	6
REVENUE GENERATED (THOUSANDS OF DOLLARS)	***60***	***79.6875***	***94.5***	***103.3125***	***105***	***98.4375***	***82.50***

+19.6875 +14.8125 +8.8125 +1.6875 -6.5625 -15.9375

-4.875 -6 -7.125 -8.25 -9.375

-1.125 -1.125 -1.125 -1.125

Answer continued on bottom of page 515.

7. *$f(x)$ is quadratic with degree 2.*

$p(x)$ is linear with degree 1.

$r(x)$ is cubic with degree 3.

8. *The degree of the product is the sum of the degrees of the factors. According to the laws of exponents, when you multiply like bases, you add the exponents. Multiplying terms of polynomial factors follows this pattern.*

REFLECT

- **How did you use multiplication to calculate the function values for the revenue function in a table?**
  See margin.
- **When you multiply linear or quadratic functions, how does using operations on the symbolic functions relate to using operations with table values?**
  See margin.
- **What type of function would result if you multiplied a cubic function by a linear function? A quadratic function by a cubic function?**
  See margin.

EXPLAIN

Multiplication is an operation that can be used to represent several actions, including

- equal-sized groups that are joined together;
- area;
- repeated addition; and
- scaling the size of an object

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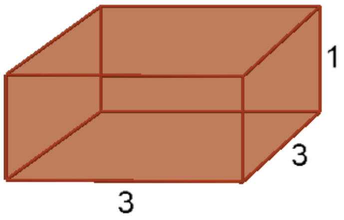


Consider a sequence of square prisms. The first prism has a length and width of 3 units and height of 1 unit. Each successive prism in the sequence has dimensions that are one unit longer than the previous prism.

You can use the verbal description of the sequence to build a table of values for the dimensions and volume for a sequence of square prisms.

PRISM NUMBER, n	LENGTH, $L(n)$		WIDTH, $W(n)$		HEIGHT, $H(n)$		VOLUME, $V(n)$
1	3	×	3	×	1	=	9
2	4	×	4	×	2	=	32
3	5	×	5	×	3	=	75
4	6	×	6	×	4	=	144
5	7	×	7	×	5	=	245

REFLECT ANSWERS:

The function values for the revenue function were generated by multiplying the values from the number of items function and the average price function.

If you multiply function values from the table, you multiply each pair or group of function values and then use the products to generate a new function rule. If you multiply two or more function rules, then you use properties of algebra to multiply polynomials instead of function values. The product is the function represented in symbolic form.

The product of a cubic (degree 3) and linear (degree 1) function would be a 4th degree function.
The product of a quadratic (degree 2) and cubic (degree 3) function would be a 5th degree function.

4. *Since the third differences are constant, r(x) is a cubic function.*

$$6a = -1.125$$
$$a = -0.1875$$

$$6a + 2b = -4.875$$
$$6(-0.1875) + 2b = -4.875$$
$$-1.125 + 2b = -4.875$$
$$2b = -3.75$$
$$b = -1.875$$

$$a + b + c = 19.6875$$
$$(-0.1875) + (-1.875) + c = 19.6875$$
$$-2.0625 + c = 19.6875$$
$$c = 21.75$$

$$d = 60$$

$$r(x) = -0.1875x^3 - 1.875x^2 + 21.75x + 60$$

From the table, you can use finite differences in $V(n)$ to write a function rule for volume in terms of the prism number. Include the 0 term so that you can use the differences to determine the function rule.

PRISM NUMBER, n	**VOLUME,** $V(n)$
0	⓪
1	9
2	32
3	75
4	144
5	245

⑨ 23 43 69 101

⑭ 20 26 32

⑥ 6 6

The third differences are constant, so $V(n)$ is a cubic function.

$6a = 6$
$a = 1$

$6a + 2b = 14$
$6(1) + 2b = 14$
$6 + 2b = 14$
$2b = 8$
$b = 4$

$a + b + c = 9$
$(1) + (4) + c = 9$
$5 + c = 9$
$c = 4$

$d = 0$

Substituting a, b, c, and d into $V(n) = an^3 + bn^2 + cn + d$ generates the function rule $V(n) = n^3 + 4n^2 + 4n$.

