Multiplying Linear Functions

5.2

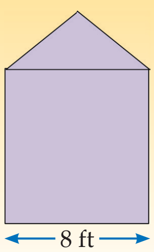
FOCUSING QUESTION How do you solve problems involving multiplication of linear functions?

LEARNING OUTCOMES

- I can use tables and equations to multiply linear functions.
- I can apply mathematics to solve problems arising in everyday life, society, and the workplace.

ENGAGE

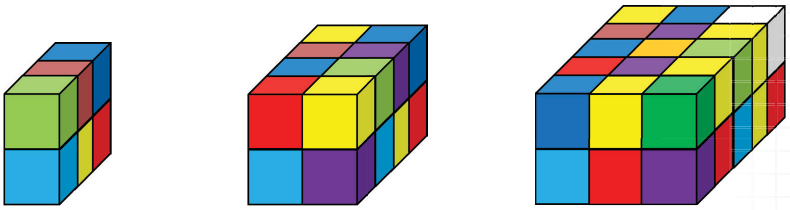
A tool shed is in the shape of a pentagonal prism, where the bases are the left and right sides of the tool shed. The floor is a rectangle that is 10 feet wide and 8 feet deep. What additional information would you need in order to calculate the volume of the tool shed?



See margin.

EXPLORE

Super Sheds offers a series of storage sheds that are in the shape of rectangular prisms.



SHED 1 **SHED 2** **SHED 3**

1. Use cubes to build Shed 4 and Shed 5.
See margin.

TEKS

AR.4A Connect tabular representations to symbolic representations when adding, subtracting, and multiplying polynomial functions arising from mathematical and real-world situations, such as applications involving surface area and volume.

MATHEMATICAL PROCESS SPOTLIGHT

AR.1A Apply mathematics to problems arising in everyday life, society, and the workplace.

ELPS

5G Narrate, describe, and explain with increasing specificity and detail to fulfill content area writing needs as more English is acquired.

VOCABULARY

polynomial, factor, product

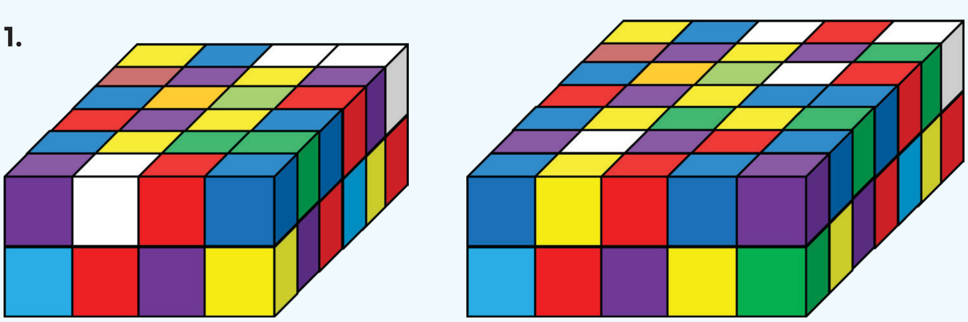
MATERIALS

- centimeter or linking cubes (70 per group of students)
- graphing technology

ENGAGE ANSWER:

The base of the pentagonal prism has two parts: the rectangle framed by the side wall and the triangle beneath the sloping parts of the roof. You also need to know the height of the side wall and the height of the triangle beneath the roof.

1.



SHED 4 **SHED 5**

2. Construct a table of values to show the relationship between the shed number, n, and each dimension of the rectangle. Work backwards in your table to determine the area of the rectangle that represents Term 0.

SHED NUMBER, n	LENGTH OF SHED, $L(n)$	WIDTH OF SHED, $W(n)$	HEIGHT OF SHED, $H(n)$
0	***2***	***0***	***2***
1	***3***	***1***	***2***
2	***4***	***2***	***2***
3	***5***	***3***	***2***
4	***6***	***4***	***2***
5	***7***	***5***	***2***
n			

3. Use patterns in the table, such as finite differences or successive ratios, to determine the function rules, $L(n)$, $W(n)$, and $H(n)$.
See margin.