Another way to determine the volume function, $V(n)$, is to multiply the function rules written in symbolic form for each dimension. Use finite differences to write function rules for length, width, and height functions.

PRISM NUMBER, n	**LENGTH,** $L(n)$
0	2
1	3
2	4
3	5
4	6
5	7

+1 +1 +1 +1 +1

WIDTH, $W(n)$
2
3
4
5
6
7

+1 +1 +1 +1 +1

HEIGHT, $H(n)$
0
1
2
3
4
5

+1 +1 +1 +1 +1

- $L(n) = n + 2$
- $W(n) = n + 2$
- $H(n) = n$

The volume of a square prism, which is also a rectangular prism, is calculated with the formula $V = Bh$, where B represents the area of the base and h represents the height of the prism. The base is a square, so the area of the base is the side length of the base squared.

$$B(n) = [L(n)]^2$$
$$B(n) = (n + 2)^2$$
$$B(n) = n^2 + 4n + 4$$

Multiply the area of the base function, $B(n)$, by the height function, $H(n)$, to determine the function rule for volume, $V(n)$.

$$V(n) = B(n) \cdot H(n)$$
$$V(n) = (n^2 + 4n + 4)(n)$$
$$V(n) = n^3 + 4n^2 + 4n$$

Either way, the function rule for the volume is the same.

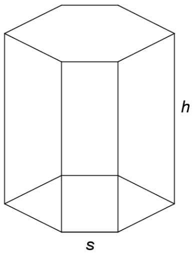
MULTIPLYING POLYNOMIAL FUNCTIONS

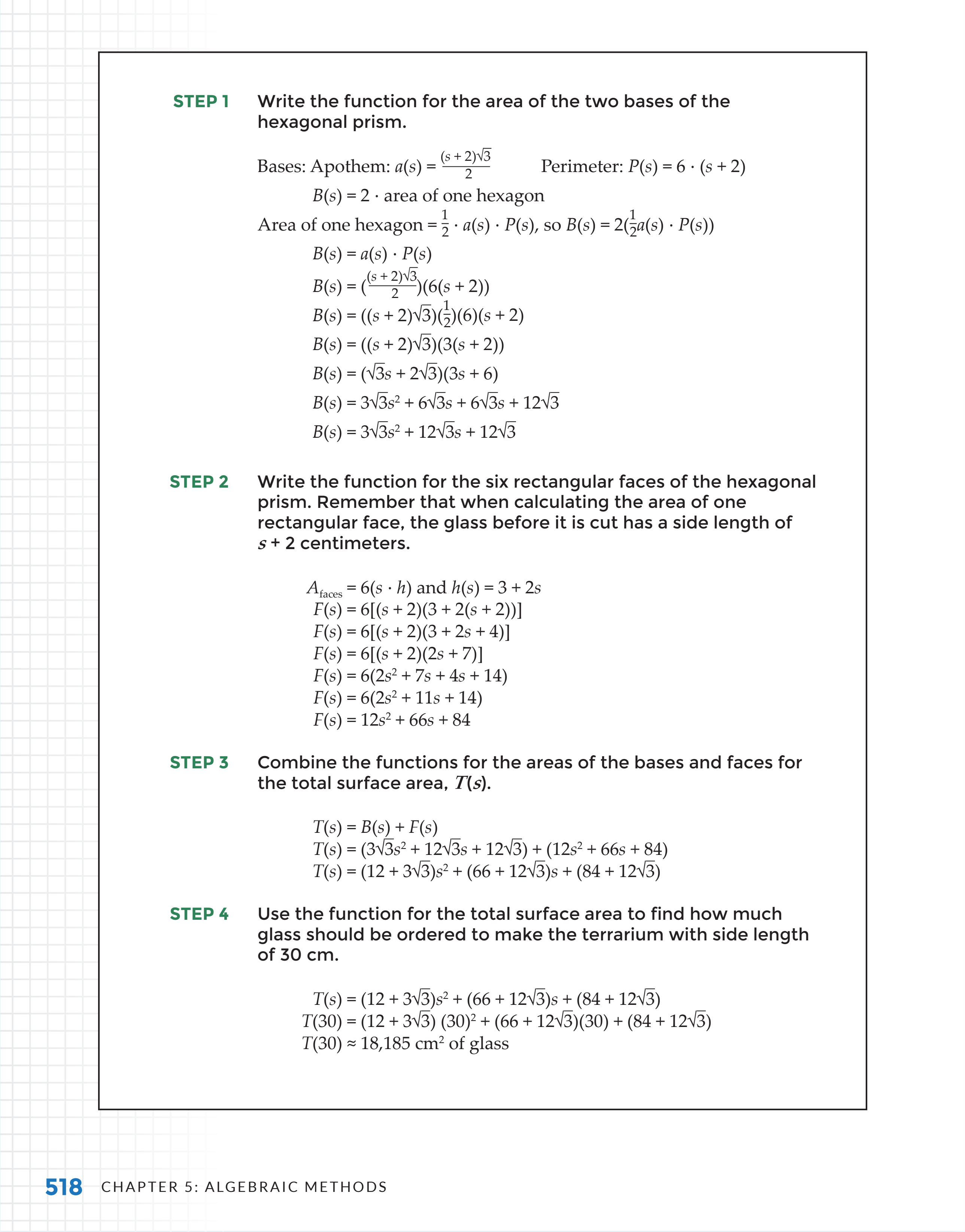
A polynomial function is a function that, when written symbolically, is composed of terms with real number coefficients, a variable, and real number exponents. Constant, linear, quadratic, and cubic functions are types of polynomial functions.

- You can multiply polynomial functions two ways:
  1. Make a table of values for each factor to generate the values of the function that is the product of the factors. Then use finite differences to generate a symbolic function rule.
  2. Multiply the symbolic function rules for the factors and use the properties of algebra to simplify the expression.

EXAMPLE 1

Terrific Terrariums Company makes custom hexagonal terrariums. The company must know the surface area of each terrarium to determine how much glass to order. The formula for the area of the hexagonal base is the product of one half the apothem a and the perimeter p. The area of the rectangular sides equals the side length times the height of the terrarium. The side length of the regular hexagonal bases is $s + 2$, which includes 2 centimeters added in for waste when the glass is cut. The height of the terrariums is always three centimeters more than twice the side length of the bases. Write the function for the total surface area of glass ordered in terms of the side length s and use it to find the surface area of glass ordered for the terrarium with a side length of 30 centimeters.





STEP 1 **Write the function for the area of the two bases of the hexagonal prism.**

Bases: Apothem: $a(s) = \frac{(s+2)\sqrt{3}}{2}$ Perimeter: $P(s) = 6 \cdot (s + 2)$

$B(s) = 2 \cdot$ area of one hexagon

Area of one hexagon $= \frac{1}{2} \cdot a(s) \cdot P(s)$, so $B(s) = 2(\frac{1}{2}a(s) \cdot P(s))$

$$B(s) = a(s) \cdot P(s)$$
$$B(s) = (\tfrac{(s+2)\sqrt{3}}{2})(6(s + 2))$$
$$B(s) = ((s + 2)\sqrt{3})(\tfrac{1}{2})(6)(s + 2)$$
$$B(s) = ((s + 2)\sqrt{3})(3(s + 2))$$
$$B(s) = (\sqrt{3}s + 2\sqrt{3})(3s + 6)$$
$$B(s) = 3\sqrt{3}s^2 + 6\sqrt{3}s + 6\sqrt{3}s + 12\sqrt{3}$$
$$B(s) = 3\sqrt{3}s^2 + 12\sqrt{3}s + 12\sqrt{3}$$

STEP 2 **Write the function for the six rectangular faces of the hexagonal prism. Remember that when calculating the area of one rectangular face, the glass before it is cut has a side length of $s + 2$ centimeters.**

$A_{\text{faces}} = 6(s \cdot h)$ and $h(s) = 3 + 2s$

$$F(s) = 6[(s + 2)(3 + 2(s + 2))]$$
$$F(s) = 6[(s + 2)(3 + 2s + 4)]$$
$$F(s) = 6[(s + 2)(2s + 7)]$$
$$F(s) = 6(2s^2 + 7s + 4s + 14)$$
$$F(s) = 6(2s^2 + 11s + 14)$$
$$F(s) = 12s^2 + 66s + 84$$