4. What types of functions are $L(n)$, $W(n)$, and $H(n)$? Explain how you know.
See margin.

A constant function is a special type of linear function in which the function value stays the same regardless of the value of the independent variable. A **constant function** has a constant rate of change of 0.

5. Add a column to your table for values of $V(n)$, the volume in cubic units of each shed. Calculate each shed's volume.

SHED NUMBER, n	LENGTH OF SHED, $L(n)$	WIDTH OF SHED, $W(n)$	HEIGHT OF SHED, $H(n)$	VOLUME OF SHED, $V(n)$
0	***2***	***0***	***2***	***0***
1	***3***	***1***	***2***	***6***
2	***4***	***2***	***2***	***16***
3	***5***	***3***	***2***	***30***
4	***6***	***4***	***2***	***48***
5	***7***	***5***	***2***	***70***
n	***$n + 2$***	***n***	***2***	

6. Use patterns in the table, such as finite differences or successive ratios, to determine the function rule for $V(n)$.
$V(n) = 2n^2 + 4n$

7. The volume of a rectangular prism is the product of its length, width, and height. Use the function rules for $L(n)$, $W(n)$, and $H(n)$ to calculate $V(n)$ using multiplication.
See margin.

3. *$L(n) = n + 2$*

$W(n) = n$

$H(n) = 2$

4. *$L(n)$ and $W(n)$ are linear functions because there is a constant rate of change.*

$H(n)$ is a constant function because the function value is always 2.

7. *$V(n) = (n + 2)(n)(2)$*

$V(n) = 2n(n + 2)$

$V(n) = 2n^2 + 4n$

500 CHAPTER 5: ALGEBRAIC METHODS

8. How does the function rule for $V(n)$ you wrote from number patterns in the table compare to the function rule you wrote for $V(n)$ from the function rules for $L(n)$, $W(n)$, and $H(n)$? Use graphing technology to explain your answer.
See margin.

9. If the height of the shed increased by one layer of cubes each time, how would the volume function be different?
See margin.

10. How do the degrees of the polynomial factors relate to the degree of the polynomial product? Explain the implications of your mathematical reasoning.
See margin.

REFLECT

- **How did you use multiplication to calculate the function values for the volume function in a table?**
  See margin.
- **When you multiply two or more linear functions, how does using operations on the symbolic functions relate to using operations with table values?**
  See margin

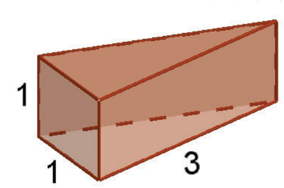
EXPLAIN

Watch Explain and You Try It Videos

or click here

Multiplication is an operation that can be used to represent several actions including:

- equal-sized groups that are joined together;
- area;
- repeated addition; and
- scaling the size of an object.



Consider a sequence of triangular prisms that have right triangles as their base. The first prism has a length of 3 units, width of 1 unit, and height of 1 unit. Each successive prism in the sequence has dimensions that are one unit longer than the previous prism.

You can use the verbal description of the sequence to build a table of values for the dimensions and volume for a sequence of triangular prisms.

8. *The function rules are equivalent. If you make a graph or a table of each function rule, the graphs coincide and the table shows equal function values for each independent variable value.*

9. *The height function would not be a constant function, but would be a linear function with a non-zero rate of change. Multiplying the height function (say, $H(n) = n$) by the length function and width function would generate a volume function that is a cubic function.*

10. *The degree of the product is the sum of the degrees of the factors. According to the laws of exponents, when you multiply like bases, you add the exponents. Multiplying polynomial factors follows this pattern.*

REFLECT ANSWERS:

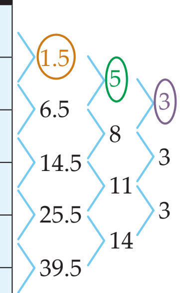
The function values for the volume function were generated by multiplying the values from each dimension function (length, width, and height) together.