STEP 3 **Combine the functions for the areas of the bases and faces for the total surface area, $T(s)$.**

$$T(s) = B(s) + F(s)$$
$$T(s) = (3\sqrt{3}s^2 + 12\sqrt{3}s + 12\sqrt{3}) + (12s^2 + 66s + 84)$$
$$T(s) = (12 + 3\sqrt{3})s^2 + (66 + 12\sqrt{3})s + (84 + 12\sqrt{3})$$

STEP 4 **Use the function for the total surface area to find how much glass should be ordered to make the terrarium with side length of 30 cm.**

$$T(s) = (12 + 3\sqrt{3})s^2 + (66 + 12\sqrt{3})s + (84 + 12\sqrt{3})$$
$$T(30) = (12 + 3\sqrt{3})\ (30)^2 + (66 + 12\sqrt{3})(30) + (84 + 12\sqrt{3})$$
$$T(30) \approx 18{,}185 \text{ cm}^2 \text{ of glass}$$

ADDITIONAL EXAMPLE

Ship-Shape Shipment Co. is ordering cardboard for their most popular rectangular prism shipping containers. They must know the surface area of the container in to determine how much cardboard to purchase. The length of their containers is five more than twice the width, w, in feet. The height of their containers is three times the width. Write the function for the total surface area in terms of the width, w, and use it to find the surface area of cardboard ordered for a shipping container with a width of 4.5 feet.

The width can be represented by w. The length of the container, $L(w)$, is $2w + 5$. The height of the container, $H(w)$, is $3w$. Surface area of a prism can be calculated with $SA = Ph + 2B$, where P is the perimeter of the base, and B is the area of the base.

$P(w) = 2(L(w) + w) = 2((2w + 5) + w) = 2(3w + 5) = 6w + 10$

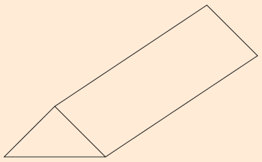
$B(w) = L(w) \cdot w = (2w + 5) \cdot w = 2w^2 + 5w$

$S(w) = P(w) \cdot h + 2B(w) = (6w + 10) \cdot 3w + 2(2w^2 + 5w) = 22w^2 + 40w$

A container with a width of 4.5 feet would have a surface area of $S(w) = 22(4.5^2) + 40(4.5) = 625.5\ ft^2$.

YOU TRY IT! #1

A milk chocolate bar is packaged in a box shaped like a triangular prism. The two bases are congruent equilateral triangles and the three faces are congruent rectangles. This bar is available in 1.23 ounces, 2.46 ounces, 3.69 ounces and 4.92 ounces sizes. Using the values in the table, write a function for the length of the boxes, the area of the triangular base, and the volume of the boxes. If the chocolatier plans to put a new 11.07 ounce bar on the market, what would be the volume of the packaging box if the triangle side length is 9 inches?



BAR SIZE, OZ	TRIANGLE SIDE LENGTH, IN., s	BOX LENGTH, IN. $L(s)$	AREA OF TRIANGULAR BASE, IN.^{2} $A(s)$	VOLUME OF BOX, IN.^{3} $V(s)$
0	0	0	0	0
1.23	1	4.2	0.433	1.819
2.46	2	8.4	1.732	14.549
3.69	3	12.6	3.897	49.104
4.92	4	16.8	6.928	116.394

See margin.

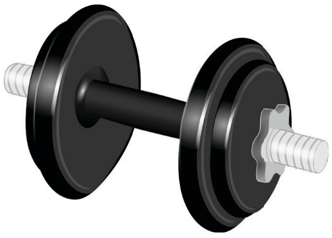
YOU TRY IT! #1 ANSWER:

The length function is linear, $L(s) = 4.2s$. *The area of the triangular base is a quadratic function,* $A(s) = 0.433s^2$. *The volume function is a cubic function, the product of the area of the base and the length:* $V(s) = A(s) \cdot L(s) = (0.433s^2)(4.2s) = 1.8186s^3$. $V(9) = 1.8186(9)^3 = 1325.76 in.^3$.