If you multiply function values from the table, you multiply each pair or group of function values and then use the products to generate a new function rule. If you multiply two or more function rules instead of numerical function values, then you use the properties of algebra and the laws of exponents to generate a new function rule. The product is the function represented in symbolic form.

PRISM NUMBER, n	LENGTH, $L(n)$	WIDTH, $W(n)$	HEIGHT, $H(n)$		VOLUME, $V(n)$
1	3 ×	1 ×	1 ×	$\frac{1}{2}$ =	1.5
2	4 ×	2 ×	2 ×	$\frac{1}{2}$ =	8
3	5 ×	3 ×	3 ×	$\frac{1}{2}$ =	22.5
4	6 ×	4 ×	4 ×	$\frac{1}{2}$ =	48
5	7 ×	5 ×	5 ×	$\frac{1}{2}$ =	87.5

From the table, you can use finite differences in $V(n)$ to write a function rule for volume in terms of the prism number. Include the 0 term so that you can use the differences to determine the function rule.

PRISM NUMBER, n	VOLUME, $V(n)$
0	0
1	1.5
2	8
3	22.5
4	48
5	87.5



The third differences are constant, so $V(n)$ is a cubic function.

$$6a = 3$$
$$a = 0.5$$

$$6a + 2b = 5$$
$$6(0.5) + 2b = 5$$
$$3 + 2b = 5$$
$$2b = 2$$
$$b = 1$$

$$a + b + c = 1.5$$
$$(0.5) + (1) + c = 1.5$$
$$1.5 + c = 1.5$$
$$c = 0$$

$$d = 0$$

Substituting a, b, c, and d into $V(n) = an^3 + bn^2 + cn + d$ generates the function rule $V(n) = 0.5n^3 + n^2$.

Another way to determine the volume function, $V(n)$, is to multiply the function rules written in symbolic form for each dimension. Use finite differences to write function rules for length, width, and height functions.

502 CHAPTER 5: ALGEBRAIC METHODS

First Differences	PRISM NUMBER, n	LENGTH, $L(n)$	First Differences
	0	2	
+1	1	3	+1
+1	2	4	+1
+1	3	5	+1
+1	4	6	+1
+1	5	7	+1

WIDTH, $W(n)$	First Differences
0	
1	+1
2	+1
3	+1
4	+1
5	+1

HEIGHT, $H(n)$	First Differences
0	
1	+1
2	+1
3	+1
4	+1
5	+1

- $L(n) = n + 2$
- $W(n) = n$
- $H(n) = n$

The volume of a triangular prism is calculated with the formula $V = \frac{1}{2}lwh$, so multiply the functions together to determine the function rule for volume, $V(n)$.

$$V(n) = \frac{1}{2}L(n) \cdot W(n) \cdot H(n)$$
$$V(n) = \frac{1}{2}(n + 2)(n)(n)$$
$$V(n) = \frac{1}{2}n^2(n + 2)$$
$$V(n) = \frac{1}{2}n^3 + n^2$$

Either way, the function rule for the volume is the same.

MULTIPLYING LINEAR FUNCTIONS

A linear function with a constant rate of change is one type of polynomial function.

- You can multiply linear functions two ways:
  1. Make a table of values for each factor to generate the values of the function that is the product of the factors. Then, use finite differences to generate a symbolic function rule.
  2. Multiply the symbolic function rules and use the properties of algebra and laws of exponents to simplify the expressions.

503

INSTRUCTIONAL HINT

Student's answers may vary slightly from those given in the tables, dependent upon what is used for the value of pi, i.e. 3.14, $\frac{22}{7}$, symbol π on the graphing calculator.

EXAMPLE 1

The label of a cylindrical oatmeal container is rectangular, with the length of the label equal to the circumference of the container and the width of the label equal to the height of the container. The containers come in several sizes for home and restaurant use, as shown in the table. Use the patterns in the table values to write a function rule for the label area of the oatmeal containers. Dimensions are given in inches, and circumference and label area are rounded to the nearest tenth.

CONTAINER NUMBER, n	DIAMETER OF CONTAINER, $D(n)$	HEIGHT OF CONTAINER, $H(n)$	CIRCUMFERENCE OF CONTAINER, $C(n)$	LABEL AREA, IN SQ. IN., $L(n)$
1	3	4	9.4	37.7
2	4	6	12.6	75.4
3	5	8	15.7	125.7
4	6	10	18.8	188.5
5	7	12	22.0	263.9

STEP 1 Find the finite difference in the values for the surface area, including a zero term.

CONTAINER NUMBER, n	DIAMETER OF CONTAINER, $D(n)$	HEIGHT OF CONTAINER, $H(n)$	CIRCUMFERENCE OF CONTAINER, $C(n)$	LABEL AREA, IN SQ. IN., $L(n)$
0	2	2	6.3	12.6
1	3	4	9.4	37.7
2	4	6	12.6	75.4
3	5	8	15.7	125.7
4	6	10	18.8	188.5
5	7	12	22.0	263.9

First differences: 25.1, 37.7, 50.3, 62.8, 75.4

Second differences: 12.6, 12.6, 12.6, 12.6

Step 2 The second differences are constant, so the function is quadratic. Use the information to determine a, b, and c to write the function rule for $L(n)$.

$2a = 12.6$
$a = 6.3$

$a + b = 25.1$
$6.3 + b = 25.1$
$b = 18.8$

$c = 12.6$

So the function rule is $L(n) = 6.3n^2 + 18.8n + 12.6$.

ADDITIONAL EXAMPLE

Sue's Kite Sailing Shop sells traditionally shaped kites of various sizes. In testing which size kites sail the best in local weather conditions, Sue makes multiple kites to trial. The table shows the dimensions of the kites in inches and the resulting area of each kite in square inches. Use the table values to write a function rule for the area. Verify your rule by writing the function rule for $A(k)$ as half of the product of the functions for diagonal 1 and diagonal 2.

KITE NUMBER, k	DIAGONAL 1, $d_1(k)$	DIAGONAL 2, $d_2(k)$	AREA, $A(k)$
1	2	1	1
2	3	4	6
3	4	7	14
4	5	10	25
5	6	13	39

The set of data represents a quadratic function because the second finite differences in y-values are constant, and the differences in x-values are 1. The function rule for $A(k)$ is $\frac{3}{2}n^2 + \frac{1}{2}n - 1$. As half of the product of $d_1(k)$ and $d_2(k)$, $A(k) = \frac{1}{2}(n + 1)(3n - 2) = \frac{3}{2}n^2 + \frac{1}{2}n - 1$. The function rules are the same.

504 CHAPTER 5: ALGEBRAIC METHODS

ALTERNATE METHOD: COMBINING $D(n)$ AND $H(n)$

STEP 1 **Use the formula for the lateral area of a cylinder and the function rules for the diameter and height of the oatmeal containers to determine the function rule for the surface area of the labels symbolically.**

The first differences for the diameter values are increasing by a constant 1, and the value for the zero term is 2. So the linear function is $D(n) = n + 2$. The values for the height are increasing by a constant 2, and the value for the zero term is 2. So the linear function is $H(n) = 2n + 2$.

The formula for the lateral area of a cylinder is $LA = C \cdot h = \pi \cdot d \cdot h$.

$L(n) = \pi(n + 2)(2n + 2) = \pi(2n^2 + 6n + 4)$. Rounding π to approx. 3.14,
$L(n) = 3.14(2n^2 + 6n + 4) = 6.28n^2 + 18.84n + 12.56$.

STEP 2 **Compare the function rule for $L(n)$ you wrote from number patterns in the table to the function rule you wrote for $L(n)$ from the function rules for $D(n)$ and $H(n)$.**

They are the same except for the differences in rounding.