EXAMPLE 2

A physical trainer has planned a schedule of strength training exercises for his clients. The training cycle lasts x number of months. The increase in the number of repetitions of a few of the exercises, $r(x)$, depends on the months the clients have been in the training cycle. At each month, the total number of repetitions, $t(x)$ is the product of the number of months at that point in the cycle and the formula for the increase in number of repetitions by month. Using the table, write a function for the total number of repetitions planned for each month.

x	$r(x)$	$t(x)$
0	2	0
1	3	3
2	6	12
3	11	33
4	18	72
5	27	135



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ADDITIONAL EXAMPLE

Every year Lisa decides to add money to her savings account to save for a trip to the beach. And, every year opportunities and emergencies happen causing Lisa to stop putting money into her savings account. This year Lisa has a renewed fervor to save up for her beach vacation. The time, in weeks, that Lisa deposits money into her savings account is represented by $t(x) = 2x - 1$, where x is the actual number of weeks. The weekly savings rate for Lisa's savings account is $r(x) = 1.5x + 7$. The total amount she has saved is the product of the time Lisa has saved, $t(x)$, and the weekly savings rate, $r(x)$. Write a function rule for the amount of money is Lisa's savings account, $m(x)$. Use technology to graph the function rule. If Lisa's car beach trip will cost $600, but she stops making deposits at the end of February when she has to pay for car repairs, will she have enough for her trip yet?

The function $m(x) = t(x) \cdot r(x)$ *becomes* $(2x - 1)(1.5x + 7)$ *which simplifies to* $3x^2 + 12.5x - 7$. *It will take Lisa 15 weeks to save over $600, so if she stops saving at the end of February, which is about 9 weeks into the year, she will not yet have enough money for her beach trip.*

STEP 1 Find the finite differences in the function values for $t(x)$.

x	$r(x)$	$t(x)$
0	2	0
1	3	3
2	6	12
3	11	33
4	18	72
5	27	135

3 9 21 39 63

6 12 18 24

6 6 6

STEP 2 Use the patterns in the differences and the table values to write the function for the total number of repetitions planned for the end of each month.

Since the third differences are constant, the function $t(x)$ is cubic.

$$\begin{aligned} 6a &= 6 \\ a &= 1 \end{aligned} \qquad \begin{aligned} 6a + 2b &= 6 \\ 6 + 2b &= 6 \\ 2b &= 0 \\ b &= 0 \end{aligned} \qquad \begin{aligned} a + b + c &= 3 \\ (1) + (0) + c &= 3 \\ 1 + c &= 3 \\ c &= 2 \end{aligned} \qquad d = 0$$

With these values for a, b, c, and d, the function rule is $t(x) = x^3 + 2x$.

YOU TRY IT! #2

Statistics on weight loss programs suggest that the amount of weight lost by most people follows a parabolic curve opening downward. This is explained by initial enthusiasm and then some success that creates a motivation to continue the program. After a while, when motivation decreases, the program is gradually abandoned. The time on the program is logged as the function, $t(x) = x + 1$, where x is the actual number of months. A possible function rule for a monthly weight loss rate is $r(x) = -0.28x + 2$. The amount of weight loss is the product of the time on the program $t(x)$ and the monthly weight loss rate $r(x)$. Write a function rule for the amount of weight loss $a(x)$ by multiplying function rule for the time on the program $t(x)$ and the function rule for the monthly weight loss rate, $r(x)$. Use technology to graph the function rule to predict the number of months when weight loss typically stops.

See margin.

YOU TRY IT! #2 ANSWER:

The function $a(x) = t(x) \cdot r(x)$ becomes $(x + 1)(-0.28x + 2)$. This can be simplified to

$a(x) = -0.28x^2 + 1.72x + 2$. A graph of the function $a(x)$ shows a y-value of zero at an x-value of a little more than 7, so the typical person stops losing weight in just over 7 months.

EXAMPLE 3

Propane space heaters are a versatile solution for heating a variety of rooms, including garages and workshops. A table from the manufacturer's catalog shows specifications for determining the right size heater by the various sizes of space to be heated. Use the table to write functions for the dimensions of the rooms and then multiply the functions to write a function for the volume of the rooms.