YOU TRY IT! #1

The Sign of the Times Company markets a variety of rectangular signs made of a durable weatherproof material mounted on a frame. The cost of this weatherproof material, purchased in square feet, must be taken into consideration when the company determines the prices of the signs. The table shows the dimensions of the signs in feet and the resulting area of the face of the signs in square feet. Use color tiles to build each sign. Use the table values to write a function rule for the surface area. Verify your rule by writing the function rule for $A(n)$ as a product of the functions for length and width.

SIGN NUMBER, n	LENGTH, $L(n)$	WIDTH, $W(n)$	AREA, $A(n)$
0	1	2	2
1	2	3	6
2	3	4	12
3	4	5	20
4	5	6	30
5	6	7	42

See margin.

YOU TRY IT! #1 ANSWER:

The set of data represents a quadratic function because the second finite differences in y-values are constant, and the differences in x-values are 1. The function rule for $A(n)$ is $n^2 + 3n + 2$. As a product of $L(n)$ and $W(n)$, $A(n) = (n + 1)(n + 2) = n^2 + 3n + 2$. The function rules are the same.

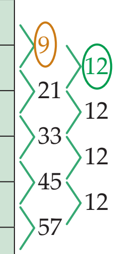
EXAMPLE 2

To wrap a box, the "rule of thumb" to determine the amount of wrapping paper is to add 3 inches to each of the dimensions of the box. For boxes in the shape of a cube, use the patterns found in the table of values to write a function for the area of wrapping paper required, based on x, the edge length of the boxes. Edge lengths are given in feet, and areas are in square feet. Then write a function rule as a product of the functions for wrapping paper edge length and area of each face of the wrapping paper, and compare it to the function rule determined from the table of values (remember to convert the 3 inches to feet).

EDGE LENGTH OF BOX, x	EDGE LENGTH OF PAPER, $E(x)$	AREA OF PAPER FOR EACH FACE, $A(x)$	NUMBER OF FACES	TOTAL AREA OF PAPER, $T(x)$
0	0.25	0.0625	6	0.375
1	1.25	1.5625	6	9.375
2	2.25	5.0625	6	30.375
3	3.25	10.5625	6	63.375
4	4.25	18.0625	6	108.375
5	5.25	27.5625	6	165.375

Step 1 Determine the finite differences between successive y-values for $T(x)$.

EDGE LENGTH OF BOX, x	TOTAL AREA OF PAPER, $T(x)$
0	0.375
1	9.375
2	30.375
3	63.375
4	108.375
5	165.375



STEP 2 Determine whether or not the differences are constant.

The differences in x, Δx, are all 1, so they are constant.
The second differences of y, Δy, are all 12, and are constant. So $T(x)$ is a quadratic function.

ADDITIONAL EXAMPLE

For an expert gift wrapper in a department store, wrapping paper waste must be at an all time minimum to save in costs. Barbara tells her employees that the standard calculation, or the "rule of thumb," to determine how much wrapping paper to use on any shape of package is to add one inch to each of the dimensions of the package. Most the packages being wrapped in Barbara's store are various sizes of rectangular prisms. If the length of the packages are 4 times the width of the package, and the height is always 2 inches, write a function for the area of wrapping paper required, based on w, the measurement of the width of the package, using the formula for surface area, $SA = 2(lw + wh + hl)$. Edge lengths are in inches and areas in square inches for this problem.

The function rule for the area of the wrapping paper, $A(w)$, in terms of the width w, is

$A(w) = 2[(4w + 1)(w + 1) + (w + 1)(3) + (3)(4w + 1)]$

$A(w) = 8w^2 + 40w + 14.$

506 CHAPTER 5: ALGEBRAIC METHODS

STEP 3 Determine a, b, and c from the patterns in the table.

$2a = 12$, so $a = 6$ $a + b = 9$, so $b = 3$ $c = 0.375$

With those values, the function rule is $T(x) = 6x^2 + 3x + 0.375$.

STEP 4 Write a function rule for the total area of wrapping paper needed $T(x)$ as a product of the function rules for the edge length of the wrapping paper $E(x)$ and the area of each face of the wrapping paper $A(x)$, and the number of faces.

$T(x) = 6(A(x)) = 6(E(x)^2)$, and $E(x) = x + 0.25$, with 3 inches converted to 0.25 feet.

$T(x) = 6(x + 0.25)^2$

$T(x) = 6(x^2 + 0.5x + 0.0625)$

$T(x) = 6x^2 + 3x + 0.375$

STEP 5 Compare the function rule for $T(x)$ you wrote from number patterns in the table to the function rule you wrote for $T(x)$ from the product of the function rules for $E(x)$ and $A(x)$.

The function rules are the same for each method. The domain contains all real numbers greater than 0 and allows the function rule to apply to any size box in the shape of a cube.

YOU TRY IT! #2

Apply the "rule of thumb" for wrapping a box in the shape of a rectangular prism, with the length twice the width and the height one half the width. Determine the function rule for the area of wrapping paper needed using the formula for surface area, $SA = 2(lw + wh + hl)$.

See margin.

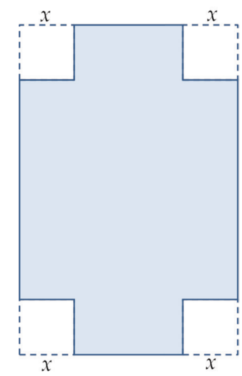
YOU TRY IT! #2 ANSWER:

The function rule for the area of the wrapping paper, $A(w)$, in terms of the width w, is

$$A(w) = 2[(2w + 0.25)(w + 0.25) + (w + 0.25)(\tfrac{1}{2}w + 0.25) + (\tfrac{1}{2}w + 0.25)(2w + 0.25)]$$

$A(w) = 7w^2 + 3.5w + 0.375$.

EXAMPLE 3



A metal baking pan is constructed by cutting a square out of each corner of a rectangular sheet of aluminum measuring 12 inches by 18 inches. The sides created are then folded up to form the height of the pan. Use the table of values to write a function rule, $V(x)$, for the volume of the baking pan. Use your function rule to predict the volume of a pan that has a square of 2.5 inches cut out of each corner.

SIDE LENGTH OF SQUARE, IN INCHES, x	WIDTH OF PAN, IN INCHES, $w(x)$	LENGTH OF PAN, IN INCHES, $l(x)$	HEIGHT OF SIDES, IN INCHES, $h(x)$	VOLUME OF PAN, IN CU. IN., $V(x)$
0	12	18	0	0
1	10	16	1	160
2	8	14	2	224
3	6	12	3	216
4	4	10	4	160
5	2	8	5	80

First differences: 160, 64, -8, -56, -80
Second differences: -96, -72, -48, -24
Third differences: 24, 24, 24

STEP 1 **Write a function rule for the volume of the pan, $V(x)$.**

The third differences in the y-values are all 24, so $V(x)$ is a cubic function.

$6a = 24$
$a = 4$

$6a + 2b = -96$
$24 + 2b = -96$
$2b = -120$
$b = 60$

$a + b + c = 160$
$4 + -60 + c = 160$
$-56 + c = 160$
$c = 216$

$d = 0$

$V(x) = 4x^3 - 60x^2 + 216x.$

STEP 2 **Substitute $x = 2.5$ into the function to determine the volume of the resulting pan.**

$V(x) = 4x^3 - 60x^2 + 216x$
$V(2.5) = 4(2.5)^3 - 60(2.5)^2 + 216(2.5)$
$V(2.5) = 4(15.625) - 60(6.25) + 216(2.5)$
$V(2.5) = 62.5 - 375 + 540$
$V(2.5) = 227.5$

The volume of the resulting pan will be 227.5 cubic inches.