Image source: pixabay.com

STEP 1 **Determine the function rules for the length, width, and height of various rooms listed in the table.**

ROOM NUMBER, x	LENGTH OF ROOM, IN FEET, $l(x)$	WIDTH OF ROOM, IN FEET, $w(x)$	HEIGHT OF CEILING, IN FEET, $h(x)$
1	15	11	7
2	16	12	8
3	17	13	9
4	18	14	10

Each of the dimensions appears to be a linear function since there is a constant increase of 1 in the table values. Using the table to find the zero term for each, the length function is $l(x) = x + 14$; the width function is $w(x) = x + 10$; and the function for the height is $h(x) = x + 6$.

STEP 2 **Determine the function rule for the volume as the product of the function rules for length, width, and height. Simplify using the properties of algebra.**

$$V(x) = l(x) \cdot w(x) \cdot h(x)$$
$$V(x) = (x + 14)(x + 10)(x + 6)$$
$$V(x) = (x^2 + 10x + 14x + 140)(x + 6)$$
$$V(x) = (x^2 + 24x + 140)(x + 6)$$
$$V(x) = x^3 + 24x^2 + 140x + 6x^2 + 144x + 840$$
$$V(x) = x^3 + 24x^2 + 6x^2 + 140x + 144x + 840$$
$$V(x) = x^3 + 30x^2 + 284x + 840$$

So the standard formula $V = lwh$ is written as a function, $V(x) = x^3 + 30x^2 + 284x + 840$, to determine the appropriate heater size for a room.

ADDITIONAL EXAMPLE

Bakery Supplies 4 Less sells flour in cylindrical containers of various sizes for home and restaurant use. Use the table of dimensions below to write functions for the dimensions of the flour containers and then use the formula $V = \pi r^2 h$ to multiply the functions to write a function for the volumes of the containers.

The radius is $r(x) = 2x + 2$.

The height is $h(x) = 3x + 5$.

The volume of a container is $V(x) = \pi(2x + 2)^2(3x + 5) = (12x^3 + 44x^2 + 52x + 20)\pi$.

CONTAINER NUMBER, x	RADIUS, IN INCHES, $r(x)$	HEIGHT, IN INCHES, $h(x)$
1	4	8
2	6	11
3	8	14
4	10	17
5	12	20

YOU TRY IT! #3

To track the profits from the sales of an economy truck, the marketing department has built functions from data on the past several years. Each year the number sold has followed the function $n(x) = -0.1x^2 + 0.8x + 5$, in millions of trucks, where x represents the number of years after the first year the truck was put on the market. The price of the truck has risen according to the function $p(x) = 1.2x + 21$, in thousands of dollars. The function for the total revenue generated $r(x)$, in millions of dollars, is the product of the number of trucks sold $n(x)$ and the price of each truck $p(x)$. The total profit each year, also in millions of dollars, has been about 22% of the revenue generated. Using symbolic multiplication of the functions, write the function for total profit $T(x)$ from the sales of the trucks. Simplify the expression using the properties of algebra. Then compare the projected profit from this year's sales, 10 years after the year the truck was put on the market, to the first year.

See margin.

PRACTICE/HOMEWORK

Use the scenario and table below to answer questions 1 – 4.

CRITICAL THINKING

Ms. Ritas is playing a number game with her students. When a student called out a number, Ms. Ritas would fill in the columns to show the functional relationship between the number the student called out and the value of the number.

STUDENT NUMBER, n	VALUE FOR PUZZLE 1, $f(n)$	VALUE FOR PUZZLE 2, $g(n)$	PRODUCT OF PUZZLE 1 AND PUZZLE 2 $P(n)$
1	8	2	16
2	11	7	77
3	14	12	168
4	17	17	289
5	20	22	440

1. Use finite differences to write a function rule, $f(n)$, to represent the rule for Puzzle 1 and $g(n)$ to represent the rule for Puzzle 2.
See margin.

2. Use finite differences to write a function rule, $P(n)$, to represent the product of the puzzle values for each Student Number, n. Include the 0 term so that you can use the differences to determine the function rule.
See margin.