ADDITIONAL EXAMPLE

Maxwell is a competitive food-eating champion. His favorite food contests by far are the hot dog eating contests. The number of hot dogs Maxwell can eat is a product of the amount of time of the contest in minutes and the rate at which he can typically scarf down hot dogs in hotdogs per minute. In each contest, he eats for x minutes, and his average speed is about $1.5x - 9$. Write a function rule for the average hot dogs eaten per contest, $h(x)$, and the total hot dogs Maxwell can eat in a year, $c(x)$, if he usually enters a contest ever 4 weeks.

The function rule for Maxwell's average hotdogs eaten, h(x), in terms of minutes x, is $h(x) = x(1.5x - 9) = 1.5x^2 - 9x$. The function rule for Maxwell's yearly hot dogs eaten is $c(x) = 13(1.5x^2 - 9x) = 19.5x^2 + 117x$.

508 CHAPTER 5: ALGEBRAIC METHODS

YOU TRY IT! #3

The distance traveled by a family on vacation driving across the state is a product of the time driven each day of the weeklong trip and their speed (the rate of miles per hour). The time they drove each day averaged x hours and their average speed was approximately $65 - 2x$ mph. Write a function rule for the average distance driven each day, $d(x)$, and the total distance for the week $w(x)$.

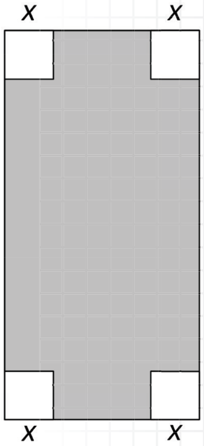
The function rule for the family's daily distance is $d(x) = 65x - 2x^2$. The function rule for the family's weekly distance is $w(x) = 7(65x - 2x^2) = 455x - 14x^2$.

PRACTICE/HOMEWORK

For questions 1 – 10 use the scenario below.

GEOMETRY

A cardboard container is constructed by cutting a square out of a rectangular piece of cardboard measuring 20 inches by 40 inches. The sides created are then folded up to form the height of the container.



1. Use graph paper and a pencil to sketch each corner. Use the paper and pencil sketches to help you fill in the table of values based on x, the side length of the removed square. The first 2 lines of the table have been completed for you.

SIDE LENGTH OF SQUARE, IN INCHES, x	WIDTH OF CONTAINER, IN INCHES, $w(x)$	LENGTH OF CONTAINER, IN INCHES, $l(x)$	HEIGHT OF SIDES, IN INCHES, $h(x)$	VOLUME OF CONTAINER, IN CU. IN., $V(x)$
0	20	40	0	0
1	18	38	1	684
2	*16*	*36*	2	*1152*
3	*14*	*34*	3	*1428*
4	*12*	*32*	4	*1536*
5	*10*	*30*	5	*1500*
6	*8*	*28*	6	*1344*
7	*6*	*26*	7	*1092*

2. Based on the finite differences in the table, what type of function is $V(x)$?
Linear Quadratic Exponential **[Cubic]**

3. Using finite differences and what you know about writing function rules from tables, fill in the blanks below to find the values of a, b, c, and d.

$6a = $ ***24***
$a = $ ***4***

$6a + 2b = $ ***-216***
24 $+ 2b = $ ***-216***
$2b = $ ***-240***
$b = $ ***-120***

$a + b + c = 684$
4 $+$ ***-120*** $+ c = $ ***684***
-116 $+ c = $ ***684***
$c = $ ***800***

$d = $ ***0***

4. Write the function $V(x)$ based on the values of a, b, c, and d.
$V(x) = 4x^3 - 120x^2 + 800x$

5. Based on the function rule for $V(x)$, what would be the volume of a container if the side length of the removed square is 7.5 inches?
937.5 cubic inches

6. Write a function $w(x)$ to represent the width of the container based on the side length of the removed square, x.
$w(x) = 20 - 2x$

7. Write a function $l(x)$ to represent the length of the container based on the side length of the removed square, x.
$l(x) = 40 - 2x$