YOU TRY IT! #3 ANSWER:

$T(x) = 0.22(r(x)) = 0.22(n(x) \cdot p(x))$

$T(x) = 0.22(-0.1x^2 + 0.8x + 5)(1.2x + 21)$

$T(x) = 0.22[(-0.12x^3 + 0.96x^2 + 6x) + (-2.1x^2 + 16.8x + 105)]$

$T(x) = 0.22(-0.12x^3 - 1.14x^2 + 22.8x + 105)$

$T(x) = -0.0264x^3 - 0.2508x^2 + 5.016x + 23.1$

$T(10) = -0.0264(10)^3 - 0.2508(10)^2 + 5.016(10) + 23.1 = 21.78$

$T(0) = 23.1$

$T(10) - T(0) = 21.78 - 23.1 = -1.32$

So profits for year 10 are projected to be down by 1.32 million dollars from the first year.

1.

	STUDENT NUMBER, n	VALUE FOR PUZZLE 1, $f(n)$		VALUE FOR PUZZLE 2, $g(n)$	
	1	8		2	
+1			+3		+5
	2	11		7	
+1			+3		+5
	3	14		12	
+1			+3		+5
	4	17		17	
+1			+3		+5
	5	20		22	

$a = 3$ $\quad a + b = 8$ $\quad a = 5$ $\quad a + b = 2$

$3 + b = 8$ $\quad 5 + b = 2$

$b = 5$ $\quad b = -3$

$f(n) = (3n + 5)$ $\quad g(n) = (5n - 3)$

3. Write a function rule, $P(n)$, by multiplying the function rules written in symbolic form for each Puzzle rule. $P(n) = f(n) \cdot g(n)$
See margin.

4. Is the function rule for the Product of Puzzle 1 and Puzzle 2, $P(n)$ the same? Verify using your graphing calculator and explain your answer.
See margin.

Use the scenario and table below to answer questions 5 – 8.

GEOMETRY

A shipping company has different size shipping boxes that are in the shape of rectangular prisms. The dimensions of the prisms are related to the prism number and shown in the table below. The volume of each prism is also shown in the table.

PRISM NUMBER, x	LENGTH, $L(x)$	WIDTH, $W(x)$	HEIGHT, $H(x)$	VOLUME, $V(x)$
1	5	3	4	60
2	6	4	5	120
3	7	5	6	210
4	8	6	7	336
5	9	7	8	504

5. Use finite differences to write a function rule, $L(x)$, to represent the rule for the length of the prism, $W(x)$, to represent the width of the prism, and $H(x)$, to represent the height of the prism.
See margin.

6. Use finite differences to write a function rule, $V(x)$, to represent the Volume of each prism Include the 0 term so that you can use the differences to determine the function rule.
See margin.

7. Write a function rule, $V(x)$, by multiplying the function rules written in symbolic form for the length, the width, and the height. $V(x) = L(x) \cdot W(x) \cdot H(x)$
See margin.

8. Is the function rule for the Volume of the prisms, $V(x)$ the same? Verify using your graphing calculator and explain your answer.
See margin.

For questions 9 – 13, determine the function rule, $A(x)$, for the area of the figure described as the product of the function rules for the dimensions given.

GEOMETRY

9. Write a function rule to represent the area of a rectangle with length, $l(x) = (\frac{1}{2}x + 4)$ and width, $w(x) = (\frac{1}{4}x - 8)$.
$A(x) = (\frac{1}{8}x^2 - 3x - 32)$

2. *See margin below.*

3. $P(n) = f(n) \cdot g(n)$

$P(n) = (3n + 5) \cdot (5n - 3)$

$P(n) = 15n^2 + 16n - 15$

4. *Yes. The function rules are equivalent. If you make a graph or a table of each function rule, the graphs coincide and the table shows equal function values for each independent variable value.*

5. *See margin below.*

6. *See margin at bottom of page 524.*

7. $V(x) = L(x) \cdot W(x) \cdot H(x)$

$V(x) = (x + 4) \cdot (x + 2) \cdot (x + 3)$

$V(x) = x^3 + 9x^2 + 26x + 24$

8. *Yes. The function rules are equivalent. If you make a graph or a table of each function rule, the graphs coincide and the table shows equal function values for each independent variable value.*

2.