8. Write a function $h(x)$ to represent the height of the container.
$h(x) = x$

9. The volume of any rectangular prism can be found using the formula $V=lwh$ where l represents the length of the base of the prism, w represents the width of the base of the prism, and h represents the height of the prism. Use the function rules for $l(x)$, $w(x)$, and $h(x)$ to write a function rule for finding the volume of a cardboard container based on the side length of the removed square, x. Write in simplified form.

$$V(x) = l(x) \cdot w(x) \cdot h(x)$$

$V(x) = 4x^3 - 120x^2 + 800x$

10. What do you notice about the function rules for $V(x)$ in problem 4 and problem 9?
They are the same.

510 CHAPTER 5: ALGEBRAIC METHODS

For questions 11 – 20, use the situation below.

GEOMETRY

A rectangular prism has a height of x cm. The length of the base is 2 cm less than the height. The width of the base is 3 cm more than the height.

11. Fill in the table of values based on the information in the situation.

HEIGHT OF PRISM, IN CM, x	LENGTH OF BASE OF PRISM, IN CM, $l(x)$	WIDTH OF BASE OF PRISM, IN CM, $w(x)$	LATERAL SURFACE AREA OF PRISM, IN CUBIC CM, $A(x)$
0	***-2***	***3***	0
1	***-1***	***4***	6
2	***0***	***5***	20
3	***1***	***6***	42
4	***2***	***7***	72
5	***3***	***8***	110
6	***4***	***9***	156

12. Why do the first 3 lines in the table not make sense in the situation?
A prism cannot have a lateral surface area of zero. The length of the base of a prism cannot have negative values or have a value of zero.

13. What type of function is $A(x)$?
Linear [Quadratic] Exponential Cubic

14. Use finite differences in the table to fill in the blanks below to find the values of a, b, and c.

$2a =$ ***8***
$a =$ ***4***

$a + b = 6$
4 $+ b =$ ***6***
$b =$ ***2***

$c =$ ***0***

15. Write the function rule for $A(x)$ using the values of a, b, and c.
$A(x) = 4x^2 + 2x$

16. Write a function $l(x)$ to represent the length of the base of the prism based on the side length of the height of the prism, x.
$l(x) = x - 2$

17. Write a function $w(x)$ to represent the width of the base of the prism based on the side length of the height of the prism, x.
$w(x) = x + 3$

18. Write a function $h(x)$ to represent the height of the prism.
$h(x) = x$

511

19. The lateral surface area of a rectangular prism can be found using the formula $A = (2l + 2w)h$, where l represents the length of the base of the prism, w represents the width of the base of the prism, and h represents the height of the prism. Use the function rules for $l(x)$, $w(x)$, and $h(x)$ to write a function rule for $A(x)$. Write the function rule in simplified form.

$$A(x) = (2 \cdot l(x) + 2 \cdot w(x)) \cdot h(x)$$

$$A(x) = (2 \cdot \underline{(x-2)} + 2 \cdot \underline{(x+3)}) \cdot \underline{x}$$

$A(x) = 4x^2 + 2x$

20. What would be the lateral surface area of a rectangular prism with a height of 10 cm?

420 cm^2

For questions 21 – 25 use the functions shown to write the simplest of form of the indicated products.

$a(x) = x$ $\quad b(x) = 10 - 2x$ $\quad c(x) = 3x + 12$ $\quad d(x) = 2x^2 + 5x$

21. $b(x) \cdot c(x)$

$-6x^2 + 6x + 120$

22. $a(x) \cdot d(x)$

$2x^3 + 5x^2$

23. $a(x) \cdot c(x)$

$3x^2 + 12x$

24. $b(x) \cdot d(x)$

$-4x^3 + 10x^2 + 50x$

25. $a(x) \cdot b(x) \cdot c(x)$

$-6x^3 + 6x^2 + 120x$

512 CHAPTER 5: ALGEBRAIC METHODS