STUDENT NUMBER, n	PRODUCT OF PUZZLE 1 AND PUZZLE 2 $P(n)$
0	-15
1	16
2	77
3	168
4	289
5	440

31, 61, 91, 121, 151

30, 30, 30, 30

$2a = 30$
$a = 15$

$a + b = 31$
$15 + b = 31$
$b = 16$

$c = -15$

$P(n) = 15n^2 + 16n - 15$

5.

PRISM NUMBER, x	LENGTH, $L(x)$	WIDTH, $W(x)$	HEIGHT, $H(x)$
1	5	3	4
2	6	4	5
3	7	5	6
4	8	6	7
5	9	7	8

+1, +1, +1, +1 (prism number); +1, +1, +1, +1 (length); +1, +1, +1, +1 (width); +1, +1, +1, +1 (height)

$a = 1$ $a + b = 5$
$1 + b = 5$
$b = 4$

$a = 1$ $a + b = 3$
$1 + b = 3$
$b = 2$

$a = 1$ $a + b = 4$
$1 + b = 4$
$b = 3$

$L(x) = (x + 4)$ $W(x) = (x + 2)$ $H(x) = (x + 3)$

10. Write a function rule to represent the area of a triangle with base, $b(x) = (4x - 5)$ and height, $h(x) = (2x + 6)$.
$A(x) = (4x^2 + 7x - 15)$

11. Write a function rule to represent the area of a square with side, $s(x) = (\frac{1}{2}x - 5)$.
$A(x) = (\frac{1}{4}x^2 - 5x + 25)$

12. Write a function rule to represent the area of a parallelogram with base, $b(x) = (x - 2)$ and height, $h(x) = (x^2 + 6x + 3)$.
$A(x) = (x^3 + 4x^2 - 9x - 6)$

13. Write a function rule to represent the area of a trapezoid with base 1, $b(x) = (7x + 8)$, base 2, $f(x) = (5x - 1)$ and height, $h(x) = (4x + 6)$.
$A(x) = (24x^2 + 50x + 21)$

For questions 14 – 20, determine the function rule, V(x), for the volume of the figure described as the product of the function rules for the dimensions given.

GEOMETRY

14. Write a function rule to represent the volume of a rectangular prism with length, $l(x) = (2x + 1)$, width, $w(x) = (x + 3)$, and height, $h(x) = (3x - 1)$.
$V(x) = (6x^3 + 19x^2 + 2x - 3)$

15. Write a function rule to represent the volume of a cube with side, $s(x) = (3x + 1)$.
$V(x) = (27x^3 + 27x^2 + 9x + 1)$

16. Write a function rule to represent the volume of a triangular prism with Base Area, $B(x) = (5x^2 + 11x + 2)$ and height, $h(x) = (4x - 3)$.
$V(x) = (20x^3 + 29x^2 - 25x - 6)$

17. Write a function rule to represent the volume of a square pyramid with Base Area, $B(x) = (x^2 + 4x + 4)$ and height, $h(x) = (6x + 3)$.
$V(x) = (2x^3 + 9x^2 + 12x + 4)$

18. Write a function rule to represent the volume of a triangular pyramid with Base Area, $B(x) = (9x^2 + 24x + 12)$ and height, $h(x) = (5x - 1)$.
$V(x) = (15x^3 + 37x^2 + 12x - 4)$

19. Write a function rule to represent the volume of a hexagonal pyramid with Base Area, $B(x) = (4x^2 - 3x - 1)$ and height, $h(x) = (3x + 6)$.
$V(x) = (4x^3 + 5x^2 - 7x - 2)$

20. Write a function rule to represent the volume of a cone with height, $h(x) = (6x - 9)$ and Base Area, $B(x) = (4x^2 - 12x + 9)\pi$.
$V(x) = (8x^3 - 36x^2 + 54x - 27)\pi$

6.

PRISM NUMBER, x	VOLUME, $V(x)$
0	24
1	60
2	120
3	210
4	336
5	504

36, 60, 90, 126, 168

24, 30, 36, 42

6, 6, 6

$6a = 6$
$a = 1$

$6a + 2b = 24$
$6(1) + 2b = 24$
$6 + 2b = 24$
$2b = 18$
$b = 9$

$a + b + c = 36$
$(1) + (9) + c = 36$
$10 + c = 36$
$c = 26$

$d = 24$

$V(x) = x^3 + 9x^2 + 26x + 24$